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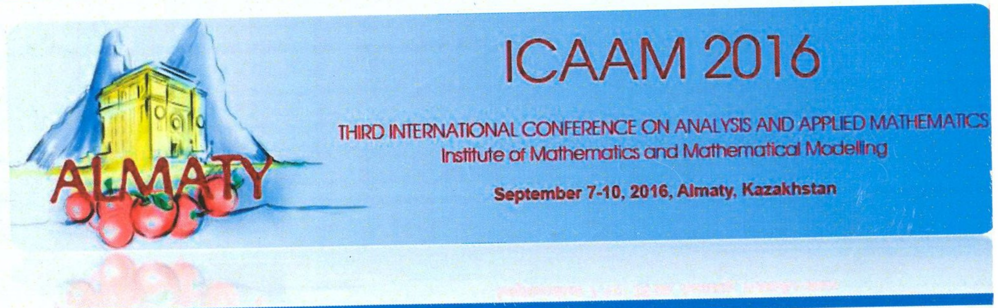
ICAAM 2016

THE ABSTRACT BOOK



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- Minisymposium:Dynamical Systems

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- Prof. **Shavkat Arifdzhanovich Alimov**, Head of the laboratory of mathematical modeling and Chief Scientist of the Malaysian Institute of Microelectronic Systems (MIMOS) in Kuala Lumpur, Malaysia
- Prof. **Ravi Agarwal**, Texas A & M University, USA
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- Prof. **Iskander A. Taimanov**, Chair of Geometry and Topology, Novosibirsk State University, Head of the Laboratory of Dynamical Systems, Sobolev Institute of Mathematics, Russia

On the solvability of a nonlocal boundary value problem for the Laplace operator with opposite flows at the part of the boundary

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Abstract: In the present paper we investigate a nonlocal boundary problem for the Laplace equation in a half-disk, with opposite flows at the part of the boundary. Our goal is to find a function $u(r,\theta) \in C^0(\bar{D}) \cap C^2(D)$ satisfying in D the equation

$$\Delta u = 0$$

with the boundary conditions

$$u(1,\theta) = f(\theta),\ 0 \leq \theta \leq \pi,$$

$$u(r,0) = 0, r \in [0,1],$$

$$\frac{\partial u}{\partial \theta}(r,0) = -\frac{\partial u}{\partial \theta}(r,\pi) + \alpha u(r,\pi),\quad r \in (0,1)$$

where $D = \{(r,\theta):\quad 0 < r < 1, 0 < \theta < \pi\}$; $\alpha < 0$; $f(\theta) \in C^2[0,\pi]$, $f(0) = 0$, $f'(0) = -f'(\pi) + \alpha f(\pi)$.

The difference of this problem is the impossibility of direct applying of the Fourier method (separation of variables). Because the corresponding spectral problem for the ordinary differential equation has the system of eigenfunctions not forming a basis. Throughout this note we mainly use techniques from our work [1]. A special system of functions based on these eigenfunctions is constructed. This system has already formed the basis. This new basis is used for solving of the nonlocal boundary value problem. The existence and the uniqueness of the classical solution of the problem are proved.

Keywords: Laplace equation, basis, eigenfunctions, nonlocal boundary value problem

2010 Mathematics Subject Classification: 33C10, 34B30, 35J, 35P10

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[1] Dildabek G., Tengayeva A., and Sadybekov M.A. "On the solvability of a nonlocal boundary problem for the Helmholtz operator in a half-disk", *Contemporary Analysis and Applied Mathematics.* Vol. 2, No. 2, pp. 305–313, 2014.

About the correctness to some nonlocal boundary value problem for the equation of the mixed type of the first kind and the second order in space

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Abstract: As it is known the Dirichlet problem for the equation of the mixed type of the first sort is incorrect. Naturally there is a problem: whether it is impossible to substitute statements of the problem of Dirichlet other conditions enveloping all boundary which ensure a problem correctness? For the first time, such problems have been offered and studied in the works T.S.Kalmenov's. In the present work for the equation of the mixed type of the first kind, the second order in space unambiguously solvability and smoothness of the generalized solution, of some nonlocal boundary value problem with constant coefficients from Sobolev spaces is studied.

Let $\Omega = \prod\limits_{i=1}^{n} (\alpha_i, \beta_i) \in R^n$. $n-$ a measured parallelepiped.

