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Interdependence of Real, Financial and Export Import Indicators in a DSGE Model of Multiple Countries

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Abstract Interdependence, which is a consequence of the international division of labor and use of the world's natural resources, increases at the global level. Macroeconomic indicators of each country are more exposed to shocks arising in the country and in partner countries. In this paper, we propose a model of dynamic stochastic general equilibrium (DSGE) of many countries. For each country, the variables of output, inflation, interest rate, exchange rate, terms of trade, as well as exports and imports for each pair of countries are included in the model. In accordance with the number of countries the model contains equations of dynamic IS and New Keynesian Phillips curves and equations of monetary policy. The estimation of the model was implemented for the economies of Kazakhstan, Russia and the EU. An asymmetrical interaction of large and small economies is taken into account. The analysis of the impact of internal and external shocks on the macroeconomic variables is performed for each country/region. Responses of indicators on various shocks are obtained. For example, a positive technology shock in the country leads to the negative reaction of output, inflation and interest rate variables, as well as having a positive impact on imports and the negative impact on exports in each partner country. Cost-push as well as monetary policy shocks reduce imports and increase exports, and this is also observed for a couple of countries where there is no such a shock. It is revealed that the value of the response does matter to the size of the economy. The model allows analyzing the effects of the macroeconomic policies of trading partners to the fluctuations of the various shocks. The model can be extended in various directions.

Keywords Interdependence • DSGE model • Impulse-response analysis • Export • Import

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1 Introduction

There is an increasing interdependence on the global level in the world which is the consequence of international labor division, use of world reserves of natural resources. Macroeconomic indicators of each country, such as GDP growth, inflation rate, exchange rate, export, import of goods are more exposed to shocks as in the country, so in the partner economies. Besides, a great influence of big economies on other countries should be expected. An effective instrument to the solution of this problem can be Dynamic stochastic general equilibrium models. Their fundamentals were laid in Kydland and Prescott (1982). They are based on microeconomic analysis of agents optimizing their behavior in the conditions of flexible prices. Price flexibility leaves opportunity only for real values to make fluctuations in the economy.

Then, elements of the Keynesian approach, containing nominal rigidities were included in the DSGE model. In Calvo (1983) a pricing mechanism as a certain stochastic process of decision-making firms to change the price or keeping it at the same level was proposed. As a result a new paradigm in the construction of models of dynamic stochastic general equilibrium appeared. They take into account the microeconomic foundations of decision-making by households, optimization behavior of monopolistically competitive firms and regulatory functions of the state. Because of the nominal rigidity of prices and wages, compliance with the results of calculations required by the model with real data of short-term macroeconomic fluctuations in the economy is achieved.

Among the most well-known DSGE models created in the last two decades and intended for policy analysis and forecasting, there is a list of developments by central banks of Europe and America (Smets and Wouters 2003; Dib 2001; Cuche-Curti et al. 2009) and central banks of developing countries (Medina and Soto 2007; Tovar 2008; Galí and Monacelli 2005) and others.

In this article a model of dynamic stochastic general equilibrium of many countries in which a way to construct models of two countries was offered (Obstfeld and Rogoff 2001; Corsetti and Pesenty 2001). For each country variables such as output, inflation, interest rate, exchange rate, trade conditions and export and import for each pair of countries are included in the model. A model contains equation of dynamic IS and New Keynesian Phillips curves and equations of monetary policy. Asymmetric interference of big and small economies is considered in the equations. The analysis of the impact of internal and external shocks on the macroeconomic performance of each country/region is done. Response on various indicators shocks is obtained. It was noted that for the size of the economy the value of the response index is important. The model can be used to analyze the impact of macroeconomic policies both within the country and abroad due to the fluctuations of the various shocks. Section 2 presents a model of dynamic stochastic general equilibrium in many countries/regions, with its mathematical reasoning. It is supplemented by the equations of exports and imports between the countries. The results of calculations on the model of the three countries/regions are presented in Sect. 3. The last section concludes.

2 Model

2.1 Households

Domestic and foreign goods are consumed in each country. For production, firms use labor. Nominal rigidity of prices is modeled with the use of mechanism by Calvo (1983). It is assumed that the world population is made up of a continuum of infinitely long-lived households indexed through $i \in [0,\ 1]$. Households in each country have the same preferences. There are K countries in the world. In the country k households are indexed as $i \in J_k$. A set of $J_k,\ k = 1,\ 2, \ldots, K$, do not cross/overlap and cover all the households. We denote n_k the set of a measure J_k, which reflects the population of the country k.

In the country k a composite index of consumption is determined by assuming that all the traded goods and trade costs are ignored:

$$C_k = \left(\sum\nolimits_{l=1}^{K} n_l^{\frac{1}{\mu}} C_{kl}^{\frac{\mu-1}{\mu}}\right)^{\frac{\mu}{\mu-1}}, \tag{1}$$

where C_{kl}—a composite index of consumer goods in the country k produced in the country l, μ—parameter. The representative household maximizes C_k while limiting

$$\sum\nolimits_{l=1}^{K} P_{kl} C_{kl} = P_k \tag{2}$$

Where P_{kl}—the index of the prices of goods from the country l in the currency of the country k, P_k—index of the prices of all goods consumed in the country k. Under the condition of maximum consumption (1) under the limit (2) we obtain

$$C_{kl} = \left(\sum_{j=1}^{K} n_j P_{kj}^{1-\mu}\right)^{\frac{\mu}{1-\mu}} \frac{n_l}{P_{kl}^{\mu}} C_k,$$

$$P_k = \left(\sum\nolimits_{l=1}^{K} n_l P_{kl}^{1-\mu}\right)^{\frac{1}{1-\mu}}. \tag{3}$$

For simplicity, we consider the case where $\mu = 1$. In the limit where μ is tending to 1, we find that the price index

$$P_k = \prod_{l=1}^{K} P_{kl}^{n_l},$$

and the index of consumer goods from country l to country k

$$C_{kl} = n_l \left(\frac{P_{kl}}{P_k}\right)^{-1} C_k, \quad k, l = 1, \ldots, K. \tag{4}$$

And a composite index of consumption in the country k will take the form of a power function

$$C_k = \prod\nolimits_{l=1}^{K} \frac{C_{kl}^{n_l}}{{n_l}^{n_l}} = \frac{C_{k1}^{n_1} C_{k2}^{n_2} \ldots C_{kK}^{n_K}}{n_1^{n_1}\, n_2^{n_2} \ldots n_K^{n_K}}, \;\; k,\ l = 1, 2, \ldots, K. \tag{5}$$

Here and below the index of period t is omitted if it is inessential. The index of consumption of goods in the country k produces in the country l:

$$C_{kl} = \left[\left(\frac{1}{n_l}\right)^{\frac{1}{\gamma}} \int_j C_k(i)^{\frac{\gamma-1}{\gamma}} di\right]^{\frac{\gamma}{\gamma-1}}, \tag{6}$$

where $C_k(i)$—consumption of good i in the country k, γ—the elasticity of substitution across two individual goods i, j produced in the country k, $\gamma > 1$. The representative household maximizes C_{kl} on $C_k(i)$, $i \epsilon J_l$ provided

$$P_{kl} C_{kl} = \int_{J_l} P_k(i) C_k(i) di, \tag{7}$$

where $P_k(i)$—the price of good i in the country k.

In the country k a representative household has a discontinued utility

$$U_{kt} = \mathbb{E}_t \left\{ \sum\nolimits_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_{ks}^{1-\rho}}{1-\rho} + \frac{\omega_k}{1-\delta} \left(\frac{M_{ks}}{P_{ks}}\right)^{1-\delta} - \vartheta_k \frac{L_{ks}^{1+\varphi}}{1+\varphi} \right] \right\}, \tag{8}$$

Where C_{ks}—real consumption, $\frac{M_{ks}}{P_{ks}}$—real money balances, P_{ks}—consumer price index in the country k, L_{ks}—is the cost of labor in the time period s. Parameter β, $0 < \beta < 1$, represents the intertemporal discount factor, parameters ρ, δ, φ define utility function elasticities of the relevant variables.

