Montenegrin Academy of Sciences and Arts

University of Montenegro

University of Evora, Portugal

Dorodnicyn Computing Centre of RAS

Moscow Institute of Physics and Technology (State University)

VI International Conference on Optimization Methods and Applications

OPTIMIZATION AND APPLICATIONS

(OPTIMA-2015)

Petrovac, Montenegro, September 2015

PROCEEDINGS

Moscow — 2015

UDC 519.658

Proceedings include abstracts of reports presented at the VI International Conference on Optimization Methods and Applications "Optimization and applications" (OPTIMA-2015) held in Petrovac, Montenegro, September 27 - October 3, 2015.
Edited by V.U. Malkova.

The Conference is organized by

The Montenegrin Academy of Sciences and Arts (MASA), Montenegro

University of Montenegro, Montenegro

University of Évora, CIMA-UE, Portugal

Dorodnicyn Computing Centre of RAS, Russia

Moscow Institute of Physics and Technology (State University), Russia

Научное издание

Organizing Committee

Milojica Jaćimović, Chair, Montenegrin Academy of Sciences and Arts, Montenegro
Yury G. Evtushenko, Chair, Dorodnicyn Computing Centre of RAS, Russia
Vladimir Bushenkov, Chair, University of Evora, Portugal

Izedin Krnic, University of Montenegro
Oleg Obradovic, University of Montenegro
Vladimir Jaćimović, University of Montenegro
Nevena Mijajlović, University of Montenegro
Vera Kovacevic-Vujcic, University of Belgrade
Vesna Dragović, Montenegrin Academy of Sciences and Arts
Angelia Nedich, University of Illinois at Urbana Champaign, USA
Alexander P. Afanasiev, Institute for Information Transmission Problems of RAS, Russia
Anatoly S. Antipin, Dorodnicyn Computing Centre of RAS, Russia
Natalia K. Burova, Dorodnicyn Computing Centre of RAS, Russia
Vladimir A. Garanzha, Dorodnicyn Computing Centre of RAS, Russia
Alexander I. Golikov, Dorodnicyn Computing Centre of RAS, Russia
Alexander Yu. Gornov, Inst. System Dynamics and Control Theory, SB of RAS, Russia
Alexander V. Lotov, Dorodnicyn Computing Centre of RAS, Russia
Evgeny G. Molchanov, Moscow Institute of Physics and Technology, Russia
Yuri Nesterov, CORE Université Catholique de Louvain, Belgium
Igor G. Pospelov, Dorodnicyn Computing Centre of RAS, Russia
Alexander A. Shananin, Moscow Institute of Physics and Technology, Russia
Gueorgui Smirnov, University of Minho, Portugal
Tatiana Tchemisova, University of Aveiro, Portugal
Alexey A. Tretyakov, Dorodnicyn Computing Centre of RAS, Russia
Ivetta A. Zonn, Dorodnicyn Computing Centre of RAS, Russia
Vitaly G. Zhadan, Dorodnicyn Computing Centre of RAS, Russia
Vladimir I. Zubov, Dorodnicyn Computing Centre of RAS, Russia

Contents

Karine Abgaryan, D.L. Reviznikov Multilevel approach to modeling and optimization of semiconductor heterostructures 12

Karine Abgaryan, Olga Volodina, Sergey Uvarov Application of computer modeling for the solution of optimizing problems of parametrical identification of potentials of interatomic interaction 14

Alexander P. Abramov Improper Vector Set Problem 16

Alexander Afanasyev, Stanislav Emelyanov, Elena Putilina How to differentiate a function knowing only its integrals 18

Kamil Aida-zade, Vagif Abdullaev On Application of the Fast Automatic Differentiation Technique in Optimal Control Problems for Discrete Processes 20

Kamil Aida-zade, Jamilya Asadova Control of the Oscillatory Process by Means of Lumped Sources 22

Kamil Aida-zade, Yegana Ashrafova Localization of Leakage Points in Pipelines of Complex Structure 24

Kamil Aida-zade, Samir Guliyev Zonal Control of Nonlinear Systems with Feedback on State and on Output 26

Elena Andreeva, Irina Bolodurina, Ulia Ivanova Optimal control of nonsmooth system of differential delay equations with application in immunology 28

Anton Anikin, Evgeniya Finkelstein The parallel version of the stochastic approximation algorithm for the reachable set of dynamic systems with discontinuous right-hand side 29

Anton Anikin, Alexander Gornov, Alexander Andrianov GPU-accelerated gradient methods for solving PageRank problem 30

Anatoly Antipin Controlled dynamic model with a boundary value problem 32

Sergey Astrakov Efficient region monitoring with sensors outer positioning 34

Artem Baklanov A Heuristic Algorithm for the Double Integrator Traveling Salesman Problem 35

Maxim Balashov Polyhedral approximations in $\mathbb{R}^n$ 36

A.I. Belokrys-Fedotov, V.A. Garanzha, L.N. Kudryavtseva Self-organization of nonlinear elastic networks for Delaunay-Voronoi meshing of implicit domains 38

A.S. Berdyshev On the existence of eigenvalues of the problem with Bitsadze-Samarskii type conditions for a mixed parabolic-hyperbolic equation 40

Olga Brezhneva, Alexey A. Tret'yakov New approach to optimality conditions for irregular optimization problems 42

Alexander Chentsov, Daniel Khachay, Michael Khachay Linear time algorithm for Generalized Traveling Salesman Problem with precedence constraints of a special type 44

Iuliia Chernova Shannon's function of the lungs' clearence time 46

Dmitry Denisov, Olga Nekrasova Determination of the optimal parameters of reinsurance 48

Vasily Dikusar, Nicholas Olenev About features of differential models for a bank 50

Vasily Dikusar, Elena Zasukhina, Sergey Zasukhin Problem of controllability of nonstationary discrete systems 52

Olga Druzhinina, Natalia Petrova, Elena Shchennikova Conditions of optimal stabilization of nonlinear dynamic models of technical systems 54

Pavel Dvurechensky, Alexander Gasnikov, Evgenia Gasnikova Modern approaches for numerical methods for finding stochastic equilibrium in Beckman equilibrium transport flows model 56

Pavel Dvurechensky, Alexander Gasnikov, Marina Kaznacheeva Optimization methods for finding equilibrium in Beckman and stable dynamics models 58

V.I. Elkin Symmetries of Affine Control Systems 60

Anton Eremeev, Julia Kovalenko On Travelling Salesman Problem with Vertex Requisitions 61

Adil Erzin, Nenad Mladenovic, Roman Plotnikov VNS for min-power symmetric connectivity problem 63

Yuri G. Evtushenko, Alexander I. Golikov On inverse linear programming problems ... 65

Yury Evtushenko, Vladimir Zubov Fast Automatic Differentiation Technique: Investigation and Applications ... 66

Andrey Fomin, Yuri Orlov, Mikhail Posypkin, Izrael Sigal Using Simulation to Study Performance of Parallel Tree Search Schedulers ... 67

Bogdan V. Ganin Piecewise quadratic functions in linear optimization problems ... 68

Alexander Gornov The library of unimodal optimization algorithms ... 70

V.A. Garanzha, L.N. Kudryavtseva Iterative solver for stiff hyperelastic deformation and springback problems ... 71

Alexander Gasnikov, Pavel Dvurechensky, Petro Stetsyuk, Alexandra Suvorikova, Alexey Chernov Equilibriums in multistage transport problems ... 73

Edward Gimadi Algorithms with performance guarantees for some hard discrete optimization problems ... 75

Edward Gimadi, Anna Kurochkina On a clustering problem on the real axis ... 76

Edward Gimadi, Ivan Rykov, Oxana Tsidulko, Mikhael Khachay TSP-approach to solving some problems of covering graphs by m disjoint cycles ... 77

Evgeny Golshteyn, Ustav Malkov, Nikolay Sokolov A Hybrid Method for Solving Bimatrix Games ... 79

Tatiana Gruzdeva, Andrey Orlov On Solving Bilevel Problems with a Matrix Game as the Follower Problem ... 81

Milojica Jaćimović, Nevena Mijajlović Iterative methods for solving quasi-variational inequalities ... 83

Vladimir Jaćimović, Nikola Konatar Directional control of bifurcations into targeted trajectory ... 84

M.T. Jenaliyev, M.N. Kalimoldayev Duality and optimization methods of multidimensional nonlinear dynamic systems ... 85

Vyacheslav Kalashnikov, Nataliya Kalashnykova, Mariel Leal-Coronado Monotone solutions of parametric complementarity problems 87

Maksat Kalimoldayev, Alexandra Alexeyeva, Gulshat Amirkhanova Economic soliton of (2+1)-dimensional nonlinear A4-model 89

Olga Kaneva, Dmitriy Atink Research of statistic methods in finding the solutions of a linear stochastic problems 91

Igor Kaporin Bounding Restricted Isometry in Compressed Sensing Matrices via the K-condition number 92

Alexander Kel'manov Some quadratic Euclidean 2-clustering problems: NP-hardness and efficient algorithms with performance guarantees 93

Alexander Kel'manov, Sergey Khamidullin, Vladimir Khandeev Fully polynomial-time approximation scheme for a sequence 2-clustering problem 94

Alexander Kel'manov, Vladimir Khandeev Fully polynomial-time approximation scheme for a special case of a quadratic Euclidean 2-clustering problem 96

Alexander Kel'manov, Anna Motkova An exact pseudopolynomial algorithm for a special case of a Euclidean balanced variance-based 2-clustering problem 98

Alexander Kel'manov, Artem Pyatkin NP-hardness of Quadratic Euclidean Balanced 2-Clustering Problem 99

Ruben.V. Khachaturov An example of multicriterial optimization in a pseudo-metric space of criteria using the set of equivalence method 100

V.R. Khachaturov, A.V. Zlotov Software for regional design 102

Mikhail Khachay, Katherine Neznakhina Polynomial Time Approximation Scheme for Euclidean Minimum-weight k-Size Cycle Cover Problem in $\mathbb{R}^d$ 104

Elena Khoroshilova Dynamic multi-criteria problem: method of solutions 105

Liudmila Klochkova, Vladimir Tishkin The transport informational Model for ecological Systems ("TIMES") 107

Konstantin Kobylkin Integrality gap for Hitting Set problem on geometric graphs 109

Ivan Kotliarov A mathematical model of franchisee income 111

Vladimir Krivonozhko, Finn Førsund, Andrey Lychev Smoothing the frontier in the DEA models 113

V.M. Krivtsov, V.R. Soloviev Numerical Simulation of a Surface Dielectric Barrier Discharge 114

Oleg Kuzenkov Controlled systems of measure dynamics 115

Yuriy Laptin Exact penalty functions and convex extensions of functions in decomposition schemes in variables 116

Tatyana Lebedeva, Tatyana Tyupikova The Solution of the Problem of Personal Computers Park Updating on the Basis of Modelling 118

Alexander Lotov, Elena Zasukhina Application of Edgeworth-Pareto Hull approximation in block separable nonlinear multi-objective problems 119

Elena Madaeva, Arsalan Mizhidon Identification of linear dynamic systems on the harmonic signal 120

O. Mamyrbayev, Ye. Amirgaliev, M. Kalimoldayev, A.Keylan Segmentation Methods of Speech Signal for Multimodal Speech Recognition 121

Olga Masina, Olga Druzhinina Optimal control and motion stability for technical systems described by differential equations with multiple-valued right parts 124

I.E. Mikhailov, A.L. Skiba Numerical solution of shape optimization problem for elliptic elastic plates with hole 126

Klara Mizhidon, Arsalan Mizhidon Estimation of functional optimal value for one control problem 128

Yury Morozov Proximate time-optimal control of third-order integrators with phase constraints 129

Nataliia Obrosova, Alexander Shananin About market capitalization of low competitive company in the conditions of unstable demand 131

Nicholas Olenev A dynamic model of economy with social stratification 133

Nicholas Olenev, Vsevolod Ostapov Optimal investment problems in a dynamic model of economy with venture capital 135

Andrei Orlov On the Local and Global Search in Polymatrix Games .. 137

Valeriy Parkhomenko Atmospheric general circulation and thermohaline ocean circulation coupled model numerical experiments realization . . 139

Maxim Patrin, Andrey Lychev, Vladimir Krivonozhko DEA models for the Internet Acquiring . . . 142

Alexander Pesterev Synthesis of an optimal path following controller for a wheeled robot with constrained control resource . . . 143

Lev Petrov, Alexey Stepanov Illegal migration forecasting using system dynamics . . . 145

Nikolay Pilnik, Igor Pospelov, Stanislav Radionov Model of the Russian banking system in view of banks' demand for liquidity . . . 147

Agnieszka Prusińska, Alexey A. Tret'yakov P-th order maximum principle for nonregular optimal control problems . . . 148

Vitaliy Pyankov Discrete model of economy with delays . . . 150

L.A. Rybak, D.I. Malyshev, A.A. Gunkin Application of H_∞–optimization for motion control of robotic platforms . . . 152

Alexander Salienko, Yuriy Volkov Application of splines for the hill diagram hydroturbine modeling . . . 155

Vladimir Savelyev A problem of rythmical production . . . 156

Simon Serovajsky Differentiation Functors and its Application in the Optimization Control Theory . . . 158

Ilyas Shakenov Inverse problems for two dimensional parabolic equations with infinite horizon . . . 159

Kanat Shakenov Numerical solution of the some parametric inverse problem of atmospheric optics by Monte Carlo methods . . . 160

Valery Shevchenko Lattices of tringulations of linear polyhedral cones and their f-vectors . . . 161

V.I. Shmyrev A Method of Meeting Paths for Piecewise Linear Exchange Model . . . 163

R.Yu. Simanchev, I.V. Urazova On the symmetries of the b-matching polytope . . . 164

Vladimir Skarin On an algorithm for optimal correction of inconsistent problems of convex programming . . . 166

Vladimir Srochko, Vladimir Antonik Sufficient optimality condition based on the increment formula 168

Alexander S. Strekalovskiy Merit functions for d.c. problems 169

Nikolai Tikhomirov, Damir Ilyasov The models of optimal economic development with risk criteria 171

Tatiana Tikhomirova, Valeriya Gordeeva The approach to the assessment of health loss due to premature mortality in Russia 173

Oleksandr Trofymchuk, Liudmila Ushakova, Vladimir Vasyanin On complexity of one variety of the packing problem 175

Anton Ushakov, Igor Vasilyev A parallel subgradient method for maximizing a Lagrangian dual function in the p-median problem 177

Aida Valeeva, Ruslan Valeev, Yuliya Goncharova, Ivan Koshcheev The Development of the Logistic Transport System for Solving the Problem of Homogeneous Goods Delivery 179

Alexander Vasin, Polina Kartunova Optimization of Network Structure for Tree-type Market 181

Maxim Yanulevich A Numerical Method for Solving Convex Optimal Control Problems 183

N.R. Yunicheva The study procedure of solvability of the system of interval algebraic equations for the object control problem with inaccurate data in parameters 185

Gennady Zabudsky, Tatiana Keyner Solving one location problem of rectangles in the plane with weighted points 187

Lidia Zaozerskaya On average number of feasible solutions of a class of the generalized set packing problem 188

T.S. Zarodnyuk, A.L. Beklaryan, E.A. Finkelshtein, F.A. Belousov The technology for solving the boundary value problems for systems of functional-differential equations of point type 189

Vitaly Zhadan On some variant of simplex-like algorithm for solving linear semi-definite programming problem 191

Vladimir Zubov, Alla Albu On the Effective Gradient Calculation With Help of Fast Automatic Differentiation Technique 192

Vladimir Zykin Integrity constraint of incomplete data 193

Anna Zykina, Olga Kaneva Complementarity problem in non-linear two-stage problem of stochastic programming 194

Author index 196

Multilevel approach to modeling and optimization of semiconductor heterostructures

Karine Abgaryan[1], **D.L. Reviznikov**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
kristal83@mail.ru
[2] *Moscow Aviation Institute (National Research University), Moscow, Russia;*

Multilayer structures based on wide-bandgap semiconductors have some fundamental advantages for the production of SFH-transistors. Particularly its' application provides wide possibilities of device bond structure variation and generation of two-dimensional electronic gas (2DEG) with high charge carriers density. Application of multilevel computation methods [1] to the calculation of charge density distribution and mobility as well as to the modeling of the initial phases of semiconductor heterostructures growth is considered in this project.

Three-level modeling scheme for the nanoscale semiconductor heterostructures with respect of the spontaneous and piezoelectric polarization is proposed. The scheme combines calculations on three different scales. First of all ab initio calculations [2] of interface charge density should be performed. Calculation of charge carriers distribution based on the conjugated Schrödinger and Poisson equations allows to obtain following 2DEG properties: energy levels, corresponding wavefunctions, potential energy distribution and charge carriers density distribution over the heterostructure [3]. On the third step electron mobility in 2DEG with respect to various scattering mechanisms can be calculated. The increase in the calculation process speed was achieved by the use of the approach based on the approximation of non-linear dependence of electron concentration on the potential. The efficiency of the proposed algorithm for the problems under consideration is demonstrated. In the framework of this model the values of 2DEG concentration and mobility were calculated for various Al concentrations x in AlxGa1-xN barrier. A number of computational experiments were performed to study the influence of such heterostructure characteristics as barrier thickness, barrier doping, spacer width, presence of AlN intermediate layer on the properties of 2DEG.

The developed methods of mathematical modeling open prospects for solution of the optimization problems actual for development of microwave electronics.

The work is supported by the Russian Science Fund (project no. 14-11-00782).

References

1. *K. Abgaryan* "Multiscale computational model of multy-layer semiconductor nanostructures", IV International Conference on Optimization Methods and Applications. (OPTIMA- 2013). Petrovac, Montenegro. September 2013.Abstracts. pp.13-14.
2. *I.Supryadkina, K.Abgaryan, D.Bazhanov, I.Mutigullin* "AB initio study of macroscopic polarization of AlN, GaN and AlGaN", Phys.Status Solidi. 2014. 11. No.2, 307-311.
3. *K. Abgaryan, I. Mutigullin, D. Reviznikov* "Computational model of 2DEG mobility in the AlGaN/GaN heterostructures", Phys.Status Solidi. 2015. 11, No.2, 407-411.

Application of computer modeling for the solution of optimizing problems of parametrical identification of potentials of interatomic interaction

Karine Abgaryan[1], Olga Volodina[1], Sergey Uvarov[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
kristal83@mail.ru

One of the priority directions of the current technologies in structural materials science is receiving new materials with the set of properties. The newest methods of computer modeling allow to predict structural characteristics of new materials without carrying out natural experiments and to count their properties.

In this work multilevel approach at which calculations of structure and properties of one-component and two-component materials at each level of scale are carried out with application of the corresponding approximations and methods is applied. At the nuclear and crystal level quantum-mechanical calculations are used (on the basis of the theory of functionality of electronic firmness [1]). The values received at the first level of scale are transferred to the second microscopic large-scale level. At this level molecular and dynamic approach with use of modern potentials of interatomic interaction is used. Originally the type of potential of interatomic interaction (Morse, Tersoff, Lennard-Jones, Brenner-Tersoff's potential, etc.) which will model most effectively and precisely behavior of atoms of a crystal lattice depending on type of a chemical bond (ion, covalent, metal, etc.), and also from features of the modelled material is selected. Further selection of parameters of the chosen potential of interatomic interaction is made and the optimizing problem of parametrical identification is solved [2, 3]. In the course of selection results of the calculations received at the nuclear and crystal level [1] were used. To carry out identification of parameters of the potential chosen for modeling of concrete material, criterion function of the following look is formed:

$$
\begin{aligned}
&F(\xi) = \omega_1(E_{coh}(\xi) - E_{coh}^{fpc})^2 + \omega_2(a(\xi) - a^{fpc})^2 + \\
&+\omega_3(B(\xi) - B^{fps})^2 + \omega_4(C'(\xi) - C'^{fpc})^2 + \omega_5(C_{44}(\xi) - C_{44}^{fpc})^2 + \\
&+\omega_6(\zeta(\xi) - \zeta^{fpc})^2 \to \min \qquad \xi = (\xi_1 \dots \xi_m).
\end{aligned}
\tag{1}
$$

In criterion function as the reference the values of cohesive energy

($E_{coh}(\xi)$), calculated with the help the ab-initio calculations at the first large-scale level and the values of cohesive energy (E_{coh}), calculated by means of the optimized potential of interatomic interaction and depending on its parameters are used. Values of a constant lattice – $a(\xi)$, values of the volume module of elasticity – $B(\xi)$, the module of shift $C'(\xi)$, constant elasticity $C_{44}(\xi)$ and Kleynman's constant $\zeta(\xi)$, also depend on potential parameters. Calculation of all these values was carried out on the formulas given in [4]. The contribution of each square of a difference to value of criterion function decided on the help of weight coefficients. The essence of process of identification of parameters of potential consists in finding of such set of parameters at which the calculated values of physical quantities are close to reference values that is expressed in achievement by criterion function (1) minimum value. Thus, the solution of a problem of minimization of criterion function provides finding of an optimum set of parameters of potential for the description of structure of the considered material. The Granular Radial Search and Basin-Hopping [5] method were applied to the solution of an optimizing task. The problem of parametrical identification of potentials of interatomic interaction was solved for unicomponent materials, such as Al, Si, Fe, W, and for the two-component materials AlN, Al_20_3, S_iC. Results of computer modeling will be coordinated with known tabular data.

The authors were supported by the Russian Science Fund (no. 14-11-00782).

References

1. *Kohn W., Sham L.J.* Phys. Rev. 140, A1133, 1965.
2. *Abgaryan K.K., Posypkin M.A.* "Optimization methods as applied to parametric identification of interatomic potentials", Computational Mathematics and Mathematical Physics, 2014, Vol. 54, No. 12, pp. 1929-1935.
3. *Abgaryan K.K., Volodina O. V., Uvarov S.I.* "Mathematical modeling of processes of formation of clusters of dot defects in silicon on the basis of molecular and dynamic approach", News of higher education institutions. Materials of electronic equipment. 2015, No. 1, 37-42.
4. *Powell D.* "Elasticity, Lattice Dynamics and Parameterisation Techniques for the Tersoff Potential Applied to Elemental and Type III-V Semiconductors", diss. University of Sheffield, 2006.
5. *Wales, D J, and Doye J P K* " Global Optimization by Basin-Hopping and the Lowest Energy Structures of Lennard-Jones Clusters Containing up to 110 Atoms", Journal of Physical Chemistry A, 1997, 101, 5111.

Improper Vector Set Problem

Alexander P. Abramov[1]

[1] *Computing Center RAS, Moscow, Russia;* apabramov@list.ru

The using of vector sets for a planning is one of the approaches in a problem of resource allocation among multiple users. Let the scalar b be the value of a resource, which is used for the production n types of commodities. Assume that the model is linear. Denote by x_i the output level of commodity $i = 1, \ldots, n$. Let a_i be the cost coefficient. Under the condition of total consumption of the resource, we have $\sum a_i x_i = b$. Suppose we know the vector set x^0 such that its components correspond to the desired proportions of production. In this case the solution is the vector $x^* = \omega x^0$, where the scalar ω has the value $b / \sum a_i x_i^0$.

Let us consider the problem related to a methodology for calculating vector sets. This methodology involves two steps. First the experts must determine the proportions of production for some couples of commodities. In the general case, these recommendations may contain contradictions. The second step is an algorithmic processing of the expert data. This step generates the vector set x^0.

Assume that the experts have determined the proportions of production for some couples of commodities: $x_i/x_j = r_{ij}$, $(i,j) \in R$, where r_{ij} are some positive scalars and R is a set of ordered pairs of indexes. These proportions may contain contradictions. The simplest example of such contradiction is the inequality $r_{ij} \neq 1/r_{ji}$. Such cases may occur when one expert group relates the importance of the commodity i to other commodities, and another expert group take commodity j as a basis for comparing, where $j \neq i$. It may be also violated the transitivity: $r_{ij} r_{jk} \neq r_{ik}$. Such improper problems should be regarded as an adequate to economic realities, when the proportion of the production are evaluated by independent experts on the basis of personal preferences.

To solve the improper problem of finding a vector set we will use the methodology of multi-criteria optimization. This means that we minimize the deviations of actual proportions with respect to the given ones. In fact, we have the following problem with $|R|$ objective functions:

$$\left| \frac{x_i}{x_j} - r_{ij} \right| \to \min, \qquad (i,j) \in R, \quad x_i, x_j > 0.$$

Let us remark that these objective functions estimate the absolute values of deviations. This may result to unjustifiably high importance for the criteria with large absolute values of parameters r_{ij}. To eliminate this influence we propose to use the following $|R|$ objective functions:

$$\left|\frac{1}{r_{ij}}\frac{x_i}{x_j}-1\right| \to \min, \qquad (i,j)\in R, \quad x_i, x_j > 0. \tag{1}$$

We consider the approach to the definition of a vector set such that the initial multi-criteria problem is converted into an one-criterion problem. We assume that the optimal solution is the vector which provides the minimum of the maximum value in (1). In this case the problem is formulated as follows: find the vector x^* and the scalar ω^* such that

$$\omega \to \min$$

under the conditions

$$\left|\frac{1}{r_{ij}}\frac{x_i}{x_j}-1\right| \leqslant \omega, \qquad (i,j)\in R, \quad x_i, x_j > 0.$$

The author was supported by the Russian Foundation for Basic Research (project no. 13-07-00730).

How to differentiate a function knowing only its integrals

Alexander Afanasyev[1], Stanislav Emelyanov[2], Elena Putilina[3]

[1] *The Kharkevich Institute for Information Transmission Problems of RAS, Moscow, Russia;* apa@isa.ru

[2] *The Kharkevich Institute for Information Transmission Problems of RAS, Moscow, Russia;*

[3] *The Kharkevich Institute for Information Transmission Problems of RAS, Moscow, Russia;* elena@isa.ru

In this report we propose an approach based on replacing the differentiation operation on the operation of integration, which has good conditionality. The main result is a formula representing the analyzed signal in the form of a power function, whose coefficients are calculated on the basis of repeated integrations. This formula has an approximating meaning in relation to the operation of differentiation.

The function $\varphi(t)$ approximating the function $f(t)$ is sought in the form

$$\varphi(t) = \sum_{i=0}^{n} A_i \, \frac{t^i}{i\,!}$$

Coefficients $A_0,\ A_1,\ A_2, \ldots, A_n$ have the sense of derivatives of $f(t)$. $A = (A_0, A_1, A_2, \ldots, A_n)^T$ calculated by the formula

$$A = D\left(\frac{1}{\tau}\right)\, M^{-1} D\left(\frac{1}{\tau}\right)\, F(\tau)$$

where $D(\cdot\,)$ diagonal matrix with diagonal elements $\dfrac{1}{\tau^i},\ i = 0, 1, 2, \ldots, n$.

