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Geometric characteristics of the solitonic solution in the case of finite density

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Abstract. Some exact solutions of nonlinear partial differential equations are widely investigated both mathematical and physical points of view. Physically interesting solution as solitonic is well known. Also solitonic solution have simple behavior in bumping and are stable. There are various methods for searching of these exact solutions.

Keywords: Surface, Solitonic solution, Quadratic form
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INTRODUCTION

Exact solutions of nonlinear partial differential equations are widely investigated both mathematical and physical points of view [1–4]. Physically interesting solution as soliton is well known. Also soliton solution have simple behavior in bumping and are stable. There are various methods for searching of these exact solutions [2, 3]. In (1+1)-dimensional case nonlinear partial differential equations are given as a condition of zero curvature

$$U_t - V_x + [U,V] = 0,$$

where $[U,V] = UV - VU$, matrix U is prescribed, and matrix V is expressed in the terms of the elements of the matrix U. Also the nonlinear partial differential equation is the compatibility condition the system of linear equations

$$\phi_x = U\phi, \ \ \phi_t = V\phi,$$

where ϕ is a scalar function. In this case there is a surface with immersion function $P(x,t)$ defined by the formulas $\frac{\partial P}{\partial x} = \phi^{-1}X\phi$, $\frac{\partial P}{\partial t} = \phi^{-1}Y\phi$. The surface defined by $P(x,t)$ is identified with surface in tree-dimensional space with coordinates [1] $x_j = P_j(x,t), \ \ j = 1,2,3$. Frame on the surface is given [1]

$$\frac{\partial P}{\partial x} = \phi^{-1}X\phi, \quad \frac{\partial P}{\partial t} = \phi^{-1}Y\phi, \quad N = \phi^{-1}J\phi,$$

where $J = \frac{[X,Y]}{|[X,Y]|}, \ \ |X| = \sqrt{<X,X>}$. By definition

$$<X,Y> = -\frac{1}{2}tr(XY),$$

where X,Y are some matrices. The first and second fundamental forms are given

$$I = <X,X> dx^2 + 2 <X,Y> dxdt + <Y,Y> dt^2, \tag{1}$$

$$II = <\frac{\partial X}{\partial x} + [X,U], J > dx^2 + 2 < \frac{\partial X}{\partial t} + [X,V], J > dxdt + < \frac{\partial Y}{\partial t} + [Y,V], J > dt^2. \tag{2}$$

As it is shown in the work [1] immersion function P can be defined by formula

$$P = \gamma_0\phi^{-1}\phi_\lambda + \phi^{-1}M_1\phi = \sum_{j=1}^{3} P_j f_j,$$

where M_1 is a matrix function depending on λ, x and t which are unknown variables. Here $f_j = -\frac{i}{2}\sigma_j$ is the basis of the corresponding algebra, σ_j are Pauli matrices and $[f_i, f_j] = f_k$. In this case, X and Y can be written as

$$X = \gamma_0 U_\lambda + M_{1x} + [M_1, U], Y = \gamma_0 V_\lambda + M_{1t} + [M_1, V],$$

where γ_0 is an arbitrary constant; M_{1x}, M_{1t} are derivatives of the matrix M by x and t correspondingly.

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SOLITONIC IMMERSION IN (1+1)-DIMENSION

Let the matrices X, Y, J have the form

$$X = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad Y = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \quad J = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}, \tag{3}$$

where $a_{ij},\ b_{ij},\ c_{ij}, i = \overline{1,2}, j = \overline{1,2}$ are some arbitrary variables.

In this case the matrices J are expressed through elements of the matrix X and Y in accordance with the formulas

$$c_{11} = \frac{a_{12}b_{21} - b_{12}a_{21}}{\mid [X,Y] \mid}, \quad c_{21} = \frac{a_{21}(b_{11} - b_{22}) + b_{21}(a_{22} - a_{11})}{\mid [X,Y] \mid},$$
$$c_{12} = \frac{b_{12}(a_{11} - a_{22}) + a_{12}(b_{22} - b_{11})}{\mid [X,Y] \mid}, \quad c_{22} = \frac{a_{21}b_{12} - b_{21}a_{12}}{\mid [X,Y] \mid}. \tag{4}$$