In the area $Q = \Omega \times (0,T)$ we consider a differential equation of the second order

$$(1)\quad Lu = K(x)\, u_{tt} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{i,j}(x)\, u_{x_j}\right) + \alpha\,(x,t)\, u_t + c(x,t)\, u = f(x,t),$$

where, $x_1 K(x) > 0$ at $x_1 \neq 0$ and thus $x_1 \in (\alpha_1, \beta_1)$, $\alpha_1 < 0 < \beta_1$.

We assume, that coefficients of equation (1) -are smooth enough functions and let the condition following is satisfied: $a_{i,j}\xi_i\xi_j \geq a_0|\xi|^2$, where,

$$a_{i,j}(x) = a_{j,i}(x);\ a_0 - const > 0,\ \xi \in R^n; |\xi|^2 = \sum_{i=1}^{n} \xi_i^2.$$

Problem. To find a generalized solution of equation (1) from the Sobolev space $W_2^l(Q)$, $(2 \leq l$- is integer), satisfying to nonlocal boundary conditions

$$(2)\qquad \gamma D_t^p\, u|_{t=0} = D_t^p\, u|_{t=T}$$

$$(3)\qquad \eta D_{x_i}^p\, u|_{x_i=\alpha_i} = D_{x_i}^p\, u|_{x_i=\beta_i},$$

at $p = 0, 1$; here γ *and* $\eta - const \neq 0$, where $D_t^p u = \frac{\partial^p u}{\partial t^p}$ $D_t^0 u = u$.

Keywords: second order mixed type equation of the first kind,nonlocal boundary value problem with constant coefficients, Sobolev spaces,unique solvability,existence of solution, smoothness of the generalized solution.

2010 Mathematics Subject Classification: 35M10

Mathematical modeling of non-equilibrium sorption

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Abstract: We consider the system of equations modeling the process of non-equilibrium sorption. Difference approximation of differential problem by the implicit scheme is formulated. The solution of the difference problem is constructed using the sweep method. Based on the numerical results we can conclude the following: when the relaxation time decreases to 0, then the solution of non-equilibrium problem tends with increasing time to solution of the equilibrium problem.

Keywords: The system of equations of non-equilibrium sorption, difference approximation, the implicit scheme, sweep method, numerical experiments.

2010 Mathematics Subject Classification: 35Q35, 65M06, 76S05

Exact solution of two phase spherical Stefan problem with two free boundaries

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Abstract: Solution of the heat equation in a spherical domain with two free boundaries (two-phase Stefan problem) when one of the subdomains degenerates at the initial time is considered. The use of conventional finite-difference methods in these cases is not expedient because of the degenerate domain. The solution is found in the form of combination of Integral Error functions series. [1] and then recurrent solvability of nonlinear algebraic equations for determining the coefficients of the series is proved.

The mathematical model includes the spherical heat distribution with constant coefficients for each of the phases

$$\frac{\partial\theta_1}{\partial t} = a_1^2\left(\frac{\partial^2\theta_1}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_1}{\partial r}\right), \qquad \alpha(t) < r < \beta(t), \tag{1}$$

$$\frac{\partial\theta_2}{\partial t} = a_2^2\left(\frac{\partial^2\theta_2}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_2}{\partial r}\right), \qquad \beta(t) < r < \infty, \tag{2}$$

with initial condition

$$\theta_1(r,0) = 0, \tag{3}$$

$$\theta_2(r,0) = f(r), \tag{4}$$

subjected to free boundary conditions

$$\theta_1(\alpha(t),t) = \theta_m, \tag{5}$$

$$-\lambda_1\frac{\partial\theta_1\big(\alpha(t),t\big)}{\partial r} = Q, \tag{6}$$

where $Q = \frac{P(t)}{2\pi\alpha^2(t)}$

$$\theta_1(\beta(t),t) = \theta_2(\beta(t),t) = \theta_m, \tag{7}$$

the Stefan's condition

$$-\lambda_1\frac{\partial\theta_1(\beta(t),t)}{\partial r} = -\lambda_2\frac{\partial\theta_2(\beta(t),t)}{\partial r} + L\gamma\frac{d\beta}{dt}, \tag{8}$$

as well as the condition at infinity

$$\theta_2(\infty,0) = 0. \tag{9}$$

To solve this problem we firstly reduce it to the linear problem. Then suggest exact solution in the form of Heat polynomials and series combination of integral error functions whose coefficients have to be determined.