$M = \left\{ \frac{1}{(i+j)\,!} \right\}_{(n+1)\times(n+1)}$

$F(\tau) = (f_0(\tau), f_1(\tau), f_2(\tau), \ldots, f_n(\tau))^T,$

where $f_i(\tau) = \underbrace{\int\limits_t^{t+\tau} \cdots \int}_{i} f(S_0)\, dS_0, dS_1, \ldots, dS_i$ — n-fold integral.

The formula is compared with the Taylar formula.

The theorem to estimate the accuracy of computation of the derivatives is formulated and proved.

Theorem 1.

Let $f(t) - n+1$ times continuously differentiable function. Then

$$\left| A_k - f_k^{(t)} \right| \leq L \, \frac{(n+k)\,!}{(2n+1)\,!} \, C_{n+1}^k \;\; \tau^{n+1-k},$$

$$L - \;\text{constant},$$

$$C_{n+1}^k = \frac{(n+2-k)(n+3-k)\ldots(n+1)}{k\,!}$$

The proposed approach allows to approach the problem of generalization of the differentiation operation to the case of nonsmooth functions.

References

1. *S.V. Emel'yanov, A.P. Afanas'ev, and A.S. Tarasov* "Restoration of a Signal Based on Multiple Integrals," in: Doklady Mathematics, **89**, No. 3 (2014).

On Application of the Fast Automatic Differentiation Technique in Optimal Control Problems for Discrete Processes

Kamil Aida-zade[1,2], **Vagif Abdullaev**[1,2]

[1] *Azerbaijan State Oil Academy, Baku, Azerbaijan;*
[2] *Institute of Control Systems of ANAS, Baku, Azerbaijan;*
kamil_aydazade@rambler.ru; vaqif_ab@rambler.ru

We consider the control problem for discrete processes:

$$x^{i+1} = f_i(X^i, U^i), \quad i = 1, 2, ..., N-1, \tag{1}$$

$$f_0(X^0, U^0) = 0, \tag{2}$$

$$f_N(X^N, U^N) \to \min_{u \in V}. \tag{3}$$

Here $x^i \in R^n$ is the state of the controlled discrete process at the i^{th} step, $i = 1, 2, ..., N$; $X^i = (x^1, x^2, ..., x^N) \in R^{nN}$; $u^j \in R^r$ is the state of the j^{th} control action, $j = 1, 2, ..., M$, $U = (u^1, u^2, ..., u^M) \in R^{rN}$; $X^i \subset X$ and $U^i \subset U$ are the sets of the object's and control actions' states determined by the corresponding index sets $X^i = \{x^j : j \in I^i \subset I = \{1, 2, ..., N\}\}$, $U^i = \{u^j : j \in J^i \subset J = \{1, 2, ..., M\}\}$; $f_i(\cdot, \cdot)$, $i = 0, 1, ..., N-1$, are given n–dimensional vector-functions, differentiable with respect to their arguments; $f_N(\cdot, \cdot)$ is the given differentiable function; $V \subset R^{rN}$ is the domain of admissible values of the control actions.

Depending on the structure of the index sets I^i, J^i, $i = 0, 1, ..., N-1$, the correlations (1)-(2) can describe one-step, multi-step, loaded, delayed discrete processes with initial, boundary, as well as non-separated boundary and intermediate conditions.

Depending on the structure of the index sets I^N, J^N, we obtain discrete analogues of Lagrangian, Mayer, Bolza and Moiseev functionals.

We have obtained necessary optimality conditions for the problem (1)-(3) in the general form using the Fast Automatic Differentiation technique [1]. A comparative analysis with known optimality conditions for certain classes of control problems for discrete processes, obtained by other authors, has been conducted [2-4].

We give the numerical results of solution to the optimal control problems for loaded discrete processes with unseparated intermediate and boundary conditions, based on the derived constructive formulas for the gradient of the criterion functional and on special algorithms of calculation of the state of discrete loaded processes with unseparated conditions, which are analogues of continuous processes [5, 6].

References

1. *K.R. Aida-zade, Yu.G. Evtushenko.* "Fast automatic differentiation on computers", Matem. Mod., **1**, No.1, 120–131 (1989).
2. *V.G. Boltyansky.* Optimal Control Over Discrete Systems. Nauka, Moscow, (1973).
3. *R. Gabasov, F.M. Kirillova.* Concerning theory of necessary conditions of optimality for discrete systems, Avtomat. i Telemekh., **12**, 39-47 (1969).
4. *K.B.Mansimov.* "Optimality Conditions in a Discrete Control Problem with Nonlocal Boundary Conditions", Journal of Automation and Information Sciences, **44**, No.9, 74-83 (2012).
5. *V.M. Abdullaev, K.R. Aida-zade.* "Numerical Solution of Optimal Control Problems for Loaded Lumped Systems", Comput. Math. Math. Phys., **46**, No.9, 1487-1502 (2006).
6. *V.M. Abdullaev, K.R. Aida-zade.* "On numerical solution to systems of loaded ordinary differential equations", Comput. Math. Math. Phys., **44**, No.9, 1585–1595 (2004).

Control of the Oscillatory Process by Means of Lumped Sources

Kamil Aida-zade[1,2], Jamilya Asadova[2]

[1] *Azerbaijan State Oil Academy, Baku, Azerbaijan;*
[2] *Institute of Control Systems of ANAS, Baku, Azerbaijan;*
`kamil_aydazade@rambler.ru; jamilya_babaeva@rambler.ru`

We consider the oscillatory process describing the transition from one steady-state fluid motion regime in a pipeline to another:

$$-\frac{\partial p(x,t)}{\partial x} = \frac{\partial \omega(x,t)}{\partial t} + 2a\omega + \sum_{i=1}^{L} u_i(t)\cdot\delta(x-\overline{x}_i),$$

$$-\frac{\partial p(x,t)}{\partial t} = c^2\frac{\partial \omega(x,t)}{\partial x}, \qquad x\in[0,l],\ t\in[0,T]. \tag{1}$$

Here $p(x,t)$, $\omega(x,t)$ are the pressure and fluid velocity at the point $x\in[0,l]$ at the time moment t; l is the length of the pipeline; $2a = const$ is the parameter of the process, which depends on geometrical dimensions and on parameters of the fluid itself; $\overline{x}_i\in(0,l)$ and $u_i(t)$ are the locations and magnitudes of pointwise impact on the process (pumping stations); T is the transient period.

We have boundary, initial and final conditions as follows:

$$\omega(x,0) = \varphi_{10}(x) = const,\ p(x,0) = \varphi_{20}(x), \tag{2}$$

$$\omega(0,t) = v_1(t),\ p(l,t) = v_2(t), \tag{3}$$

$$\omega(x,T) = \varphi_{1T}(x) = const,$$

$$p(x,T) = \varphi_{2T}(x), x\in[0,l], t\in[0,T]. \tag{4}$$

Here $\varphi_{10}(.)$,$\varphi_{20}(.)$,$\varphi_{1T}(.)$,$\varphi_{2T}(.)$ are given functions.

It is required to find the functions $u(t) = (u_1(t),...,u_L(t))$ and $v(t) = (v_1(t),v_2(t))$, under which the transient period T is minimal. We have the following constraints on the control actions and on the phase state:

$$\underline{v}_i \leq v_i(t) \leq \overline{v}_i,\ i = 1, 2,\ t \in [0, T], \tag{5}$$

$$\underline{\gamma}_i p(x_i - 0, t) \leq u_i(t) \leq \overline{\gamma_i} p(x_i - 0, t),\ i = 1, 2, ..., L, \tag{6}$$

$$\underline{\omega} \leq \omega(x, t) \leq \overline{\omega},\ x \in (0, l],\ t \in (0, T]. \tag{7}$$

We have carried out numerical analysis of the dependency of the optimal transient period from the length and resistance coefficient of the pipeline, from the number and locations of the pumping stations, as well as from the initial and final values of the steady-state regimes ([1]).

The investigations being conducted here generalize the results ([1]-[4]), obtained for boundary controls in case there are intermediate lumped control actions.

References

1. *K.R. Aida-zade, J.A. Asadova.* "Study of Transients in Oil Pipelines", Automation and Remote Control, **72**, No.12, 2563-2577 (2011).
2. *P.A. Revo and G.D. Chabakauri.* "The Wave Equation with a Boundary Control at the Left Endpoint and with Free Right Endpoint and the Complete Vibration Damping Problem", Differential Equations, **36**, No.6, 886-894 (2000).
3. *V.A. Il'in.* "Two-Endpoint Boundary Control of Vibrations Described by a Finite-Energy Generalized Solution of the Wave Equation", Differential Equations, **36**, No.11, 1652-1659, (2000).
4. *V.A. Il'in.* "Two-Endpoint Boundary Control of Vibrations Described by a Finite-Energy Generalized Solution of the Wave Equation", Differential Equations, **36**, No.11, 1652-1659, (2000).
5. *V.A. Il'in and E.I. Moiseev.* "Optimal Boundary Control of a String by an Elastic Force Applied at One End, the Other End Being Free", Differential Equations, **41**, No.1, 96-110, (2005).

Localization of Leakage Points in Pipelines of Complex Structure

Kamil Aida-zade[1,2], **Yegana Ashrafova**[1,2]

[1] *Institute of Control Systems of Azerbaijan National Academy of Sciences,*
[2] *Azerbaijan State Oil Academy, Baki, Azerbaijan;*
kamil_aydazade@rambler.ru; ashrafova_yegana@yahoo.com

The problem of localization of leakage points in pipelines of complex structure is considered in the work. This work differs from many previously examined cases for certain linear sections of trunk pipelines [1-3].

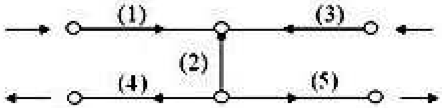

Fig.1. The scheme of pipe network with 5 nodes.

To be specific, assume that the considered pipeline consists of 5 sections (Fig.1). Suppose that at some moment $t \geq t_0$ at the point $\xi \in (0, l)$ of any k^{th} section of the pipeline network, fluid leakage with the flow rate $q^{loss}(t)$ began. Fluid motion on the k^{th} linear section of the pipeline network of the length l_k and diameter d_k can be described by the following system:

$$\begin{cases} -\frac{\partial P^k(x,t)}{\partial x} = \frac{\rho}{S^k}\frac{\partial Q^k(x,t)}{\partial t} + 2a^k \frac{\rho}{S^k} Q^k(x,t), x \in (0, l_k),\ t \in (0, T], \\ -\frac{\partial P^k(x,t)}{\partial t} = c^2 \frac{\rho}{S^k}\frac{\partial Q^k(x,t)}{\partial x} + c^2 \frac{\rho}{S^k} \sum\limits_{i=1}^{L} q_i^{loss}(t)\delta(x - \xi_i), k = \overline{1,5}, \end{cases} \tag{1}$$

where $\delta(\cdot)$ is Dirac's delta function; $P^k(x,t), Q^k(x,t)$ the pressure and flow rate at the point $x \in (0, l^k)$ on the k^{th} section of the pipeline network; c the speed of sound in the medium; $2a^k = const$. Under known leakage points and leakage flow$\xi_i, q_i^{loss}(t)$,$i = 1, ..., L$, for calculation of the fluid motion regimes in the pipeline on the time interval $[t_0, T]$, we use the following conditions:

$$Q^1(0,t) = u_1(t), Q^3(0,t) = u_3(t), P^4(l_4,t) = u_4(t), Q^5(l_5,t) = u_5(t). \tag{2}$$

The conditions (2) provide a predetermined pipeline transportation regime. In the internal nodes of the network, the following matching conditions

are fulfilled:
$P^1(l_1,t) = P^3(l_3,t) = P^2(l_2,t), Q^1(l_1,t) + Q^2(l_2,t) + Q^3(l_3,t) = 0,$

$$P^4(0,t) = P^5(0,t) = P^2(0,t), Q^4(0,t) + Q^5(0,t) + Q^2(0,t) = 0 \qquad (3)$$

and the initial state of the process:

$$Q^k(x,0) = Q_0^k(x), P^k(x,0) = P_0^k(x), k \in K = \{1,3,4,5\},\ x \in [0,\ l_k]. \qquad (4)$$

The problem lies in finding the leakage points$\xi = (\xi_1, ..., \xi_L)$,$\xi_i \in [0, l_k]$,$i = \overline{1, L}, k = \overline{1,5}$, and the corresponding amount of loss of raw stock $q^{loss}(t) = (q_1^{loss}(t), ..., q_L^{loss}(t))$ for $t \in [t_0, T]$ using the given mathematical model and observed information. With the purpose of solving the problem, we consider the following functional determining the deviation of the observed regimes at the given points of the pipeline section from the calculated regimes:

$$J(\xi, q^{loss}(t)) = \sum_{k\in\{1,3\}} \int_\tau^T [Q^k(t) - Q_{mes}^k(t)]^2 dt + \sum_{k\in\{4,5\}} \int_\tau^T [P^k(t) - P_{mes}^k(t)]^2 dt +$$

$$+\varepsilon_1 \left\| q^{loss}(t) - \tilde{q} \right\|_{L_2[\tau,T]}^2 + \varepsilon_2 \left\| \xi - \tilde{\xi} \right\|_R^2 \to \min,$$

where $Q^k(t) = Q^k(l_k, t; \xi, q^{loss}), P^k(t) = P^k(0, t; \xi, q^{loss}), k \in K = \{1,3,4,5\}$ is the solution to the problem (1)-(4) under any given$(\xi, q^{loss}(t))$; $\tilde{\xi}, \tilde{q} \in R^L, \varepsilon_1,\ \varepsilon_2$ the regularization parameters. If the number of the section on which leakage has taken place is not known beforehand, then it is necessary to solve the aforementioned problem for all the sections. It is clear that the case under which the minimal value of the functional has been obtained corresponds to the solution to the given problem.

References

1. *I. A. Charnyi.* Unsteady Motion of Real Fluid in Pipes, Nedra, Moscow, 199p. (1975)[in Russian].
2. *K.R. Aida-zade, J.A. Asadova.* Study of Transients in Oil Pipelines, Automation and Remote Control, Vol. 72, No. 12, p. 2563-2577(2011).
3. *K.R. Aida-zade, E.R. Ashrafova.* Localization of the points of leakage in an oil main pipeline under non-stationary conditions, Journal of Engineering Physics and Thermophysics, Vol. 85, No. 5, Springer, p. 1148-1156(2012).

Zonal Control of Nonlinear Systems with Feedback on State and on Output

Kamil Aida-zade[1]**, Samir Guliyev**[2]

[1] *Azerbaijan State Oil Academy, Baku, Azerbaijan;*
`kamil_aydazade@rambler.ru`
[2] *Institute of Control Systems of ANAS, Baku, Azerbaijan;*
`copal@box.az`

In the work, we investigate the optimal feedback control problems for objects described by systems of nonlinear ordinary differential equations

$$\dot{x}(t) = f(x(t), u(t), p), t \in (t_0, T], \tag{1}$$

under different forms of feedback both on state and on output, on different classes of zonal control functions. It is assumed that the initial state of the object $x(t_0)$ can take values from an a-priori known set X^0 with the density (weighting) function $\rho_{X^0}(x)$, and the time-constant parameters p of the object take values from the given set P with the density (weighting) function $\rho_P(p)$.

Let the quality of control for each given specific initial point $x^0 \in X^0$ and the values of the parameters $p \in P$ be evaluated by the following functional:

$$I(u; T, x^0, p) = \int_{t_0}^{T} f^0(x(t), u(t), p)dt + \Phi(x(T), T). \tag{2}$$

Given that the initial state and values of the parameters of the object are unspecified, the quality of control of the object will be evaluated by the following functional averaged over all $x^0 \in X^0$ and $p \in P$:

$$J(u, T) = \int_{X^0} \int_{P} I(u, T; x^0, p) \cdot \rho_{X^0}(x) \cdot \rho_P(p) dp dx^0 / (\mathrm{mes} X^0 \cdot \mathrm{mes} P). \tag{3}$$

Control of the dynamics of the process (1) is accomplished based on the presence of feedback of the object's current state $x(t)$ or the state of the object's output $y(t)$, which is determined by the known nonlinear function of its state $x(t)$: $y(t) = G(x(t))$, $y \in R^{\nu}$. Here the ν-dimensional vector-function of observation $G(x)$ is continuously differentiable with respect to each variable on the set X. According to these types of feedback, the synthesized control functions will have, in the general case, the representations: $u = V(x, K)$ and$u = W(y, K)$. Here K are the parameters of the synthesized functions; $V(.,.)$ and $W(.,.)$ are some functions of the object's state $x(t)$ and of the object's output $y(t)$, respectively. In both cases, for structural construction of the synthesized functions, we introduce the concept of zonality, that is constancy of the values of the control's synthesized parameters in each of the subsets (zones), which are obtained by splitting (partitioning) the set of all possible states of the object or the set of all possible values of the object's output. The control functions' values are also determined by the type of feedback and the class of functional dependency of the control from the current observed value of the state vector or the output vector. We analyze the cases of continuous and discrete feedback, for which we consider piecewise constant and piecewise linear functional dependencies of the parameters of the zonal control functions from the state vector or the output vector.

In all the statements of the considered problems the synthesized control functions are determined by finite constant vectors and matrices, which eventually lead to finite optimization problems. For solution to these problems, it is efficient to use numerical first-order optimization algorithms: gradient projection or conjugate gradient projection method [1]. With this purpose, we derive formulas for the gradient of the criterion functional. We have developed the corresponding software and carried out numerical experiments on several test problems. We have also made a comparison between the solutions to the feedback zonal control problems under the feedback on the object's output and state [2,3].

References

1. *F.P. Vasilyev* Optimization Methods, Moscow (2002).
2. *S.Z. Guliyev.* "Synthesis of Control in Nonlinear Systems with Different Types of Feedback and Strategies of Control", Journal of Automation and Information Sciences, **45**, No.7, 74–86 (2013).
3. *K.R. Aida-zade and S.Z. Guliyev.* "Zonal Control of Lumped Systems on Different Classes of Feedback Functions", Third International Conference on Optimization Methods and Applications "Optimization and Applications" (OPTIMA-2012), Costa da Caparica, Portugal, 153-156 (2012).

Optimal control of nonsmooth system of differential delay equations with application in immunology

Elena Andreeva, Irina Bolodurina[1], Ulia Ivanova

[1] *Orenburg State University, Russia;* ipbolodurina@yandex.ru

The main purpose of the study is to provide a broad overview of important ideas in analysis and search of optimal control nonsmooth system of differential equations with delay, presented in the form of

$$\dot{x}(t) = \begin{cases} f_1(t, x(t-\tau), x(t), u(t)), S(t,x) < 0, \\ f_2(t, x(t-\tau), x(t), u(t)), S(t,x) \geq 0, \end{cases} \tag{1}$$

where $x \in R^n, u \in R^m$ - vectors of the state and control respectively, τ - constant lag, $S(t,x)$ - scalar function continuously differentiable by the set of arguments and determining the violation of smoothness of the right part of the system (1). The main focus of this work is the problem of necessary optimality conditions and their application to constructions numerical methods of search for the solution to practical tasks in the field of immunology, described by a system of differential equations of form (1). The research examined the different immunological problems of optimal control in modeling the process of antigens reproduction and neutralization, proliferation and differentiation of plasmacells and antibodies. The above tasks differ from each other in various features, such as the dimensionality of phase area, the types of nonlinearities, the restrictions, multiextremality, types of the gaps in the system, as well as the presence of lags in the phase variables etc. For the problems considered specialized methods have been developed for the search of optimal control, a variety of necessary optimality conditions, the algorithms and the software have been obtained allow to find the optimum control and give it a meaningful interpretation.

The parallel version of the stochastic approximation algorithm for the reachable set of dynamic systems with discontinuous right-hand side

Anton Anikin, Evgeniya Finkelstein

Institute for System Dynamics and Control Theory SB RAS, Irkutsk, Russia;
finkel@icc.ru

The paper considers a class of nonlinear dynamical systems with discontinuous right-hand side: $\dot{x} = f(x(t), u(t), t)$, $pr : g(t, x) < 0$ with the initial conditions $x(t_0) = x^0$ and controls $u_i \in [\underline{u_i}, \overline{u_i}]$ on time interval $t \in [t_0, t_1]$. $g(x, t)$ is event-driven continuously differentiable function defined by a predicate and determines the changing time of the system dynamics depending on the state or time. Questions of the solutions existence for differential equations with discontinuous right-hand sides are devised in works of N.N. Krasovskii, A.I. Subbotin, A.F. Filippov, etc.

Theoretical and numerical analysis of such systems is significantly complicated. In particular, for the reachable set (RS) approximation attainability (MD) one can not use most of the developed methods. Based on the multistart idea method of RS stochastic approximation easier than other ones can be adopted for discontinuous systems. The main difficulty arising in this approach is the integration, it is repeated many times and should be made with high accuracy. Among the methods of integration, that do not use event-function, the simplest version of Euler's method with iterations and fixed small, not more than 10^{-6}, step has shown high efficiency. In this case one integration requires essential time and solution is a parallel implementation of the approximation method. Nvidia CUDA technology makes parallel computing on graphics accelerators available. The resulting implementation of RS stochastic approximation method was tested on the Intel Core i5-2500K CPU and Nvidia GeForce GTX 580 GPU. The parallel version for two CPU cores allow to obtain double acceleration comparing with the use of single-threaded version. Using a graphics adapter with 512 cores allows to provide calculation about 50 times faster.

The work was partly supported by the Russian Foundation for Basic Research (project no. 14-01-31296).

GPU-accelerated gradient methods for solving PageRank problem

Anton Anikin[1], Alexander Gornov[1], Alexander Andrianov[3]

[1] *Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* anton.anikin@htower.ru, gornov@icc.ru

[2] *Keldysh Institute of Applied Mathematics RAS, Moscow, Russia;* and@a5.kiam.ru

Optimization problems with large and very large dimensions ("Huge-Scale optimization problems") occur naturally in a wide range of scientific fields. One of such modern problem is web page ranking, which involve finding PageRank vector:

$$\begin{gathered} P^T x = x \\ P \in R^{n\times n}, \;\; x \in R^n \\ \langle x, e\rangle = 1, \;\; e = (1, ..., 1)^T \\ x_i \geq 0, \;\; i = 1, ..n \end{gathered} \tag{1}$$

where P - stochastic matrix specifying the web graph.

The original problem can be reduced to the problem of convex optimization by different ways [1,2]. In the current work we consider following unconstrianed minimization problem:

$$f(x) = \frac{1}{2}||Ax||_2^2 + \frac{\gamma^-}{2}\sum_{i=1}^{n}(-x_i)_+^2 + \frac{\gamma^+}{2}(\langle x, e\rangle - 1)^2 \to \min \tag{2}$$

where:

$(x)_+ = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$ - the penalty function for negative elements;

γ^- - the penalty parameter for negative elements;

γ^+ - the penalty parameter for the violation of constraint $\langle x, e\rangle = 1$.

The considered problem has a high computational complexity associated with large dimensions, of the order of 10^6 variables and above for modern practical statements. Therefore, many papers present different approaches and technologies to accelerate the minimization algorithms for problems of this class. The current work discusses the author's experience of applying the power of modern graphics accelerators (GPU) in solving the problem of Pagerank.

The main time-consuming operation for considered problem is the procedure of multiplying sparse matrix A to a dense vector ($y = Ax$). This operation is necessary for computing the function value, and also for calculating its gradient. In [1] proposes an approach for fast update of y components, which is effective when used in the subgradient optimization method. But this technique imposes substantial restrictions on the structure of the matrix A, and in the worst case cost of subgradient calculating will be equal to the cost of computing the full gradient. Therefore this paper presents author's GPU-implementations of "full-gradient" optimization methods – the classic variant of the conjugate gradient method (Fletcher-Reeves version) and author's modification of Barzilai-Borwein method (BB+P), which uses "Polyak step" [3] on iteration in the case of incorrect choice of minimization step by traditional BB algorithm.

Considered methods are implemented by authors with C++ language and Nvidia CUDA technology, tested and degugged on several computational systems with modern CPUs ang GPUs. The results of numerical experiments for number of PageRank problems, constructed from Stanford University's collection [4] of network-graphs (up to 10^6 variables) are presented. GPU-versions of the implemented methods significantly accelerate (up to 20 times) the solution finding for considered optimization problems.

The authors were supported by Russian Foundation for Basic Research (project no. 13-01-00470)

References

1. *Nesterov Y.* "Subgradient methods for huge-scale optimization problems", Mathematical Programming, Vol. 146, No. 1-2. pp. 275-297 (2014).
2. *A.V. Gasnikov, D.Yu. Dmitriev* "On efficient randomized algorithms for finding the PageRank vector", Computational Mathematics and Mathematical Physics, Vol. 55, No. 3, pp. 349-365 (2015)
3. *B.T. Polayk* "Minimization of nonsmooth functionals," Journal of Computational Mathematics and Computational Physics, Vol. 9, No. 3 pp. 509–521 (1969) (in russian).
4. *Stanford Large Network Dataset Collection.* — URL: snap.stanford.edu/data

Controlled dynamic model with a boundary value problem

Anatoly Antipin[1]

[1] *Computing Center of RAS, Moscow, Russia;*
`asantip@yandex.ru`

The paper deals with the dynamic control system, consisting of two components. One of them is a finite-dimensional boundary value problem. This problem describes the stable equilibrium state of finite-dimensional object. Another component of the system is controlled dynamics. It describes the transition process for system from an arbitrary disturbed state in the equilibrium state. The model of terminal object has the form

$$\begin{cases} x^* \in \mathrm{Argmin}\{f(x) \mid Ax \leq y^*,\ x \in \mathrm{R}^n\}, \\ \langle p - p^*, Ax^* - y^* \rangle \leq 0,\ \ p \in \mathrm{R}^m_+, \\ \langle y - y^*, p^* \rangle \leq 0,\ \ y \in \mathrm{Y}. \end{cases} \tag{1}$$

The first two problems this system are equivalent to the problem of computing the saddle point of the Lagrangian for convex programming. Here (p^*, x^*) is a saddle point of Lagrange function $l(p; x, y) = f(x) + \langle p, Ax - y \rangle$, $y \geq 0$ is a parameter. The latter problem is variational inequality, which connects the multipliers of Lagrangian p^* with vector of right-hand side of constraints y^*. The boundary value problem is an equilibrium model of interaction between two participants with partially conflicting interests. It is assumed that (1) under the action of perturbations lose the equilibrium. In this case, there is the necessity, using control tools, return back the object in a state of equilibrium. This can be done using the following dynamic model

$$\begin{cases} \dfrac{d}{dt}x(t) = D(t)x(t) + B(t)u(t)\ \text{ for almost all }\ t \in [t_0, t_1], \\ x(t_0) = x_0,\ x(t_1) = x_1^* \in \mathrm{X}_1 \subseteq \mathrm{R}^n,\ u(\cdot) \in \mathrm{U}, \\ x_1^* \in \mathrm{Argmin}\{\langle f(x_1) \mid A_1 x_1 \leq y_1^*,\ x_1 \in \mathrm{X}_1\}, \\ p_1^* \in \mathrm{Argmax}\{\ \langle p_1, A_1 x_1^* - y_1^* \rangle \mid p_1 \geq 0\}, \\ y_1^* \in \mathrm{Argmax}\{\ \langle p_1^*, y_1 \rangle \mid y_1 \in \mathrm{Y}_1\}. \end{cases} \tag{2}$$

The set of admissible controls is expected to be integrally bounded: $\mathrm{U} = \{u(\cdot) \in \mathrm{L}_2^r[t_0, t_1] \mid \|u(\cdot)\|^2_{\mathrm{L}_2^r} \leq \mathrm{const}\}$; $\mathrm{X}_1 \subseteq \mathrm{R}^n$ is the attainability set, $\mathrm{Y}_1 \subset \mathrm{R}^m_+$ is a convex closed set, $f(x_1)$ is a convex differentiable function, A_1 is a fixed $m \times n$ matrix, $D(\cdot), B(\cdot)$ are continuous matrices, x_0 is a given initial value.