The representative household i maximizes utility (8) under the budget constraints

$$P_{kt} C_{kt} + M_{kt} + B_{kt} + P_{kt} \tau_{kt} \le W_{kt} L_{kt} + (1 + i_{kt-1}) B_{kt-1} + \\ + M_{kt-1} + \Pi_{kt}, \quad t = 0, 1, \ldots$$

Here for the country k and period t: W_{kt}—nominal wage in a perfect labor market, the same for all households, i_{kt-1}—nominal interest rate for the time interval from t − 1 to t for a one-period risk free corporate bonds B_{kt-1} in the domestic currency are indicated. Money M_{kt} does not give a nominal income. Π_{kt}—income of a representative household, τ—the real undistorted lump-sum taxes. The index of household $i \epsilon J_k$ is omitted for simplicity. For each country $k = 1,\ldots,K$ optimality conditions of the first order are true:

$$\frac{C_{kt}^{-\rho}}{P_{kt}} = \beta(1 + i_{kt})\mathbb{E}_t\left[\frac{C_{kt+1}^{-\rho}}{P_{kt+1}}\right], \tag{9}$$

$$\vartheta_k \frac{L_{kt}^{\varphi}}{C_{kt}^{-\rho}} = \frac{W_{kt}}{P_{kt}}, \tag{10}$$

$$\frac{\omega\left(\frac{M_{kt}}{P_{kt}}\right)^{-\delta}}{C_{kt}^{-\rho}} = \frac{i_{kt}}{1 + i_{kt}}. \tag{11}$$

2.2 Firms

It is assumed that each household is also a producer of the good $i \in J_k$. The goods are thought to be differentiated, thus each such a firm has a market power. In simple words output of each firm $i \in J_k$. is defined by the production function

$$Y_{kt}(i) = A_{kt}L_{kt}(i). \tag{12}$$

The value A_{kt} sets the influence of shock performance. It is assumed that in different countries these values may be correlated. In the case of accounting energy costs as in Mukhamediyev (2014) production function might look like

$$Y_{kt}(i) = A_{kt}min\{L_{kt}(i),\ O_{kt}(i)\},$$

Where $O_{kt}(i)$—the cost of oil as an energy resource, L_{kt}—labor costs. For a country that produces oil, its production sector should be separately disclosed. Here A_{kt} reflects technological shocks. Behavior of A_{kt} is described by an autoregressive process

$$lnA_{kt} = \rho_{ak}lnA_{kt-1} + \varepsilon_{akt}, \quad \varepsilon_{akt} \sim\ i.i.d.\left(0,\ \sigma_{ak}^2\right).$$

Since the goods are assumed to be diversified, the firm may change the price of its goods to a certain limit, i.e. there is a monopolistic competition. Labor markets in the countries are isolated. Firms hire labor in their countries. For the production function (12) the firm's profit

$$\Pi_{kt}(i) = P_{kt}(i)\left[Y_{kt}(i) - \frac{W_{kt}}{A_{kt}P_{kt}(i)}Y_{kt}(i)\right].$$

The coefficient

$$MC_{kt} = \frac{W_{kt}}{A_{kt}P_{kt}(i)}$$

is the real marginal cost of the firm. The optimal production volume of a good $i \in J_k$ is determined by the condition for maximization:

$$\frac{\partial \Pi_{kt}(i)}{\partial Y_{kt}(i)} = P_{kt}(i) + Y_{kt}(i)\frac{\partial P_{kt}(i)}{\partial Y_{kt}(i)} - \frac{W_{kt}}{A_{kt}} = 0.$$

Let's consider the situation with flexible prices. Then all the firms in each period t optimally adjust their prices and set them the same, $P_{kt}(i) = P_{kt}$. Since only the firm i produces this product, then the equilibrium output should be equal to the global demand for it, that is $Y_{kt}(i) = C_t^w(i)$. Note that

$$\frac{Y_{kt}(i)}{P_{kt}}\frac{\partial P_{kt}}{\partial Y_{kt}(i)} = \frac{C_t^w(i)}{P_{kt}}\frac{\partial P_{kt}}{\partial C_t^w(i)} = -\frac{1}{\eta},$$

where η—the elasticity of demand for good at a price. Therefore, real marginal cost of production in the case of flexible prices is the same for all manufacturers in all countries:

$$\widetilde{MC} = \frac{\eta - 1}{\eta}.$$

Conditions of cleaning of market of commodity i is equilibrium of supply of this good to the total demand for all countries:

$$Y_{kt}(i) = \sum_{l=1}^{K} n_l C_{lt}(i), \quad i \in J_k,\ k = 1, \ldots, K.$$

It is believed that consumption is distributed across all the countries in proportion to the population.

Given the law of one price (6) it follows

$$C_{lmt} = n_m \left(\frac{P_{lmt}}{P_{lt}} \right)^{-1} C_{lt} = n_m \left(\frac{P_{kmt}}{P_{kt}} \right)^{-1} C_{lt}.$$

With S_{klt} we denote the terms of trade between countries k and l:

$$S_{klt} = \frac{P_{kkt}}{P_{klt}}.$$

Each country consumes exactly its real income. Formally we believe that $S_{kk} = 1$. It follows that

$$C_{kt} = \frac{P_{kkt} Y_{kt}}{P_{kt}} = \frac{P_{kkt} Y_{kt}}{\prod_{l=1}^{K} P_{klt}^{n_l}} = \prod_{l=1}^{K} S_{klt}^{n_l} Y_{kt}. \tag{13}$$

Now, using the production function (12) and optimality conditions (9)–(11) we can calculate equilibrium output under flexible prices.

$$\widetilde{Y}_{kl} = A_{kt}^{\frac{\varphi+1}{\varphi+\rho}} \chi^{\frac{-1}{\varphi+\rho}} \left(\frac{\eta - 1}{\eta} \right)^{\frac{1}{\varphi+\rho}} \prod_{l=1}^{K} \left[S_{klt}^{\frac{-\rho(n_l - 1)}{\varphi+\rho}} \right] \tag{14}$$

It depends positively on the overall performance and condition of country's trade with other countries, because $n_l < 1$.

2.3 Rigid Prices

Let's assume that in addition to monopolistic competition there is also nominal rigidity of prices. For the country k we present Euler equation in the following form:

$$C_{kt}^{-\rho} = \beta(1 + i_{kt}) P_{kt} \mathbb{E}_t \left[\frac{C_{kt+1}^{-\rho}}{P_{kt+1}} \right].$$

We insert in it real consumption C_{kt} from equation (13):

$$\prod_{l=1}^{K} \left(S_{klt}^{-\rho n_l} \right) Y_{kt}^{-\rho} = \beta(1 + i_{kt}) P_{kt} \mathbb{E}_t \left[\frac{1}{P_{kt+1}} \prod_{l=1}^{K} \left(S_{klt+1}^{-\rho n_l} \right) Y_{kt+1}^{-\rho} \right].$$

In the steady state economy we denote output as $\overline{Y}_k$., by $\overline{S}_{kl}$—the terms of trade of country k with country l, by $\bar{\imath}_k$—nominal interest rate, by $\overline{P}_k$– price index of commodities in the country k. Let's write this equation for the steady state.

We do log-linearization of both equations. Let's denote: $s_{klt} = \ln S_{klt}$, $y_{kt} = \ln Y_{kt}, \bar{s}_{kl} = \ln \bar{S}_{kl}$, $\bar{y}_k = \ln \bar{Y}_k$, $p_{kt} = \ln P_{kt}$,$\bar{p}_k = \ln \bar{P}_k$. Using the properties of logarithms, we obtain the following equation:

$$\sum_{l=1}^{K} \rho n_l s_{klt} - \rho y_{kt} + \sum_{l=1}^{K} \rho n_l \bar{s}_{kl} + \rho \bar{y}_k = \ln(1 + i_{kt}) - \ln\left(1 + \bar{i}_k\right) +$$

$$+P_{kt} - \mathbb{E}_t\left[p_{kt+1}\right] - \sum_{l=1}^{K} \rho n_l \mathbb{E}_t[s_{klt}] - \rho \mathbb{E}_t\left[y_{kt+1}\right] + \sum_{l=1}^{K} \rho n_l \mathbb{E}_t[\bar{s}_{kl}] + \rho \bar{y}_k.$$

We denote variable deviations from their values in the steady state: $\hat{y}_{kt} = y_{kt} - \bar{y}_k$, $\hat{i}_{kt} = i_{kt} - \bar{i}_k$. Note that the difference $\pi_{kt+1} = p_{kt+1} - p_{kt}$ is the rate of inflation in period t + 1. Then the equation is transformed to:

$$\hat{y}_{kt} = \mathbb{E}_t\left[\hat{y}_{kt+1}\right] + \frac{1}{\rho}\mathbb{E}_t\left[\pi_{kt+1} - \hat{\imath}_{kt}\right] + \sum_{l=1}^{K} n_l \mathbb{E}_t[\Delta s_{klt+1}], \quad k = 1,,,K. \tag{15}$$

This equation is equation of the dynamic IS curve. It sets the aggregate demand in the country k. In the period t aggregate demand increases if the expected outcome in period $t+1$ will be higher than its steady state. Expectation of inflation increase will also increase demand on domestic goods. But the expected improvement in terms of trade with other countries, that is positive value of Δs_{klt+1}, will lead to increase of current aggregate demand as prices of domestic goods will become relatively higher than prices of imported goods and incomes of the countries will increase.

