

About Identification of Oil Layers Form

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Abstract. The work is devoted to defining ranges of distribution of a numerical solution in a well to a zone of a layer using mathematical model. The mathematical model is obtained on the basis of the laws of conservation of mass. To solve a problem of concerning pressure it is applied limiting method. The received results can be applied at drawing up of effective computing algorithms.

Introduction

The problem of isothermal filtration theory of fluid in a porous medium was considered, allowing self-similar solution in two dimensions and built an efficient computational algorithm with relation to the pressure in the presence of a free surface between immiscible fluids. The study design consists of: deriving the equations using the velocity potential, composite type system of equations is given a more convenient form with relation to the pressure and saturation, with relative saturation shows the use of similar variables and bringing to the problem of the Stefan type, then built a computational algorithm for the numerical implementation on a computer.

Formulation of the Problem

Let ρ_α , μ_α and P_α accordingly, the density, the coefficient of fluid pressure for each phase: water ($\rho_\epsilon, \mu_\epsilon, P_\epsilon$) and oil (ρ_H, μ_H, P_H). As in [1], the potentials Φ_α are introduced by the next formulas

$$\Phi_\epsilon = p_\epsilon + \rho_\epsilon \cdot g \cdot h, \quad \Phi_H = p_H + \rho_H \cdot g \cdot h, \quad (1)$$

where h – the height of a point above the fixed level, g – acceleration of gravity. The generalized Darcy law for each of the phases under these assumptions takes the form [1]:

$$\vec{g} = -k_\alpha \cdot \nabla \Phi_\alpha, \quad (\alpha = \epsilon, H) \quad (2)$$

where $k = K(x, y, \Phi_\alpha) \cdot \tilde{k}(s)$ - filtration coefficient. In the case of accounting capillary pressure, the pressures P_H and P_ϵ are linked each other by the relation of Laplace

$$p_H(x, y, t) - p_\epsilon(x, y, t) = p_k(s), \quad (3)$$

where $p_k(s)$ - capillary pressure, and for the hydrophilic layer $\frac{dp_k}{ds} < 0$. Regarding the saturation of each phase based on the continuity equation, we have:

$$\frac{\partial s}{\partial t} = \text{div}(k_\alpha \cdot \nabla \Phi_\alpha), \quad (\alpha = \epsilon, H) \quad (4)$$

and the relation

$$s_e + s_n = 1 \quad (5)$$

By introducing of function of current ψ , as in [1]:

$$\frac{\partial \psi}{\partial y} = \mathcal{G}_1, \quad \frac{\partial \psi}{\partial x} = -\mathcal{G}_2 \quad (6)$$

and differentiation (3) with (4) we obtain the following system of equations

$$\begin{cases} \frac{\partial s}{\partial t} = \frac{\partial}{\partial x} \left[a \cdot \frac{\partial s}{\partial x} + c \left(f_1 + \frac{\partial \psi}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[a \cdot \frac{\partial s}{\partial y} + c \left(f_2 - \frac{\partial \psi}{\partial x} \right) \right], \\ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = [F_1(x, y, s, \psi_x, \psi_y) + F(x, y, z)] \cdot (k_e + k_n), \end{cases} \quad (7)$$

where $a = -c \cdot k_n \cdot \frac{dp_k}{ds} \geq 0$, $c = \frac{k_e}{k_n + k_e} \equiv c(s) \geq 0$,

$$f_1 = g \cdot k_n \cdot (\rho_e - \rho_n) \cdot \frac{\partial h}{\partial x}, \quad f_2 = g \cdot k_n \cdot (\rho_e - \rho_n) \cdot \frac{\partial h}{\partial y},$$

$$F_1 \equiv \left\{ \frac{\partial \psi}{\partial x} \cdot \frac{\partial}{\partial x} \left(\frac{1}{k_n + k_e} \right) - \frac{\partial \psi}{\partial y} \cdot \frac{\partial}{\partial y} \left(\frac{1}{k_n + k_e} \right) \right\}, \quad (8)$$

$$F_2 \equiv g \cdot \left\{ \frac{\partial h}{\partial y} \cdot \frac{\partial s}{\partial x} - \frac{\partial h}{\partial x} \cdot \frac{\partial s}{\partial y} \right\} \cdot (c_n \cdot \rho_n + c_e \cdot \rho_e), \quad c_\alpha = \frac{k_\alpha}{k_e + k_n}, \quad (\alpha = e, n)$$

To determine functions $s(x, y, t)$ and $\psi(x, y, t)$ on the boundary Γ considered that the following conditions are met:

$$s|_\Gamma = \tilde{g}(t, \sigma) \quad (9)$$

$$\left. \frac{\partial \psi}{\partial n} \right|_\Gamma = k_n \cdot \frac{\partial p_n}{\partial \sigma} + k_e \cdot \frac{\partial p_e}{\partial \sigma} + g \cdot \frac{\partial h}{\partial \sigma} \cdot (k_n \cdot \rho_n + k_e \cdot \rho_e) \equiv \theta(t, \sigma),$$

where σ - parameter of Γ .