Then the first fundamental form (1) of the two-dimensional surface is $I = Edx^2 + 2Fdxdt + Gdt^2$, where

$$E = -\frac{1}{2}(a_{11}^2 + 2a_{12}a_{21} + a_{22}^2), \quad F = -\frac{1}{2}(a_{11}b_{11} + a_{12}b_{21} + a_{21}b_{12} + a_{22}b_{22}), \tag{5}$$

$$G = -\frac{1}{2}(b_{11}^2 + 2b_{12}b_{21} + b_{22}^2). \tag{6}$$

As example of the solitonic equation which reduced to the immersion we consider nonlinear Schrödinger equation

$$i\psi_t + \psi_{xx} + 2\beta|\psi|^2\psi = 0,$$

where $\beta = +1,\ \psi$ is a complex function. In this case, matrices U, V have the form [3]

$$U = \frac{\lambda\sigma_3}{2i} + U_0,\ U_0 = i\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix},$$

$$V = \frac{i\lambda^2}{2}\sigma_3 + i|q|^2\sigma_3 - i\lambda\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix} + \begin{pmatrix} 0 & \bar{q}_x \\ -q_x & 0 \end{pmatrix}. \tag{7}$$

We present the lemma.

Lemma 1. *The second fundamental form in sense of Fokas-Gelfand corresponding to the solitonic solution q of the nonlinear Schrödinger equation in the case of finite density has the form*

$$II = Ldx^2 + 2Mdxdt + Ndt^2, \tag{8}$$

where

$$\begin{aligned}
L = &-\tfrac{1}{2}\{a_{11x}c_{11} + a_{12x}c_{21} + a_{21x}c_{12} + a_{22x}c_{22} - \lambda i(a_{21}c_{12} - a_{12}c_{21}) \\
&+ iq(a_{12}c_{11} + a_{22}c_{12} - a_{11}c_{12} - a_{12}c_{22}) + i\bar{q}(a_{21}c_{22} + a_{11}c_{21} - a_{22}c_{21} - a_{21}c_{11})\}, \\
M = &-\tfrac{1}{2}\{a_{11t}c_{11} + a_{12t}c_{21} + a_{21t}c_{12} + a_{22t}c_{22} + i(\lambda^2 + 2|q|^2)(a_{21}c_{12} - a_{12}c_{21}) \\
&+ (q_x + \lambda iq)(a_{11}c_{12} + a_{12}c_{22} - a_{12}c_{11} - a_{22}c_{12}) \\
&+ (\bar{q}_x - \lambda i\bar{q})(a_{11}c_{21} + a_{21}c_{22} - a_{21}c_{11} - a_{22}c_{21})\}, \\
N = &-\tfrac{1}{2}\{b_{11t}c_{11} + b_{12t}c_{21} + b_{21t}c_{12} + b_{22t}c_{22} + i(\lambda^2 + 2|q|^2)(b_{21}c_{12} - b_{12}c_{21}) \\
&+ (q_x + \lambda iq)(b_{11}c_{12} + b_{12}c_{22} - b_{12}c_{11} - b_{22}c_{12}) \\
&+ (\bar{q}_x - \lambda i\bar{q})(b_{11}c_{21} + b_{21}c_{22} - b_{21}c_{11} - b_{22}c_{21})\}.
\end{aligned} \tag{9}$$

Proof. We present the matrices (3), (7) in the form (2). After some algebraic operations we obtain (8), (9). □

ONESOLITONIC SOLUTION OF THE NONLINEAR SCHRÖDINGER EQUATION CORRESPONDING TO THE SURFACE

We consider a particular case of the immersion at $\gamma_0 = 1, \quad M_1 = 0$. In this case, we get

$$X = U_\lambda = \frac{1}{2i}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, Y = V_\lambda = -i\begin{pmatrix} -\lambda & \bar{q} \\ q & \lambda \end{pmatrix}, J = \begin{pmatrix} 0 & -\frac{\bar{q}}{\sqrt{q\bar{q}}} \\ \frac{q}{\sqrt{q\bar{q}}} & 0 \end{pmatrix}, \tag{10}$$

and $P = \phi^{-1}\phi_\lambda$. In order to calculate the explicit expressions for the immersion function P we consider a onesolitonic solution of the nonlinear Schrödinger equation in the case of finite density in the form

$$q(x,t) = \rho\frac{1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\}}{1+exp\{\nu_1(x-vt-x_0)\}}, \tag{11}$$

where $v = -\omega cos\frac{\theta}{2}, x_0 = \frac{1}{\nu_1}lni\gamma_1;\ \ \omega, \theta, \gamma_1, \nu_1$ are some parameters of the model.