It is well known that for each control $u(\cdot) \in \mathrm{U}$ and given initial condition x_0 there exists a unique trajectory $x(\cdot)$ which satisfies the identity

$$x(t) = x(t_0) + \int_{t_0}^{t} (D(\tau)x(\tau) + B(\tau)u(\tau))d\tau, \quad t_0 \leq t \leq t_1,$$

and belongs to the linear variety of absolutely continuous functions. Further, we denote this class as $\mathrm{AC}^n[t_0, t_1] \subset \mathrm{L}_2^n[t_0, t_1]$.

To solve the problem, we construct iterative sequences $x^k(\cdot) \in \mathrm{AC}^n[t_0, t_1]$, which, however, may have weak limit points $x^*(\cdot) \in \mathrm{L}_2^n[t_0, t_1]$ lying outside of this class $\mathrm{AC}^n[t_0, t_1]$. Dual method has the form

$$\begin{cases} \bar{p}_1^k = \pi_+(p_1^k + \alpha(A_1 x^k(t_1) - y_1^k)), \\ \bar{\psi}^k(t) = \psi^k(t) + \alpha(D(t)x^k(t) + B(t)u^k(t) - \dfrac{d}{dt}x^k(t)), \\ y_1^{k+1} = \pi_{\mathrm{Y}_1}(y_1^k + \alpha \bar{p}_1^k), \\ (x^{k+1}(t_1), x^{k+1}(t), u^{k+1}(t)) \in \mathrm{Argmin} \Big\{ \dfrac{1}{2}|x(t_1) - x^k(t_1)|^2 + \alpha f(x(t_1)) \\ +\alpha\langle \bar{p}_1^k, A_1 x(t_1) - y_1^{k+1}\rangle + \dfrac{1}{2}\|x(t) - x^k(t)\|^2 + \dfrac{1}{2}\|u(t) - u^k(t)\|^2 \\ + \alpha \displaystyle\int_{t_0}^{t_1} \langle \bar{\psi}^k(t), D(t)x(t) + B(t)u(t) - \dfrac{d}{dt}x(t)\rangle dt \Big\}, \\ p_1^{k+1} = \pi_+(p_1^k + \alpha(A_1 x^{k+1}(t_1) - y_1^{k+1})), \\ \psi^{k+1}(t) = \psi^k(t) + \alpha(D(t)x^{k+1}(t) + B(t)u^{k+1}(t) - \dfrac{d}{dt}x^{k+1}(t)). \end{cases}$$

The convergence of this process for all components of the solution of the original problem is proved.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-01-06045-a), and the Program for Support of Leading Scientific Schools (project No. NSh-4640.2014.1).

Efficient region monitoring with sensors outer positioning

Sergey Astrakov[1,2]

[1] *Design Technological Institute of Digital Techniques, Novosibirsk, Russia;*
[2] *Novosibirsk State University, Novosibirsk, Russia;* astrakov90@gmail.com

Wireless sensor network (WSN) consists of sensors that are placed in the monitoring region and use wireless communication to exchange information. The main sensor functions are to collect data, primary processing and transmission of collected data. In this case, the sensing area of each sensor is usually represented as a disk with a certain radius centered at the location of the sensor, and it is said that the sensor covers this disk. The mail issue in WSN is to save the energy of the sensors. A cover of plane region S is such a set of disks C, where each point of the region belongs to at least one of disk. The cover density is defined as the ratio of the area of all disks in C to the area of S. Under the regular cover means a cover of the plane region by disks in which the whole region can be divided into regular polygons (tiles), forming a regular lattice. In this case, all the polygons should be covered equally.

Usually, researchers determine a min-density cover with the discs of one, two and three radii [1]. In the paper [2] we perform similar investigations of a band, however, requiring external monitoring of the area. This means that, due to some inaccessibility, we cannot place sensors inside the controlled area. In this paper, we consider the external monitoring restricted areas. It is proved that the lowest density of the outer coating of the circle and the square is achieved by 4 circles and is equal to 3 and $3\pi/8 \approx 2,356$ respectively. Additionally, examples of effective external coatings 3D regions are offered.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-07-00139).

References

1. *V. Zalubovsky, A. Erzin, S. Astrakov, H. Choo.* "Energy-efficient Area Coverage by Sensors with Adjustable Ranges," Sensor, No. 9, 2446–2460 (2009).
2. *S. Astrakov, A. Erzin.* "Efficient Band Monitoring with Sensors Outer Positioning," Optimization, No. 10, 1367–1378 (2013).

A Heuristic Algorithm for the Double Integrator Traveling Salesman Problem

Artem Baklanov[1]

[1] *N.N. Krasovskii Institute of Mathematics and Mechanics, Yekaterinburg, Russia;* `artem.baklanov@gmail.com`

Modern engineering challenges dictate a consideration of dynamic versions of the classic the traveling salesman problem (TSP). Namely, trajectories of 'cities' and 'the traveling salesman' satisfy dynamics in terms of differential equations.

As the first approximation, dynamics of some objects can be considered as a linear system. Namely, it is possible to obtain a good approximation of spacecraft dynamics (in deep space) by using the double integrator model. In this regard, the TSP for controlled objects described by linear differential equations is important. A special case of such problem is the TSP for the double integrator [1].

We considered a double integrator which has to visit a set of given stationary points at a minimum travel time. Control constraints are defined in terms of a convex compact set. We obtained an upper bound for the minimum travel time, by developing the method of transformation of the original problem into a generalised traveling salesman problem. This transformation is based on a discretisation of sets of admissible visiting velocities. To solve time-optimal two-point problems, we use the duality of optimal control problems and convex programming [2].

Note that STOP-GO-STOP [1] heuristic algorithm was proposed. It was shown that in the worst case scenario STOP-GO-STOP provides solution with total time $T\sqrt{2n}$, where n is a number of nodes to visit and T is the total time provided by the discretisation algorithm.

The authors were supported by the Russian Foundation for Basic Research (project no. 14-08-00419 A).

References

1. *K. Savla, F. Bullo, E. Frazzoli.* "Traveling Salesperson Problems for a double integrator," IEEE Trans. Automat. Contr., **54**, No. 4, 788–793 (2009).
2. *N.N. Krasovskii.* Theory of motion control, Nauka, Moscow (1968) (In Russian).

Polyhedral approximations in $\mathbb{R}^n$

Maxim Balashov[1]

[1] *Moscow Institute of Physics and Technology, Dolgoprudny, Russia;*
`balashov73@mail.ru`

Let $\mathbb{R}^n$ be a real n-dimensional Euclidean space with the inner product (x, y), for any $x, y \in \mathbb{R}^n$.

The modulus of convexity $\delta_A(\varepsilon)$ of the set A, introduced by B.T. Polyak [1,2,3], will be the most important tool for us.

Let E be a Banach space and let a subset $A \subset E$ be convex and closed. *The modulus of convexity* $\delta_A : [0, \operatorname{diam} A) \to [0, +\infty)$ is the function defined by

$$\delta_A(\varepsilon) = \sup \left\{ \delta \geq 0 \;\middle|\; B_\delta\left(\frac{x_1 + x_2}{2}\right) \subset A,\ \forall x_1, x_2 \in A : \ \|x_1 - x_2\| = \varepsilon \right\}.$$

We shall consider the standard polyhedral approximation of a closed convex compact $A \subset \mathbb{R}^n$ on a grid $\mathbb{G} = \{p_k\}_{k=1}^N$ of unit vectors from $\mathbb{R}^n$ with step $\Delta \in (0, \frac{1}{2})$:

$$\hat{A} = \{x \in \mathbb{R}^n \mid (p_k, x) \leq s(p_k, A),\ \forall k,\ 1 \leq k \leq N\}.$$

Here $s(p, A)$ is the supporting function of the set A, i.e.

$$s(p, A) = \max_{x \in A}(p, x).$$

The main result [3] is that

$$h(A, \hat{A}) \leq \frac{8}{7}\varepsilon(\Delta)\Delta,$$

where $\varepsilon(\Delta)$ is a solution of the equation $\frac{\delta_A(\varepsilon)}{\varepsilon} = \frac{\Delta}{4 - \Delta^2}$ and

$$h(A, \hat{A}) = \max_{\|p\|=1} |s(p, \hat{A}) - s(p, A)|$$

is the standard Hausdorff distance between the sets A and $\hat{A}$.

We also shall consider polyhedral approximations in some special situation of presupporting function.

For a positively uniform function $f : \mathbb{R}^n \to \mathbb{R}$ define the set

$$O_f = \{p \in \mathbb{R}^n \mid \|p\| = 1,\ \mathrm{conv} f(p) = f(p)\}.$$

We shall postulate that $\mathrm{conv} f(p)$ is a proper function and $f(p)$ itself is not convex.

Let $R \geq r > 0$ be such constants that $r\|p\| \leq f(p) \leq R\|p\|$ for all p. Suppose also that there exists $\delta > 0$ such that for any $p \in O_f$ and for all $q \in B_\delta(p)$ we have

$$|f(q) - f(p) - (f'(p), q - p)| \leq w(\|q - p\|),$$

where $f'(p)$ is the Frechet derivative, $w(t) > 0$ for $t \in (0, \delta)$ and $\lim\limits_{t \to +0} \frac{w(t)}{t} = 0$.

Let

$$A = \{x \in \mathbb{R}^n \mid (p, x) \leq f(p),\ \forall p \in \mathbb{R}^n\},$$

$$\hat{A} = \{x \in \mathbb{R}^n \mid (p, x) \leq f(p),\ \forall p \in \mathbb{G}\}.$$

Then for sufficiently small step Δ we have the estimate

$$h(A, \hat{A}) \leq \frac{R}{r} \frac{w(\Delta)}{1 - \frac{\Delta^2}{2}}.$$

The work was supported by grant RFBR 13-01-00295-a.

References

1. B. T. Polyak, Existence theorems and convergence of minimizing sequences in extremum problems with restrictions, Soviet Math, 7 (1966), 72-75.

2. M. V. Balashov, D. Repovš, Uniform convexity and the splitting problem for selections, J. Math. Anal. Appl. 360:1 (2009), 307-316.

3. M. V. Balashov, D. Repovš, Polyhedral approximations of strictly convex compacta, J. Math. Anal. Appl. 374 (2011), 529-537.

Self-organization of nonlinear elastic networks for Delaunay-Voronoi meshing of implicit domains

A.I. Belokrys-Fedotov[1], **V.A. Garanzha**[2], **L.N. Kudryavtseva**[3]

Dorodnicyn Computing Center Russian Academy of Sciences, Moscow Institute of Physics and Technology, Moscow, Russia;
[1]belokrys.fedotov@gmail.com, [2]garan@ccas.ru, [3]arnir@rambler.ru

In 2004 G. Strang and P. Persson suggested simple algorithm (Matlab code of their algorithm is less than one page) which allows to triangulate implicit planar domains. The idea of the method is based on using scattered point data set as an input. Edges of the Delaunay triangulation form spatial network with topology depending on point positions. Each edge is considered as a nonlinear elastic strut meaning that expanding forces appear when length of the edge is below the target one. Points which go outside the domain are projected back to its boundary thus forming impearmable barrier for expansion. This elastic network is relaxed attaining certain equilibrium. The remarkable feature of the above algorithm is that during the relaxation process topological irregularities (vertices with valence not equal to 6) are eventually moved to the boundary which resembles expulsion of dislocation from crystal lattice. The resulting mesh tends to be topologically regular.

We present algorithm which generalizes the idea of Strang and Persson and allows to construct 3d tetrahedral meshes in implicit domains with piecewise-regular boundary. Elastic energy of network in our approach is combination of expansion potential and sharpening potential. The latter is applied to boundary face and allows to reproduce sharp edges on the domain boundary without their explicit definition.

Implicit functions defining the computational domain admit heterogeneous and incomplete representation which is illustrated in Fig.1. In this example “elephant” is defined by tesselation, “pavilion” is defined by planar cross-sections, while “ball” is defined analytically. Boolean operations are used to create the final domain. Surface mesh with reconstructed sharp edges and volume mesh are shown.

Fig.2 illustrates typical behaviour of algorithm when after elastic relaxation sharp edges are reconstructed and mesh quality is improved. In this example the body is defined analytically using boolean operations.

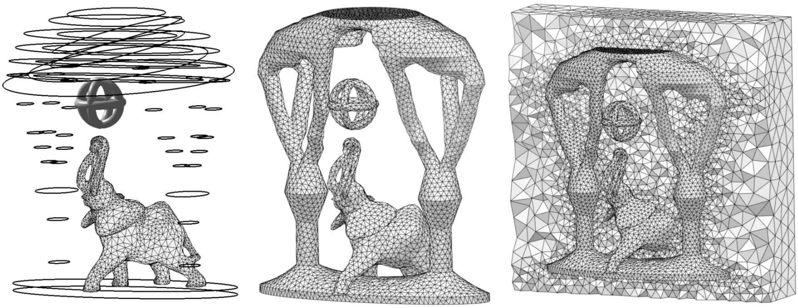

Fig.1. Heterogeneous representation of computational domain:

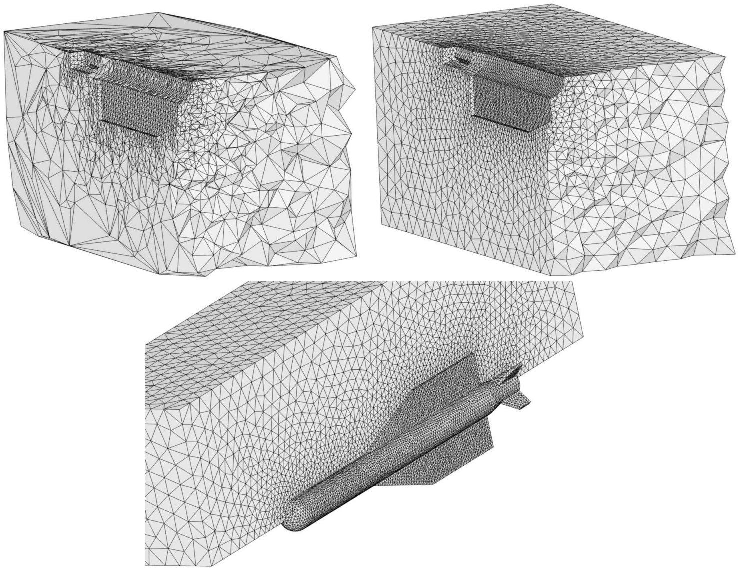

Fig.2. Initial Delaunay mesh, mesh after self-organization and sharpening, and full body.

This work is supported by the Russian government under grant "Measures to Attract Leading Scientists to Russian Educational Institutions" (contract No. 11.G34.31.0072) and RFBR grant 14-07-00805.

On the existence of eigenvalues of the problem with Bitsadze-Samarskii type conditions for a mixed parabolic-hyperbolic equation

A.S. Berdyshev[1]

[1] *Kazakh National Pedagogical University named after Abai, Almaty, Kazakhstan;* berdyshev@mail.ru

Since second half of the seventies of the last century, along with solvability questions intensively studied the spectral properties of local and nonlocal boundary value problems for mixed type equations.

Research on boundary value problems and their spectral properties for mixed parabolic-hyperbolic and mixed-composite type equations are given in [1]. Questions of unique solvability and Volterra property of the analogue of the generalized Tricomi problem (the problem B) for a mixed parabolic-hyperbolic equation of the second and third orders were studied in [1-3].

Completeness and Riesz basis property of the root functions of a class of nonlocal boundary value problems for a mixed parabolic-hyperbolic equations of second order proved in [1], and for the third order [4]. Existence theorems of eigenvalues of the local boundary value problems for parabolic-hyperbolic equation of the second and third orders were proved in [1].

Unique solvability and Volterra property of a class of problems with Bitsadze-Samarskii type conditions (nonlocal conditions) for a mixed parabolic-hyperbolic equation with a non-characteristic line of type changing studied in [1,5]. From the above, arises a question: is it possible to formulate a nonlocal problem for a parabolic-hyperbolic equation that has its eigenvalues.

The main result of this message is the statement and establishment of the existence of the problem eigenvalues with the Bitsadze-Samarskii type conditions for a mixed parabolic-hyperbolic equation:

$$Lu = f(x, y),$$

where

$$Lu = \begin{cases} u_x - u_{yy}, & y > 0 \\ u_{xx} - u_{yy}, & y < 0, \end{cases}$$

in a finite simply connected domain Ω of the surface of independent variables x,y, bounded at y¿0 by segments AA_0, A_0B_0, BB_0 straight lines $x = 0, y = 1, x = 1$ respectively, and at $y < 0$ by characteristics $: x+y = 0$ and $: x - y = 1$ of the Eq.(1). Let us straight line BD is given by equation $y = a(x-1), 0 < a < 1,\ \frac{a}{1+a} \le x \le 1$, and located inside the characteristic triangle $0 \le x + y \le x - y \le 1$.

Problem B. To find a solution of Eq. (1), satisfying conditions

$$u|_{AA_0 \cup A_0B_0} = 0,$$

$$[u_x + u_y]\,[\theta_1(t)] + \mu(t)\,[u_x + u_y]\,\lfloor\theta_1^*(t)\rfloor = 0, 0 < t < 1$$

where $\theta_1(t), (\theta_1^*(t))$ is affix of the intersection point of the characteristic BC (straight line BC) with the characteristic, starting from the point $(t, 0), 0 < t < 1, \mu(t)$ is given function.

Under certain constraints for given problems established the unique solvability and existence of eigenvalues of the problem B.

References

1. *Berdyshev A.S.* Boundary Value Problems and their spectral properties for equations of mixed parabolo-hyperbolic and mixed- composite types. Institute of Information and Computational. Monograph. Almaty, 2015. -224 p. (in Russian)
2. *Salakhytdynov M.S., Berdyshev A.S.* "Boundary Value Problems for parabolo-hyperbolic equation in a domain with deviation from characteristics," Doclady AN Russia. -1992. -. 327, 3. -P. 303-305.(in Russian)
3. *Berdyshev A.S., Cabada A., Karimov E.T, Akhtaeva N.S.* "On the Volterra property of a boundary problem with integral gluing condition for mixed parabolic- hyperbolic equation," Boundary value problems. -2013. -P.1-14. doi: 10.1186/1687-2770-2013-94.
4. *Berdyshev A.S.* "On the basis of the system of root functions of a nonlocal problem for a third-order equation with a parabolic-hyperbolic operator," Differential equations. -2000. -.36, 3. - P.372-276.(in Russian)
5. *Berdyshev A.S.* "On the Volterra property of some problems with Bitsadze-Samarskii type conditions for a mixed parabolic-hyperbolic equation," Siberian Mathematical Journal. -2005. .46, 3. -P.500-510.(in Russian)

New approach to optimality conditions for irregular optimization problems

Olga Brezhneva[1], **Alexey A. Tret'yakov**[2]

[1] *Miami University, Oxford OH, USA;* brezhnoa@miamioh.edu
[2] *Dorodnicyn Computing Center of the Russian Academy of Sciences, Moscow, Russia; System Research Institute, Polish Academy of Sciences, Warsaw, and University of Podlasie, Siedlce, Poland;* tret@uph.edu.pl

In this talk, we present a new approach to deriving optimality conditions for irregular nonlinear optimization problems in the form

$$\underset{x\in\mathbb{R}^n}{\text{minimize}}\ f(x) \quad \text{subject to} \quad g(x) = (g_1(x), \ldots, g_m(x)) \le 0, \tag{1}$$

where f and g_i are sufficiently smooth functions and classical regularity assumptions are not satisfied at a solution x^* of problem (1). The approach is based on the p-regularity theory described, for example, in [1, 2].

Without loss of generality, we assume that the set of indices of the active at x^* constraints is $I(x^*) = \{1, \ldots, m\}$.

Let $H_g(x^*) = \{h \in \mathbb{R}^n \mid \langle g_i'(x^*), h\rangle \le 0,\ i \in I(x^*)\}$. For $h \in H_g(x^*)$, let $I_1(x^*, h) = \{i \in I(x^*) \mid \langle g_i'(x^*), h\rangle = 0\}$. Assuming $|I_1(x^*, h)| = m_1 \ne 0$, we construct s special *acute cones* in such a way that each index from $I_1(x^*, h)$ is used in defining at least one cone. We also require that all cones are different so that $s \le m_1$. For defining a cone with number k $(k = 1, \ldots, s)$ we use indices $i_1, \ldots, i_{r_k} \in I_1(x^*, h)$ chosen in such a way that vectors $g_{i_1}'(x^*), \ldots, g_{i_{r_i}}'(x^*)$ generate an *acute cone* and $i_j \ne i_l$, if $j \ne l$. As a result, there exist $\gamma_i \in \mathbb{R}^n$ such that $\langle g_j'(x^*), \gamma_i\rangle < 0$, $j = i_1, \ldots i_{r_k}$, and for all $j \in J_k(x^*, h) = I_1(x^*, h)\backslash\{i_1, \ldots, i_{r_k}\}$ we have $-g_j'(x^*) = \alpha_{ji_1} g_{i_1}'(x^*) + \ldots + \alpha_{ji_{r_k}} g_{i_{r_k}}'(x^*)$, where $\alpha_{ji_1} \ge 0, \ldots, \alpha_{ji_{r_k}} \ge 0$. Then for every index $j \in J_k(x^*, h)$, we can introduce new mappings:

$$\tilde{g}_j(x) = g_j(x) + \alpha_{ji_1} g_{i_1}(x) + \ldots + \alpha_{ji_{r_k}} g_{i_{r_k}}(x). \tag{2}$$

Assume that there exists $h \in H_g(x^*)$ such that $\langle \tilde{g}_j''(x^*)h, h\rangle \le 0, \quad j \in J_k(x^*, h)$. Notice that otherwise, x^* is an isolated feasible point for (1). Define $I_0^{1\,k}(x^*, h) = \{i_1, \ldots, i_{r_k}\}$, $I_0^1(x^*, h) = \bigcup_{k=1}^{s} I_0^{1\,k}(x^*, h)$, $I_0^{2\,k}(x^*, h) = \{i \in J_k(x^*, h) \mid \langle \tilde{g}_i''(x^*)h, h\rangle = 0\}$, and $I_0^2(x^*, h) = \bigcup_{k=1}^{s} I_0^{2\,k}(x^*, h)$.

D e f i n i t i o n 1. We say that mapping $g(x)$ is *tangent 2–regular* at a point $x^* \in \mathbb{R}^n$ along a vector $h \in H_g(x^*)$ if for any $\xi \in \mathbb{R}^n$ satisfying

$$\begin{array}{lll} \langle g'_{i_k}(x^*), \xi\rangle \le 0, & k = 1, \ldots, r_i, & i = 1, \ldots, s, \\ \langle \tilde{g}''_{i_{r_i}+j}(x^*)h, \xi\rangle \le 0, & j = 1, \ldots, m_1 - r_i, & i = 1, \ldots s, \end{array} \tag{3}$$

there exists a set of feasible points x in the form $x = x^* + \alpha h + \omega(\alpha)\xi + \eta(\alpha)$, where $\alpha > 0$ is sufficiently small, $\|\eta(\alpha)\| = o(\omega(\alpha))$, $\omega(\alpha) = o(\alpha)$ and $\alpha^2/\omega(\alpha) \to 0$ as $\alpha \to 0$.

Theorem 1. *Let $f \in C^1(\mathbb{R}^n)$ and $g \in C^2(\mathbb{R}^n)$. Assume that $g(x)$ is tangent 2-regular at a point x^* along a vector $h \in H_g(x^*)$ and $\langle f'(x^*), h\rangle = 0$. Then there exists $\lambda^*(h) = (\lambda_i^*(h))_{i \in I_0^1(x^*,h) \bigcup I_0^2(x^*,h)} \ge 0$ such that*

$$f'(x^*) + \sum_{i \in I_0^1(x^*,h)} \lambda_i^*(h) g'_i(x^*) + \sum_{i \in I_0^2(x^*,h)} \lambda_i^*(h) \tilde{g}''_i(x^*)h = 0. \tag{4}$$

Notice that since $I_0^1(x^*, h) \subset I_1(x^*, h)$, $I_0^2(x^*, h) \subset I_1(x^*, h)$ and for every $j \in J_k(x^*, h)$, (2) holds, then (4) can be rewritten as

$$f'(x^*) + \sum_{i \in I_1(x^*,h)} \lambda_i g'_i(x^*) + \sum_{i \in I_1(x^*,h)} \gamma_i g''_i(x^*)h = 0, \quad \lambda_i \ge 0,\ \gamma_i \ge 0.$$

Theorem 1 can be illustrated by an example with $f(x) = x_1 + x_2 - x_3 + x_1^2 + x_2^2 + x_3^2$ and $g(x) = \big(-x_1,\ -x_2,\ x_2 - x_1^2 + x_2^2 + x_3^2\big) \le 0$. In this example, $x^* = 0$, and $g(x)$ is tangent 2-regular at a point x^* along a vector $h = (1, 0, 1)^T$.

Similarly to Definition 1, for $p > 2$, we can define $g(x) : \mathbb{R}^n \to \mathbb{R}^m$ to be tangent p-regular at $x^* \in \mathbb{R}^n$ along a vector $h \in H_g(x^*)$ and formulate optimality conditions similar to ones given in Theorem 1.

The second author was supported by the Russian Foundation for Basic Research (project no. 14-07-00805) and by National Leading Schools (project no. 4640.2014.1).

References

1. *A. A. Tret'yakov.* "Necessary and sufficient conditions for optimality of pth order," USSR Comput. Math. and Math. Phys., **24**, No. 2, 123–127 (1984).
2. *A. A. Tret'yakov, J. E. Marsden.* "Factor–analysis of nonlinear mappings: p–regularity theory," Communications on Pure and Applied Analysis, **2**, No. 4, 425–445 (2003).