In accordance with the mechanism of price correction Calvo (1983) manufacturer i changes the price in each period with the probability of $1 - \theta$, maximizing the expected profit at the price $P_t(i)$:

$$\mathbb{E}_t\left\{\sum_{s=t}^{\infty} \theta^{s-t} \beta^{s-t} \left(\frac{C_s^w}{C_t^w}\right)^{-\rho} \left[\frac{P_{kt}(i)}{P_{kks}} Y_{ks}(i) - MC_{ks} Y_{ks}(i)\right]\right\}.$$

Here $\beta^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}$ is the stochastic discount factor which is the marginal rate of substitution of global consumption between s and t, MC_{ks}—marginal costs of production in the country k of period s. With a probability θ^{s-t} the producer price in the period $s > t$ is equal to $P_{kt}(i)$, $i \epsilon J_k$.

The profit of the firm in the period s that set a price in the period t, equals:

$$\Pi_{ks}(i) = P_{kt}(i) Y_{ks}(i) - W_{ks} \frac{P_{kks}}{A_{ks} P_{kks}} Y_{ks}(i) = P_{kt}(i) Y_{ks}(i) - MC_{ks} Y_{ks}(i) P_{kks}.$$

We divide on price P_{kks} and find the real profit of the firm $i\epsilon J_k$ in the period s ($s > t$).

$$\frac{\Pi_{ks}(i)}{P_{kks}} = \frac{P_{kt}(i)}{P_{kks}} Y_{ks}(i) - MC_{ks} Y_{ks}(i).$$

We put expression (7) for the global demand for good $i\epsilon J_k$ produced in the country k in the targeted function of the firm instead of the $Y_{ks}(i)$ and write the necessary condition of the maximum of this function by equating to zero its derivative of $P_{kt}(i)$. After some transformations we obtain:

$$P_{kt}(i)\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\mathbb{E}_t\left[MC_{ks}\left(\frac{P_{kks}}{P_{kkt}}\right)^{\eta}\left(\frac{P_{kks}}{P_{ks}}\right)^{-1}C_s^w 1-\rho\right] =$$

$$= P_{kkt}\frac{\eta-1}{\eta}\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\mathbb{E}_t\left[\left(\frac{P_{kks}}{P_{kkt}}\right)^{\eta-1}\left(\frac{P_{kks}}{P_{ks}}\right)^{-1}C_s^w 1-\rho\right]. \quad (16)$$

A positive value of θ corresponds to rigid prices. We carry out log-linearization of the equation (16). After transformation we obtain:

$$\hat{p}_{kt}(i) - \hat{p}_{kkt} = (1-\theta\beta)\mathbb{E}_t\sum_{s=t}^{\infty}(\theta\beta)^{s-t}[\widehat{mc}_{ks} + \hat{p}_{kks}],$$

$$\hat{p}_{kt}(i) - \hat{p}_{kkt} = (1-\theta\beta)\widehat{mc}_{kt} +$$
$$+ \theta\beta\mathbb{E}_t\left[\hat{p}_{kt+1} - \hat{p}_{kkt+1} + \hat{\pi}_{kkt+1}\right]. \quad (17)$$

In this situation, true the equation:

$$\hat{p}_{kt}(i) = \frac{1}{1-\theta}\hat{p}_{kkt} - \frac{\theta}{1-\theta}\hat{p}_{kkt-1}.$$

We put this expression in the equation (17) and get:

$$\pi_{kkt} = \beta\mathbb{E}_t[\pi_{kkt+1}] + \frac{(1-\theta\beta)(1-\theta)}{\theta}\widehat{mc}_t. \quad (18)$$

This is a New Keynesian Phillips curve for the country k. Here $\widehat{mc}_t = mc_t - \widetilde{mc}_t$. Note the differences in the determination of inflation rate in the Phillipps equation and in the equation of the dynamic IS curve. In equation (16) π_{kt} is determined by the consumer price index, but in the equation (18) π_{kkt} represents growth rate of prices of goods produced in the country k.

From production under flexible prices, we now turn to the issue of deviations of output under the rigid prices: $x_{kt} = \hat{y}_{kt} - \hat{\tilde{y}}_{kt}$. We find the ratio

$$\frac{MC_{kt}}{\widetilde{MC}_{kt}} = \frac{\eta W_{kt}}{(\eta-1)A_{kt}P_{kkt}} = \frac{\eta W_{kt}\prod_{l=1}^{K}P_{klt}^{n_l}}{(\eta-1)A_{kt}P_{kkt}} = \frac{\eta W_{kt}}{A_{kt}(\eta-1)P_{kt}}\prod_{l=1}^{K}S_{klt}^{-n_l},$$

$$\frac{MC_{kt}}{\widetilde{MC}_{kt}} = \frac{\eta\chi}{A_{kt}(\eta-1)}\left(\frac{Y_{kt}}{A_{kt}}\right)^{\varphi}\left(\prod_{l=1}^{K}S_{klt}^{-n_l}\right)^{\rho} Y_{kt}^{\rho}\prod_{l=1}^{K}S_{klt}^{-n_l} =$$

$$= \frac{\eta\chi}{(\eta-1)}\left(\frac{1}{A_{kt}}\right)^{\varphi+1}\prod_{l=1}^{K}S_{klt}^{(\rho-1)n_l}Y_{kt}^{\rho+\varphi}.$$

We transform equation (10) for the flexible prices using equations (12) and (13).

$$\vartheta_k\frac{L_{kt}^{\varphi}}{C_{kt}^{-\rho}} = \frac{W_{kt}}{A_{kt}P_{kkt}}A_{kt}\frac{P_{kkt}}{P_{kt}}, \qquad \vartheta_k L_{kt}^{\varphi}\left(Y_{kt}\prod_{l=1}^{K}S_{klt}^{n_l}\right)^{\rho} = \frac{\eta}{(\eta-1)}A_{kt}\prod_{l=1}^{K}S_{klt}^{n_l},$$

$$\vartheta_k\left(\frac{Y_{kt}}{A_{kt}}\right)^{\varphi}Y_{kt}^{\rho} = \frac{\eta-1}{\eta}A_{kt}\prod_{l=1}^{K}S_{klt}^{(1-\rho)n_l}.$$

So, the issue under the flexible prices:

$$\widetilde{Y}_{kt}^{\varphi+\rho} = \frac{1}{\vartheta_k}\frac{\eta-1}{\eta}A_{kt}^{\varphi+1}\prod_{l=1}^{K}S_{klt}^{(1-\rho)n_l}. \tag{19}$$

Then, we obtain

$$\frac{MC_{kt}}{\widetilde{MC}_{kt}} = \left(\frac{Y_{kt}}{\widetilde{Y}_{kt}}\right)^{\varphi+\rho}.$$

We carry out log-linearization:

$$\widehat{mc}_{kt} = (\varphi+\rho)(y_{kt}-\hat{y}_{kt}) = (\varphi+\rho)x_{kt}.$$

We rewrite equation (15) of the dynamic IS curve using deviation of output under the rigid prices from deviation under flexible prices:

$$x_{kt} = \mathbb{E}_t[x_{kt+1}] + \frac{1}{\rho}\left(\mathbb{E}_t[\pi_{kt+1}] - \hat{i}_{kt}\right) + \sum_{l=1}^{K}n_l\mathbb{E}_t[\Delta s_{klt+1}] - \hat{\tilde{y}}_{kt} + \mathbb{E}_t\left[\hat{\tilde{y}}_{kt+1}\right], \tag{20}$$

And also equation (18) of the neoclassical Phillipps curve:

$$\pi_{kkt} = \beta\mathbb{E}_t[\pi_{kkt+1}] + \frac{(1-\theta\beta)(1-\theta)}{\theta}(\varphi+\rho)x_t + u_{kt}, \tag{21}$$

where u_{kt} is the auto regression process

$$u_{kt} = \rho_{uk} u_{kt-1} + \upsilon_{kt}, \qquad \upsilon_{kt} \sim i.i.d.\left(0,\ \sigma^2_{\upsilon k}\right).$$

