

Variation Calculus and methods of optimization

Educational Manual

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Some theoretical foundations of optimal control problem are expounded in the educational
manual, methods of variation calculus, maximum principle, dynamical programming for
solving of typical problems of economic macromodels.

The tasks for independent work with solving of concrete examples, brief theory and
solution algorithms of the problems, term tasks on sections of optimal control and variation
calculus are presented in the appendix.

It is intended as an educational manual for students of the high schools training on
specialties: «mathematics», «mechanics», «mathematical and computer modeling» and
«informatics». It will be useful for the post-graduate students and scientific workers of the
economic, multi-technical and naturally-technical specialties.

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