Theorem 2. *Onesolitonic solution of the nonlinear Schrödinger equation in the case of finite density corresponds a surface in sense of Fokas-Gelfand with following coefficients of the first fundamental form*

$$E = \frac{\nu_1^2 exp^2\{\nu_1(x-vt-x_0)\}}{(1+exp\{\nu_1(x-vt-x_0)\})^4}\left[\frac{4\rho^2x^2}{(\lambda-\bar{\lambda}_1)^4}(2-e^{i\theta}-e^{-i\theta}) + \frac{4(e^{i\theta}-1)^2[1+\nu_1 x(1-e^{i\theta}exp^2\{\nu_1(x-vt-x_0)\})]^2}{(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4}\right],$$

$$F = \frac{2\rho^2\nu_1^2 vxexp\{\nu_1(x-vt-x_0)\}(e^{i\theta}+e^{-i\theta}-2)}{(\lambda-\bar{\lambda}_1)^4(1+exp\{\nu_1(x-vt-x_0)\})^3} + \frac{4\nu_1^3 vxexp^2\{\nu_1(x-vt-x_0)\}(e^{i\theta}-1)^2(e^{i\theta}-exp^2\{\nu_1(x-vt-x_0)\}-1)}{(\lambda-\bar{\lambda}_1)^2(1+exp\{\nu_1(x-vt-x_0)\})^4(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4}$$

$$\times(1+\nu_1 x-\nu_1 xe^{i\theta}exp^2\{\nu_1(x-vt-x_0)\}), \tag{12}$$

$$G = \frac{v^2\nu_1^2 exp\{\nu_1(x-vt-x_0)\}}{(\lambda-\bar{\lambda}_1)^4(1+exp\{\nu_1(x-vt-x_0)\})^4}\Big[\rho^2(e^{i\theta}-e^{-i\theta})^2(1+2exp\{\nu_1(x-vt-x_0)\})^2$$

$$+\rho^2(e^{i\theta}-e^{-i\theta}-2)^2 + \frac{4\nu_1^2x^2(e^{i\theta}-1)^2(e^{i\theta}exp\{\nu_1(x-vt-x_0)\}-1)^2}{(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4}\Big],$$

where $\lambda_1 = const$.

Proof. Solution of the linear system we find in the form

$$\psi = \phi e^{-(\frac{\lambda\sigma_3}{2i}x+\frac{i\lambda^2}{2}\sigma_3 t)}. \tag{13}$$

Taking into account (13) and applying (7) we have

$$\psi_x = \left(\frac{\lambda\sigma_3}{2i}+U_0\right)\psi - \psi\frac{\lambda\sigma_3}{2i} = \frac{\lambda\sigma_3}{2i}\psi - \psi\frac{\lambda\sigma_3}{2i} + U_0\psi = \left[\frac{\lambda\sigma_3}{2i},\psi\right] + U_0\psi. \tag{14}$$

We take

$$\psi = I - \frac{\tilde{A}}{\lambda-\lambda_1^*}, \quad \tilde{A} = \begin{pmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & \tilde{d} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \lambda_1^* - const. \tag{15}$$

We substitute (15) to (14)

$$\psi_x = U_0 - \frac{U_0\tilde{A}}{\lambda-\lambda_1^*} - \frac{1}{2i}[\sigma_3,\tilde{A}] - \frac{\lambda_1^*}{2i(\lambda-\lambda_1^*)}[\sigma_3,\tilde{A}]. \tag{16}$$