Linear time algorithm for Generalized Traveling Salesman Problem with precedence constraints of a special type

Alexander Chentsov[1], **Daniel Khachay**[2], **Michael Khachay**[3]

[1] *Krasovsky Institute of Mathematics and Mechanics, Ekaterinburg, Russia;* `chentsov@imm.uran.ru`

[2] *Ural Federal University, Ekaterinburg, Russia;* `daniil.khachay@gmail.com`

[3] *Krasovsky Institute of Mathematics and Mechanics, Ekaterinburg, Russia;* `mkhachay@imm.uran.ru`

We consider Generalized Asymmetric Traveling Salesman Problem (GATSP) with the following statement. There are given n finite disjunct sets $M_1, \dots, M_n$ (we call them *megapolices*), such that $M_j = \{g_{j1}, \dots, g_{jp}\}$, and a starting point $x_0 \notin \cup M_j$. There are given costs $\hat{c}(x_0, g_{j\tau})$ and $\check{c}(g_{j\tau}, x_0)$ of transition from x_0 to any point $g_{j\tau}$ and vice versa, transition costs $c(g_{l\sigma}, g_{j\tau})$ for any $j, l \in \mathbb{N}_n = \{1, \dots, n\}$, $j \neq l$ and $\sigma, \tau \in \mathbb{N}_p$, and visiting costs (costs of inner jobs) $c'(g_{j\tau})$. It is required to find a permutation $\pi : \mathbb{N}_n \to \mathbb{N}_n$ defining the order of visiting of the given megapolices and a finite sequence $g_{\pi(1)\tau(1)}, \dots, g_{\pi(n)\tau(n)}$ such that

$$\hat{c}(x_0, g_{\pi(1)\tau(1)}) + \sum_{i=1}^{n-1} \left(c'(g_{\pi(i)\tau(i)}) + c(g_{\pi(i)\tau(i)}, g_{\pi(i+1)\tau(i+1)}) \right) + \check{c}(g_{\pi(n)\tau(n)}, x_0) \to \min \quad (1)$$

Actually, the costs $\hat{c}(x_0, g_{ij})$, $\check{c}(g_{ij}, x_0)$, $c(g_{il}, g_{jm})$, and $c'(g_{ij})$ can depend on several additional parameters, e.g. a list of visited or unvisited megapolises. To put it simple, hereinafter we will skip them.

This problem has many applications is practice. Among them the well known dismountling problem for retired Nuclear Power Plants. For all of these applications, approximate solutions are not allowed. Therefore, to find an exact solution, we use the classic dynamic programming (DP) approach (see e.g. [1],[2]). Unfortunately, in general case, an exact solution can not be found efficiently unless $P = NP$ since GATSP is NP-hard [3]. In particular, running time of the DP algorithm is $\Omega(np^2 2^n)$ in the worst case. Nevertheless, taking into account some kind of additional constrains, e.g. precedence constrains defined on the set of megapolices can speed up the DP scheme significantly [4]. The results presented extend the approach introduced in [5], [6].

Consider the following precedence constrains. For a fixed natural number k, every feasible permutation P should satisfy the equation

$$\forall i, j \in \mathbb{N}_n \ (j \geq i + k) \Rightarrow (\pi(i) < \pi(j)). \tag{2}$$

Theorem 1. *For any $k \in \mathbb{N}_n$ and p, time complexity of the DP is $O(n \cdot p^2 k^2 2^{k-2})$.*

Constraints (2) can be relaxed in the following way. Suppose, to any i a parameter $k(i)$ is assigned such that

$$\forall i, j \in \mathbb{N}_n \ (j \geq i + k(i)) \Rightarrow (\pi(i) < \pi(j)). \tag{3}$$

Theorem 2. *Running time of the DP algorithm for GATSP with precedence constrains (3) is $O(p^2 \sum_{i=1}^{n} k^*(i)(k^*(i)+1)2^{k^*(i)-2})$, where $k^*(i) = \max\{k(j)\colon i - k(j) + 1 \leq j \leq i\}$.*

The authors were supported by the Russian Science Foundation (project no. 14-11-00109).

References

1. *R. Bellman.* “Dynamic programming treatment of the traveling salesman problem,“ J. Assoc. Comput. Mach., **9**, pp. 61–63. (1963).
2. *M. Held and R. Karp.* “A dynamic progrmming approach to sequencing problems,“ J. Soc. Indust. Appl. Math., **10**, No. 1., pp. 196–210. (1962).
3. *M. Garey and D. Johnson.* “Computers and Intractability: A Guide to the Theory of NP-completeness,“ W. H. Freeman and Co., San Francisco, Calif. (1979).
4. *G. Steiner.* “On the complexity of dynamic programming for sequencing problems with precedence constraints, “ Annals of Operations Research, **26**, No. 1, pp. 103–123 (1990).
5. *E. Balas.* “New classes of efficiently solvable generalized Traveling Salesman Problems,“ Annals of Operations Research, **86**, pp. 529–558, (1999).
6. *E. Balas and N. Simonetti.* “Linear time dynamic-programming algorithms for new classes of restricted TSPs: a computational study,“ INFORMS J. on Computing, **13**, No. 1, pp. 56–75 (2000).
7. *A.G. Chentsov, D.M. Khachay, and M.Y. Khachay.* “Exact linear time algorithm for some case of Generalized Traveling Salesman Problem,“ Trudy Instituta Matematiki i Mechaniki(in Russian), **21**, No.3 (2015).

Shannon's function of the lungs' clearence time

Iuliia Chernova[1]

[1] *Lomonosov Moscow State University, Moscow, Russian Federation;*
ger_julia@mail.ru

Lungs are modeled by full dichotomy tree oriented to its root called I-tree and denoted $D^{-1}(b, r, n, l)$ with the following parameters
— $l \in \mathbb{N}$ - depth of the tree;
— $n \in \mathbb{N}$ - each edge (bronchus) from D^{-1} is splitted into n equaled parts, called *lash* and indexed by values i from 1 to n;
— $b, r \in \mathbb{N}$ - two values $2^{l-j}b$ and $2^{l-j}r$ are assinged to each edge with depth j and called *capacity* and *swapping measure* of the edge's lash.

The *process of transportation* of substance is the process defined on defined I-tree $D^{-1}(b, r, n, l)$ with the set of the restrictions described in [1].

In terms of the process of transportation additional characteristics is added to the I-tree – amount V' of substance distributed over the tree. That I-tree is denoted $D^{-1}(b, r, n, l, V')$

Let function $L(b, r, n, l, V')$ is the maximum amount of time required for clearing of I-tree $D^{-1}(b, r, n, l, V')$ with a random initial distribution of substance. That function is usually called Shannon's complexity function.

Theorem 1. *Function $L(b, r, n, l, V')$ is defined by the following*
1) if $(2^l - 1)bn \le V' \le V$ than $L(b, r, n, l, V') = \left]\frac{b}{r}\right[(2nl - 1)$;
2) if $0 < V' < (2^l - 1)bn$ then
i) if $r = 1$ then
a) if $V' \le bn$ then

$$L(b, 1, n, l, V') = \begin{cases} V' + b(n-1), & \text{if } l = 1 \text{ and } n = \left]\frac{V'}{b}\right[, \\ 2V' -]\frac{V'}{b}[+ nl - 1, & \text{otherwise}; \end{cases}$$

b) if $bn < V' \le bn + (l-1)n$ then

$$L(b, 1, n, l, V') = V' + n(l + b - 1) - 1$$

c) if $V' > bn + (l-1)n$ *then*

$$L(b,r,n,l,V') = \begin{cases}]\frac{b_{h_3}}{2^{l-1}}[+2b(nl-1), & \text{if } k_3 = 1 \quad h_3 = 1, \\ 2\left(]\frac{b_{h_3}}{2^{l-k_3}}[+(b-1)(n(l-k_3+1)-h_3)+nl-\frac{3}{2}\right), & \text{otherwise}; \end{cases}$$

ii) if $r > 1$ *then*

a) if $V' \leq nl$ *then* $L(b,r,n,l,V') = V' + nl - 1$*;*

b) if $V' > nl$ *then*

$$L(b,r,n,l,V') = \begin{cases}]\frac{b_{h_3}}{2^{l-1}r}[+2]\frac{b}{r}[(nl-1), & \text{if } k_3 = 1 \quad h_3 = 1, \\ 2\left(]\frac{b_{h_3}}{2^{l-k_3}r}[+(]\frac{b}{r}[-1)(n(l-k_3+1)-h_3)+nl-\frac{3}{2}\right), & \text{otherwise}; \end{cases}$$

where

$$k_3 = 1 + [l - log_2(\frac{V'-nl}{(b-1)n} + 1)], \qquad h_3 = n -]\frac{V'-nl-(2^{l-k_3}-1)(b-1)n}{2^{l-k_3}(b-1)}[+1,$$

$$b_{h_3} = V' - nl - (2^{l-k_3} - 1)(b-1)n - 2^{l-k_3}(b-1)(n-h_3) + 1.$$

Let V is maximum possible value of V', $L_m(b,r,n,l)$ and $L(b,r,n,l)$ are middle value (1) and maximum value (2) over all $L(b,r,n,l,V')$

$$L_m(b,r,n,l) = \frac{1}{V} \cdot \sum_{V'=1}^{V} L(b,r,n,l,V') \tag{1}$$

$$L(b,r,n,l) = \max_{1 \leq V' \leq V} L(b,r,n,l,V') \tag{2}$$

Theorem 2. $L_m(b,r,n,l) \sim 2n]\frac{b}{r}[l$ *if* $l \to \infty$*.*

Theorem 3. $L_m(b,r,n,l) \sim L(b,r,n,l)$ *if* $l \to \infty$*.*

Results of the further research are included to [2], [3] and to the set of the other author's articles.

References

1. *Yu.G. Geraskina.* "One automaton model in biology," Discrete Mathematics and Applications, V. 17, No. 4, 375–394 (2007).
2. *Yu.G. Geraskina.* "On start states of an automaton model of lung in pure environment," Discrete Mathematics and Applications, V. 18, No. 5, 517–534 (2008).
3. *Yu.G. Chernova.* "Characterization of states of an automaton model of lung in pure environment," Intelligent Systems, V. 17, No. 1-4, 206–213 (2013).

Determination of the optimal parameters of reinsurance

Dmitry Denisov[1], Olga Nekrasova[2]

[1] *Lomonosov Moscow State University, Faculty of computational mathematics and cybernetics, Moscow, Russian Federation;* dvden@cmc.msu.ru

[2] *Lomonosov Moscow State University, Faculty of computational mathematics and cybernetics, Moscow, Russian Federation;* necros_0889@mail.ru

Determination of the optimal allocation of risks is an important task of the theory and practice of actuarial calculations. The optimal parameters of reinsurance depend on the situation on the insurance market, which defines the possible values for these parameters, including at the administrative level.

In the reinsurance theory there are two types of reinsurance: excess of loss and excess of loss ratio for the portfolio as a whole. In the first case, the reinsurance operation is applied to each policies of the portfolio and in the second - to the total risk. We consider the combination of these two types of reinsurance under restriction on loss ratio of portfolio. The relation between the limits of the retention for individual and total losses and problem of minimization of the reinsurance premiums were investigated in [1].

For a discrete process of changes in capital of the insurance company $U_{k+1} = U_k + c - W_k$, where c - the amount of the annual premiums, U_k - capital at the end of the year k, W_k - total loss,

$$W_k = \sum_{j=1}^{N_k} X_{jk},$$

X_{jk} - individual payment for the end of the year j, operation of individual loss reinsurance is formulated as a definition of actual payment

$$Y_{jk} = \begin{cases} X_{jk}, X_{jk} \leqslant d_X \\ d_X, X_{jk} > d_X \end{cases}, \qquad (1)$$

In this case, the operation of total loss reinsurance is defined as following

$$Z_k = \begin{cases} W_k, W_k \leqslant \min(d_W, a \cdot c_r) \\ d_W, W_k > \min(d_W, a \cdot c_r) \end{cases}, \qquad (2)$$

Where a - ultimate loss ratio for the portfolio, c_r - retained after reinsurance premium. The problem of optimization of parameters of reinsurance corresponds to the solution of the following problem

$$E[Z_k/c_r] \to \min$$

$$P(c_r - Z_k < 0) \leqslant q_k, \tag{3}$$

where q_k - the limit value of probability of a negative financial result.

The solution to this problem is investigated in dependence on the frequency of payments and the parameters of individual losses.

References

1. *H.G. Verbeek.* "On optimal reinsurance", The ASTIN Bulletin - Publication of the ASTIN Section of the Permanent Committee for International Actuarial Congresses, Vol. IV, Part I. 29–38 (1966).
2. *Jun Cai and Ken Seng Tan and Chengguo Weng and Yi Zhang.* "Optimal reinsurance under VaR and CTE risk measures", Insurance: Mathematics and Economics, 185–196, (2008).
3. *Yichun Chi and Ken Seng Tan.* "Optimal reinsurance with general premium principles", Insurance: Mathematics and Economics, vol. 52, issue 2, 180–189, (2013).

About features of differential models for a bank

Vasily Dikusar[1], Nicholas Olenev[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
`dikussar@yandex.ru, nolenev@yahoo.com`

Problems with a small parameter in the derivative refer to stiff equations [1-5]. It is difficult to formulate a precise definition of stiffness, but the main idea is that the equation includes some terms that can lead to rapid variation in the solution.

In applying explicit methods to solve stiff problems step size is limited to numerical stability rather than accuracy. Very strong stability of the differential equation is a disadvantage in terms of the accuracy of the numerical solution using the classical explicit methods [1-5].

As an example let consider the linear problem in general form:

$$\dot{y}(t) = Ay(t) + g(t), g \in R^n, A \in R^{n\times n}, y \in R^n, y(0) = y_0, \qquad (1)$$

where A – is a constant value matrix. Let $\lambda_1, \lambda_2, ..., \lambda_n$ are eigenvalues of the matrix A. The problem (1) is called stiff if

1. There are λ_i for which $Re\lambda_i \ll 0$.

2. There are λ_i such that they are small in comparison with the absolute value of the eigenvalues satisfying item 1.

3. There is no λ_i with a large positive value of real part.

4. There is no λ_i with a large imaginary part, for which the condition $Re\lambda_i \ll 0$ is not satisfied.

The stiffness of the system for nonlinear problems described in terms of the eigenvalues of the Jacobi matrix along the curve of the exact solution. The stiffness of the nonlinear problem can be completely described in terms of stiffness for a linear problem with variable coefficients

$$\dot{\Delta}(t) = M(t)\Delta(t), t \geq t_*, \Delta(t_*) = \bar{y}(t_*) - y(t_*). \qquad (2)$$

The mathematical model of a typical bank can be rewrited in the following notation [6]:

$$\dot{x}_1 = -x_1\left(\frac{u_1}{\tau}+\frac{u_2}{\Delta}\right)+x_2\left(\frac{r(t)}{q(t)}+\frac{1}{\theta(t)}+\frac{1-u_2}{\Delta}\right)-$$
$$-x_3\left(r_1(t)-\frac{1}{\eta(t)}+\frac{u_3}{\Delta_1}\right)+\frac{u_3kI_0}{\Delta_1}-p(t)C(t)+\Phi(t);$$

$$\dot{x}_2 = x_1\frac{u_2}{\Delta}+x_2\left(\frac{\dot{q}(t)}{q(t)}+\frac{1}{\theta(t)}-\frac{1-u_2}{\Delta}\right);$$
$$\dot{x}_3 = -x_3\left(\frac{1}{\eta(t)}+\frac{u_3}{\Delta}+\frac{u_3kI_0}{\Delta_1}\right), t\in[0,T]. \tag{3}$$

For the numerical solution of the problem of eigenvalues [7] distributed computing [8] and GRID-technologies are used.

The authors were supported by the Russian Foundation of Basic Research (project no. 15-07-08952).

References

1. *Shalashilin V.I., Kuznetsov E.B.* Metod prodolzheniya resheniya po parametru i nailuchshaya parametrizatsiya, M.: URSS. 1999.
2. *Rakitskiy Yu. V., Ustinov S.M., Chernorutskiy I.G.* Chislennyie metodyi resheniya zhestkih sistem, M.: Nauka, 1979.
3. *Lebedev V.I.* Funktsionalnyiy analiz i vyichislitelnaya matematika. M.: Fizmatlit. 2000.
4. *Hairer E., Wanner G.* Solving ordinary differential equations II: Stiff and differential-algebraic problems (second ed.), Berlin: Springer-Verlag, 1996.
5. *Fedorenko R.P.* Vvedenie v vyichislitelnuyu fiziku. M.: MFTI, 1994.
6. *Romanyuk D. V.* Model formirovaniya kreditno-depozitnoy politiki banka, Preprint WP/97/027: M.. TsEMI RAN. 1997.
7. *Dikusar V.V., Koshka M., Figura A.* Kachestvennyie i kolichestvennyie metodyi v zadachah optimalnogo upravleniya s fazovyimi i smeshannyimi ogranicheniyami. M.: MFTI, 2001.
8. *Dolmatova A.I., Olenev N.N.* Dynamics of social stratification modeling: parallel algorithms and programs. M.:Dorodnicyn Computing Centre, 2014.

Problem of controllability of nonstationary discrete systems

Vasily Dikusar[1], Elena Zasukhina[2], Sergey Zasukhin[3]

[1] *Computing Center RAS, Moscow, Russia;* dikussar@ccas.ru
[2] *Computing Center RAS, Moscow, Russia;* zasuhina@ccas.ru
[3] *Moscow Institute of Physics and Technology, Moscow, Russia;* svz1989@gmail.com

Problems concerning controllability, controllability criteria and methods of determining programmed control are discussed. The problem of complete controllability criterion for discrete unsteady control systems was studied for example in [1]. Complete controllability criteria for linear nonstationary discrete systems are offered.

As an example consider linear nonstationary discrete controlled system:

$$X_{k+1} = P_k X_k + Bu_k + F_k, \quad k = 0, 1, 2, \ldots, \tag{1}$$

where $X_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^T$, $P_k \in R^{n\times n}$, $F_k \in R^n$, $k = 0, 1, 2, \ldots$, $B \in R^n$, u_k, $k = 0, 1, 2, \ldots$, – control (all values are real).

D e f i n i t i o n 1. The system (1) is completely controllable on segment $[0, N]$ $(n \le N)$ if for any initial state $X(0)$ and any final state $X(N)$ there exists control $u = (u_0, u_1, \ldots, u_{N-1})^T$ leading the system (1) from the state $X(0)$ to the state $X(N)$ in N steps i.e $X_0 = X(0)$, $X_N = X(N)$. Any control solving this problem will be called as programmed control.

Consider a matrix $D(N) \in R^{n\times N}$:

$$D(N) = \left\| \prod_{i=1}^{N-1} P_i B,\, \prod_{i=2}^{N-1} P_i B,\, \ldots,\, P_{N-1}B,\, B \right\|.$$

The following theorem is valid.

Theorem 1. *In order that the system (1) is completely controllable it is necessary and sufficient that the matrix $A(N) = D(N)D^T(N)$ is positive definite. And the programmed control is determined by relations:*

$$U = D^T(N)C + V, \tag{2}$$

$$C = A^{-1}(N)(X(N) - \prod_{i=0}^{N} P_i X(0) - \sum_{k=1}^{N-1} \prod_{i=r}^{N-1} P_i F_{k-1} - F_{N-1},$$

$$D(N)\,V = 0, \quad V = (v_0, v_1, \ldots, v_{N-1})^T. \tag{4}$$

The proof of the theorem 1 is based on the idea that the matrix $A(N)$ is positively definite if and only if the rows of the matrix $D(N)$ are linearly independent [2].

Remark 1. If the system (1) in completely controllable on the segment $[0, N]$ then it is completely controllable on the segment $[0, M]$, $M > N$.

Remark 2. It is not difficult to see that in stationary case $P_i = A$, $i = 0, 1, 2, \ldots$, the theorem 1 is entirely equivalent to Kalman criterion [3].

Remark 3. It is clear that programmed control is not determined uniquely. Therefore we can select programmed control which will be optimal in some sense. Then a problem of optimal control reduces to a problem of constrained optimization:

$$\Phi(X(0), X(N), F_0, F_1, \ldots, F_{N-1}, V) \to \min, \quad D(N)\,V = 0.$$

All these results can be extended to following cases:

- on each step control is a vector i.e.

  $$B \in R^{n\times m}, u_k = (u_{k1}, u_{k2}, \ldots, u_{km})^T, k = 0, 1, 2, \ldots;$$

- the system of control is not stationary i.e. the system (1) becomes

  $$X_{k+1} = P_k X_k + B_k u_k + F_k, \quad k = 0, 1, 2, \ldots,$$

  where $B_k \in R^{n\times m}$, $u_k = (u_{k1}, u_{k2}, \ldots, u_{km})^T$, $k = 0, 1, 2, \ldots$.

References

1. *V. V. Dikusar, N. V. Zubov, M. V. Krylova.* "Complete controllability criterion for nonstationary discrete control systems," Trudy ISA RAS, Dynamics of heterogeneous systems, **42**, No. 2, 35-39 (2009).
2. *F.R. Gantmakher* The theory of matrices, Nauka, Moscow (1967).
3. *R. Kalman, P. Falb, M. Arbib.* Topics in mathematical system theory, Mir, Moscow (1971).

Conditions of optimal stabilization of nonlinear dynamic models of technical systems

Olga Druzhinina[1], **Natalia Petrova**[2], **Elena Shchennikova**[3]

[1,2] *Dorodnicyn Computing Center of RAS, Russia;* ovdruzh@mail.ru
[3] *Mordovian State University by name of N.P.Ogarev, Russia;* schennikova.e@yandex.ru

The search of conditions of optimal stabilization and construction of corresponding algorithms is actual problem in research of behavior of nonlinear controlled systems [1–4]. Fundamental approach to optimal stabilization for systems of ordinary differential equations was developed by V.V. Rumyantsev [1] with use of condition of minimization for a functional characterizing the quality of control.

In the present work we consider two types of dynamical models of technical controlled systems: the model technical manipulator and the model of functioning of cascade power system. Indicated models are described by nonlinear multiply connected systems of ordinary differential equations. The conditions of optimal stabilization to respect to all phase variables and to respect to a part phase variables are obtained. For research of models method of Lyapunov functions and method of multi-level stabilization are used. We consider also the continuously-discrete systems with construction of piesewisely continuous control. The results of present work continue researches [4–8] and can find application in problems of analysis of controlled technical systems.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-08-00710).

References

1. *V.V. Rumyantsev.* "On optimal stabilization of controlled systems", Appl. Math. Mech., **34**, No. 3, 440–456 (1970).
2. *V.V. Rumyantsev, A.S. Oziraner.* Stability and Stabilization of Motion with Respect to Part of Variables, Nauka, Moscow (1987).
3. *A.S. Andreev, S.P. Bezglasny.* "On stabilization of controlled systems with guaranteed estimate of control quality", Appl. Math. Mech., **61**, No. 1, 44–51 (1997).

4. *O.V. Druzhinina, A.A. Shestakov.* "Generalized direct Lyapunov method for research of stability and attraction in general time systems", Sbornik Mathematics, **193**, No. 10, 17–48 (2002).
5. *O.V. Druzhinina, N.P. Petrova.* "Optimal stabilization and estimation of linearization errors for nonlinear dynamic models of technical manipulators", Proc. of V International conference on optimization methods and applications "Optimization and applications" held in Petrovac, Montenegro, 2014. Dorodnicyn Computing Center of RAS, Moscow, 2014, pp. 57–58.
6. *O.V. Druzhinina, E.V. Shchennikova.* "Stabilization of multiply connected continuously-discrete system on the basis of synthesis of piesewisely continuous control", Science Intensive Technologies, **16**, No. 1, 3–9 (2015).
7. *O.V. Druzhinina, O.N. Masina, E.V. Shchennikova.* "Optimal stabilization of programmed motion of manipulation systems", Dynamics of complex systems, **5**, No. 3, 58–64 (2011).
8. *E.V. Shchennikova.* Stabilitylike Properties of Nonlinear Controlled Systems, Russian University of Peoples Friendship, Moscow, (2001).

Modern approaches for numerical methods for finding stochastic equilibrium in Beckman equilibrium transport flows model

Pavel Dvurechensky[1], Alexander Gasnikov[2], Evgenia Gasnikova[3]

[1] *Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany; Institute for Information Transmission Problems RAS, Moscow, Russia*
pavel.dvurechensky@wias-berlin.de

[2] *Moscow Institute of Physics and Technology, Dolgoprudny, Russia;*
gasnikov@yandex.ru

[3] *Moscow Institute of Physics and Technology, Dolgoprudny, Russia;*
egasnikova@yandex.ru

We consider the problem of finding the stochastic equilibrium in Beckmann [1] model for congested traffic flows. Let $\Gamma = (V, E)$ be the graph of transport network, where V is the set of vertices and $E \in V \times V$ is the set of edges. We denote $n = |V| \cong 10^4$, $m = |E| \cong 4|V|$. Also we assume that the set of all pairs origin-destination OD satisfies $n \ll |OD| \ll n^2$. Let P be the set of all paths in Γ. To define B-model we also need x_p – the flow on the path p, $f_e(x) = \sum_{p \in P} \delta_{ep} x_p$ – the flow on the edge e, where δ_{ep} equals 1 if $e \in p$ and 0 otherwise. Let $\tau_e(f_e(x))$ be the cost for choosing the edge e. Let $G_p(x) = \sum_{e \in E} \tau_e(f_e(x))\delta_{ep}$ be the cost for the path p. Also let $d_w, w \in OD$ be the demand corresponding to the pair of origin and destination w. Then the feasible set of flow distribution is given by $X = \{x \geq 0 : \sum_{p \in P_w} x_p = d_w, w \in OD\}$, where P_w is the set of all paths corresponding to w. The problem of finding the stochastic equilibrium in Beckmann model is equivalent to the solution of the problem

$$\min_{f = \Theta x, x \in X} \left\{ \Phi(f(x)) = \frac{1}{|E|} \sum_{e \in E} \sigma_e(f_e(x)) + \gamma \sum_{w \in OD} \sum_{p \in P_w} x_p \ln(x_p/d_w) \right\},$$

where $\sigma_e(f_e(x)) = \int_0^{f_e(x)} \tau_e(z)dz$, $\partial\Phi(f(x))/\partial x_p = G_p(x), p \in P$, $\Theta = \|\delta_{ep}\|_{e \in E, p \in P}$, $\gamma > 0$.

We compare different modern approaches (semi-stochastic gradient descent (e.g. [2]), randomized dual coordinate ascent (e.g. [3]), APPROX, ALPHA [4] etc.) to the special sum-type convex optimization problem with entropy regularization. Close problems arise in Machine Learning, but in this paper we restrict ourselves mainly by traffic flow applications. The novelty of our approach is the following: we consider conditional optimization problems, our gradient step can't be done explicitly (so we have to use the concept of inexact oracle). It seems that in this short paper we first time explain the nature of modern sum-type convex optimization methods in perspective of comparison this methods with each other and we try to explain them from unified point of view.

Details can be found at http://arxiv.org/abs/1505.07492.

The research was partially supported by RFBR, research project No. 13-01-12007-ofi_m and 15-31-20571- mol_a_ved.