In addition to the boundary conditions (9) it is also known initial distribution of water saturation $s(x, y, t)$ in the reservoir:

$$s(x, y, 0) = \tilde{g}_0(x, y) \quad (10)$$

Theorem. Under certain values of pressure and $\lim_{t \rightarrow \infty} \tilde{g}(t, \sigma) = \beta > 0$ the following relations are valid: $R(t) = D_*(a, b, \beta) \cdot t^{1/2}$ and $s(x, y, t) = \nu[\xi, \beta]$, where a, b - positive constants.

Proof. In what way is easy to find, function $\nu[\xi, \beta]$, $\xi = \frac{ax + by}{\sqrt{t + 1}}$, and parameter $D_*(a, b, \beta)$ are determined by the following conditions

$$\frac{d^2 v}{d\xi^2} + \frac{\xi}{2} a(v) \frac{dv}{d\xi} = 0, \quad a = \Phi'(v), \quad \xi \in (0, D_*), \quad (11)$$

$$v(0, \beta) = \beta, \quad v(D_*, \beta) = 0, \quad (12)$$

$$\frac{dv}{d\xi}(D_*, \beta) = -\frac{1}{2} D_* \quad (13)$$

We show that for each $D > 0$ there is at least one function $V(\xi)$, that satisfies the equation (11) and the boundary conditions (12). Further, calculating the derivative $dV/d\xi$ at the point $\xi = D$ and putting it into the left side of (13), we obtain an equation which solution D_* determines the solution of the problem (12) - (13). To determine the function $V(\xi)$ consider the linear boundary value problem:

$$\frac{d^2 \tilde{V}}{d\xi^2} + \frac{\xi}{2} a[g(\xi)] \frac{d\tilde{V}}{d\xi} = 0, \quad \tilde{V}(0) = \beta, \quad \tilde{V}(D) = 0,$$

where argument in coefficient is arbitrary non-negative function $g(\xi)$, continuous on the interval $(0, D)$ and there is limited by constant β . The decision of the last is given by

$$\tilde{V}(\xi) = -\beta \cdot \frac{\int_0^D \exp\left(-\int_0^\tau \frac{s}{2} a[g(s)] ds\right) d\tau}{\int_0^D \exp\left(-\int_0^\tau \frac{s}{2} a[g(s)] ds\right) d\tau}. \quad (14)$$

The right side of the written out expression is a continuous operator $\Psi(g)$, defined on the set \mathfrak{R} of functions g with the previously described properties, and displays this set in itself. Furthermore, since the derivatives $\tilde{V}(\xi)$ of functions are uniformly bounded:

$$\left| \frac{d\tilde{V}}{d\xi}(\xi) \right| \leq \beta \left(\int_0^D \exp\left(-\frac{a_0}{4} s^2\right) ds \right)^{-1},$$

where $a_0 = \min_{s \in (0, \beta)} \{a(s), a^{-1}(s)\}$, the operator $\Psi(g)$ is completely continuous on the set \mathfrak{R} . By the theorem of Schauder there is at least one fixed point V of the operator $\Psi : V = \Psi(V)$. The function $V(\xi)$ satisfies the equation (11) and the conditions (12). The equation $\frac{dv}{d\xi}(D) = -\frac{1}{2} D$ has at least one solution $D_* > 0$, because for $V(\xi)$ there is the representation analogous to (14), which are easily deduced inequality

$$-\beta e^{-\frac{a_0 D^2}{4}} \left(\int_0^D e^{-\frac{\tau^2}{4a_0}} d\tau \right)^{-1} \leq \frac{dV}{d\xi}(D) \leq -\beta e^{-\frac{D^2}{4a_0}} \left(\int_0^D e^{-\frac{a_0 \tau^2}{4}} d\tau \right)^{-1}.$$