On the other hand, from (15) it follows

$$\psi_x = -\frac{\tilde{A}_x}{\lambda-\lambda_1^*}. \tag{17}$$

From (16) and (17), we get

$$-\frac{\tilde{A}_x}{\lambda-\lambda_1^*} = U_0 - \frac{U_0\tilde{A}}{\lambda-\lambda_1^*} - \frac{1}{2i}[\sigma_3,\tilde{A}] - \frac{\lambda_1^*}{2i(\lambda-\lambda_1^*)}[\sigma_3,\tilde{A}]. \tag{18}$$

Thus,

$$\tilde{A}_x = U_0\tilde{A} + \frac{\lambda_1^*}{2i}[\sigma_3,\tilde{A}], U_0 = \frac{1}{2i}[\sigma_3, A]. \quad (19)$$

We note, that

$$[\sigma_3,\tilde{A}] = \sigma_3\tilde{A} - \tilde{A}\sigma_3 = 2\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}. \quad (20)$$

Then by substituting (20) to (35) we get

$$U_0 = \frac{1}{i}\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}. \quad (21)$$

We put (20) to (19) and get

$$\begin{pmatrix} \tilde{a}_x & \tilde{b}_x \\ \tilde{c}_x & \tilde{d}_x \end{pmatrix} = \frac{1}{i}\begin{pmatrix} \tilde{b}\tilde{c} & \tilde{b}\tilde{d} \\ -\tilde{c}\tilde{a} & -\tilde{c}\tilde{b} \end{pmatrix} + \frac{\lambda_1^*}{i}\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}. \quad (22)$$

From (7) and (21) we get

$$i\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix} = \frac{1}{i}\begin{pmatrix} 0 & b \\ -c & 0 \end{pmatrix} \Rightarrow \begin{cases} i\bar{q} = \frac{1}{i}b \\ iq = -\frac{1}{i}c \end{cases} \Rightarrow \begin{cases} b = -\bar{q}, \\ c = q. \end{cases} \quad (23)$$

Thus, we found the matrix $\tilde{A}$ in the explicit form with components (22). By using (11), we get

$$\tilde{a} = -\frac{i\nu_1 x exp\{\nu_1(x - vt - x_0)\}(e^{i\theta} - 1)}{(1 + exp\{\nu_1(x - vt - x_0)\})(1 + e^{i\theta} exp\{\nu_1(x - vt - x_0)\})} - \lambda_1^*. \quad (24)$$

From (22) it follows $\tilde{a} = -\frac{i\tilde{c}_x}{c} - \lambda_1^* \Rightarrow \tilde{a} = -\frac{1}{i}\int \bar{q}q dx$. By using (11), we get

$$\tilde{a}_x = \frac{1}{i}\tilde{b}\tilde{c} \Rightarrow \tilde{a}_x = \frac{1}{i}(-\bar{q})q, \quad (25)$$

then

$$\tilde{a} = -\frac{iq_x}{q} - \lambda_1^*. \quad (26)$$

Consequently, from (22), (23) we can write

$$\tilde{d} = \frac{i\tilde{b}_x}{\tilde{b}} - \lambda_1^* \Rightarrow \tilde{d} = \frac{i(-\bar{q})_x}{(-\bar{q})} - \lambda_1^* \Rightarrow \tilde{d} = \frac{i\bar{q}_x}{\bar{q}} - \lambda_1^*. \quad (27)$$

Using (11) we get

$$\tilde{d} = \frac{i\nu_1 x exp\{\nu_1(x - vt - x_0)\}(e^{-i\theta} - 1)}{(1 + exp\{\nu_1(x - vt - x_0)\})(1 + e^{-i\theta} exp\{\nu_1(x - vt - x_0)\})} - \lambda_1^*. \quad (28)$$