References

1. *A. Gasnikov et al.* "Intriduction to mathematical modelling of transport flows," MCCME, 2013.
2. *J.Konecny, P. Richtrik* "Semi-stochastic gradient descent methods", e-print, 2013, http://www.maths.ed.ac.uk/ richtarik/
3. *Q. Zheng, P. Richtrik, T.Zhang* "Randomized dual coordinate ascent with arbitrary sampling", e-print, 2015. arXiv:1411.5873
4. *Z. Qu, P. Richtarik* "Coordinate Descent with Arbitrary Sampling I: Algorithms and Complexity", e-print, 2014. arXiv:1412.8060

Optimization methods for finding equilibrium in Beckman and stable dynamics models

Pavel Dvurechensky[1], **Alexander Gasnikov**[2], **Marina Kaznacheeva**[3]

[1] *Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany; Institute for Information Transmission Problems RAS, Moscow, Russia*
pavel.dvurechensky@wias-berlin.de
[2] *Moscow Institute of Physics and Technology, Dolgoprudny, Russia;*
gasnikov@yandex.ru
[3] *Moscow Institute of Physics and Technology, Dolgoprudny, Russia;*
marina_dec@mail.ru

We consider the problem of finding an equilibrium in Beckmann (B) [1] and stable dynamics (SD) models [2]. These two models play an important role in congested traffic flow modeling. Also they are important for optimization of railroad freight. In a recent work [3] the authors consider these two models and show that stable dynamics model can be obtained from the B-model with a particular cost functions on the links as a limit when some parameter tends to 0. In some applications one need to mix both models, so that some edges have B-type cost function and some have SD-type cost function. In this work we extend [3] and develop efficient methods for calculating the equilibrium in Beckmann model and in the mix of B- and SD-model.

Let $\Gamma = (V, E)$ be the graph of transport network, where V is the set of vertices and $E \in V \times V$ is the set of edges. We denote $n = |V| \cong 10^4$, $m = |E| \cong 4|V|$. Also we assume that the set of all pairs origin-destination OD satisfies $n \ll |OD| \ll n^2$. Let P be the set of all paths in Γ. To define B-model we also need x_p – the flow on the path p, $f_e(x) = \sum_{p \in P} \delta_{ep} x_p$ – the flow on the edge e, where δ_{ep} equals 1 if $e \in p$ and 0 otherwise. Let $\tau_e(f_e(x))$ be the cost for choosing the edge e. Let $G_p(x) = \sum_{e \in E} \tau_e(f_e(x)) \delta_{ep}$ be the cost for the path p. Also let $d_w, w \in OD$ be the demand corresponding to the pair of origin and destination w. Then the feasible set of flow distribution is given by $X = \{x \geq 0 : \sum_{p \in P_w} x_p = d_w, w \in OD\}$, where P_w is the set of all paths corresponding to w. The problem of finding the equilibrium flow distribution is equivalent to the solution of the problem $\min \left\{ \Phi(f(x)) = \sum_{e \in E} \sigma_e(f_e(x)) : f = \Theta x, x \in X \right\}$, where $\sigma_e(f_e(x)) = \int_0^{f_e(x)} \tau_e(z) dz$, $\partial \Phi(f(x)) / \partial x_p = G_p(x), p \in P$, $\Theta = \|\delta_{ep}\|_{e \in E, p \in P}$.

We propose to solve this problem using Conditional Gradient Descent (Frank-Wolfe, FW) (see e.g. [4]). Every step needs calculation of $\nabla \Phi(f(x))$ which can be done by Dijkstra algorithm. In the work [3] authors consider cost functions parametrized by parameter μ, e.g. $\tau_e(f_e) = \bar{t_e} \left(1 - \mu_e \ln(1 - f_e/\bar{f_e})\right)$. Then the SD-model can be obtained from B-model as a limit when $\mu_e \to +0 \forall e \in E$. For some applications such limit needs to be taken not for all edges but only for a subset of them. Also important is the situation when for some edges μ_e is taken very small. In such situations the function $\Phi(f(x))$ become non-smooth or starts to have large Lipschitz constant of the gradient and FW-algorithm becomes inapplicable.

We propose for the described case to construct a dual problem for the considered one. We obtain a non-smooth problem on a simple set. To calculate its subgradient we need to calculate shortest paths (in terms of cost $\tau_e(f_e)$. We apply primal-dual method from [5]. Also we use the fact that the goal function in the dual problem is the sum of large number of similar functions and propose a randomized method for solving it. The main advantage that calculating the subgradient of one random function in the sum is much cheaper than calculating the full subgradient. We provide complexity analysis in terms of arithmetic operations for both proposed method.

The research was partially supported by RFBR, research project No. 13-01-12007-ofi_m and 15-31-20571-mol_a_ved.

References

1. *M. Beckmann, C.B. McGuire, C.B. Winsten.* "Studies in the economics of transportation," RM-1488. Santa Monica: RAND Corporation, 1955.
2. *Yu. Nesterov, A. de Palma.* "Stationary Dynamic Solutions in Congested Transportation Net-works: Summary and Perspectives," Networks Spatial Econ, **3**, No.3, 371–395 (2003)
3. *A. Gasnikov, Yu. Dorn, Yu. Nesterov, S. Shpirko.* "On three-stage stable dynamics model for road traffic flows," Math Mod., **26**, No.6, 34–70 (2014)
4. *M. Jaggi.* "Revisiting Frank-Wolfe: Projection-Free Sparse Convex Optimization," 2013.
5. *Yu. Nesterov* "Primal-dual subgradient methods for convex problems," Mathematical Programming., **120**, No. 1, 221–259 (2009).

Symmetries of Affine Control Systems

V.I. Elkin[1]

[1] *Institution of Russian Academy of Sciences*
Dorodnicyn Computing Centre of RAS , Moscow, Russia; `elk_v@mail.ru`

Affine control systems are nonlinear systems that are linear in controls; that, is, these are the systems of the form

$$\dot{y} = f_0(y) + f(y)u, \quad y \in M \subset R^n, \quad u \in R^r. \tag{1}$$

Here, y are the phase variables; u are the controls; and M is the phase space that is a domain. We assume that f_0 is a smooth vector field; f is an n-by-r matrix in which the columns f_α, $\alpha = 1, 2, \ldots, r$, are smooth vector fields; and $\mathrm{rank} f(y) = \mathrm{const}$. A solution (or phase trajectory) of system (1) is defined as a continuous piecewise smooth function $y(t)$ for which there exists a piecewise continuous control $u(t)$ such that $y(t)$ and $u(t)$ satisfy (1).

A diffeomorphism $\psi\colon M \to M$ is a symmetry of system (1) whenever $y(t)$ is a solution of system (1), $y'(t) = \psi(y(t))$ is also a solution of system (1). Knowledge of symmetries is helpful in finding new solutions of control systems from already known solutions. Moreover, symmetries play an important role in decomposition of control systems. For example, some group of symmetries determines reduction of system (1) by substitution of variables to the system

$$\dot{z}_1 = v_1, \tag{2}$$

$$\dot{z}_2 = h_0(z_2) + h(z_2)v_2, \tag{3}$$

where z_1, z_2 — new phase variables, v_1, v_2 — new controls. The decomposition (2) (3) separates the trivial part (2). This is helpful in decomposing any control problem related to system (1) into two problems—a trivial problem related to trivial system (2) and, in general, a nontrivial problem related to system (3).

The author was supported by the Russian Foundation for Basic Research (project no. 13-01-00866).

On Travelling Salesman Problem with Vertex Requisitions

Anton Eremeev[1], **Julia Kovalenko**[2]

[1] *Sobolev Institute of Mathematics, Laboratory of Discrete Optimization, Omsk, Russia;* eremeev@ofim.oscsbras.ru
[2] *Omsk Law Academy, Omsk, Russia;* julia.kovalenko.ya@yandex.ru

We consider the Travelling Salesman Problem (TSP) with Vertex Requisitions [2]: given a complete digraph $G = (X, U)$, where $X = \{x_1, \dots, x_n\}$ is the set of vertices, $U = \{(x, y) : \; x, y \in X, x \neq y\}$ is the set of arcs with non-negative weights $\rho(x, y)$, $(x, y) \in U$. Besides that, a family of vertex subsets (requisitions) $X^i \subseteq X$, $i = 1, \dots, n$, is given, such that $1 \leq |X^i| \leq 2$ for all $i = 1, \dots, n$.

Let F denote the set of the bijections from $X_n = \{1, \dots, n\}$ to X that satisfy the conditions $f(i) \in X^i$, $i = 1, \dots, n$, for all $f \in F$. The problem asks for a mapping $f^* \in F$, such that $\rho(f^*) = \min\limits_{f \in F} \rho(f)$, where $\rho(f) = \sum\limits_{i=1}^{n-1} \rho(f(i), f(i+1)) + \rho(f(n), f(1))$ for $f \in F$.

The TSP with Vertex Requisitions is strongly NP-hard [2], and this problem does not admit a fully polynomial time approximation scheme unless P=NP. In [2], A.I. Serdyukov developed exact algorithm for solving the problem, based on enumeration of all perfect matchings in a special bipartite graph.

Let us construct a bipartite graph $\bar{G} = (X_n, X, \bar{U})$ where the subsets of vertices of bipartition X_n, X have equal size and the set of edges is $\bar{U} = \{(i, x) : i \in X_n, \; x \in X^i\}$. There is a one-to-one correspondence between the set F of feasible solutions to TSP with Vertex Requisitions and the set of perfect matchings in graph $\bar{G}$. An edge $(i, x) \in \bar{U}$ is called *special*, if (i, x) belongs to all perfect matchings in graph $\bar{G}$. A maximal (by inclusion) bi-connected subgraph with at least two edges is called a *block*. Every perfect matching in graph $\bar{G}$ is defined by a combination of maximal matchings chosen in each block (one matching per block) and the set of all special edges [2].

It was proved in [2] that the set of feasible solutions in almost all instances of the TSP with Vertex Requisitions has at most n elements and these instances are solvable in $O(n^2)$ time.

In this work we propose a modification of algorithm [2] using the approach form [1]. The modification speeds up the evaluation of objective function during the process of perfect matching enumeration. Such modification realizes some preliminary computations of objective function for block contacts [1]. As a result it is shown that almost all instances of the TSP with Vertex Requisitions are solvable in $O(n\ln(n))$ time.

Using a connection with perfect matchings in bipartite graph $\bar{G}$ and preliminary computations of objective function the following results are obtained. A mixed integer linear programming model is formulated for the TSP with Vertex Requisitions. The model consists of $O(n)$ Boolean variables, $O(n^2)$ real variables and constraints. A method of neighbourhood construction for the local search algorithm of the considered problem is proposed. A neighbourhood of the solution to the TSP with Vertex Requisitions is defined through neighbourhood of the corresponding perfect matching.

The authors are supported by the RSF Grant (project no. 15-11-10009).

References

1. *A.V. Eremeev, J.V. Kovalenko.* "On complexity of optimal recombination for one scheduling problem with setup times," Diskretnyi analys i issledovanie operacii, **19**, No. 3, 13-26 (2012). (In Russian)
2. *A.I. Serdyukov.* "On travelling salesman problem with prohibitions," Upravlaemye systemi, **17**, 80-86 (1978). (In Russian)

VNS for min-power symmetric connectivity problem

Adil Erzin[1], **Nenad Mladenovic**[2], **Roman Plotnikov**[3]

[1] *Sobolev Institute of Mathematics, Novosibirsk, Russia;*
adilerzin@math.nsc.ru
[2] *University of Valenciennes and Hainaut-Cambresis, Famars, France*
nenadmladenovic12@gmail.com
[3] *Novosibirsk State University, Novosibirsk, Russia;* nomad87@ngs.ru

Elements of many communication networks use wireless communication for data exchange. Herewith energy consumption of a network's element is proportional to d^s, where $s \geq 2$, and d is a transmission range [1, 3]. In some networks, e.g., in wireless sensor networks, each element (sensor) has a limited energy, and its efficient use permits to extend the lifetime of a whole network [2, 3]. For the rational energy usage, modern sensor can adjust its transmission range. Then the problem under consideration is to find a transmission range for each element that supports a strongly connected subgraph in order to minimize the energy consumption. The problem can be formulated in the following way.

Given a simple undirected weighted graph $G = (V, E)$ with a vertex set V, $|V| = n$, and an edge set E, find a spanning tree T^* of G which is the solution to the following problem:

$$W(T) = \sum_{i \in V} \max_{j \in V_i(T)} c_{ij} \to \min_{T}, \tag{1}$$

where $V_i(T)$ is the set of vertices adjacent to a vertex i in the tree T, and $c_{ij} \geq 0$ be the weight of the edge $(i, j) \in E$.

Any feasible solution of (1), i.e., a spanning tree in G, is called a *communication tree* (subgraph). It is known that (1) is strongly NP-hard [1] and if N $\neq$ NP, then the problem is inapproximable within the ratio $1 + \frac{1}{260}$ [3].

In [1] it is shown that a minimal spanning tree is a 2-approximation solution to the problem (1). In [3] a more precise ratio estimate for the minimal spanning tree is reported. In [3] a set of heuristic algorithms is proposed and their a posteriori analysis is performed. Since we were not completely satisfied with the results obtained, in [3] we proposed a new hybrid heuristics that combines genetic algorithm with the variable neighborhood search [4]. There two new local search heuristics for the problem (called LI and VND) are proposed. They are then used as a mutation operator within the genetic algorithm (GA). The computational results show the high efficiency of the proposed hybrid heuristic.

In this paper we propose the different heuristics based on the variable neighborhood search (VNS) [4]. Contribution of this paper may be summarized as follows.

- New local search that is based on "elementary tree transformation" [4] is proposed. In terms of solution quality it significantly outperforms the previous one (named as LI), but uses more computation time.
- Several Basic VNS and General VNS based heuristics are proposed and tested. Some of those new heuristics give results of better quality than recent state-of-the-art (hybrid heuristic [2]), especially for solving more realistic large size problems.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-07-00139) and the Ministry of Education and Science of the Republic of Kazakhstan (project no. 0115PK00550).

References

1. *E. Althaus, et al.* "Power efficient range assignment for symmetric connectivity in static ad hoc wireless networks", Wireless Networks, **12**, No. 3, 287–299 (2006).
2. *A. Erzin, R. Plotnikov.* "Using VNS for the optimal synthesis of the communication tree in wireless sensor networks", Electronic Notes in Discrete Mathematics, No. 47, 21–28 (2015).
3. *A. Erzin, R. Plotnikov, Y. Shamardin.* "On some polynomially solvable cases and approximate algorithms in the optimal communication tree construction problem", Journal of Applied and Industrial Mathematics, No. 7, 142–152 (2013).
4. *P. Hansen, N. Mladenovic.* "Variable neighborhood search: Principles and applications", European Journal of Operational Research, No. 130, 449–467 (2001).

On inverse linear programming problems

Yuri G. Evtushenko[1], **Alexander I. Golikov**[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
`evt@ccas.ru, gol-a@yandex.ru`

A method for solving the inverse linear programming problem is proposed. For a given linear programming (LP) problem we adjust the cost coefficients as less as possible (under L_p measure) so that a known feasible solution becomes the optimal one. The first inverse LP problem within this formulation under L_1 and L_∞ measure was considered in [1,2]. The inverse LP problem under the L_1 as well as L_∞ norm is also a linear programming problem. Here we consider the Euclidean vector norm L_2 [3]. In such a case the inverse LP problem is reduced to unconstrained minimization of convex piecewise quadratic function. The Generalized Newton method can be used for unconstrained minimization this function and it converges globally in a finite number of steps.

The authors were supported by the Russian Foundation for Basic Research (project no. 14-07-00805) and by the Leading Scientific Schools Grant no. 4640.2014.1

References

1. *J. Zhang Z. Liu* "Calculating some inverse linear programming problems," J. of Comp. and Appl. Mathematics, vol. 72. pp. 261–273. (1996).
2. *R.K. Ahuja, J.B. Orlin* "Inverse Optimization," Operations Research, vol. 49. pp. 771-783. (2001).
3. *G.A. Amirhanova, A.I. Golikov, Yu.G. Evtushenko* "On inverse linear programming problem," Trudy Instituta Matematiki i Mekhaniki UrO RAN, no. 3. (2015).

Fast Automatic Differentiation Technique: Investigation and Applications

Yury Evtushenko[1], **Vladimir Zubov**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* evt@ccas.ru
[2] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* zubov@ccas.ru

A new and unified methodology for computing first order derivatives of functions obtained in complex multistep processes is developed on the basis of general expressions for differentiating a composite function. From these results, we derive the formulas for fast automatic differentiation (FAD) of elementary functions, for gradients arising in optimal control problems, nonlinear programming and gradients arising in discretizations of processes governed by partial differential equations. In the proposed approach we start with a chosen discretization scheme for the state equation and derive the exact gradient expression. Thus a unique discretization scheme is automatically generated for the adjoint equation. For optimal control problems, the proposed computational formulas correspond to the integration of the adjoint system of equations that appears in Pontryagin's maximum principle. This technique appears to be very efficient, universal, and applicable to a wide variety of distributed controlled dynamic systems and to sensitivity analysis.

The application of the FAD-technique to complex optimal control problems is discussed. The examples of solved problems with the help of FAD-technique are presented.

This work was supported by the Russian Science Foundation, project no. 14-11-00782.

Using Simulation to Study Performance of Parallel Tree Search Schedulers

Andrey Fomin[1], **Yuri Orlov**[1], **Mikhail Posypkin**[2,3], **Izrael Sigal**[3]

[1] *Moscow State University, Moscow, Russia;*
[2] *Insitute for Information Transmission Problems of RAS, Moscow, Russia;*
[3] *Computing Centre of RAS, Moscow, Russia;*

Parallel computing is one of the most common ways to cope with high computational complexity of deterministic global optimization. Many optimization methods relies on *branch-and-bound (B&B)* scheme when the initial problem is partitioned into smaller sub-problems forming so-called *B&B tree*. Since the structure of the tree is not known in advance the static distribution is usually not efficient. To overcome this problem parallel B&B solvers use dynamic load balancing to distribute the computational load among processors. Though experimental evaluation on real problems and parallel platforms is an indispensable tool for performance study it can be resources consuming for large scale HPC systems.

In this paper we propose an alternative approach based on simulation. The developed tool simulates both a parallel system and a tree search process. This approach allows to easily run many virtual experiments on thousands of processors and branch-and-bound trees of various sizes. In BNB-Solver [1] library used for our experiments schedulers interact via a strictly defined interface with a solver and a parallel platform. The simulator transparently substitutes the real parallel system and the real solver. Thus we can conveniently evaluate the performance of scheduling algorithms incorporated to the BNB-Solver library. Besides the simulator we also developed a graphical front-end that visualizes the processors load and communication among processors.

Supported by Ministry of Science and Education of Republic of Kazakhstan, project 0115PK00554 and by the Russian Foundation for Basic Research (project no. 14-07-00805)

References

1. *Y. Evtushenko , M. Posypkin , I. Sigal* “A framework for parallel large-scale global optimization” //Computer Science-Research and Development. Vol. 23. No. 3-4. 211–215 (2009).

Piecewise quadratic functions in linear optimization problems

Bogdan V. Ganin[1]

[1] *MIPT, CCAS, Moscow, Russia;* ganin@phystech.edu

Let the primal LP problem be written in the following form:

$$f_* = \min_{x, x_0 \in Z} \; x_0, \tag{P'}$$

$$Z = \{x \in R^n, x_0 \in R^1 \;:\; Ax = b, \; -c^\top x + x_0 = 0, \; x \geq 0_n\}.$$

Let $\hat{z}$ be a fixed arbitrary vector in R^{n+1}, $\hat{z}^\top = [\hat{x}^\top, \hat{x}_0]$. Then the problem (P') reduces to the following unconstrained maximization problem:

$$I' = \max_{p \in R^m, p_0 \in R^1} \; S(\overline{p}, \beta', \hat{z}), \quad \overline{p}^\top = [p^\top, p_0], \tag{1}$$

where β' is a fixed scalar, and the function S is determined by:

$$S(\overline{p}, \beta', \hat{z}) = b^\top p - \frac{1}{2} \|(\hat{x} + A^\top p - \; p_0 c)_+\|^2 - \frac{1}{2} \; (\hat{x}_0 + \; p_0 - \beta')^2. \tag{3}$$

There exists β'_* such that for all $\beta' \geq \beta'_*$ the solution $p(\beta')$, $p_0(\beta')$ to problem (1) define the projection of the point $\hat{z}$ on the solution set of the problem (P') by formulas:

$$\begin{bmatrix} \hat{x}^* \\ \hat{x}_0^* \end{bmatrix} = \begin{bmatrix} (\hat{x} + A^\top p(\beta') - \; p_0(\beta') c)_+ \\ \hat{x}_0 + \; p_0(\beta') - \beta' \end{bmatrix}. \tag{4}$$

It turns out that a threshold value of the parameter β' is smaller than in method from [1] for LP problems, where the optimum value of the objective function is a strictly positive.

Also, it is possible to identify a class of problems where we can take $\hat{x}_0 = \beta'$ in (4) and the function S becomes independent of the parameter:

$$S(\overline{p}) = b^\top p - \frac{1}{2} \|(\hat{x} + A^\top p - \; p_0 c)_+\|^2 - \frac{1}{2} \; p_0^2. \tag{5}$$

The method above based on the unconstrained minimization of convex piecewise quadratic function can be used to find the sparsest solution of a large underdetermined system of linear equations. In [2] it was suggested that such a problem can be effectively solved by minimizing the l_1-norm of the vector being the solution of this system of equations:

$$f_* = \min_{x \in X} \|x\|_1, \quad X = \{x \in R^n \ : \ Ax = b\}. \tag{P_1}$$

Using change of variables $x = x_+ - x_-$, where $x_+, x_- \geq 0_n$, the problem (P_1) can be written in the form of the standard LP problem:

$$f_* = \min_{x \in X} x_+ + x_-, \quad X = \{x \in R^n \ : \ A(x_+ - x_-) = b, \ x_+, x_- \geq 0_n\},$$

which reduces to the unconstrained maximization:

$$\max_{p \in R^m} b^\top p - \frac{1}{2}\|(A^\top p - \beta)_+\|^2 - \frac{1}{2}\|(-A^\top p - \beta)_+\|^2, \tag{6}$$

where β is a parameter.

In this work it was estimated the density of the solution found using method above. The generalized Newton method with Armijo's rule for step size was used for solving unconstrained maximization problem (6).

References

1. *A. I. Golikov,; Yu. G. Evtushenko,* "Finding the Projection of a Given Point on the Set of Solutions of a Linear Programming Problem," Proceedings of the Steklov Institute of Mathematics, No. 263, S68–S83, (2008).
2. *D. L. Donoho.* "For most large underdetermined systems of linear equations, the minimal l_1 solution is also the sparsest solution," Comm. Pure Appl. Math., No. 59(7), 907–934, July, (2006).

The library of unimodal optimization algorithms

Alexander Gornov

Institute for System Dynamics and Control Theory SB RAS, Irkutsk, Russia;
gornov@icc.ru

Over the past decades, the theory of finite-dimensional optimization has accumulated a huge store of approaches and methods for a local extremum search. The basis of modern optimization software traditionally consists of a set of algorithms that allows to implement a good working multimethod computational schemes. Software libraries of algorithms implemented in common standards can also simplify the creation of necessary service and computing process management tools, such as interactive subsystems, calculations scheduler, verification and test units. In our opinion, the relevance of this task of selection and implementation of such algorithms collection is defined by the huge-scale problems difficulty and the importance of the unimodal, optimization problem which is widely used as an auxiliary in the algorithms of global optimization.

The report considers a library of algorithms of local optimization, focused on a class of unimodal functions. Our goal was to implement in a single software system as much algorithms as possible. All the algorithms are local, some of them are well known and bases on strict mathematical theory, and some implement new heuristic approaches. Currently, the library includes about 50 methods implementation, and we the work can not be considered complete. In particular, it includes algorithms modifications requiring quadratic memory. They are regularized Newton method, quasi-Newton BFGS and DFP algorithm, non-gradient Powell-Brent method, the method of ellipsoids and other. Also there are some more efficient methods: conjugate gradient (23 versions), the L-BFGS method, Polyaks method, barzilai-borwein method, several versions for the the gravity centers method, several methods proposed by Yu.E.Nesterov.

The report shows results of computational experiments for several classes of problems: problems of small (up to 10 000 variables), medium (between 10 000 and 100 000 variables), large (over one million variables) dimension and degenerate or close to them problems.

The work was partly supported by the Russian Foundation for Basic Research (project no. 13-01-00470 and no. 15-07-03827).

Iterative solver for stiff hyperelastic deformation and springback problems

V.A. Garanzha[1], **L.N. Kudryavtseva**[2]

Dorodnicyn Computing Center Russian Academy of Sciences, Moscow
Institute of Physics and Technology, Moscow, Russia;
[1]`garan@ccas.ru`, [2]`arnir@rambler.ru`

Many problems in computational geometry can be solved via construction of spatial homeomorphisms with controlled properties. 2d and 3d manifold flattening and parameterization, deformation and morphing are widely used in computer graphics, computational biology and anatomy, molecular biology, pattern recognition, GIS, geology and stratigraphy, mesh generation, shape optimization, reverse engineering and other fields.

Viable solution to the above problems is provided by hyperelastic deformations of specially devised polyconvex materials. Existence theory for polyconvex stored energy functionals developed by John Ball assumes: (a) the set of admissible mappings is not empty; (b) minimizing sequence is constructed by the so-called direct method. From the computational point of view above statements are translated into: (a) mesh untangling problem (construction of admissible deformation) and, (b) mesh optimization problem.

Mesh untangling is hard and inherently nonlinear problem. In 1997 S.A. Ivanenko have shown that untangling problem can be solved using only finite number of steps where nonlinear variational problem is solved exactly. We show that this result can be extended to the case of inexact solution of variational problem.

Untangling and construction of optimal discrete deformations for real-life problems can become extremely stiff problem and requires powerful preconditioning. We present new nonlinear preconditioning strategy which combines advantages of simple iterative scheme due to (Charakhchyan, Ivanenko, 1988) and implicit solver due to (Garanzha, Kaporin, 1999) and is applicable to very stiff deformation problems. New adaptive untangling procedure based on dynamic extraction of subdomains with tangled mesh allows to reduce computational cost for untangling.

Thick near-wall prismatic layers and large offsets can be constructed using springback procedure when thin highly compressed layer of hyperelastic material is attached to fixed surface and is allowed to expand. This procedure allows to construct one-cell-wide layers and offsets with thickness comparable to the characteristic size of the body. Resulting mesh layer does not contain inverted cells. Self-intersection zones can be easily eliminated by cutting off excessive thickness. This procedure generally does not lead to offset thickness reduction due to local dents or elevations which is quite different from the advancing front collision detection technique. When small surface elements are present discrete variational problem can become quite stiff due to very large target height-to-base ratio of expanding elasting cells. Problem with convergence of iterative procedure is manifested as final thickness falling short of target values. We show how new preconditioning strategy performs on the variational springback problem and illustrate its behaviour on hard real-life test cases. Fig.1 illustrates hard untangling test when large rigid cube is rotated inside elastic one.