It reflects the impact of the shocks in the production costs. By log-linearization from (19) we obtain for flexible prices

$$\widetilde{y}_{kt} = -\frac{1}{\varphi+\rho}\ln\chi + \frac{1}{\varphi+\rho}\ln\left(\frac{\eta-1}{\eta}\right) + \frac{\varphi+1}{\varphi+\rho}a_{kt} + \frac{1-\rho}{\varphi+\rho}\sum_{l=1}^{K} n_l s_{klt}.$$

We calculate the sum of the last two terms in equation (20).

$$\mathbb{E}_t\left[\hat{\widetilde{y}}_{kt+1}\right] - \hat{\widetilde{y}}_{kt} = \mathbb{E}_t\left[\hat{y}_{kt+1}\right] - \hat{y}_{kt} =$$
$$= \frac{\varphi+1}{\varphi+\rho}\mathbb{E}_t[\Delta a_{kt+1}] + \frac{1-\rho}{\varphi+\rho}\sum_{l=1}^{K} n_l \mathbb{E}_t[\Delta s_{klt+1}].$$

By the definition terms of trade of the country k with the country l:

$$S_{klt} = \frac{P_{kkt}}{\xi_{klt}P_{lkt}} = \frac{P_{kkt}}{P_{klt}P_{kkt}}.$$

Taking the logarithm of both sides of equation we get:

$$s_{klt} = -e_{klt} - p_{lkt} + p_{kkt} = p_{kkt} - p_{klt}.$$

Hence, we have:

$$\mathbb{E}_t[\Delta s_{klt+1}] = \mathbb{E}_t[\pi_{kkt+1}] - \mathbb{E}_t[\pi_{klt+1}].$$

From formula (3) the below ratio follows

$$\mathbb{E}_t[\pi_{kt+1}] = \mathbb{E}_t[\pi_{kkt+1}] - \sum_{l=1}^{K} n_l \mathbb{E}_t[\Delta s_{klt+1}].$$

Then, the equation of the dynamic curve IS can be written as:

$$x_{kt} = \mathbb{E}_t[x_{kt+1}] + \frac{1}{\rho}\left(\mathbb{E}_t[\pi_{kkt+1}] - \hat{i}_{kt}\right) + \frac{\varphi+1}{\varphi+\rho}\mathbb{E}_t[\Delta a_{kt+1}] +$$
$$+ \frac{\varphi(1-\rho)}{\rho(\varphi+\rho)}\sum_{l=1}^{K} n_l \mathbb{E}_t[\Delta s_{klt+1}]. \qquad (22)$$

To eliminate currency speculation conditions of uncovered interest arbitrage must be satisfied:

$$1 + i_{kt} = (1 + i_{lt}) \frac{\mathbb{E}_t[\varepsilon_{klt+1}]}{\varepsilon_{klt}}, \quad k \neq l.$$

Log-linearization will bring this equation to the form

$$i_{kt} = \mathbb{E}_t[\Delta e_{klt+1}] + i_{lt}, \qquad k \neq l.$$

We write this equation for the period t – 1, equation from it and from the equation of terms of trade in log form, we obtain:

$$\Delta s_{klt} = \hat{i}_{lt-1} - \hat{i}_{kt-1} + \pi_{kkt} - \pi_{llt}, \quad k \neq l. \tag{23}$$

Note that, $\Delta s_{lkt} = -\Delta s_{klt}$, and the relationship between the increments of logarithms in terms of trade between the two countries

$$\Delta s_{lmt} = \Delta s_{kmt} - \Delta s_{klt}. \tag{24}$$

Consequently, independent Δs_{klt} of all $K - 1$, for example, $\Delta s_{1lt}, \quad l = 2, \ldots, K$. Other values Δs_{mlt} are expressed through them. Equations defining the movement of interest rates should be added to equations (21)–(23). According to the rule of monetary policy Taylor (1993) interest rates are set by central banks in accordance with the formula of the following form:

$$i_{kt} = \psi_{\pi k}\pi_{kkt} + \psi_{xt}x_{kt} + \psi_{ik}\hat{i}_{kt-1} + \upsilon_{kt}, \qquad k = 1, \ldots, K. \tag{25}$$

It is assumed that dynamics υ_{kt} is defined exogenously with the autoregression process of the first order:

$$\upsilon_{kt} = \rho_\upsilon \upsilon_{kt-1} + \nu_{\upsilon kt}, \nu_{k\upsilon t} \sim i.i.d.\left(0, \sigma^2_{k\upsilon}\right).$$

After the global financial crisis that began in 2007 there are debates in the literature on how to formally take into account the systemic stability factors in the response function of the monetary authorities. Changes in the monetary policy of central banks are discussed in the article of Di Giorgio (2014).

Thus, the model K of countries is described by $4K - 1$ equations (21)–(23), (25) and contains the same number of variables given the connection (24) between the increments of logarithms in terms of trade.

2.4 Export and Import of Goods

Goods are considered diversified, i.e. each firm produces its product and its production meets the world demand. Therefore, in each period t export of good

j from country k to the country l equals the volume of consumption of this product in the country l:

$$Ex_{klt}(j) = C_{lkt}(j), j \in J_k.$$

Total index of consumption in the country l, produced in the country k, according to the formulas (5) and (13) equals

$$C_{lkt} = n_k \left(\frac{P_{lkt}}{P_{lt}}\right)^{-1} C_{lt} = n_k \left(\frac{P_{lkt}}{P_{lt}}\right)^{-1} \prod_{m=1}^{K} S_{lmt}^{n_m} Y_{lt}.$$

Consider that

$$\frac{P_{lt}}{P_{lkt}} = \prod_{m=1}^{K} \left(\frac{P_{lmt}}{P_{lkt}}\right)^{n_m} = \prod_{m=1}^{K} \left(\frac{P_{kmt}}{P_{kkt}}\right)^{n_m} = \prod_{m=1}^{K} (S_{kmt})^{-n_m}.$$

Hence, for export from country k to the country l it is fair

$$Ex_{klt} = n_k \prod_{m=1}^{K} S_{kmt}^{-n_m} \prod_{m=1}^{K} S_{lmt}^{n_m} Y_{lt},$$

which corresponds to the macroeconomic theory of direct dependence of the export of the country from the production volume abroad. Also, export depends on the ratio of the terms of trade of the countries k and l with other countries. The value n_k reflects production capacity of the country k. Let ex_{klt} denote logarithm of export Ex_{klt}. Then,

$$ex_{klt} = \ln n_k - \sum_{m=1}^{K} n_m s_{kmt} + \sum_{m=1}^{K} n_m s_{lmt} + y_{lt}$$

For the expectation of the export growth in log form, the following is true

$$\mathbb{E}_t[\Delta ex_{klt+1}] = -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] + \sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] + \mathbb{E}_t[y_{lt+1}] - y_{lt}.$$

Let's use the relation between the rate of inflation on consumer price index and on producer price index

$$\mathbb{E}_t[\pi_{lt+1}] = \mathbb{E}_t[\pi_{llt+1}] - \sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}],$$

Which follows from the equation (3) and from the equation of the dynamic IS curve (15):

$$\mathbb{E}_t\left[y_{lt+1}\right] - y_{lt} = \mathbb{E}_t\left[\hat{y}_{lt+1}\right] - \hat{y}_{lt} = -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] - \frac{1}{\rho}\mathbb{E}_t\left[\pi_{lt+1} - \hat{i}_{lt}\right] =$$

$$= \left(\frac{1}{\rho} - 1\right)\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] - \frac{1}{\rho}\mathbb{E}_t\left[\pi_{llt+1} - \hat{i}_{lt}\right].$$

Consequently, for the expected rate of export growth from country k to the country l, an equation is obtained

$$\mathbb{E}_t[\Delta ex_{klt+1}] = -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] -$$

$$-\frac{1}{\rho}\sum\nolimits_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] - \frac{1}{\rho}\mathbb{E}_t\left[\pi_{llt+1} - \hat{i}_{lt}\right]. \tag{26}$$

Import $Im_{klt}(j)$ of the product $j \in J_k$ from the country l to the country k equals the consumption of this good in the country k:

$$Im_{klt}(j) = C_{klt}(j), j \in J_l.$$

Similarly, the index of consumption of all goods in the country k produced in the country l, according to the formulas (5) and (13) equals

$$C_{klt} = n_l \left(\frac{P_{klt}}{P_{kt}}\right)^{-1} C_{kt} = n_l \left(\frac{P_{klt}}{P_{kt}}\right)^{-1} \prod_{m=1}^{K} S_{kmt}^{n_m} Y_{kt}.$$

Also, we have

$$\frac{P_{kt}}{P_{klt}} = \prod_{m=1}^{K}\left(\frac{P_{kmt}}{P_{klt}}\right)^{n_m} = \prod_{m=1}^{K}\left(\frac{P_{lmt}}{P_{llt}}\right)^{n_m} = \prod_{m=1}^{K}(S_{lmt})^{-n_m}.$$

After substitution we get that import from the country l to the country k

$$Im_{klt} = n_l \prod_{m=1}^{K} S_{lmt}^{-n_m} \prod_{m=1}^{K} S_{kmt}^{n_m} Y_{kt}$$

or in log form

$$im_{klt} = \ln n_l - \sum_{m=1}^{K} n_m s_{lmt} + \sum_{m=1}^{K} n_m s_{kmt} + y_{kt}.$$

Import to the country k positively depends on the production volume in this country and negatively depends on ratios of terms of trade of the countries k and l with other countries.