From (22), (23) it follows

$$\tilde{d}_x = -\frac{1}{i}\tilde{c}\tilde{b}. \quad (29)$$

Moreover from (28), (29) follow

$$\tilde{d} = \frac{1}{i}\int q\bar{q}dx \quad (30)$$

Taking into account (30), we get (25) in the form

$$\tilde{d} = -\tilde{a}. \quad (31)$$

Thus, the matrix $\tilde{A}$ for onesolitonic solution (11) of the nonlinear Schrodinger equation takes the form

$$\tilde{A} = \begin{pmatrix} -\frac{i\nu_1 x exp\{\nu_1(x-vt-x_0)\}(e^{i\theta}-1)}{(1+exp\{\nu_1(x-vt-x_0)\})(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})} - \lambda_1^* & -\rho\frac{1+e^{-i\theta}exp\{\nu_1(x-vt-x_0)\}}{1+exp\{\nu_1(x-vt-x_0)\}} \\ \rho\frac{1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\}}{1+exp\{\nu_1(x-vt-x_0)\}} & \frac{i\nu_1 x exp\{\nu_1(x-vt-x_0)\}(e^{-i\theta}-1)}{(1+exp\{\nu_1(x-vt-x_0)\})(1+e^{-i\theta}exp\{\nu_1(x-vt-x_0)\})} - \lambda_1^* \end{pmatrix}. \quad (32)$$

Then we take $\phi = I - \frac{A}{(\lambda-\bar{\lambda}_1)^2}$, where λ_1 is constant, then from (10) we get

$$P = \phi^{-1}\phi_\lambda = \left(I + \frac{\tilde{A}}{\lambda - \lambda_1}\right)\frac{\tilde{A}}{(\lambda-\bar{\lambda}_1)^2}. \tag{33}$$

On the other hand, we have

$$P = \sum_{j=1}^{3} P_j f_j = -\frac{i}{2}\sum_{j=1}^{3} P_j \sigma_j = \begin{pmatrix} -\frac{i}{2}P_3 & -\frac{i}{2}P_1 - \frac{1}{2}P_2 \\ -\frac{i}{2}P_1 + \frac{1}{2}P_2 & \frac{i}{2}P_3 \end{pmatrix}. \tag{34}$$

From (33), (34) with help of (29) we get $P_3 = \frac{2i\tilde{a}}{(\lambda-\bar{\lambda}_1)^2}$. Now taking into account (31) we find P_3 in the explicit form for nonlinear Schrodinger equation in the case of finite density

$$P_3 = \frac{2\nu_1 x exp\{\nu_1(x-vt-x_0)\}(e^{i\theta}-1)}{(\lambda-\bar{\lambda}_1)^2(1+exp\{\nu_1(x-vt-x_0)\})(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})} - \frac{2i\lambda_1^*}{(\lambda-\bar{\lambda}_1)^2}. \tag{35}$$

From (33), (34) we get $P_2 = \frac{\tilde{c}-\tilde{b}}{(\lambda-\bar{\lambda}_1)^2}$. Thus,

$$P_1 = \frac{i(\tilde{c}+\tilde{b})}{(\lambda-\bar{\lambda}_1)^2}, \quad P_2 = \frac{(\tilde{c}-\tilde{b})}{(\lambda-\bar{\lambda}_1)^2}, \quad P_3 = \frac{2i\tilde{a}}{(\lambda-\bar{\lambda}_1)^2}.$$

From (33), (11) by using known formulas

$$sh\zeta = \frac{e^{\zeta}-e^{-\zeta}}{2}, ch\zeta = \frac{e^{\zeta}+e^{-\zeta}}{2}, cos\zeta = \frac{e^{i\zeta}+e^{-i\zeta}}{2}, sin\zeta = \frac{e^{i\zeta}-e^{-i\zeta}}{2i}, \tag{36}$$

where $\zeta = \nu_1(x-vt-x_0)$ and we get the explicit values P_1, P_2, P_3 of the matrixes P

$$\begin{aligned} P_1 &= \frac{i\rho(e^{i\theta}-e^{-i\theta})exp\{\nu_1(x-vt-x_0)\}}{(\lambda-\bar{\lambda}_1)^2(1+exp\{\nu_1(x-vt-x_0)\})}, \\ P_2 &= \frac{\rho(2+e^{i\theta}exp\{\nu_1(x-vt-x_0)\}+e^{-i\theta}exp\{\nu_1(x-vt-x_0)\})}{(\lambda-\bar{\lambda}_1)^2(1+exp\{\nu_1(x-vt-x_0)\})}. \end{aligned} \tag{37}$$