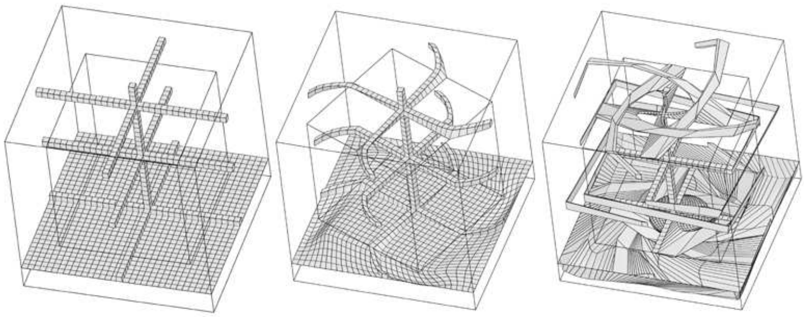

Fig. 1. Elastic deformation of mesh cell due to rotation of rigid interior cube by angle $\pi/8$ and π, respectively.

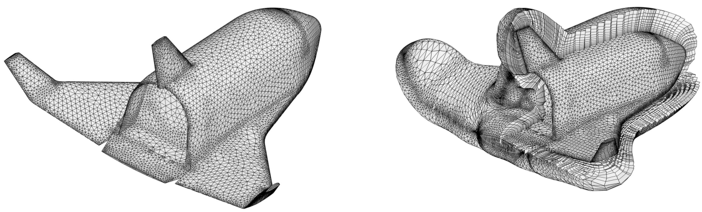

Fig. 2. Surface mesh for TSAGI SRV and prismatic layer.

This work is supported by the Russian government under grant "Measures to Attract Leading Scientists to Russian Educational Institutions" (contract No. 11.G34.31.0072) and RFBR grant 14-07-00805.

Equilibriums in multistage transport problems

Alexander Gasnikov[1], **Pavel Dvurechensky**[2], **Petro Stetsyuk**[3], **Alexandra Suvorikova**[4], **Alexey Chernov**[5]

[1] *MIPT, Dolgoprudny, Russian Federation;* gasnikov@yandex.ru
[2] *WIAS, Berlin, Germany; IITP RAS, Moscow, Russia;* pavel.dvurechensky@wias-berlin.de
[3] *Glushkov Institute of Cybernetics NAS Ukraine, Kiev, Ukraine;* stetsyukp@gmail.com
[4] *IRTG 1792, Berlin, Germany; IITP RAS, Moscow, Russia;* a.suvorikova@gmail.com
[5] *MIPT, Dolgoprudny, Russian Federation;* alexmipt@mail.ru

Search of equilibrium in multistage models of transport flows results in the solving of saddle-point problem (1) with the convex-concave structure:

$$\min_{\substack{\sum_{j=1}^n x_{ij} = L_i, \\ \sum_{i=1}^n x_{ij} = W_j, \\ i,\, j = 1,\dots,n}} \max_{y \in Q} \left\{ \sum_{i,j=1}^n x_{ij} \ln x_{ij} + \sum_{i,j=1}^n c_{ij}(y) x_{ij} + g(y) \right\}, \qquad (1)$$

where $c_{ij}(y)$ and $g(y)$ are concave smooth functions.

The dual problem for (1) is written in the form (2).

$$\max_{y \in Q} \max_{\lambda,\mu} \left\{ \langle \lambda, L \rangle + \langle \mu, W \rangle - \sum_{i,j=1}^n \exp(-c_{ij}(y) - 1 + \lambda_i + \mu_j) + g(y) \right\} \qquad (2)$$

Internal maximization problem for (λ, μ) can be explicitly made in terms of μ when λ is fixed and vice verse. This approach is well-known as balancing method for calculation of matrix of correspondences from the entropy model. It can be considered as method of simple iteration for explicit expressions for (λ, μ) from extremum conditions: $\lambda = \Lambda(\mu)$, $\mu = M(\lambda)$. Operator $(\lambda, \mu) \to (\Lambda(\mu), M(\lambda))$ is contracting one in Birkhoff-Gilbert metric and thus have geometric rate of convergence (one need to perform $N = O\left(\ln(\sigma^{-1})\right)$ iterations to guarantee this accuracy σ). That operator demonstrates fast convergence on practice and thus allows effective solution of internal problem. Due to the lack of knowledge about overall problem (for example, unknown Lipschitz constant) Nesterov's universal method with inexact oracle (see def. 1) is used.

D e f i n i t i o n 1. (δ, L)-oracle provides $(F(y), G(y))$ and that $(F(y), G(y))$ satisfy (3) for any $y, y' \in Q$.

$$0 \le f(y') - F(y) - \langle G(y), y' - y \rangle \le \frac{L}{2}\|y' - y\|^2 + \delta \tag{3}$$

The problem with Hölder's gradient of the goal function ($v \in [0,1]$; $\|\nabla f(y') - \nabla f(y)\|_* \le L_v \|y' - y\|^v$) can be considered as a smooth problem with inexact oracle for any δ with

$$L = L_v \left[\frac{L_v(1-v)}{2\delta(1+v)}\right]^{\frac{1-v}{1+v}} \tag{4}$$

Theorem 1. *There is one-parametric family of universal gradient methods (with parameter* $p \in [0,1]$*) which require no more than* $N_p(\varepsilon)$ *iterations (5) to solve the problem with accuracy* ε *if* $\delta \le O\left(\varepsilon / N_p(\varepsilon)^p\right)$.

$$N_p(\varepsilon) = O\left(\inf_{v \in [0,1]} \left(\frac{L_v R^{1+v}}{\varepsilon}\right)^{\frac{2}{1+2pv+v}}\right) \tag{5}$$

The time required for solving the problem (2) will be $O(N_1(\varepsilon) T \ln(\varepsilon^{-1}))$ where T is the time required to solve auxiliary problem by balancing method with relative error 1%. Experiments show that it is required $T \approx 1s$ on modern PC for $n = 10^2$.

The crucial feature of this method is its adaptability (it does not require knowledge of Lipschitz constant) and self-tuning to the optimum smoothness of the function.

Authors acknowledges Yu.E. Nesterov for the set of valuable comments.

The research was partially in MIPT with support of Russian Foundation for Basic Research (project no. 15-31-20571-mol_a_ved, 14-01-00722-a, 13-01-12007-ofi_m)

References

1. *A.V. Gasnikov, A.V. Chernov, P.E. Dvurechensky, P.I. Stetsyuk, A.L. Suvorikova.* "Universal method of Nesterov with inexact oracle in application of searching equillibriums in multistage transport problems," http://arxiv.org/.

Algorithms with performance guarantees for some hard discrete optimization problems

Edward Gimadi[1]

[1] *Novosibirsk University and Sobolev Institute of Mathematics, Novosibirsk, Russia;* gimadi@math.nsc.ru

We make several observations on efficient approximation algorithms with proven guarantees for some discrete optimization problems that mainly associated with rooting, covering and clustering. The problems considered are NP-hard in general case. We pay particular attention to the results achieved in recent years through research in Novosibirsk State University and Sobolev Institute of Mathematics. Most of the problems considered is to find in the complete weighted graph multiple discrete disjoint structures (subgraphs) with the extremal total weight of edges within them. We will touch on the following problems.

1. m-Peripatetic Salesman Problem (m-PSP): finding several Hamiltonian circuits in complete weighted graph.
2. Random MIN m-PSP with different weight functions of their routes on instances unbounded from above.
3. m-Capacitated 2-PSP with capacity restrictions.
4. Combinatorial algorithms with performance guarantees for finding several Hamiltonian circuits in a complete directed weighted graph.
5. Diameter-bounded (from belou) Minimum Spanning Tree Problem.
6. TSP-approach to construction an approximation algorithm for solving the problem m-CYCLES (CHAINS) COVER: covering a complete graph by m disjoint cycles (chains) with extremal total weight of edges.
7. Euclidean MAX m-CYCLES COVER.
8. Random MIN m-CYCLES COVER on instances $UNI(0,1)$.
9. Metric and Quadratic Euclidean MAX m-Weighted Clique Problems: 2-approximation algorithm.
10. Euclidean MAX Vector Subset Problem: a randomized algorithm.
11. Clustering Problem in network models.
12. Clustering Problem on the real axis.

The work was supported by the Russian Foundation for Basic Research (projects no. 15-01-00976, no. 13-07-00070).

On a clustering problem on the real axis

Edward Gimadi[1], **Anna Kurochkina**[2]

[1] *Novosibirsk University and Sobolev Institute of Mathematics, Novosibirsk, Russia;* `gimadi@math.nsc.ru`

[2] *SibGUTI, Novosibirsk, Russia;* `alkurochkin@ngs.ru`

Clustering is the classification of patterns (observations, data items, or feature vectors) into groups (clusters). The clustering problem has been addressed in many contexts and by researchers in many disciplines. However, clustering is a difficult problem combinatorially.

In this report, we consider a special case of Euclidean clustering problem, namely the problem of clustering on the real axis. Let the set $X = \{x_i\}$ of n elements (points) on the real axis and a natural number $m < n$ of clusters be given. Each point $x \in X$ is described by the weight $w(x)$ and by a cost function $f(x)$ for its assigning as the center of some cluster. Each cluster C_k has the capacity W_k, $k = 1, \ldots, m$. Let the distance function $\rho(x, y)$ between points x, y on real axis be given.

The problem is to find a partition of the set $X = \{x_i\}$ on disjoint subsets (clusters) $C_1, C_2, \ldots, C_m$ such that

$$\sum_{k=1}^{m} \min_{y \in C_k} \Big(f(y) + \sum_{x \in C_k} w(x)\rho(x, y) \Big) \to \min_{\{C_k\}}, \tag{1}$$

under constraints

$$\sum_{x \in C_k} w(x) \le W_k, \; k = 1, \ldots, m. \tag{2}$$

Our results:

1) We prove that the problem (1)-(2) is NP-hard, even for the linear or quadratic distance function.

2) The example is presented that shows nonoptimality of the decision for the problem with connected clusters $C_1, C_2, \ldots, C_m$.

3) For solving the special problem with connected clusters the exact algorithm is constructed. The algorithm implements the dynamic programming technique and runs in time $O(mn2^m)$, i.e. in the case of fixed parameter m it has linear time complexity depending on n.

TSP-approach to solving some problems of covering graphs by m disjoint cycles

Edward Gimadi[1], Ivan Rykov[1], Oxana Tsidulko[1], Mikhael Khachay[2]

[1] *Novosibirsk University and Sobolev Institute of Mathematics, Novosibirsk, Russia;* gimadi@math.nsc.ru, rykov@ngs.ru, tsidulko.ox@gmail.com
[2] *IMM Uro RAN, Ekaterinburg, Russia;* mkhachay@imm.uran.ru

Given a complete edge-weighted graph G, the m-CYCLES COVER problem is to find m cycles of extremal total weight, such that every vertex in G belongs to exactly one cycle. The problem is strong NP-hard even in Metric or Euclidean cases [1]. The TSP-approach to construct approximation algorithms for m-CYCLES COVER problem consists in reordering some spanning cyclic configuration of the TSP approximate solution into a spanning subgraph for m-CYCLES COVER problem.

Approximation algorithm for our problem finds an approximate solution (Hamiltonian cycle) $\widetilde{H} = (1, \ldots, n)$ for the TSP and solves the auxiliary problem $\sum_{k=1}^{m} \Big(w(u_{k-1}+1, u_k) - w(u_k, u_k+1) \Big) \to \min_u (\max_u)$ for all feasible partitions $u = (u_1, u_2, \ldots, u_m)$ of the cycle $\widetilde{H}$ into a set of disjoint segments (chains) $C_k = (u_{k-1}+1, \ldots, u_k)$, $k = 1, \ldots, m$, such that $1 \le u_1 < u_2 < \ldots < u_m \le n$, and each segment contains at least 2 edges; $u_0 = u_m$. As a resulting solution for m-CYCLES COVER problem we take the set of cycles obtained by adding edges $(u^*_{k-1}+1, u^*_k)$ to the chains C_k, $k = 1, \ldots, m$, where u^* is the solution of the auxiliary problem.

Let us introduce some examples of implementation of this approach.

1. Euclidean MAX m-CYCLES COVER. Suppose, that the lengths (number of vertices) $L_1, \ldots, L_m$ of the cycles may be given or not.

Build a Hamiltonian cycle $\widetilde{H}$ using the asymptotically optimal algorithm for Euclidean MAX TSP [2,3].For the auxiliary problem, set arbitrary u_i with $L_i = u_{i+1} - u_i \ge 3$; if the length L_i of a cycle is given, set $u_{i+1} = u_i + L_i$. Adding the corresponding edges to these segments, we get the solution of the problem. Note, that cyclically shifting the edges to be deleted from $\widetilde{H}$, we can obtain n feasible solutions and then choose the best one as an answer.

If $m = o(n)$ the algorithm for Euclidean MAX m-CYCLES COVER is asymptotically optimal and runs in $O(n^3)$ time.

2. Random MIN m-CYCLES COVER. Build a Hamiltonian cycle $\widetilde{H}$ using asymptotically optimal algorithm for Random MIN TSP on $UNI(0,1)$ random inputs [4]. For the auxiliary problem contract procedure with parameter

$p = \frac{1}{n^{1/3}}$ that finds m edges $(i, i+1)$ (one in each segment $(u_{k-1}+1, u_k)$), such that the random graph G'_p contains a pair of edges $(u_{k-1}+1, i+1)$ and (i, u_k), that convert the segment to a cycle. For the Random MIN m-CYCLES COVER problem on $UNI(0,1)$ random inputs an asymptotically optimal solution can be obtained for $m \leq \frac{n^{1/3}}{\ln n}$ in $O(n^3)$ time.

3. Symmetric MAX m-CYCLES COVER. We use 3/4-approximation algorithm from [5] to find Hamiltonian cycle $\widetilde{H}$. Then we remove m edges, that don't belong to the 2-factor and the perfect matching. Adding the edges to convert the segments to cycles we will only improve the ratio 3/4 for the Symmetric MAX m-CYCLES COVER problem.

Similar considerations can be used to modify other known algorithms for the TSP (see, for example,[6]) into algorithms solving the m-CYCLES COVER problem.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-01-00976, 13-07-00070, 14-11-00109).

References

1. *M. Yu. Khachai, E. D. Neznakhina* "Approximability of the Problem about a Minimum Weight Cycle Cover of a Graph," Doklady Akademii Nauk, **461**, No 6, 644–649 (2015).
2. *A.I. Serdyukov* "Asymptotically optimal algorithm for solving the Euclidean maximum TSP," Upravlyaemye sistemy, No 27, 79–87 (1987).
3. *E.Kh. Gimadi* "A new version of the asymptotically optimal algorithm for solving the Euclidean TSP," Baikal XII international conference proceedings. Metody optimizacii i ih prilojenia. **1**, Irkutsk, 117–124 (2001.
4. *A. M. Frieze* "On random symmetric traveling salesman problems," Mathematics of Operations Research, **29**, No 4, 878–890 (2004).
5. *A. I. Serdyukov* "Some Extremal Bypasses in Graphs," Upravlyaemye Sistemy (Inst. Mat., Novosibirsk), No 17, 76–79 (1978).
6. *E.Kh. Gimadi , A. I. Serdyukov* "On some results for the maximum TSP," Discretnii analys i issledovanie operacii, Vol 2. Novosibirsk: Inst. Mat SB RAS, **8**, No 1, 22–39 (2001).

A Hybrid Method for Solving Bimatrix Games

Evgeny Golshteyn[1], **Ustav Malkov**[2], **Nikolay Sokolov**[3]

[1] *CEMI RAS, Moscow, Russia;* golshtn@cemi.rssi.ru
[2] *CEMI RAS, Moscow, Russia;* ustav-malkov@yandex.ru
[3] *CEMI RAS, Moscow, Russia;* sokolov_nick@rambler.ru

At the previous OPTIMA–2013 conference, we presented the talk "A Numerical Method for Solving Bimatrix Games". The then proposed algorithm (2LP-method) first selects an ordered sequence of initial points (pure strategies). For a current initial point (for example, a vector y^0 with components $y_1^0 = 1$, $y_j^0 = 0$, $j = 2, \ldots, n$) the method successively solves (for $t = 0, 1, 2, \ldots$) two linear programming (LP) problems:

$$\alpha - x'(A + B)y^t \to \min,\ x'B \le \alpha,\ x'\,1_m = 1,\ x \in E_m^+,\ \alpha \in E_1, \qquad (1)$$

$$\beta - (x^{t+1})'(A + B)y \to \min,\ Ay \le \beta,\ y'\,1_n = 1,\ y \in E_n^+,\ \beta \in E_1. \qquad (2)$$

Here (α^{t+1}, x^{t+1}) is an optimal solution of problem (1) for a fixed vector $y = y^t$; (β^{t+1}, y^{t+1}) is an optimal solution to problem (2) for a fixed vector $x = x^{t+1}$; 1_k is the unit vector (composed of k ones, where k equals either m, or n); finally, $'$ means transposition.

The iterations stop when either one finds a Nash equilibrium point (x, y) where the Nash function becomes zero: $F(x, y) = \alpha + \beta - x'(A+B)y - \alpha - \beta = 0$, or the Nash function is stabilized at a nonzero value. In the latter case, the algorithm restarts from the next initial point. On practice, the minimum value of the Nash function achieved by having applied the algorithm to the sequence of starting points is sufficiently close to zero.

The well-known Lemke–Howson method (LH-algorithm) [1] also fails for some problems to find a Nash equilibrium point after a reasonable running time. For instance, we generated several games with linearly dependent matrices A and B, which proved to be unsolvable by the 2LP-algorithm, and their solution by the LH-method took unacceptably long time.

Now we propose a *hybrid approach* that makes search of the Nash points in two steps. First, making use of the 2LP-method we find a solution with a low value of the Nash function, and then we transform the latter into a starting point for the LH-algorithm.

At a Nash point of the bimatrix game, the following relationships must hold:

$$Ay + u = 1_m, \quad x'B + v = 1_n, \quad x, u \in E_m^+, \quad y, v \in E_n^+, \qquad (3)$$

together with the complementarity conditions

$$x_i\, u_i = 0, \quad i = 1, \ldots, m, \quad y_j\, v_j = 0, \quad j = 1, \ldots, n.$$

In the resulting solution obtained by the 2LP-algorithm with a nonzero value of the Nash function, one or more complementarity conditions are usually violated. When the first step of the hybrid algorithm is over, in order to start a procedure of Lemke–Howson kind, we combine the bases of problems (1)–(2) to compose a basis for relationships (3). Next, we try to reduce the number of violated complementarity relationships by pivoting, that is, by entering non-basic variables instead of the basic ones involved in the flawed complementarity conditions. If no Nash equilibrium has been achieved, we introduce an additional column generated in a special way so that after entering it to the basis, at least one variable of each pair engaged in the complementarity conditions becomes zero. After that, the LH-procedure is exercised in order to make this column leave the basis. If the latter occurs, a Nash equilibrium point is obtained.

Our hybrid approach has successfully solved several bimatrix games that proved to be failure for both pure 2LP and LH algorithms.

References

1. *C.E. Lemke, J.T. Howson, Jr.* "Equilibrium points of bimatrix games", Journal of the Society for Industrial and Applied Mathematics, No. 12, 778–780 (1964).

On Solving Bilevel Problems with a Matrix Game as the Follower Problem

Tatiana Gruzdeva[1], **Andrey Orlov**[1]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* {gruzdeva,anor}@icc.ru

Let us formulate the simplest case of the bilevel programming problems with an equilibrium as follows:

$$\left.\begin{array}{c} \langle c, x\rangle + \langle d_1, y\rangle + \langle d_2, z\rangle \uparrow \max\limits_{x,y,z}, \\ x \in X = \{x \in I\!R^m \mid Ax \le a,\ x \ge 0,\ \ \langle b_1, x\rangle + \langle b_2, x\rangle = 1\}, \\ (y,z) \in C(\Gamma M(x)), \end{array}\right\} \qquad (\mathcal{BP}_{\Gamma M})$$

where $C(\Gamma(x))$ is the set of saddle points of the game

$$\left.\begin{array}{ll} \langle y, Bz\rangle \uparrow \max\limits_{y}, & y \in Y(x) = \{y \mid y \ge 0, \langle e_{n_1}, y\rangle = \langle b_1, x\rangle\}, \\ \langle y, Bz\rangle \downarrow \min\limits_{z}, & z \in Z(x) = \{z \mid z \ge 0, \langle e_{n_2}, z\rangle = \langle b_2, x\rangle\}. \end{array}\right\} \qquad (\Gamma M(x))$$

$c, b_1, b_2 \in I\!R^m$, $y, d_1 \in I\!R^{n_1}$; $z, d_2 \in I\!R^{n_2}$; $a \in I\!R^p$; $b_1 \ge 0,\ b_1 \ne 0,\ \ b_2 \ge 0,\ b_2 \ne 0$; A, B are matrices and $e_{n_1} = (1, ..., 1)$, $e_{n_2} = (1, ..., 1)$.

In order to elaborate numerical methods for the solving of $(\mathcal{BP}_{\Gamma M})$ we need to reformulate it as single level problem. We can replace a game at the lower level by its optimality conditions [1]. Hence, it is possible to formulate the following equivalent to $(\mathcal{BP}_{\Gamma M})$ optimization problem:

$$\left.\begin{array}{c} f(x,y,z) \stackrel{\triangle}{=} \langle c, x\rangle + \langle d_1, y\rangle + \langle d_2, z\rangle \uparrow \max\limits_{x,y,z,v},\ \ (x,y,z) \in S \\ \langle b_1, x\rangle(Bz) \le v e_{n_1},\ \ \langle b_2, x\rangle(yB) \ge v e_{n_2}. \end{array}\right\} \qquad (\mathcal{PM})$$

where $S = \{x, y, z \ge 0 \mid Ax \le a,\ \ \langle b_1, x\rangle + \langle b_2, x\rangle = 1,\ \ \langle e_{n_1}, y\rangle = \langle b_1, x\rangle,$ $\langle e_{n_2}, z\rangle = \langle b_2, x\rangle\}$. It can readily be seen, that $(\mathcal{PM})$ is a global optimization problem with a nonconvex feasible set and we can apply the Global Search Theory (GST) for the problems with d.c. constraints [2, 3].

The group of $(n_1 + n_2)$ bilinear constraints generates the basic nonconvexity in the problem $(\mathcal{PM})$. We proposed to reduce the bilinear constraints to a single, but nondifferentiable, constraint:

$$F = \max\left\{\max_{1\le i\le n_1} \langle e_{n_1}, y\rangle(Bz)_i - v; v - \min_{1\le j\le n_2} \langle e_{n_2}, z\rangle(yB)_j\right\} \le 0, \qquad (1)$$

and we obtained explicit decomposition of function (1) by the difference of two convex functions: $F(y,z,v) = g(y,z,v) - h(y,z)$.

The Local Search Algorithm for problem with d.c. constraint consists of two following procedures. Starting from given point (x, y, z, v) : $(x, y, z) \in S$, $F(y, z, v) \geq 0$, first procedure constructs a point $(\bar{x}, \bar{y}, \bar{z}, \bar{v})$ such that $(\bar{x}, \bar{y}, \bar{z}) \in S$, $F(\bar{y}, \bar{z}, \bar{v}) = 0$, $f(\bar{x}, \bar{y}, \bar{z}) \geq f(x, y, z)$. Second procedure starts at a point $(\bar{x}, \bar{y}, \bar{z}, \bar{v})$ and constructs a sequence $\{(x^r, y^r, z^r, v_r)\}$ such that

$$(x^r, y^r, z^r) \in S,\ F(y^r, z^r, v_r) \geq 0,\ f(x^r, y^r, z^r) \geq \rho,$$
$$g(y^{r+1}, z^{r+1}, v_{r+1}) - \langle \nabla h(y^r, z^r), (y^{r+1}, z^{r+1}) \rangle - \delta_r \leq$$
$$\leq \inf_{x,y,z,v} \{g(y, z, v) - \langle \nabla h(y^r, z^r), (y, z) \rangle \mid (x, y, z) \in S,\ f(x, y, z) \geq \rho_r\}.$$

The Global Search Procedure for problem $(\mathcal{PM})$ may be represented as follows. Let there be given an approximate critical point (x^k, y^k, z^k, v_k) in $(\mathcal{PM})$. For escaping it one needs to realize the following key stages.

1) Choose a number $\beta \in [\inf(g, S), \sup(g, S)]$ and construct an approximation $A_k = \{p^i = (y^i, z^i) \in I\!R^{n_1+n_2} \mid h(p^i) = \beta,\ i = 1, \ldots, N\}$ It is possible to choose $\beta_0 = h(y^k, z^k)$.

2) Compute $\rho_k = f(x^k, y^k, z^k)$ and starting from point (x^k, y^i, z^i, v_k) obtain a critical point $(\tilde{x}^i, \tilde{y}^i, \tilde{z}^i, \tilde{v}_i)$ to $(\mathcal{P})$ by the Local Search Algorithm.

3) Choose from the set of points $(\tilde{x}^i, \tilde{y}^i, \tilde{z}^i, \tilde{v}_i)$, $i = 1, \ldots, N$, the triplet $(\tilde{x}, \tilde{y}, \tilde{z})$ best with respect to the goal function $f(\cdot)$.

4) If the value of the goal function at the point $(\tilde{x}, \tilde{y}, \tilde{z})$ turns out to be better than in a current point (x^k, y^k, z^k) then $(x^{k+1}, y^{k+1}, z^{k+1}, v_{k+1}) := (\tilde{x}, \tilde{y}, \tilde{z}, \tilde{v})$ and the process is repeated.

First results of our computational simulation demonstrate efficiency of the above technique.

This research was supported by the Russian Science Foundation (project No. 15-11-20015).

References

1. *A.V. Orlov.* “Bilevel problems with matrix and bimatrix games at the lower level and d.c. constraint optimization,” Proceedings of the II International Conference OPTIMA-2011, 171–174 (2011).
2. *A.S. Strekalovsky.* “Minimizing Sequences in Problems with d.c. Constraints,” Comput. Math. Math. Phys., **45**, 418–429 (2005).
3. *T.V. Gruzdeva, A.S. Strekalovsky.* “ Local Search in Problems with Nonconvex Constraints,” Comput. Math. Math. Phys., **47**, 381–396 (2007).

Iterative methods for solving quasi-variational inequalities

Milojica Jaćimović[1], **Nevena Mijajlović**[1]

[1] *Department of Mathematics, University of Montenegro, Podgorica;*
`milojica@jacimovic.me`, `nevenamijajlovic@hotmail.com`

In our talk we will consider quasi-variational inequality:

find $x_* \in C(x_*)$ such that

$$\langle F(x_*), y - x_* \rangle \geq 0, \quad \forall y \in C(x_*), \qquad (QVI)$$

where $F : H \mapsto H$ is an operator of a Hilbert space H and $C : H \to 2^H$ is a multifunction with nonempty closed and convex values. For $C(x) = C \subseteq H$ we have variational inequality

$$\text{find } x \in C : \langle F(x_*), y - x_* \rangle \geq 0, \quad \forall y \in C. \qquad (VI)$$

The theory, as well as solution methods of the variational inequality, have been well documented in the literature.