Expectation of the growth rate of import in the log form is true

$$\mathbb{E}_t[\Delta im_{klt+1}] = -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] + \sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] + \mathbb{E}_t[y_{kt+1}] - y_{kt}.$$

The sum of the two terms in this equation

$$\mathbb{E}_t[y_{kt+1}] - y_{kt} = \mathbb{E}_t[\hat{y}_{kt+1}] - \hat{y}_{kt} =$$
$$= -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] + \frac{1}{\rho}\mathbb{E}_t[\pi_{kt+1} - \hat{i}_{kt}] =$$
$$= \left(\frac{1}{\rho} - 1\right)\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] - \frac{1}{\rho}\mathbb{E}_t[\pi_{kkt+1} - \hat{i}_{kt}] .$$

Then for the expectation of the growth rate of import to the country k from the country l, we obtain

$$\mathbb{E}_t[\Delta im_{klt+1}] = -\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{lmt+1}] +$$
$$+ \frac{1}{\rho}\sum_{m=1}^{K} n_m \mathbb{E}_t[\Delta s_{kmt+1}] - \frac{1}{\rho}\mathbb{E}_t[\pi_{kkt+1} - \hat{i}_{kt}] . \tag{27}$$

From the formulas (26) and (27) it is not hard to notice that export from the country k to the country l coincides with import to the country l from the country k.

3 Estimations on Model of Three Countries

The model was estimated based on the statistical data of Kazakhstan (country H), Russia (country F), and European Union (country G). Statistical data for constructing a model of dynamic stochastic equilibrium for three countries were collected based on data of IFS International Monetary Fund, The World Bank, Agency of statistics of Kazakhstan, and National Bank of Kazakhstan. In particular, proportionately to the population, the below values are considered

$$n_H = 0.03, \quad n_F = 0.22, \; n_G = 0.75.$$

Indeed, Kazakhstan is a relatively small country. In another approach, instead of the population, respective volumes of GDP could be taken. The model parameters were

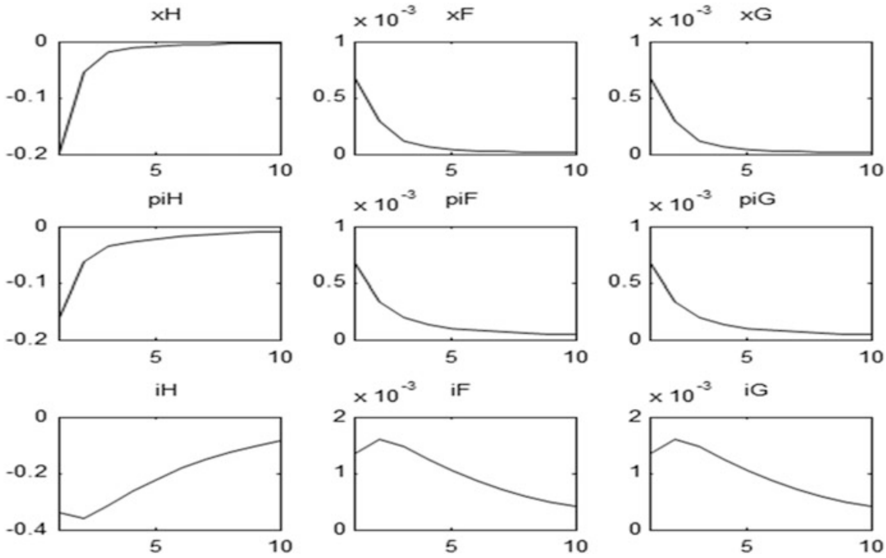

Fig. 1 Impact of technological shock in the country H. Note: xH, xF, xG—deviations of the output under rigid prices from output under flexible prices in logs, piH, piF, piG—inflation rates of the producer price index, iH, iF, iG—deviations of interest rate from its value in steady state for countries H, F, G respectively

mainly estimated with Bayesian method with the use of Metropolis-Hastings algorithm. In equations for technological shocks, parameters $\rho_{aFG}, \rho_{aGF}, \rho_{aGF}$ are taken as equal to zero. This takes into account that technological innovations arising in a large country G, quickly penetrate into both countries F and H, and technological innovations appeared in the country of a medium size F, penetrate only to the country H, and there is no flows of innovation in the opposite direction from small to big country. Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 show how macroeconomic indicators react to shocks in this country and in the other countries.

A positive technological shock in a country H reduces marginal costs of production of goods. It gives to firms the opportunity to reduce prices for domestically produced goods, which in short term leads to reduction of interest rate and deterioration of terms of trade with countries F and G and contributes to reduction of output in the country H before these parameters return to the values in the steady state (Figs. 1, 2, and 3).

Negative response of output to positive shock of overall productivity in the country is noted in the literature, for instance in Gunter (2009). At the same time, terms of trade of the countries F and G with the country H improve. This leads to increase in income of firms, higher prices, output and interest rates in these countries. However, we note that the impact of the shock in the F and G are three orders of magnitude weaker than in the country H, which is due to the relative size of these countries.

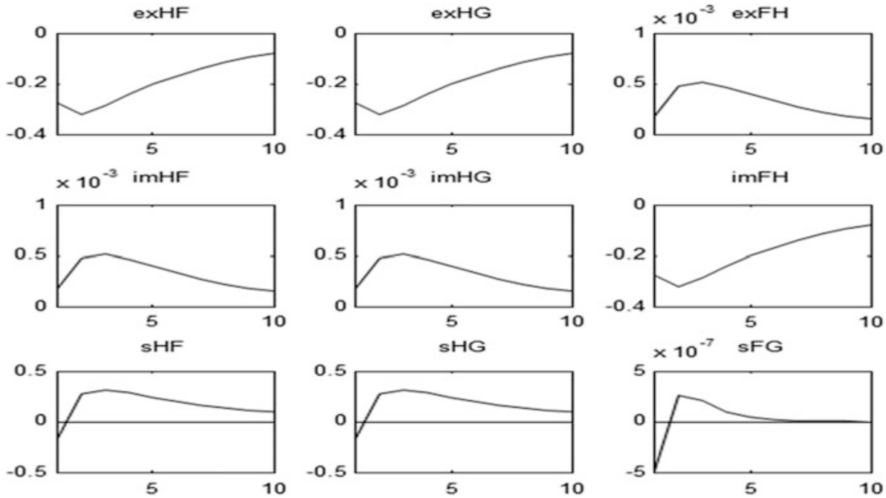

Fig. 2 Impact of technological shock in the country H. Note: Increment of logarithms: exHF—export from the country H to the country F, exHG—export from the country H to the country G, exFH—export from the country F to the country H, imHF—import from the country F to the country H, imHG—import from the country G to the country H, imFH—import from the country F to the country H, sHF—terms of trade of the country H with the country F, sHG—terms of trade of the country H with the country G, sFG—terms of trade of the country F with the country G