Now we calculate the coefficients of the first fundamental form, i.e.,

$$E = P_{1x}^2 + P_{2x}^2 + P_{3x}^2. \tag{38}$$

In order to do it we calculate P_{1x}, P_{2x}, P_{3x}. Now we calculate the first derivative squared and put to (38), then

$$\begin{aligned} E \quad = \quad & \frac{\nu_1^2 exp^2\{\nu_1(x-vt-x_0)\}}{(1+exp\{\nu_1(x-vt-x_0)\})^4}\left[\frac{4\rho^2 x^2}{(\lambda-\bar{\lambda}_1)^4}(2-e^{i\theta}-e^{-i\theta}) \right. \\ & \left. + \frac{4(e^{i\theta}-1)^2[1+\nu_1 x(1-e^{i\theta}exp^2\{\nu_1(x-vt-x_0)\})]^2}{(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4}\right]. \end{aligned}$$

Similarly, according to the formulas

$$F = P_{1x}P_{1t} + P_{2x}P_{2t} + P_{3x}P_{3t}, \quad G = P_{1t}^2 + P_{2t}^2 + P_{3t}^2,$$

we get the values

$$\begin{aligned} F &= \frac{2\rho^2\nu_1^2 v x exp\{\nu_1(x-vt-x_0)\}(e^{i\theta}+e^{-i\theta}-2)}{(\lambda-\bar{\lambda}_1)^4(1+exp\{\nu_1(x-vt-x_0)\})^3} + \frac{4\nu_1^3 v x exp^2\{\nu_1(x-vt-x_0)\}(e^{i\theta}-1)^2(e^{i\theta}-exp^2\{\nu_1(x-vt-x_0)\}-1)}{(\lambda-\bar{\lambda}_1)^2(1+exp\{\nu_1(x-vt-x_0)\})^4(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4} \\ &\quad \times(1+\nu_1 x - \nu_1 x e^{i\theta}exp^2\{\nu_1(x-vt-x_0)\}), \\ G &= \frac{v^2\nu_1^2 exp\{\nu_1(x-vt-x_0)\}}{(\lambda-\bar{\lambda}_1)^4(1+exp\{\nu_1(x-vt-x_0)\})^4}\left[\rho^2(e^{i\theta}-e^{-i\theta})^2(1+2exp\{\nu_1(x-vt-x_0)\})^2 \right. \\ &\quad \left. + \rho^2(e^{i\theta}-e^{-i\theta}-2)^2 + \frac{4\nu_1^2 x^2(e^{i\theta}-1)^2(e^{i\theta}exp\{\nu_1(x-vt-x_0)\}-1)^2}{(1+e^{i\theta}exp\{\nu_1(x-vt-x_0)\})^4}\right]. \end{aligned} \tag{39}$$

Using (35), (37), we find the coefficients of the second fundamental form L, M, N. In order to get it we have to calculate

$$\mathbf{n} = \frac{\mathbf{r}_x \times \mathbf{r}_t}{\sqrt{\Lambda}}, \quad \sqrt{\Lambda} = \sqrt{EG - F^2}. \tag{40}$$

We substitute the values (35), (37) to (40) and find the components of the vector $\mathbf{n}$. We calculate, with help of (39), the value

$$\sqrt{\Lambda} = (EG - F^2)^{\frac{1}{2}}.$$

Now we find

$$P_{1xx}, P_{2xx}, P_{3xx}.$$

Then we can find L. Similarly we find M, N. Then we can find Gaussian and the main curvature K and H. Now from (5), (6) using (10) for this case γ_0, $M_1 = 0$ we get the coefficients of the first fundamental form corresponding (11) as

$$E = \frac{1}{4}, \quad F = -\frac{\lambda}{2}, \quad G = \lambda^2 + \bar{q}q.$$

Accordingly, from (9) using (10), we find the coefficients of the second fundamental form. Now we can calculate

$$\Lambda = EG - F^2 = \frac{1}{4}\bar{q}q.$$

Theorem is proved. □

CONCLUSION

Thus, we have investigated the solitonic immersion in (1+1)-dimension. As example, (1+1)-dimensional nonlinear Schrödinger equation is considered. The first fundamental form with coefficients (12) for integrable surface corresponding to onesolitonic solution of the nonlinear Schrödinger equation in the case of finite density is found. Gaussian and the main curvature of the surface are found.

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