In recent years the theory of quasi-variational inequalities attracted a growing attention. This theory includes variational inequalities and many others important problems of interest as particular cases and it provides a mathematical tools for studying a wide range of the problems of theory of games, equilibrium programming, structural mechanics.

Let us note that from the point of theory of existence and solution methods, quasi-variational inequalities do not have an extensive literature. The reason is because quasi-variational inequalities require simultaneously solving of variational inequality and fixed point problemof multifunction. As a consequence, we have that the theory of quasivariational inequalities contains many questions to be answered. For example, the solution methods of variational inequalities are not always convenient (and they can not be adapted) for solving quasi-variational inequalities. In our talk we will present some recently developed methods for solving quasivariational inequalities. We will suggest first and second order gradient mathod, proximal method and extra-gradient method. For every method we will find rate of convergence.

Directional control of bifurcations into targeted trajectory

Vladimir Jaćimović[1], **Nikola Konatar**[1]

[1] *Faculty of Mathematics and Natural Sciences, University of Montenegro, Dorda Vaingtona bb, 81000 Podgorica, Montenegro*

`vladimir@jacimovic.me; konatarn@yahoo.com`

Bifurcation as a phenomenon has been studied in the recent decades, and the possibility of controlling a system in which a bifurcation occurs has attracted the attention of many researchers. Our goal in researching the problem of bifurcation control is to find suitable control directions for a system undergoing bifurcation in order to steer it into a favorable solution.

We will work with impulsive controls, and will show a method for finding desirable control directions. This method is based on a bifurcation theorem for abstract problems with constraints, represented by inequalities on initial and terminal conditions.

We demonstrate our method with two examples of controlled dynamical systems. These examples will show how directions of control are chosen, and the impact that the initial conditions of the desired solution have on this choice.

Duality and optimization methods of multidimensional nonlinear dynamic systems

M.T. Jenaliyev[1], M.N. Kalimoldayev[1]

[1] *Institute of Information and Computational Technologies Almaty, Kazakhstan;* muvasharkhan@gmail.com

[1] *Institute of Information and Computational Technologies Almaty, Kazakhstan;* mnk@ipic.kz

In this work, it is suggested to use an iterative algorithm based on the Krotov optimality principle in solving of optimal control problems of the complex nonlinear systems. As it is known, [1–2] the general scheme of this principle is to transition from the original optimization problem $\{J[v] \to \min\limits_{v}\}$ to a certain dual problem $\{L[\varphi,\, v] \to \max\limits_{\varphi}\}$.

The key idea of the transition to dual problem based on the following transformation of the optimal control problem:

$$J[x,u] = \int_0^T f^0(x,u,t)dt + F(x(T)) \to \min_{\{x,u\}\in D}, \tag{1}$$

$$D = \{x(t), u(t)|\ x'(t) = g(x,u,t),\ x(0) = x_0,\ t \in (0,T)\}, \tag{2}$$

using the Krotov function $\varphi(x,t)$ to a problem:

$$L[\varphi; x, u] = -\int_0^T R(\varphi; x(t), u(t), t)d\,t + G(\varphi; x(T)) \to \max_{\varphi\in\Phi}, \tag{3}$$

where $$R(\varphi; x, u, t) = -f^0(x,u,t) + \frac{\partial\varphi(x,t)}{\partial t} + \sum_{i=1}^{n} \frac{\partial\varphi(x,t)}{\partial x^i} g^i(x,u,t),$$

$$G(\varphi; x) = F(x) + \varphi(x,T) - \varphi(x_0, 0),$$

$$L[\varphi; x, u] = J[x,u] \ \text{ at } \ \{x,u\} \in D, \tag{4}$$

We define the class of functions Φ in the following way: in order for $\forall\,\varphi \in \Phi$ functionals (3), $R(\varphi; x, u, t)$ and $G(\varphi; x)$ would be defined and the relation (4) is true. An iterative algorithm for solving the problem (1)–(2), based on the given transformation.

The suggested algorithm is applied for the numerical solution of optimal control problems of complex power system, [3]:

It is required to minimize the functional

$$J(\nu) = J(\nu_1, ..., \nu_l) = 0.5 \sum_{i=1}^{l} \int_0^T \left(w_{si} S_i^2 + w_{\nu i} \nu_i^2 \right) dt + \Lambda(\delta(T),\, S(T)), \qquad (5)$$

under the conditions

$$\frac{d\,\delta_i}{d\,t} = S_i, \quad \frac{d\,S_i}{d\,t} = \frac{1}{H_i}[-D_i S_i - f_i(\delta_i) - N_i(\delta) + M_i(\delta) + u_i],\ i = \overline{1,l}, \qquad (6)$$

where

$$f_i(\delta_i) = P_i[\sin(\delta_i + \delta_i^F - \alpha_i) - \sin(\delta_i^F - \alpha_i)],$$

$$N_i(\delta) = \sum_{j=1,\, j\neq i}^{l} \overline{N}_{ij}(\delta_1, ..., \delta_l) = \sum_{j=1,\, j\neq i}^{l} \Gamma_{ij}^1[\sin(\delta_{ij} + \delta_{ij}^F) - \sin\, \delta_{ij}^F],$$

$$M_i(\delta) = \sum_{j=1,\, j\neq i}^{l} \overline{M}_{ij}(\delta_1, ..., \delta_l) = \Gamma_{ij}^2[\cos(\delta_{ij} + \delta_{ij}^F) - \cos\, \delta_{ij}^F],$$

$$\Gamma_{ij}^1 = P_{ij}\cos\,\alpha_i,\ \Gamma_{ij}^2 = P_{ij}\sin\,\alpha_i,\ P_{ij} = P_{ji},\ \Gamma_{ij}^k = \Gamma_{ji}^k,\ k = 1, 2.$$

The control will be found in the form of:

$$u_i = \nu_i - M_i(\delta),\ i = \overline{1,l},\ \text{where the functions } \nu_i \text{ to be determined.} \qquad (7)$$

For given numerical data for problem (5)–(7) numerical experiments has been conducted.

References

1. *V.F. Krotov, V.I. Gurman* Methods and problems of control optimality Nauka. Moscow (1973).
2. *V.I. Gurman* Principle in control problems. Nauka. Moscow (1997).
3. *Blekhman I.I., Fradkov A.L.* On general definitions of synchronization// In: Selected topics in vibrational mechanics. Ed. I.I. Blekhman, Singapore, World Scientific, 2004. – P. 179–188.

Monotone solutions of parametric complementarity problems

Vyacheslav Kalashnikov[1,2,3], **Nataliya Kalashnykova**[4], **Mariel Leal-Coronado**[1]

[1] *Tecnológico de Monterrey (ITESM), Monterrey, Mexico;* kalash@itesm.mx
[2] *CEMI RAS, Moscow, Russia;* slavkamx@mail.ru
[3] *Sumy State University, Sumy, Ukraine;* slavkamx@gmail.com
[4] *Universidad Autónoma de Nuevo León (UANL), Monterrey, Mexico;* nkalash2009@gmail.com

In many applications, it is important to know if solutions to parametric complementarity problems are monotone with respect to parameters involved (*see*, for example, [1] and [2]). This paper establishes conditions of various types that guarantee the latter property. In the majority of them, the crucial requirement is that the mappings defining the problem be monotone (in certain sense) by the state variables and antitone with respect to parameters.

Consider a nonlinear complementarity problem with parameters: given a parameter vector $u = (u_1, u_2, ..., u_m) \in R^m$, find a point $x \in R^n$ such that

$$x \geq 0,\ Ax + Bu + \varphi(x, u) \geq 0, \quad \text{and}$$
$$x^T(Ax + Bu + \varphi(x, u)) = 0; \tag{1}$$

here A, B are given $n \times n$ and $n \times m$ real matrices, and $\varphi : R^n \times R^m \to R^n$ is a nonlinear function. We say that $a \geq b$ if $a_i \geq b_i,\ i = 1, ..., n$.

D e f i n i t i o n 1 [3]. A matrix A is called:

- a Z-matrix if its non-diagonal elements are non-positive;
- a P-matrix if all its principal minors are positive;
- an M-matrix if it is a Z-matrix and a P-matrix.

In order not to restrict our research to the case of equal numbers of decision variables and parameters, we will use not the concept of monotonicity defined by the inner product of the vector-function and the vector of parameters, but the component-wise monotonicity notion (*cf.*, [4]) given below.

D e f i n i t i o n 2. A mapping $f : R^n \to R^m$ is called *monotone* [*antitone*] if $x_1 \geq x_2$ implies $f(x_1) \geq f(x_2)$ $\quad [f(x_1 \leq f(x_2))]$.

Theorem 1. *Let A be a positive definite M-matrix, B a non-positive one, and $\varphi(x, u)$ a differentiable function monotone by x and antitone with respect* *to u. Moreover, suppose $\varphi'_x = \varphi'_x(x,u)$ to be a positive definite M-matrix for each x and u. Then the solution $x = x(u)$ to problem* (1) *is monotone by u.*

A symmetrical result concerning the antitone behavior of solutions of the complementarity problem (1) is obtained readily by the theorem below.

Theorem 2. *Let A be a positive definite M-matrix, B a non-negative one, and $\varphi(x,u)$ a differentiable function monotone by both x and u. Moreover, suppose $\varphi'_x = \varphi'_x(x,u)$ to be a positive definite M-matrix for each x and u. Then the solution $x = x(u)$ to problem* (1) *is antitone by u.*

The paper also comprises monotonicity results for linear and implicit parametric complementarity problems, as well as for more general variational inequality problems.

The authors were supported by the Research Department of the Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM), Campus Monterrey, and by the SEP-CONACYT project CB-2013-01-221676, Mexico. The second author was also supported by the PAICYT project No. CE250-09.

References

1. *G. Isac, V.A. Bulavsky, V.V. Kalashnikov.* Complementarity, Equilibrium, Efficiency and Economics, Kluwer Academic Publishers, Dordrecht/Boston/London (2002).
2. *M.M. Kostreva.* “Elasto-hydrodynamic lubrication: A nonlinear complementarity problem,” International Journal for Numerical Methods in Fluids, **4**, No. 4, 377–397 (1984).
3. *A. Berman, D. Plemmons.* Nonnegative Matrices in Mathematical Sciences, Academic Press, New York (1979).
4. *J.M. Ortega, W.C. Rheinboldt.* Iterative Solution of Nonlinear Equations in Several Variables, Academic Press, New York/ London (1970).

Economic soliton of (2+1)-dimensional nonlinear A4-model

Maksat Kalimoldayev[1], **Alexandra Alexeyeva**[2], **Gulshat Amirkhanova**[3]

[1] *Institute of Information and Computational Technologies of SC MES RK, Almaty, Kazakhstan;* mnk@ipic.kz

[2] *Institute of Mathematics and Mathematical Modelling of SC MES RK, Almaty, Kazakhstan;* alexandra-aleks@mail.ru

[3] *Institute of Information and Computational Technologies of SC MES RK, Almaty, Kazakhstan;* gulshat.aa@gmail.com

In this article, the (2+1)-dimensional nonlinear A4-model, generalizing the Korteweg-de Vries equation, was considered. In this paper we explained the economic meaning of (2+1)-dimensional nonlinear A4-model. The definition of economic soliton as a solution to the mathematical model was given. Economic sense of the method of direct scattering problem, corresponding to the positive analytical approach to the economy, is described.

Korteweg-de Vries equation is called universal mathematical model because it describes many of the physical problems of nonlinear waves in different physical environments.Multidimensional analogues of Korteweg-de Vries equation is also universal. Alexeyeva A. presented the class of the spatially two-dimensional nonlinear mathematical models A1-A14 and AI-AXII [1], generalizing the classical Korteweg-de Vries equation [2].

Consider the (2+1)-dimensional nonlinear A4-model has the form

$$\Psi_t + V_{xxy} + 3[V^2]_y = 0 \ ,$$

where $V_x = \Psi_y$ and complex valued function $\Psi = \Psi(x, y, t) \in C^\infty(R^1 \times R^1 \times R^1_+)$ and decreases with all its partial derivatives faster than any power of $|x|^{-1}$.

We constructed hierarchy of auxiliary linear systems for the (2+1)-dimensional nonlinear A4-model. Also we showed the compatibility condition for these systems which connencts the auxiliary linear systems with this model. We found soliton solutioans of the (2+1)-dimensional nonlinear A4-model.

We solved the (2+1)-dimensional nonlinear A4-model by the method of direct scattering problem.

Spatially (2+1)-dimensional nonlinear A4-model adequately describes the objective reality of any economic phenomenon or process. It is known [3] that the behavior of the microsubjects of economy has wave character. Here, the function $\Psi = \Psi(x, y, t)$ is the wave function describing the state of the economy microsubject, x, y are the potentials of collective economic interactions; t is the time factor, which serves as a transformation parameter.

If to apply the method of direct scattering problem to the economic problems, it will conform to the implementation of a positive approach to economic policy.

In particular, the requirement to assess the consequences of any predefined permanent economic activities, such as budget, formally corresponds to the solution of the direct problem of scattering. I.e. data recovery in the scattering matrix for a given function $\Psi = \Psi(x, y, t)$ in advance on the collective interactions of x and y. This interpretation of the direct scattering problem method is consistent with the positive analytical approach in the economy.

Thus, we examined the (2+1)-dimensional nonlinear A4-model, generalizing the Korteweg-de Vries equation. We considered the auxiliary linear system for it, determined the conditions of zero curvature, which connects the auxiliary linear system with this model. We explained the economic meaning of (2+1)-dimensional nonlinear A4-model. We defined an economic soliton as solution of this mathematical model. We explained the economic meaning of the method of direct scattering problem corresponding to the positive analytical approach in the economy.

References

1. *A.V. Alexeyeva.* "(2+1)-dimensional analogs of the Korteweg-de Vries equation", International Journal of Contemorary Mathematics.New Delhi - vol.3, No. 1-2, 47–55 (2012).
2. *M. Ablowitz, X. Sigur.* Solitons and the inverse scattering method, Moscow (1987).
3. *T.V.Ogorodnikova.* "The formalization of the wave soliton of the collective economic behavior", Izvestiya of the Irkutsk State Academy of economics, Issue 3, 12–15 (2004).

Research of statistic methods in finding the solutions of a linear stochastic problems

Olga Kaneva[1], **Dmitriy Atink**[2]

[1] *Omsk State Technical Univercity, Omsk, Russia;* `okaneva@yandex.ru`
[2] *Omsk State Technical Univercity, Omsk, Russia;* `dmitryatink@gmail.com`

The work is dedicated to creating software for solving linear stochastic problems of the form:

$$F^0 = M\left(\sum_{j=1}^{n} c_j x_j\right) \to max, \tag{1}$$

$$F^i = P\left\{\sum_{j=1}^{n} a_{ij} x_j \leq b_i\right\} \geq \alpha_i, i = 1, ..., m, \tag{2}$$

$$\sum_{j=1}^{n} d_{il} x_j \leq f_l, l = 1, ..., k, \quad x_j \geq 0, j = 1, ..., n. \tag{3}$$

In the analysis of modern mathematical packages, in which it is possible to solve optimization problems, means for solving the problem (1.1)-(1.3) has not been found, and it was decided to develop of such software. A feature of developed software is the integration with popular free math software SciLab. This allowed to use the tools of SciLab for optimization tasks and use all modern means of high-level language C♯ and .Net. At this stage, implement the solution of problem (1)-(3) with indirect method (through the construction and solve of deterministic equivalent), as well as the stochastic method of reducing residual[2], which refers to direct methods for solving stochastic problem.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-41-04436).

References

1. *D.B. Yudin.* Mathematical methods of control under conditions of incomplete information, Krosand, Moscow (2010).
2. *Yu.M. Ermoliev.* Methods of stochastic programming, Nauka, Moscow (1976).

Bounding Restricted Isometry in Compressed Sensing Matrices via the K-condition number

Igor Kaporin[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;*
igorkaporin@mail.ru

The construction and analysis of compressed sensing (CS) matrices is the key point in the CS research [1]. We consider an alternative reformulation of the restricted isometry property (RIP) required for a rectangular $m \times n$ CS matrix A. Here $m < n$, and it is necessary to measure the degree of linear independence of any subset of $k < m$ columns $[a_{j(1)}, \ldots, a_{j(k)}] = A_J$ of A. Instead of using the spectral condition number of $A_J^T A_J$ (i.e., the ratio of its extreme eigenvalues), we propose to use its K-condition number (i.e., the ratio of their arithmetic to the geometric mean values taken in the kth power). For the latter quantity, there exists a lower bound which holds for any CS matrix. In particular, assuming $AA^T = \frac{n}{m} I_m$ and $(A^T A)_{ii} = 1$, we prove that there always exist such $J = \{j(1), \ldots, j(k)\}$ that

$$K(A_J^T A_J) \equiv \frac{\left(k^{-1}\mathrm{trace}(A_J^T A_J)\right)^k}{\det(A_J^T A_J)} \geq \prod_{i=1}^{k-1} \frac{1 - \frac{i}{n}}{1 - \frac{i}{m}} \approx \exp\left(\frac{k^2}{2m} - \frac{k^2}{2n}\right).$$

It must be stressed that the result is completely deterministic. The proof is based on an appropriate generalization of the Binet-Cauchy determinant identity. Note that standard RIP estimates in terms of the spectral condition number follow from this result by inequalities presented in [2]. The next step in the research might be a performance analysis for certain CS solvers in terms of this K-condition number characterization (thus circumventing the standard RIP condition).

The author was supported by the Russian Foundation for Basic Research (project no. 14-07-00805). and by the grant NSh-4640.2014.1.

References

1. *D.L. Donoho.* “Compressed sensing,” IEEE Trans. on Information Theory, **52**, No. 4, 1289–1306 (2006).
2. *I. Kaporin.* Preconditioning of iterative methods for the solution of linear systems, Doctoral dissertation, Moscow, 2011, 216 p.

Some quadratic Euclidean 2-clustering problems: NP-hardness and efficient algorithms with performance guarantees

Alexander Kel'manov[1]

[1] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* kelm@math.nsc.ru

We consider some quadratic Euclidean 2-clustering problems induced by actual issues such that data analysis, pattern recognition, statistics, computational geometry, and approximation theory. These problems are also important in a wide range of natural-science and engineering applications. The purpose of the report is overviewing new (previously unstudied), known (weakly studied) and recent results on computational complexity of these problems, and on efficient algorithms with performance guarantees for these problems.

Below is a list of considered problems.

Problem 1. *Minimum Sum-of-Squares 2-Clustering with Given Center of one cluster.*

Problem 2. *Minimum Sum-of-Squares 2-Clustering problem on Sequence with Given Center of one cluster.*

Problem 3. *Quadratic Euclidean Max-Cut.*

Problem 4. *Quadratic Euclidean Min-Sum All-Pairs 2-Clustering.*

Problem 5. *Euclidean Balanced Variance-based 2-Clustering.*

Problem 6. *Euclidean Balanced Variance-based 2-Clustering with Given Center of one cluster.*

We focus on the 2-clustering problems (for the finite set and finite sequence of points in the Euclidean space) with the given center of one cluster. So, the desired center of one of the clusters is given (without loss of generality at 0) as an input, while the center of the second cluster is unknown (a variable for optimizing). Two variants of the problems are analyzed, where the cardinalities of the clusters are either the parts of input or unknown.

The work was supported by the the Russian Foundation for Basic Research (projects no. 15-01-00462, no. 13-07-00070).

Fully polynomial-time approximation scheme for a sequence 2-clustering problem

Alexander Kel'manov[1], **Sergey Khamidullin**[2], **Vladimir Khandeev**[3]

[1] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* kelm@math.nsc.ru

[2,3] *Sobolev Institute of Mathematics, Novosibirsk, Russia;* kham@math.nsc.ru, khandeev@math.nsc.ru

We consider following strongly NP-hard [1]

Problem (Minimum sum-of-squares 2-clustering problem on sequence with given center of one cluster and cluster cardinalities). *Given a sequence* $\mathcal{Y} = (y_1, \ldots, y_N)$ *of points from* $\mathbb{R}^q$*, and some positive integer numbers* $T_{\min}$*,* $T_{\max}$ *and* M*. Find a subset* $\mathcal{M} = \{n_1, \ldots, n_M\} \subseteq \mathcal{N} = \{1, \ldots, N\}$ *such that*

$$\sum_{j \in \mathcal{M}} \|y_j - \overline{y}(\mathcal{M})\|^2 + \sum_{i \in \mathcal{N} \setminus \mathcal{M}} \|y_i\|^2 \to \min,$$

where $\overline{y}(\mathcal{M}) = \frac{1}{|\mathcal{M}|} \sum_{i \in \mathcal{M}} y_i$*, under constraints*

$$1 \le T_{\min} \le n_m - n_{m-1} \le T_{\max} \le N, \ m = 2, \ldots, M,$$

on the elements of $(n_1, \ldots, n_M)$*.*

The problem is to find a partition of a finite Euclidean sequence $\mathcal{Y}$ of points into two clusters minimizing the sum over the both clusters of the intracluster sums of squared distances from the elements of the cluster to its center. The center $\overline{y}(\mathcal{M})$ of the first cluster $\{y_j \,|\, j \in \mathcal{M}\}$ is defined as the mean values of all points in a cluster. The center of the second cluster $\{y_i \,|\, i \in \mathcal{N} \setminus \mathcal{M}\}$ is given in advance and is equal to 0. Additionally, the partition has to satisfy the following condition: for all points that are in the first cluster the difference $n_m - n_{m-1}$ between the indices of two consequent points from this cluster is bounded from below and above by some constants $T_{\min}$ and $T_{\max}$. This problem is actual, in particular, in the noise-proof analysis of time series.

In [2], a 2-approximation algorithm for the problem is proposed. The running time of the algorithm is $\mathcal{O}(N^2(N+q))$ for the case when $T_{\min} < T_{\max}$, and $\mathcal{O}(qN^2)$ for the case when $T_{\min} = T_{\max}$.

A pseudopolynomial algorithm which finds an optimal solution for the case of integer components of the points in the input set and fixed space dimension, was proposed in [3]. The running time of the algorithm is $\mathcal{O}(N^3(MD)^q)$, where D is the maximum absolute coordinate value of the points in the input set.

In this paper, we prove that the general case of this problem does not admit fully polynomial time approximation scheme (FPTAS), unless P=NP. In addition, for the case of fixed space dimension, we present FPTAS with $\mathcal{O}(N^4(1/\varepsilon)^{q/2})$-time complexity, where ε is an arbitrary relative error.

The authors were supported by the Russian Foundation for Basic Research (projects no. 15-01-00462, no. 13-07-00070).

References

1. *A.V. Kel'manov, A.V. Pyatkin.* "On Complexity of Some Problems of Cluster Analysis of Vector Sequences," J. Appl. Indust. Math., **7**, No. 3, 363–369 (2013).
2. *A.V. Kel'manov, S.A. Khamidullin.* "An Approximation Polynomial-Time Algorithm for a Sequence bi-Clustering Problem," Computat. Math. and Math. Phys., **55**, No. 6, 1068–1076 (2015).
3. *A.V. Kel'manov, S.A. Khamidullin, V.I. Khandeev.* "An exact pseudopolynomial algorithm for a sequence partitioning problem," Automation and Remote Control, 2015 (accepted).

Fully polynomial-time approximation scheme for a special case of a quadratic Euclidean 2-clustering problem

Alexander Kel'manov[1], **Vladimir Khandeev**[2]

[1] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* kelm@math.nsc.ru

[2] *Sobolev Institute of Mathematics, Novosibirsk, Russia;* khandeev@math.nsc.ru

We consider the following strongly NP-hard [1]

Problem (Minimum sum-of-squares 2-clustering problem with given center of one cluster and cluster cardinalities). *Given a set $\mathcal{Y} = \{y_1, \dots, y_N\}$ of points from $\mathbb{R}^q$ and a positive integer number M. Find a partition of $\mathcal{Y}$ into clusters $\mathcal{C}$ and $\mathcal{Y} \setminus \mathcal{C}$ such that*

$$\sum_{y \in \mathcal{C}} \|y - \overline{y}(\mathcal{C})\|^2 + \sum_{y \in \mathcal{Y} \setminus \mathcal{C}} \|y\|^2 \to \min,$$

where $\overline{y}(\mathcal{C}) = \frac{1}{|\mathcal{C}|} \sum\limits_{y \in \mathcal{C}} y$ is the centroid of $\mathcal{C}$, under constraint $|\mathcal{C}| = M$.

In [2], a 2-approximation algorithm for the problem is proposed. The running time of the algorithm is $O(qN^2)$. In [3], a polynomial-time approximation scheme (PTAS) with a $O(qN^{2/\varepsilon+1}(9/\varepsilon)^{3/\varepsilon})$-time complexity, where ε is an arbitrary relative error, is constructed.

A randomized algorithm for the problem is presented in [4]. The running time of the algorithm for the fixed failure probability, relative error of the solution, and for the certain value of parameter k is $\mathcal{O}(2^k q(k+N))$. The algorithm has also been proven to be asymptotically exact and to have $\mathcal{O}(qN^2)$-time complexity for the special values of the parameters.

A pseudopolynomial algorithm which finds an optimal solution in the case of integer components of the points in the input set and fixed space dimension is proposed in [5]. The running time of the algorithm is $\mathcal{O}(N(MD)^q)$, where D is the maximum absolute coordinate value of the points in the input set.

In this paper (see also [6]) we prove that, unless P=NP, in the general case of the problem there is no fully polynomial-time approximation scheme (FPTAS). In addition, we present such a scheme for the case of fixed space dimension.

The running time of the algorithm is $\mathcal{O}(N^3(1/\varepsilon)^{q/2})$, where ε is an arbitrary relative error.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-07-00070, no. 15-01-00462).

References

1. *A.V. Kel'manov, A.V. Pyatkin.* "Complexity of Certain Problems of Searching for Subsets of Vectors and Cluster Analysis,"Computat. Math. and Math. Phys., **49**, No. 11, 1966–1971 (2009).
2. *A.V. Dolgushev, A.V. Kel'manov.* "An Approximation Algorithm for Solving a Problem of Cluster Analysis,"J. Appl. Indust. Math., **5**, No. 4, 551–558 (2011).
3. *A.V. Dolgushev, A.V. Kel'manov, V.V. Shenmaier.* "PTAS for one problem of cluster analysis," Intelligent Information Processing: Proc. of the 9th Intern. Conf., Montenegro, Budva, 242–244 (2012). (In Russian).
4. *A.V. Kel'manov, V.I. Khandeev.* "A Randomized Algorithm for Two-Cluster Partition of a Set of Vectors," Computat. Math. and Math. Phys, **55**, No. 2, 330-339 (2015).
5. *Alexander Kel'manov, Vladimir Khandeev.* "An exact pseudopolynomial algorithm for a bi-partitioning problem," Proceedings of V International Conference Optimization and applications (OPTIMA-2014), Petrovac, Montenegro, September 28 – October 4, 108-109 (2014).
6. *A.V. Kel'manov, V.I. Khandeev.* "FPTAS for a special case of a quadratic Euclidean 2-clustering problem," Computat. Math. and Math. Phys, (2015), (accepted).