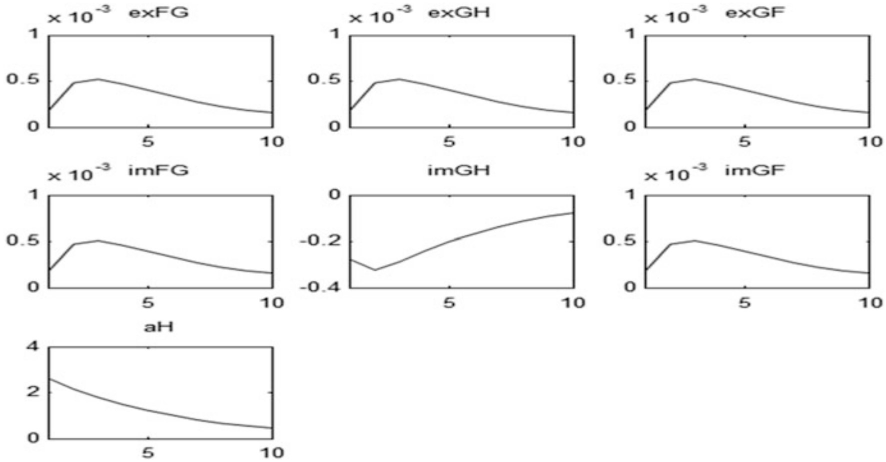

Fig. 3 Impact of technological shock in the country H. Note: Increment of logarithms: exFG—export from the country F to the country G, exGH—export from the country G to the country H, exGF—export from the country G to the country H, imFG—import from the country G to the country F, imGH—import from the country H to the country G, imGF—import from the country F to the country G, aH—total factor productivity in the country H

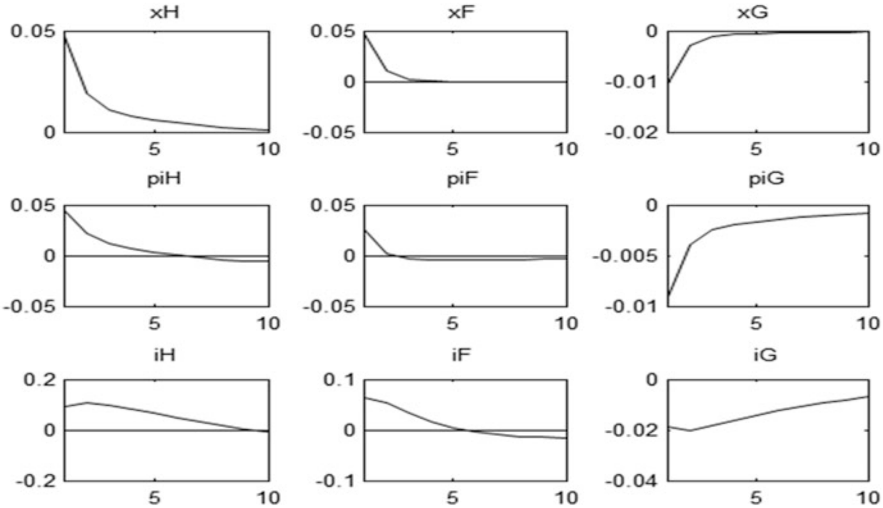

Fig. 4 Impact of technological shock in the country G. Note: xH, xF, xG—deviation of the output under the rigid prices from the output under flexible prices in logarithms, piH, piF, piG—inflation rates of the producer price index, iH, iF, iG—deviation of interest rate from its value in the steady state for the countries H, F, G respectively

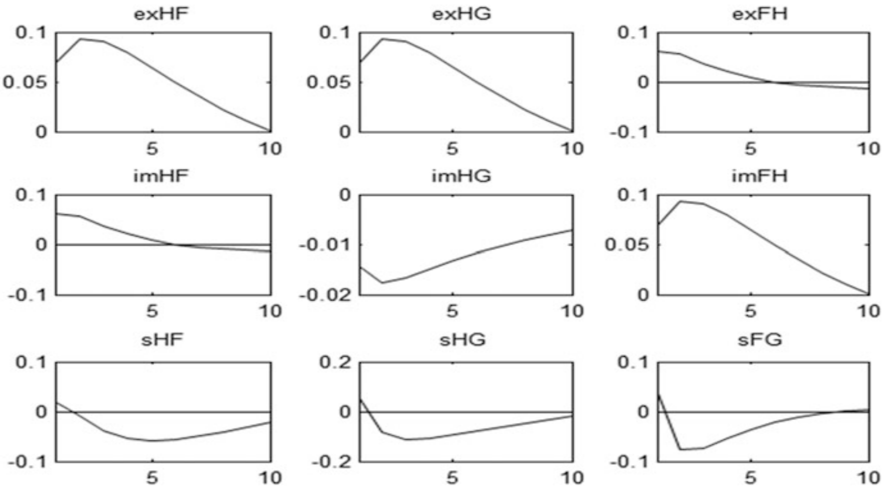

Fig. 5 Impact of technological shock in the country G. Note: Increment of logarithms: exHF—export from the country H to the country F, exHG—export from the country H to the country G, exFH—export from the country F to the country H, imHF—import from the country F to the country H, imHG—import from the country G to the country H, imFH—import from the country F to the country H, sHF—terms of trade of the country H with the country F, sHG—terms of trade of the country H with the country G, sFG—terms of trade of the country F with the country G

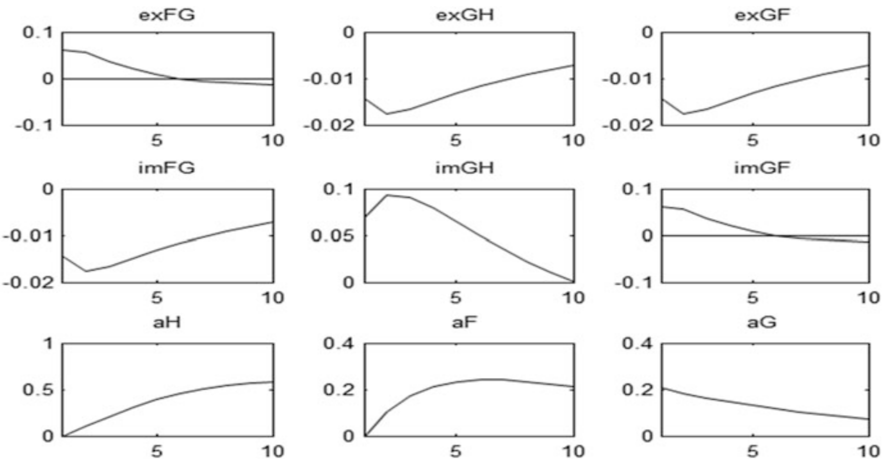

Fig. 6 Impact of technological shock in the country G. Note: Increment of logarithm: exFG—export from the country F to the country G, exGH—export from the country G to the country H, exGF—export from the country G to the country H, imFG—import from the country G to the country F, imGH—import from the country H to the country G, imGF—import from the country F to the country G, aH—total factor productivity in the country H

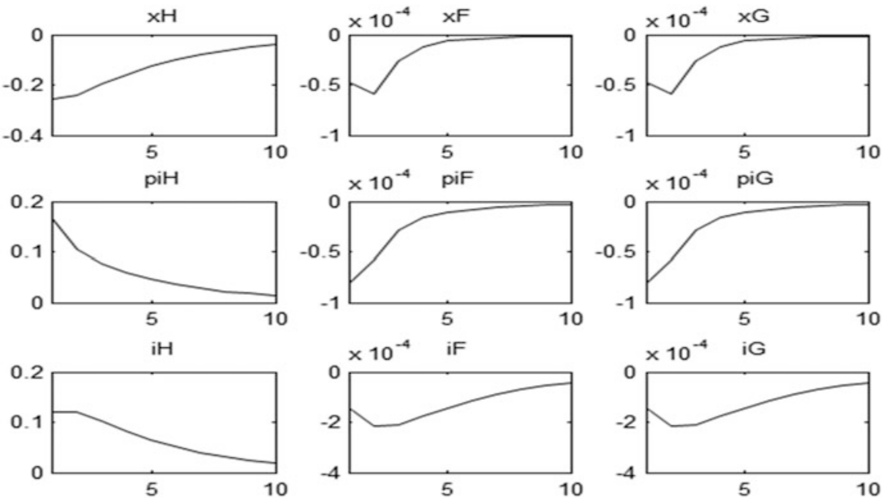

Fig. 7 Impact of production costs shock in the country H. Note: xH, xF, xG—deviations of output under rigid prices from output under flexible prices in logarithms, piH, piF, piG—inflation rates of producer price index, iH, iF, iG—deviations of interest rate from its value in the steady state for the countries H, F, G respectively

