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Inverse Coefficient Problems for Mathematical Models of One-Dimensional Heat Transfer with a Preservation of Medium Temperature Condition

Problems of determining coefficients or the right-hand side of a differential equation simultaneously with its solution are called inverse problems of the mathematical physics. In this paper we consider one family of problems modeling the process of determining the function of temperature distribution and time varying structure of a homogeneous bar by a given law of changing of medium temperature. In the process of mathematical modeling there arises an inverse problem for a heat equation where alongside with a solution of the problem it is required to find unknown coefficient depending only on a time variable.

The solvability of various inverse problems for parabolic equations was studied in papers of Anikonov Yu.E., Belov Yu.Ya., Bubnov B.A., Prilepko A.I., Kostin A.B., Monakhov V.N., Kozhanov A.I., Kaliyev I., Sabitov K.B., Pyatkov S.G., Ashyralyev A., Kerimov N.B., Ismailov M.I. and many others.

Unlike the preceding works, we study the inverse problem for a heat equation subject to boundary conditions with respect to a spatial variable under which the system of eigenfunctions of the corresponding spectral problem for an ordinary differential operator does not form a basis.

In the domain $\Omega = \{(x,t) : 0 < x < 1, 0 < t < T\}$ consider a problem on finding unknown coefficient $p(t)$ of the heat equation

$$u_t = u_{xx}(x,t) - p(t)u(x,t) + f(x,t) \tag{1}$$

and its solution satisfying the initial condition

$$u(x,0) = \varphi(x),\ 0 \le x \le 1, \tag{2}$$

the nonlocal boundary condition

$$u_x(0,t) = u_x(1,t) + \alpha u(1,t),\ \ u(0,t) = 0,\ \ 0 \le t \le T, \tag{3}$$

and the overdetermination conditions

$$\int_0^1 u(x,t)dx = E(t),\ E(t) \ne 0, 0 \le t \le T, \tag{4}$$

where $E(t) \in W_2^1(0,T)$. Here the parameter α is any positive number, and $f(x), \varphi(x)$ and $E(t)$ are given functions. At $\alpha = 0$ the boundary conditions (3) are well- known and called in literature as Samarskii-Ionkin conditions.

The direct problem (1)-(3) in case when $p(t) \equiv 0$ was investigated in [1].

The most close to the theme of the present paper is [2]. The existence of classical solution of an inverse problem analogous to our investigated problem has been justified in this paper. However, due to the fact that boundary conditions in [2] are regular, but not strengthened regular, from the input data of the problem there have been required the improvement of the smoothness and satisfaction to additional conditions:

$$(A_1)\ \varphi \in C^2[0,1]; \varphi'(0) - \alpha\varphi(0) = 0,\ \varphi(0) = \varphi(1);$$

$$\varphi_0 > 0, \varphi_{2n-1} \geq 0,\ n = 1, 2, 3....,\ \text{if}\ \alpha < 0;$$

$$\varphi_1 < 0, \varphi_{2n-1} \leq 0,\ n = 2, 3....,\ \text{if}\ \alpha > 0.$$

$$(A_2)\ E(t) \in C^1[0,T];\ E(0) = \int_0^1 \varphi(x)dx;\quad E(t) > 0,\ \ \forall t \in [0,1];$$

$$(A_3)\ f(x,t) \in C^2[\bar{D_T}]; f(x,t) \in C^2[0,1],\ \forall t \in [0,1];$$

$$f_x(1,t) - \alpha f(0,t) = 0,\ f(0,t) = f(1,t);$$

$$f_0(\tau) > 0, f_{2n-1}(\tau) \geq 0,\ n = 1, 2, 3....,\ \text{if}\ \alpha < 0;$$

$$f_{2n-1}(\tau) \leq 0,\ n = 2, 3....,\ \text{if}\ \alpha > 0.$$

In the present paper these conditions are completely removed and it is shown that the inverse problem has the unique generalized solution.

IIn this work, we have considered one family of problems of modeling the process of determining the distribution function of temperature and time varying structure of homogeneous bar of a given law changes in the average temperature. So there is an inverse problem for the heat equation in which together with finding the solution of the equation required to find unknown coefficient depending only on the time variable. The specific features of the considered problems is that the system of eigenfunctions of the multiple differentiation operator subject to boundary conditions of the initial problem does not have the basis property.

We have proved the unique existence of a generalized solution to the mentioned problem. The main result of the paper is theorem on the existence and uniqueness of a generalized solution of the problem (1)-(4).

Theorem *If functions φ and f belong to classes $\varphi \in W_2^2(0,1)$, $f \in L_2(\Omega)$, $E(t) \neq 0$ and $E(t) \in W_2^1(0,T)$, then a unique generalized solution $u(x,t) \in W_2^{2,1}(\Omega)$, $p(t) \in L_2(0,1)$ of the problem (1)-(4) exists.*

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On a Problem for Wave Equation with Data on the Whole Boundary

In this paper we propose a new formulation of boundary value problem for a one-dimensional wave equation in a rectangular domain in which boundary conditions are given on the whole boundary.

Let $\Omega \subset R^2$ be a rectangular domain, bounded by following lines: $AB : 0 \leq x \leq \ell$, $t = 0$, $BC : x = \ell$, $0 \leq t \leq T$, $CD : 0 \leq x \leq \ell, t = T$ and $AD : x = 0$, $0 \leq t \leq T$.

We consider a nonhomogeneous wave equation in Ω

$$u_{tt} - u_{xx} = f(x,t). \tag{1}$$

It is well known that the Dirichlet problem for the wave equation (1) in a rectangular domain is ill-posed [1]. Specifically, in case of our domain Ω it is easy to see that the homogeneous equation (1) with Dirichlet conditions

$$u|_{AB\cup BC\cup AD} = 0, \tag{2}$$

$$u|_{CD} = 0 \tag{3}$$

has countable number of nontrivial solutions of the form

$$u_{mn}(x,t) = sin\frac{m\pi x}{\ell}sin\frac{n\pi t}{T},\ m,n = 1,2,...$$

when the conditions $n\ell = mT$ hold.

The Dirichlet problem for a wave equation is one of the most difficult models of mathematical physics. The wave equation describes almost all types of small vibrations in distributional mechanical systems such as longitudinal sound vibrations in gas, fluid, solids; transverse waves in strings and etc. Components of electromagnetic vectors and