Keywords: Stefan problem, degenerate domain, Integral Error functions.
Mathematics subject classification: 80A22

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Solvability of a stationary problem of magnetohydrodynamics

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Abstract: This work is devoted to study the following stationary problem of magnetohydrodynamics consisting in finding the functions $\vec{v}(x)$, $p(x)$, $\vec{H}(x)$ and $\vec{E}(x)$ in $\Omega \subset R^3$:

$$-\nu\Delta\vec{v} + (\vec{v}\cdot\nabla)\vec{v} - \frac{\mu}{\rho}(\vec{H}\cdot\nabla)\vec{H} + \frac{1}{\rho}\nabla(p(x) + \frac{\mu}{2}|\vec{H}|^2) = \vec{f}(x), \quad x \in \Omega, \tag{1}$$

$$div\vec{v}(x) = 0, \quad x \in \Omega, \tag{2}$$

$$rot\vec{H}(x) - \sigma(\vec{E}(x) + \mu[\vec{v}\times\vec{H}]) = \vec{j}(x), \quad x \in \Omega, \tag{3}$$

$$div\mu\vec{H}(x) = 0, \quad x \in \Omega, \tag{4}$$

$$rot\vec{E}(x) = 0, \quad x \in \Omega, \tag{5}$$

$$\vec{v}(x)|_S = 0, \vec{E}_\tau(x)|_S = 0, \vec{H}\cdot\vec{n}|_S = 0. \tag{6}$$

Here $\vec{n}$ is the unit outward normal to S, and $\vec{E}_\tau = \vec{E} - \vec{n}\left(\vec{n}\cdot\vec{E}\right)$. $\Omega \subset R^3$ is the bounded domain with smooth boundary S.

Using the results in [1]- [3], we prove unique solvability of (1)-(6) in Sobolev and Hölder spaces.

Keywords: magnetohydrodynamics, generalized solution, stationary problem

2010 Mathematics Subject Classification: 76W99, 35D30, 60G10

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Mathematical modelling and control theory analysis of artificial satellite

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Abstract: In this paper, an analysis of Mathematical Control Theory is carried out which concludes controllability of artificial satellites. Control Theory is application-oriented mathematics that deals with the basic principles underlying the analysis and design (control) systems. To control means that one has to influence the behaviour of the system in a desirable way. A preliminary discussion is provided which acts as stairs for reaching final conclusions and includes an analytical approach to model the physical system into dynamical equations of motions which is termed as Mathematical Modelling. Since most of the real life systems are nonlinear in nature therefore an approach for linearization is described. By doing so, we will able to apply numerous linear analysis methods to study further behaviour of the system. Then a mathematical approach used to compute Transition Matrix for getting solution of linear system by using variation of parameter method. Throughout the analysis we are looking for minimum norm (optimal) steering controls,we will also introduce linear oprator technique for calculating the steering controls of the system, after then we made some interesting conclusions by using Kalman's Controllability test. Finally, a graphical simulation is provided using MATLAB which includes computations of controllability Grammian and controller with related MATLAB codes.

Keywords: Control Theory, Transition Matrix, Optimal Control, Kalman's Controllability test.

Differential operators with singular coefficients

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Abstract: Classical theory of symmetric ordinary differential operators allows to work with differential expressions of the form

(4) $$\ell_{2n}[y] := \sum_{k=0}^{n}(-1)^{n-k}(p_k\, y^{(n-k)})^{(n-k)} + i\sum_{k=0}^{n-1}(-1)^{n-k-1}\{(q_k\, y^{(n-k-1)})^{(n-k)} + (q_k\, y^{(n-k)})^{(n-k-1)}\},$$

provided that the coefficients p_k, q_k and $(p_0)^{-1}$ are locally integrable functions. This theory was developed in the works of D. Shin, N. Glazman, A. Zettle, N. Everit et al.

Our goal is to extend the frames of the classical theory and to define operators associated with non-symmetric differential expressions

(5) $$\tau(y) = \sum_{k,s=0}^{n}\left(\tau_{ks}y^{(n-k)}\right)^{(n-s)},$$

whose coefficients τ_{ks} are distributions of finite order singularity (depending on the indices k, s).

We shall discuss several approaches to this problem. The most important one is based on the so-called regularization procedure. This procedure provides several advantages. In particular, one can write out useful asymptotic formulae for the solutions of the equation

$$\ell_{2n}[y] - \lambda y = 0.$$

We shall also discuss similar problems for partial differential operators.

The talk is based on the joint papers with prof. K.A. Mirzoev and A.M. Savchuk.