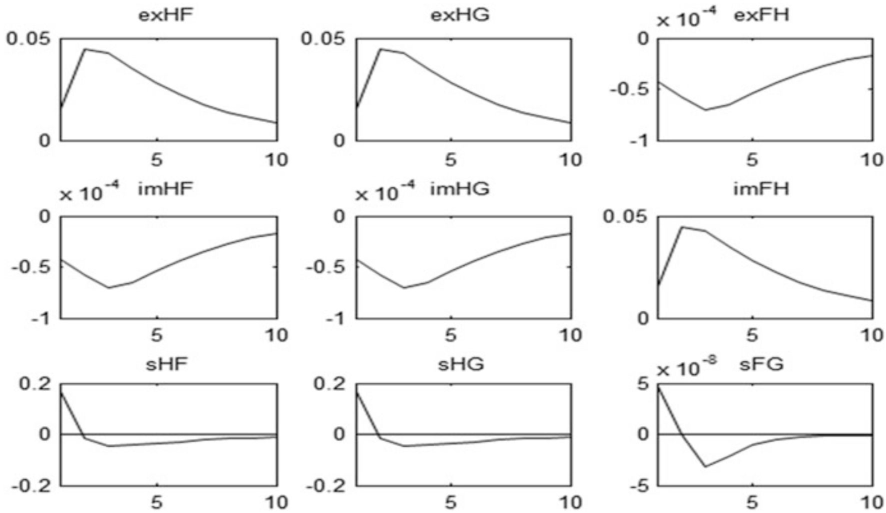

Fig. 8 Impact of production costs shocks in the country H. Note: Incremental logarithms: exHF—export from the country F, exHG—export from the country H to the country, exFH—export from the country F to the country H, imHF—import from the country F to the country H, imHG—import from the country G to the country H, imFH—import from the country F to the country H, sHF—terms of trade of the country H with the country F, sHG—terms of trade of the country H but with the country G, sFG—terms of trade of the country F with the country G

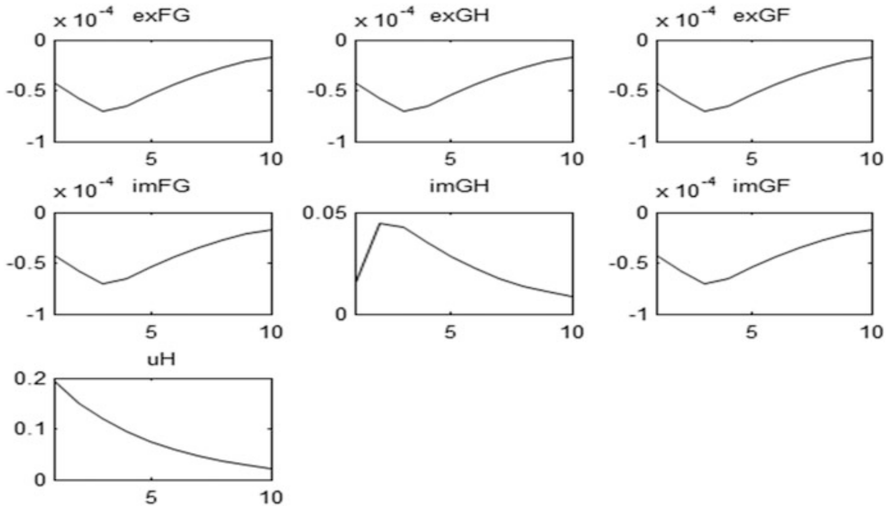

Fig. 9 Impact of shock of production costs in the country H. Note: Incremental logarithms: exFG—export from the country F to the country G, exGH—export from the country G to the country H, exGF—export from the country G to the country H, imFG—import from the country G to the country F, imGH—import from the country H to the country G, imGF—import from the country F to the country G, uH—costs of production in the country H

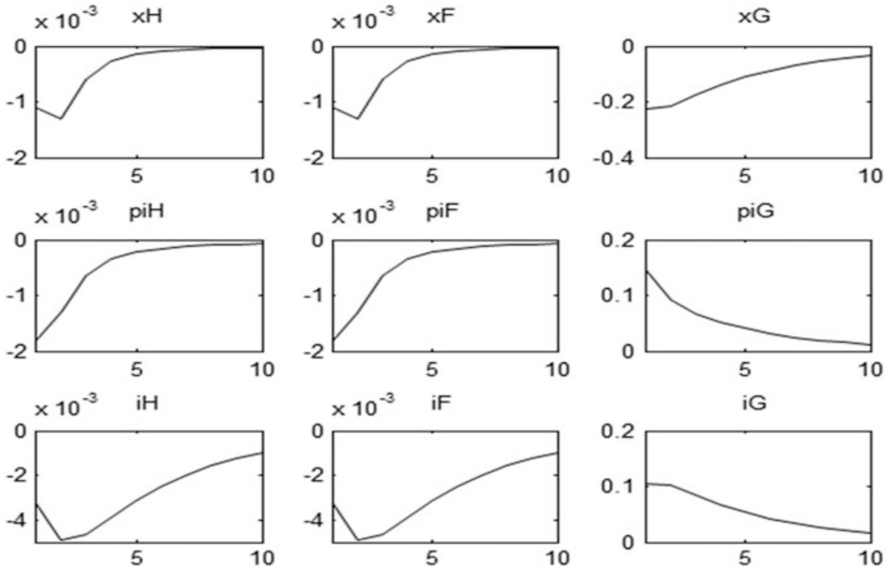

Fig. 10 Impact of shock of production costs in the country G. Note: xH, xF, xG—deviations of output under the rigid prices from output under the flexible prices in logarithms, piH, piF, piG—inflation rates of the producer price index, iH, iF, iG—deviation of interest rate from its value in steady state for countries H, F, G respectively

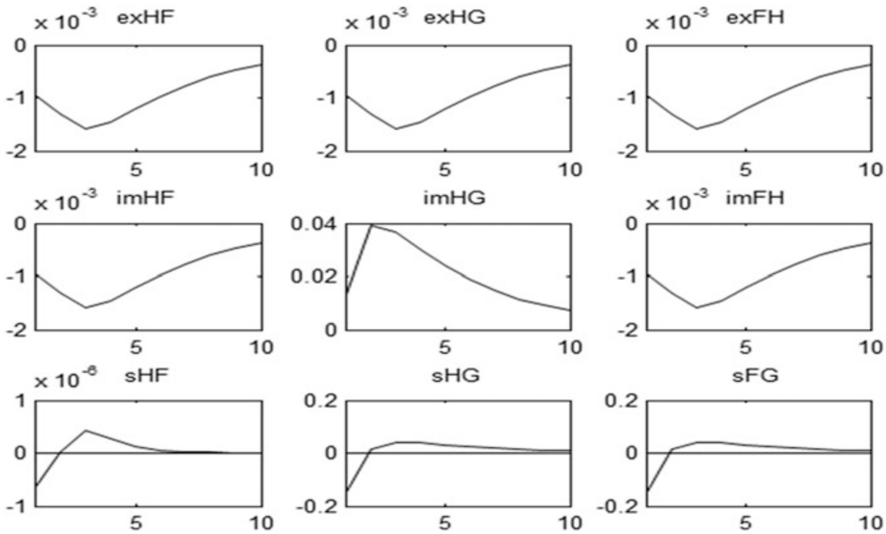

Fig. 11 Impact of shock of production costs in the country G. Note: Increment logarithms: exHF—export from the country H to the country F, exHG—export from the country H to the country G, exFH—export from the country F to the country H, imHF—import from the country F to the country H, imHG—import from the country G to the country H, imFH—import from the country F to the country H, sHF—terms of trade of the country H with the country F, sHG—terms of trade of the country H with the country G, sFG—terms of trade of the country F with the country G

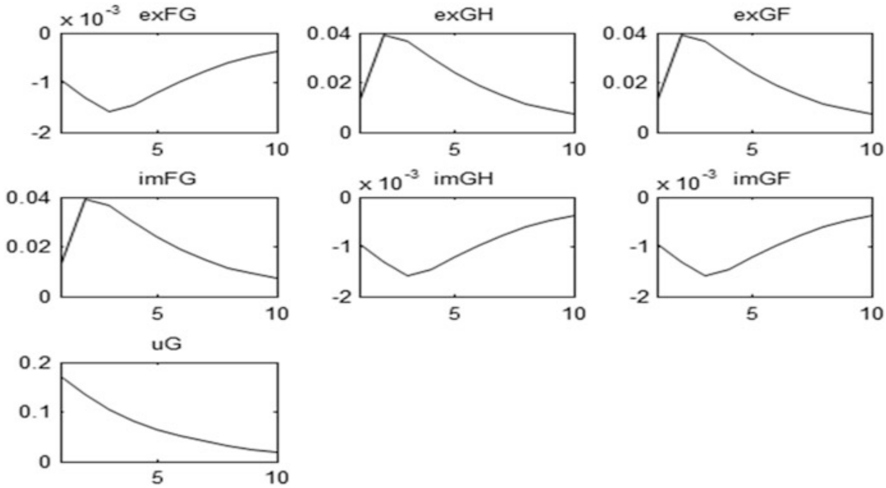

Fig. 12 Impact of production costs shock in the country G. Note: Increment logarithms: exFG—export from the country F to the country G, exGH—export from the country G to the country H, exGF—export from the country G to the country H, imFG—import from the country G to the country F, imGH—import from the country H to the country G, imGF—import from the country F to the country G, uH—production costs in the country G