Uniqueness found self-similar solution follows from the fact that the function $U_*(x, t)$ equal to $\Phi[\theta_*(x, t)]$ at $0 < x < R_*(t)$ and minus one by $x > R_*(t)$ is the only bounded generalized solution of the Stefan problem with the data in Theorem 3. The continuity $D_*(\beta)$ from the parameter β follows from the theorem on the continuous dependence of the solutions of ordinary differential equations on a parameter. The proof of the last statement of the theorem follows from the equality

$$\frac{1}{2}D_*^2(\beta) + \int_0^{D_*(\beta)} \xi \Phi[\nu(\xi, \beta)] d\xi = \beta, \quad (15)$$

which is obtained after multiplying the equation (11) by ξ and integrating over ξ from 0 to D_* using the conditions (12) and (13).

To construct a computational algorithm used the following approach. The mathematical formulation of the problem. Consider a two-dimensional rectangular grid nodes, wherein p nodes horizontally, vertically q nodes. Data points are of the form $t_{ij}(x, y, M)$, where (x, y) – its coordinates, M – magnitude.

This method does not require to determine the grid from the manifold and divides all the data on set of $p \times q$ subsets of K – taxons within each subset's points will be closer to a node in the mesh y^{ij} , than to any other node.

Let this fact as follows:

$$K_{ij} = \left\{ t \in P_k, \|y^{ij} - t\|^2 \leq \varepsilon \right\} \quad (16)$$

This grid can be deformed in two ways - stretch it along and bend across. In one case, it seeks to maintain its length, and in the other - a flat shape. Considered grid has the following properties: tensile property, this property provides a uniform grid; property of smoothness; property close to the data points. To mesh has both these properties, it is necessary to add to the minimized criterion the measure of the total grid stretching, measure of bending and measure total aggregate measure of proximity. Adding together all three of these measures, we obtain a general criterion by which the grid, on the one hand, will be attracted to data points, the other - to strive to minimize their tension and take the most smooth shape (become more regular).

Summary

We received the following quality features:

$$D = \frac{D_1}{|P_k|} + \lambda \frac{D_2}{pq} + \mu \frac{D_3}{pq} \rightarrow \min, \quad (17)$$

where $|P_k|$ – a number of points X ; λ, μ – elasticity coefficients responsible for the tension and curvature of the grid respectively; λ – the number of executed iterations, D_1, D_2, D_3 – terms are responsible for the properties of the grid (Figure 1).

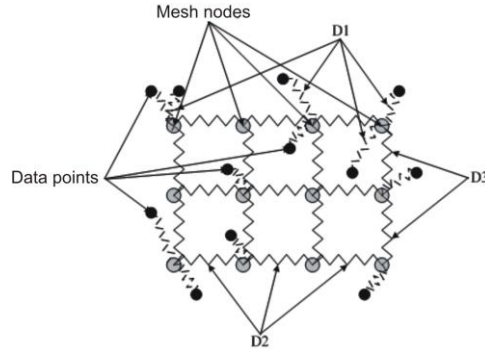


Figure 1. Overlay points on the grid.

As a measure of the grid closeness to the data point, select the value of the square of the distance from the point to the nearest grid point. The property measures the grid closeness to the data points represented as:

$$D_1 = \sum_{i,j} \sum_{t_n \in K_{ij}} \|t_n - y^{ij}\|^2 \quad (18)$$

The higher the average length of an edge, the stronger net "stretched." Thus, in the minimizing functional must enter the difference between the positions of neighboring nodes:

$$D_2 = \sum_{i=1}^p \sum_{j=2}^{q-1} \|y^{ij} - y^{i,j+1}\|^2 + \sum_{i=1}^{p-1} \sum_{j=1}^q \|y^{ij} - y^{i+1,j}\|^2 \quad (19)$$

Measure Stretching of the Mesh

Note that the summation boundaries are chosen so that the edge was not included in the amount twice in the functional D2. The degree of curvature is determined by evaluating the magnitude of the second derivative, by using the second difference. As a result, we get the following functional:

$$D_3 = \sum_{i=1}^p \sum_{j=2}^{q-1} \|2y^{ij} - y^{i,j-1} - y^{i,j+1}\|^2 + \sum_{i=2}^{p-1} \sum_{j=1}^q \|2y^{ij} - y^{i-1,j} - y^{i+1,j}\|^2 \quad (20)$$

Measure of Grid Smoothness

The resulting method allows you to restore the division border between water and oil.

References

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