Keywords: differential operators with distribution coefficients, differential operators with singular coefficients, regularization method

2010 Mathematics Subject Classification: 34E15

Existence of eigenvalues of problem with shift for an equation of parabolic-hyperbolic type

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Abstract: In the paper a spectral problem for an operator of parabolic-hyperbolic type of I kind with non-classical boundary conditions is considered. The problem is considered in a standard domain. The parabolic part of the space is a rectangle. And the hyperbolic part of the space coincides with a characteristic triangle. We consider a problem with the local boundary condition in the domain of parabolicity and with the boundary condition with displacement in the domain of hyperbolicity.

Let $\Omega \in R^2$ be a finite domain bounded for $y > 0$ by the segments AA_0, A_0B_0, B_0B, $A = (0,0)$, $A_0 = (0,1)$, $B_0 = (1,1)$, $B = (1,0)$, and for $y < 0$ by the characteristics $AC:\ x + y = 0$ and $BC:\ x - y = 1$ of an equation of the mixed parabolic-hyperbolic type

(1) $$Lu = \left\{\begin{array}{l} u_x - u_{yy}, y > 0 \\ u_{xx} - u_{yy}, y < 0 \end{array}\right\} = f(x,y).$$

Problem S. *Find a solution to Eq.* (1) *satisfying boundary conditions*

(2) $$u|_{AA_0\cup A_0B_0} = 0,$$

(3) $$\alpha u(\theta_0(t)) = \beta u(\theta_1(t)),\quad 0 \le t \le 1,$$

where $\theta_0(t) = \left(\frac{t}{2}, -\frac{t}{2}\right)$, $\theta_1(t) = \left(\frac{t+1}{2}, \frac{t-1}{2}\right)$.

We prove the strong solvability of the considered problem. The main aim of the paper is the research of spectral properties of the problem. The existence of eigenvalues of the problem is proved.

Keywords: spectral problem, equation of parabolic-hyperbolic type, boundary condition with shift

2010 Mathematics Subject Classification: 35M10, 35M12, 35P

The method of accelerated convergence for constructing conditional-periodical solutions

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Abstract: In the present paper we take quasi-linear system of differential equations

(1) $$\frac{dx}{dt} = Ax + \varepsilon f(t, x),$$

where $x = colon(x_1, x_2)$, $A = (a_{jk}), j = k = 1, 2$,
$f(t, x) = colon(f_1(t, x_1, x_2), f_2(t, x_1, x_2))$
conditionally-periodic by t with frequency basis $\omega_1, \omega_2, ..., \omega_n$; analytical by t and x in the domain
$D = \{(t, x) \subset C^3 : ||x|| \leq h, ||Im\omega t|| \leq q\}$ function, $det|A - \lambda E| = 0$ has purely imaginary roots $i\sigma_1, i\sigma_2$, and rational numbers σ_1, σ_2 non-co-measurable with $\omega_1, \omega_2, ..., \omega_n$, ϵ is a small parameter.

In order to find a conditionally-periodic solutions of (1) the method of accelerated convergence [1] is applied. As an initial approximate conditionally-periodic solutions of the system (1) $x^{(0)}(t, \varepsilon) = 0 := colon(0, 0)$ is chosen. Its residual denoted by $x^{(1)}(t, \varepsilon)$ and take this function as a first approximation to the original conditionally-periodic solutions of the system (1).

Keywords: quasi-linear system of differential equations, conditionally-periodic solution, small parameter

2010 Mathematics Subject Classification: 34B15

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Martinet-Ramis modulus for one Quadratic System

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Abstract: There are considered a quadratic system

1) $$\begin{cases} \dot{p} = p(1 - v) \\ \dot{v} = v(p - v) \end{cases}$$

This system is in some sense, limit system for well known Jouanolou system [1]. The system (1) has a saddlenode singularity at the origin. In this work we calculate first coefficients of Martinet-Ramis' modulus [2].

Martinet-Ramis' modulus (C, ϕ) (for saddlenode singular point) [2] are constructed by transformations reducing initial system to its (orbital) formal normal form. Solutions of the system (1) with given initial conditions can be found as a series (with respect to initial condition). Using these solutions it is possible to find coefficients of the normalizing transformations, and then to determinate coefficients of modulus.

As a result we get

Theorem 1. *Let (C, ϕ) be Martinet-Ramis modulus for (1). Then:* $C = 0$, $\phi(z) = z + 2\pi i z^2 + (2\pi i - 4\pi^2)z^3 + ...$

Corollary 2. *The system (1) is not analytically orbital equivalent to its formal normal form.*

Keywords: Martinet-Ramis modulus, saddlenode, central manifold

2010 Mathematics Subject Classification: 34C20, 37F75

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