Changes in export and import of countries are defined by changes in output, terms of trade which in turn depend on changes in interest rates and inflation. Export from the country H to each of the other two countries decreases, but from countries F and G to the country H increases. Imports are growing in all cases except for the import into countries F and G from the country H. The response of export and import in all cases are three orders of magnitude weaker than response of export from the country H and import to the country H. In other words, the influence of technological shock in the country H on export and import of the countries F and G is negligible.

For the technological shock in the country G responses of the variables are presented in Figs. 4, 5, and 6. Because of the positive technological shock in the country G, marginal costs are reducing and firms are capable to reduce prices for their goods. This leads to decrease of interest rate and deterioration of terms of trade with the countries H and F and helps to reduce output in the country G in short term, then indicators return to the values in steady state.

On the contrary, terms of trade of the countries H and F with the country G experience positive jumps. Because of growth of firms' incomes there is an increase in prices, output and interest rates in countries H and F. In contrast to previous cases of technological shocks in countries H and F all the responses of variables are of the same order. Indeed, technological shocks in a big country have a significant impact on macroeconomic indicators, both in the country and in other countries.

Changes in interest rates and inflation affect the terms of trade. And changes in output and terms of trade affect changes in export and import among all the countries. Export from the country G to the countries H and F falls and export from countries H and F to other countries increases. Accordingly, import to the country G from the countries H and F reduces and in all other cases it increases. Variable responses of export and import on technological shock appeared in the country G, are of the same order for all the countries H, F and G.

A variable of total factor productivity in the country G first experiences a positive jump and then returns to a steady state. Due to the assumption of penetration of innovations from the country G to both countries H and F variable deviations of total factor productivity for the countries H and F increase from the values of steady state starting from zero and then also return to the zero level.

For the shock of production cost in the country H responses of the variables are presented in Figs. 7, 8, and 9. In the country H marginal costs increase and firms are forced to raise prices for their products. There is a reduction of output, which is then returned to the value of steady state of the economy. This leads to higher interest rates and a rise in prices of goods produced in the country H. Due to changes in interest rates and inflation terms of trade with countries F and G improve.

And for these countries F and G due to the reduction of incomes of the firms, there is a decrease in prices, terms of trade conversely, deteriorate. In these countries there is a decrease of interest rate, output and inflation. In these countries there is a decrease of interest rate, output and inflation. But it should be noted that the impact of shock of production costs in the country H on the indicators of the countries F and G is small, it is four orders weaker as compared to the responses of indicators in the country H.

But changes in the output in terms of trade affect changes in export and import among all the countries. Export from the country H and import to the country H from the countries F and G increase. But in all other cases export and import between countries experience decline, but slightly by four orders of magnitude weaker than for the country H.

Responses of variables on shock of production costs in the country G are presented on Figs. 10, 11, and 12. In a country G marginal costs increase due to which the firms increase prices on produced products. There is a reduction in production, which is then returned to the value in the steady state of the economy. There is a growth in interest rates and the rise in prices of goods produced in the country G. Increase of interest rate and the rates of inflation improve terms of trade of the country G with the countries H and F. For these countries H and F due to the reduction of income of the firms, terms of trade conversely deteriorate. In the countries H and F there is reduction of interest rate, output and inflation. Note that the effect of the shock of production costs in the country G on the performance of countries H and F is also slight, weaker by three orders of magnitude as compared to the responses of the relevant indicators in the country G.

In countries H and F there is lowering the interest rate, output and inflation. Note that the effect of the shock of production costs in the country G on the performance of countries H and F are also slightly weaker than three orders of magnitude as compared to the responses of the relevant indicators in the country G.

Changes in the production and trading conditions affect changes in exports and imports among all countries. Only for exports from the country G to the other two countries and imports to the country G out of them are deviations from the values in the steady state and positive. And for export from the countries H and F to the other two countries and import from the countries H and F deviations from the steady state are negative and are three orders of magnitude weaker than in the cases in which deviations in export and import are positive.

But a production costs shock in the country G for the countries H and F is three orders of magnitude weaker than for the country G (Fig. 12). There noticeable differences of the effects of production costs shock from the effects of technological shock. It comparatively weaker passes to other countries compared to technological shock.

4 Conclusion

The principal feature of models of dynamic stochastic general equilibrium is the inclusion of equations with expectations of future values of variables. It is equation of the New Keynesian Phillips curve and the dynamic IS curve. Ignoring, for example, inflation expectations could lead to misleading results about the behavior of the economy in response to various internal and external shocks. The complexity of constructing DSGE models is in the need for a thorough mathematical basis, a rigorous derivation of the model equations and formula.

In this article the dynamic stochastic equilibrium model for several countries is presented, its mathematical basis is provided. This model is concretized for the case of three countries (regions). With the use of IFS data of the International Monetary Fund, the World Bank, the National Statistics Agency of Kazakhstan, the National Bank of Kazakhstan a model of the three countries is estimated based on statistics of Kazakhstan, Russia and the European Union using the methods of calibration and Bayesian approach. Forecasts of the effects of internal and external shocks to the main macroeconomic indicators of the three countries/regions are obtained including exports and imports. The model can be developed in various ways, e.g., by the inclusion of consumption and oil production.

Thus, in the conditions of interdependence of economic development of countries, the application of the models of dynamic stochastic general equilibrium will allow to forecast the impact of changes in economic policy on the consequences of internal and external shocks for the macroeconomic indicators of the country.

References

Agency of Statistics of the Republic of Kazakhstan. [online] Accessed August 20, 2015, from www.stat.gov.kz

Calvo, G. A. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics, 12*, 383–398.

Corsetti, C., & Pesenty, P. (2001). Welfare and macroeconomic interdependence. *The Quarterly Journal of Economics, 116*(2), 421–446.

Cuche-Curti, N. A., Dellas, H., & Natal, J. M. (2009). DSGE-CH: A dynamic stochastic general equilibrium model for Switzerland. *Swiss National BankEconomic Studies*, 5. [online] Accessed August 20, 2015, from https://www.snb.ch/n/mmr/reference/economic_studies_2009_05/source/economic_studies_2009_05.n.pdf

Di Giorgio, G. (2014). Monetary policy challenges: How central banks changed their modus operandi. *Eurasian Economic Review, 4*(1), 25–43.

Dib, A. (2001). *An estimated Canadian DSGE model with nominal and real rigidities* (Bank of Canada Working Paper). Canada.

Galí, J., & Monacelli, T. (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies, 72*(3), 707–734.

Gunter, U. (2009). *Macroeconomic Interdependence in a two-country DSGE model under diverging interest-rate rules* (Working Paper No: 0903, pp. 1–38). Austria: University of Vienna.

Kydland, F. E., & Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica, 50*(6), 1345–1370.

Medina, J., & Soto, C. (2007). *The Chilean business cycle through the lens of a stochastic general equilibrium model* (Central Bank of Chile Working Papers No. 457).

Mukhamediyev, B. (2014). Estimated DSGE model for oil producing economy of Kazakhstan. *The Macrotheme Review, 3*(3), 1–13.

National Bank of the Republic of Kazakhstan. [online] Accessed June 15, 2015, from http://www.nationalbank.kz

Obstfeld, M., & Rogoff, K. (2001).*Risk and exchange rates*. Conference paper in honor of AssafRazin, Tel-Aviv University, pp. 1–51.

Smets, F., & Wouters, R. (2003). An estimated stochastic dynamic general equilibrium model of the euro area. *Journal of the European Economic Association, 1*(5), 123–175.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy, 39*, 195–214.

Tovar, C. (2008). *DSGE models and central banks* (BIS Working Papers No. 258).