An exact pseudopolynomial algorithm for a special case of a Euclidean balanced variance-based 2-clustering problem

Alexander Kel'manov[1], **Anna Motkova**[2]

[1] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* `kelm@math.nsc.ru`

[2] *Novosibirsk State University, Novosibirsk, Russia;* `anitamo@mail.ru`

We consider the following strongly NP-hard [1]

Problem (Euclidean balanced variance-based 2-clustering with given center of one cluster and cluster cardinalities). *Given a set* $\mathcal{Y} = \{y_1, \ldots, y_N\}$ *of points from* $\mathbb{R}^q$ *and a positive integer number* M. *Find a partition of* $\mathcal{Y}$ *into clusters* $\mathcal{C}$ *and* $\mathcal{Y} \setminus \mathcal{C}$ *such that*

$$|\mathcal{C}| \sum_{y \in \mathcal{C}} \|y - \overline{y}(\mathcal{C})\|^2 + |\mathcal{Y} \setminus \mathcal{C}| \sum_{y \in \mathcal{Y} \setminus \mathcal{C}} \|y\|^2 \to \min,$$

where $\overline{y}(\mathcal{C}) = \frac{1}{|\mathcal{C}|} \sum\limits_{y \in \mathcal{C}} y$ *is the centroid of* $\mathcal{C}$, *under constrain* $|\mathcal{C}| = M$.

This problem is actual, in particular, in data analysis.

In this work we present a pseudopolynomial algorithm which finds an optimal solution for the case of integer components of the points in the input set and fixed space dimension. The running time of the algorithm is $\mathcal{O}(N(MD)^q)$, where D is the maximum absolute coordinate value of the points in the input set.

The authors were supported by the Russian Foundation for Basic Research (projects no. 15-01-00462, no. 13-07-00070).

References

1. *Kel'manov A.V., Pyatkin A.V.* "NP-hardness of Some Quadratic Euclidean 2-clustering Problems," Doklady Mathematics, 2015 (accepted).

NP-hardness of Quadratic Euclidean Balanced 2-Clustering Problem

Alexander Kel'manov[1], **Artem Pyatkin**[2]

[1] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* kelm@math.nsc.ru

[2] *Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia;* artem@math.nsc.ru

We consider the following problem. Given a finite set of points from Euclidean space find a partition of this set into two clusters minimizing the sum of weights of the clusters multiplied by their cardinalities. By the weight of the cluster we mean the sum of squared distances from the elements of the cluster to its center. The center of one cluster is given while for the second cluster it is unknown (so, it is estimated by the centroid, that is the mean value of the elements of the cluster).

Euclidean Balanced Variance-based 2-Clustering with Given Center of one cluster. *Given a set $\mathcal{Y} = \{y_1, \ldots, y_N\}$ of points from $\mathbb{R}^q$. Find: a partition of the set $\mathcal{Y}$ into two subsets $\mathcal{X}$ and $\mathcal{Z}$ such that $|\mathcal{X}| \sum_{x\in\mathcal{X}} \|x - \overline{x}(\mathcal{X})\|^2 + |\mathcal{Z}| \sum_{z\in\mathcal{Z}} \|z\|^2 \longrightarrow \min$, where $\overline{x}(\mathcal{X}) = \frac{1}{|\mathcal{X}|} \sum\limits_{x\in\mathcal{X}} x$ is the centroid of subset $\mathcal{X}$.*

This problem is actual, in particular, for solving problems in data analysis. The complexity status of this problem was open.

We prove [1] that this problem (1) is strongly NP-hard and (2) do not admit FPTAS, unless P=NP. The proof is based on polynomial-time reduction from the strongly NP-hard *Minimum Bisection* problem on cubic graphs.

The work was supported by the the Russian Foundation for Basic Research (projects no. 15-01-00462, no. 15-01- 00976, no. 13-07-00070).

References

1. *Kelmanov A.V., Pyatkin A.V.* "NP-hardness of Some Quadratic Euclidean 2-clustering Problems," Doklady Mathematics, 2015 (accepted).

An example of multicriterial optimization in a pseudo-metric space of criteria using the set of equivalence method

Ruben.V. Khachaturov[1]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* `rv_khach@yahoo.ie`

Advantages of the method of finding the set of equivalence [1, 2] for solving the multicriterial discrete optimization problems comparing to the method of finding the Pareto efficiency set are shown. An example of a set of key indicators of economic efficiency of any commercial enterprise is given and the corresponding mathematical model is formulated. In contrast to the classical problem of finding the maximum profit for any business, in this paper a multicriterial optimization problem [3, 4] is considered. In order to find the best enterprise business project, the method for solving the inverse multicriterial problems in a multidimensional pseudo-metric space is described. The solution of a concrete problem of this type is given.

The spacial distributions of optimal sets for each criterion and of the set of equivalence are shown on fig.1 and fig.2.

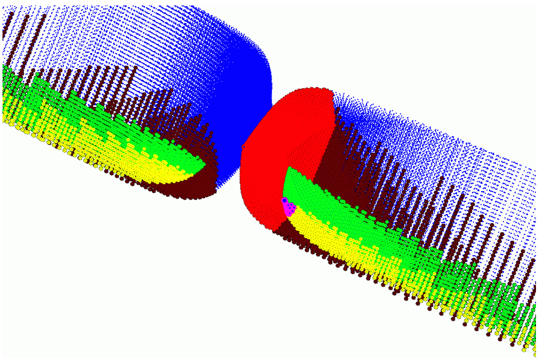

Fig. 1

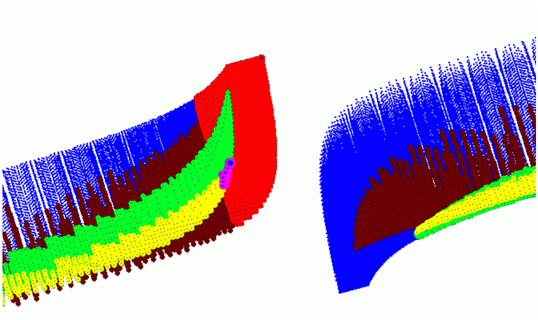

Fig. 2

References

1. *V.R. Khachaturov.* Mathematical Methods of Regional Programming, Economic-mathematical library, Nauka, Moscow (1989).
2. *V.R. Khachaturov, V.E. Veselovskiy, A.V. Zlotov, et al.* Combinatorial Methods and Algorithms for Solving the Discrete Optimization Problems of the Big Dimension, Nauka, Moscow (2000).
3. *R.V. Khachaturov.* Direct and Inverse Problems of Determining the Parameters of Multilayer Nanostructures from the Angular Spectrum of the Intensity of Reflected X-rays. Journal of Computational Mathematics and Mathematical Physics, Vol. 49, No. 10, pp. 1781-1788 (2009).
4. *R.V. Khachaturov.* Direct and Inverse Problems of Studying the Properties of Multilayer Nanostructures Based on a Two-Dimensional Model of X-Ray Reflection and Scattering. Journal of Computational Mathematics and Mathematical Physics, Vol. 54, No. 6, pp. 984-993 (2014).

Software for regional design

V.R. Khachaturov[1], **A.V. Zlotov**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* vladimir.khachaturov@rambler.ru

[2] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* zlot_a@mail.ru

In the Department of design techniques for developing systems of Computing Centre of RAS was developed methods, algorithms and programs for solving complex of problems of regional planning [1]. These problems arise primarily in the development of new regions of Siberia and the Far East. This includes the placement of infrastructure facilities for various purposes: collection and processing of natural resources, settlements of different categories, industrial facilities for various purposes, the objects of social infrastructure. At the same time it is possible to solve problems of designing circuits communications for various purposes: roads of different categories, pipe networks for various purposes, electricity grids of different voltage levels, communications and other infrastructure networks. When designing networks for different purposes it is possible to consider the heterogeneity of the territory in which they are held, and to solve the tasks of tracing the individual parts of the network. It is possible to use different forms of representation of the territory to define categorizing grid site or to set exclusion zones of various configurations for carrying out communications. It is also possible to take into account agglomeration effects from the integration of the various networks within the same corridor.When designing pipe networks, problems of hydraulic and optimization calculations are solved to select the optimal diameter of individual links of the network under constraints on the allowable pressure drop and with possibility to accommodate the necessary compressor or pumping stations. These tasks are solved also in the design of utility networks for various purposes. CPRP was tested in the design of the General schemes of arrangement of various oil and gas fields. Thus, its use has significantly reduced the capital cost of construction compared with traditional design methods. A brief descriptions of the main systems of CPRP are given below.

The design system of General schemes of development of oil and gas fields. The system is intended for the design of master plans as a separate technology systems of the arrangement well pad wells, gathering and transportation of oil and associated gas, reservoir pressure maintenance, electricity and roads), and all of them together and enables the designer mode of the computer to get the "real" construction projects. The system allows: to determine the best variants of draft of the General scheme of technological systems of arrangement, and options which are close to optimal according to various criteria; to form the structure of networks for different purposes: gathering and transportation of oil and associated gas, conduits of high and low pressure, roads, electricity; to determine the routes of communications for various purposes taking into account the heterogeneity of the territory and the presence of exclusion zones for their conduct; to calculate the optimal parameters of pipe networks: the diameters of individual sections of the network and the pressure in the network nodes; to simulate all possible solutions to find a real project in the "designer - computer" regim.

The system for allocation of objects and communications. The system is designed for solving the placement of items on the processing of raw materials, problems of construction and routing communications between the sources of raw materials, processing and consumption and the calculation of the pipe networks. The dialog mode of the system allows you to quickly view the various accommodation options with the identification and viewing of parameters of the generated project. System of network analysis and design. The system allows to solve the following network tasks: network distribution problem; the problem of the maximum flow of minimum cost; the task of designing the structure of the network; hydraulic calculations of networks.

The system of planning production, storage, transportation and distribution of petroleum products. The system allows to solve the following tasks: planning the production of various types of petroleum products; planning of petroleum products transportation by different modes of transport; calculation of storage volume in the tank farms; dynamic scheduling of production and distribution of petroleum products; calculation of the volume of supply to consumers.

References

1. *V.R. Khachaturov, A.V. Zlotov, A.N. Solomatin et al.* Planning and design of development of oil and gas producing regions and deposits. Mathematical methods, models, use. URSS, Moscow (2015).

Polynomial Time Approximation Scheme for Euclidean Minimum-weight k-Size Cycle Cover Problem in $\mathbb{R}^d$

Mikhail Khachay[1], **Katherine Neznakhina**[1]

[1] *Krasovsky Institute of Mathematics and Mechanics, Ekaterinburg, Russia;*
mkhachay@imm.uran.ru, eneznakhina@yandex.ru

For a fixed natural number k, the Minimum-weight k-Size Cycle Cover Problem (Min-k-SCCP) is studied. The problem can be treated as some generalization of the Traveling Salesman Problem (TSP) and some special case of the Vehicle Routing Problem (VRP) simultaneously. Min-k-SCCP has the following mathematical statement. For a complete weighted digraph (with loops), it is required to find a partition $\{C_1, ..., C_k\}$ of the graph by vertex-disjoint cycles having the minimum total weight.

It is known, that Min-k-SCCP is strongly NP-hard both in the general case and in special metric and Euclidean settings [1, 2]. There is 2-approximation polynomial time algorithm for the metric subclass of Min-k-SCCP [1]. For Euclidean Max-k-SCCP asymptotically correct algorithm was developed.

We extend result obtained for Euclidean Min-2-SCCP on the plane to cases of arbitrary fixed value of parameter k and d-dimensional space.

Theorem 1. *Min-k-SCCP in $\mathbb{R}^d$ has PTAS finding $(1+\frac{1}{c})$-approximation solution in $O(2^k n^{d+1}(k\log n)^{O(\sqrt{d}c)^{d-1}})$ time.*

Provided that $d = 2$, time complexity of the proposed PTAS for the Min-k-SCCP equals the complexity of PTAS proposed in [2] for the Euclidean Min-2-SCCP and it differs from it by the constant factor $2^k k^{O(c)}$.

The authors were supported by the Russian Scientific Foundation (project no. 14-11-00109).

References

1. *M. Khachay, K. Neznakhina* "Approximation of Euclidean k-size cycle cover problem," Croatian Operational Research Review, **5**, 177–188 (2014).
2. *M. Yu. Khachai, E. D. Neznakhina* "Approximability of the Problem about a Minimum-Weight Cycle Cover of a Graph," Doklady Mathematics, **461**, No. 6, 644–649 (2015).

Dynamic multi-criteria problem: method of solutions

Elena Khoroshilova[1]

[1] *Lomonosov Moscow State University, Moscow, Russia;*
khorelena@gmail.com

We study a dynamic model that describes the process of transition of a controlled object from the initial state $x_0 \in \mathrm{R}^n$ at the time t_0 in the terminal state $x(t_1) = x_1^*$ at t_1. For example, a group of people united by a common goal to implement a joint project can act as a object of control. The dynamics of the process is described by a controlled system of linear equations

$$\frac{d}{dt}x(t) = D(t)x(t) + B(t)u(t), \;\; t_0 \leq t \leq t_1, \; x(t_0) = x_0, \tag{1}$$

$$x(t_1) = x_1^* \in X_1 \subseteq \mathrm{R}^n, \; u(\cdot) \in \mathrm{U}, \tag{2}$$
$$\mathrm{U} = \{u(\cdot) \in \mathrm{L}_2^r[t_0, t_1] \mid \|u(\cdot)\|^2_{\mathrm{L}_2^r} \leq \mathrm{C}\},$$

where x_1^* is a component of the solution to the problem of multicriteria equilibrium:

$$\langle \lambda^*, f(x_1^*) \rangle \in \mathrm{Min}\{\langle \lambda^*, f(x_1) \rangle \mid x_1 \in X_1\}, \tag{3}$$
$$\langle \lambda - \lambda^*, f(x_1^*) - \lambda^* \rangle \leq 0, \quad \lambda \geq 0. \tag{4}$$

Here $f(x_1) = (f_1(x_1), f_2(x_1), ..., f_m(x_1))$ is a vector criterion; $f_i(x_1)$, $i = 1, 2, ..., m$, are convex scalar functions. The boundary value problem (3), (4) is a two-person game with Nash equilibrium. The solution of (1)-(4) is the set $(\lambda^*; x_1^*, x^*(\cdot), u^*(\cdot))$. Specifically, we are looking for a control $u^*(\cdot) \in \mathrm{U}$ such that the right end of the trajectory $x^*(\cdot)$ coincides with the component x_1^* of boundary value problem solution.

For the problem (1)-(4) we introduce a function analogous to the Lagrange function in convex programming problems:

$$\mathcal{L}(\lambda, \psi(t); x_1, x(t), u(t)) =$$

$$= \langle \lambda, f(x_1) - \frac{1}{2}\lambda \rangle + \int_{t_0}^{t_1} \langle \psi(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt}x(t) \rangle dt, \tag{5}$$

which is defined for all $(\lambda, \psi(\cdot)) \in \mathrm{R}_+^m \times \Psi_2^n[t_0, t_1]$, $(x_1, x(\cdot), u(\cdot)) \in X_1 \times \mathrm{AC}^n[t_0, t_1] \times \mathrm{U}$. Related approaches were considered in [1-2]. It is shown that the saddle point of the function (5) is a solution of (1)-(4).

To solve the problem, we use the dual extraproximal method [3]:

$$\bar{\lambda}^k = \mathrm{argmin}\left\{\frac{1}{2}|\lambda - \lambda^k|^2 - \alpha\langle \lambda, f(x_1^k) - \frac{1}{2}\lambda \rangle \mid \lambda \geq 0\right\},$$

$$\bar{\psi}^k(t) = \psi^k(t) + \alpha\left(D(t)x^k(t) + B(t)u^k(t) - \frac{d}{dt}x^k(t)\right),$$

$$(x_1^{k+1}, x^{k+1}(\cdot), u^{k+1}(\cdot)) = \operatorname{argmin}\left\{\frac{1}{2}|x_1 - x_1^k|^2 + \alpha\langle\bar{\lambda}^k, f(x_1) - \frac{1}{2}\bar{\lambda}^k\rangle\right.$$

$$+\frac{1}{2}\|x(t) - x^k(t)\|^2 + \frac{1}{2}\|u(t) - u^k(t)\|^2$$

$$\left. + \alpha\int_{t_0}^{t_1}\langle\bar{\psi}^k(t), D(t)x(t) + B(t)u(t) - \frac{d}{dt}x(t)\rangle dt\right\},$$

$$\lambda^{k+1} = \operatorname{argmin}\left\{\frac{1}{2}|\lambda - \lambda^k|^2 - \alpha\langle\lambda, f(x_1^{k+1}) - \frac{1}{2}\lambda\rangle \mid \lambda \geq 0\right\},$$

$$\psi^{k+1}(t) = \psi^k(t) + \alpha\left(D(t)x^{k+1}(t) + B(t)u^{k+1}(t) - \frac{d}{dt}x^{k+1}(t)\right),\ \alpha > 0.$$

The theorem on the convergence of the saddle-point method to the solution of the problem was proved.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-01-06045-a), and the Program for Support of Leading Scientific Schools (project no. NSh-4640.2014.1.)

References

1. *A.S. Antipin, E.V. Khoroshilova.* "Linear programming and dynamics," in: Trudy instituta matematiki i mekhaniki UrO RAN, Yekaterinburg. **19**, No. 2. 7–25 (2013)
2. *A.S. Antipin, E.V. Khoroshilova.* "Optimal Control with Connected Initial and Terminal Conditions," Proceedings of the Steklov Institute of Mathematics, **289**, Suppl. 1, S1–S17 (2015).
3. *A.S. Antipin.* "Terminal Control of Boundary Models," Comput. Math. Math. Phys., **54**, No. 2, 257–285 (2014).

The transport informational Model for ecological Systems ("TIMES")

Liudmila Klochkova[1], **Vladimir Tishkin**[2]

[1] *KIAM of RAS, Moscow, Russia;* klud@imamod.ru
[2] *KIAM of RAS, Moscow, Russia;* v.f.tishkin@mail.ru

The Transport-Informational Model for Ecological Systems ("TIMES") [1-3] is a three-dimensional grid model designed to calculate the concentration of pollutants in wind field. The model calculates pollutants spread from several sources, that favorably distinguishes it from others earlier advanced models. The program complex "TIMES" offers a number of capabilities for simulation of technogenic nature processes for atmospheric monitoring of cities and large industrial objects to users. In the code all the 3 components of wind field have been taken into account, thus the wind field model represents a 3D one. Taking into account, that the particles method describes 3D advective processes, one can, for example, describe sedimentation in wind field and in gravity field. Description of admixtures spread over high and extensive mountain ranges and behind them has become possible. Deposition data is outputted into a separated file. A pollutant deposition from the ground layer for a period is calculated as total pollutant quantity, penetrated through the lower border of the ground layer for the period. The task for sources, located at any height, including ones above the mixing layer height, has been solved. An admixture spread in different atmosphere layers occurs according to different laws. In the mixing layer the admixture gets smeared along height quickly enough. Above the mixing layer (in the free atmosphere) the admixture spreads for long distances along wind, not getting smeared practically. The admixture penetration from the atmosphere boundary layer into the free atmosphere is determined by the turbulent diffusion coefficient vertical profile at present. The turbulent diffusion coefficient in the mixing layer is accepted in limits up to 160 m/s^2 and depends on height, and in the free atmosphere it is accepted as big as 1 m/s^2. The mixing layer height can vary in space and time. Processes, caused by differences of anemometric and geostrophic winds, can be described by two models. First, as an exponential dependence of the wind speed vector on altitude, and, secondly, by the Ekhman's equations system.

The purpose of development of the mathematical model for the forecast of contaminants spread in atmosphere is the creation of support means at decisions acceptance on the environment preservation in scales of a city or industrial building up, which can be combined with geoinformation systems of monitoring (GIS).

The package "TIMES" provides the solution of the contaminants spread process equations system in wind field and its graphic display. Presence of chemical reactions and other physical-chemical processes were taken into account at the contaminants spread diffusion-convectional model (transport model) construction. The initial data for mathematical modeling are the data, received from meteostations, located inside the calculation area. The wind speed, the mixing layer top border height and other physical values are determined at the meteostations.

Apart the wind speeds field and vertical and horizontal turbulence factors are determined. The typical height of the mixing layer, within the limits of which there is intensive pollution transport, varies from 250 m to 2 km.

As far as the package has modular architecture, the impurities spread process physical model change is possible, that will certainly affect the general character of the received picture of space and time distribution.

The authors were supported by the Russian Foundation for Basic Research (Grant RFFI No 14-01-00145).

References

1. *L.V. Klochkova, D.V. Suzan, V.F. Tishkin.* "Numerical methods for description of convective transport of toxic substances in mathematical prediction of the quality of environment in a megalopolis," Journal of Computer and Systems Sciences International, **48**, No. 4, 659–671 (2009).
2. *L.V. Klochkova.* "Times: software tools for solving topical ecological problems in operation of aerospace systems," Journal of Computer and Systems Sciences International, **45**, No. 1, 64–72 (2006).
3. *L.V. Klochkova, D.V. Suzan, V.F. Tishkin.* "Mathematical modeling of exehaust gas tracks at aerospace vehicles flights in atmosphere," in: Combustion and atmospheric Pollution, Publisher, Moscow. Torus Press, 2003, pp. 595–598.

Integrality gap for Hitting Set problem on geometric graphs

Konstantin Kobylkin[1]

[1] *Krasovsky Institute of mathematics and mechanics, Ekaterinburg, Russia;*
kobylkinks@gmail.com

We consider the following geometric problem from robust network design: given a straight-line drawing of a graph $G = (V, E)$ on the plane, find the set $\mathcal{L}$ of straight lines of minimum cardinality such that each edge of E is intersected by some line from $\mathcal{L}$. This setting arises naturally for design of computer networks facing geographically localized attacks [1] where the network is of the form of planar graph while network failures are modeled as straight lines: graph edges which given straight line intersects considered broken. If G is complete bipartite, we come to known NP-hard minimum polyhedral separability problem [2,5]. In this work we analyze problem inapproximability: we prove that its integrality gap, i.e. the ratio of optima for the problem integer programming formulation and its LP-relaxation, exceeds any given constant which in some way extends MAX-SNP-hardness results of [3]. Our problem could be stated in the form of known HITTING SET problem as follows:

Problem. Given a set X_0 and a family $\mathcal{R}_0$ of its nonempty subsets, find $C \subset X_0$ which intersects each $R \in \mathcal{R}_0$ of minimum cardinality, where X_0 is the set of straight lines passing (nearly) through two distinct points from V while each set from $\mathcal{R}_0$ coincides with the set of lines from X_0 intersecting given edge of E.

Set $X_0 = \{x_1, \dots, x_n\}$ and $\mathcal{R}_0 = \{R_1, \dots, R_{|E|}\}$ for $n = 2|V|(|V| - 1)$. Let $\mathbf{X} = [\mathbf{x}_{ij}]$ be $\{0, 1\}$-matrix such that $\mathbf{x}_{ij} = 1$ iff $x_j \in R_i$. Our problem has a form of integer linear program:

$$\min\,(e_n, u) \tag{1}$$

$$\mathbf{X}u \geq e_{|E|},\ u \in \mathbb{R}^n_+ \cap \mathbb{Z}^n \tag{2}$$

where $e_m = (1, \dots, 1)^T \in \mathbb{R}^m$ for $m \in \mathbb{N}$ and $(\cdot, \cdot)$ denotes scalar product.

Consider also the following LP-relaxation:

$$\min\,(e_n, u) \tag{3}$$

$$\mathbf{X}u \geq e_{|E|},\ u \geq 0. \tag{4}$$

D e f i n i t i o n. Let k^* and k_f^* be the optima for (1) – (2) and (3) – (4) respectively. We call by *integrality gap* the supremum of ratio k^*/k_f^* over all instances of the Problem.

Let $w(\cdot)$ be inverse of Ackermann function. Using results of [4] and applying technique from [5] we come to the following

Theorem. *Integrality gap for the Problem is* $\Omega(w(f(|E|)))$ *where* f *is some monotonically increasing function.*

This can be considered as an extension of MAX-SNP-hardness results from [3] which claim the absence of PTAS for the Problem in a sense that it is unlikely to approximate the Problem in polynomial time within better than a constant factor at least based on (version of) relaxation (3) – (4). Note that those example graphs which provide claimed lower bound are in fact 2-colorable. Perhaps k-colorable graphs will give widely implied $\Omega(\log |V|)$ lower bound.

The author was supported by the Russian Scientific Foundation (project no. 14-11-00109).

References

1. *A. Efrat, E. Ezra, R. Pinchasi, S. Sankararamaan.* "Hitting Set Algorithms for Fast Data Recovery in the Face of Geographic Correlated Attacks," Proc. of 11th Int. Conf. on Design of Reliable Communication Networks. 2015.
2. *K. Kobylkin.* "Lower Bounds for the Number of Hyperplanes Separating Two Finite Sets of Points," Proceedings of the Steklov Institute of Mathematics, **289**, Suppl. 1, 126-138 (2015).
3. *M. Poberii.* "MAX-SNP hardness of MIN-PC and MASC-GP(n) problems," Pattern Recognition and Image Analysis, **21**, No. 2, 199-201 (2011).
4. *N. Alon.* "A Non-linear Lower Bound for Planar Epsilon-nets," Discrete and Computational Geometry, **47**, No. 2, 235-244 (2012).
5. *N. Megiddo.* "On the Complexity of Polyhedral Separability," Discrete and Computational Geometry, **3**, No. 1, 325-337 (1988).

A mathematical model of franchisee income

Ivan Kotliarov[1]

[1] *National Research University Higher School of Economics, St. Petersburg, Russia;* ivan.kotliarov@mail.ru

According to [1] franchisee revenue can be modeled according to the following formula:

$$\pi = \alpha e_f + \beta e_F + \varepsilon,$$

e_f — franchisees individual efforts (outlet management etc);
e_F — franchisors individual efforts (franchisee selection, franchise brand management, general marketing strategy for franchise network, franchisee training etc);
ε — a random variable with mean 0;
α, β — multiplicators that show productivity of respective efforts.

There are two problems with this model:
– It supposes no cooperation between franchisee and franchisor;
– Effectiveness of franchisors efforts does not depend on franchisee effectiveness.

Let us try to modify this model in order to eliminate these problems:

$$\pi = m\bar{P} + M\bar{\Pi} + \varepsilon, \tag{1}$$

m — franchisees individual entrepreneurial abilities (as an independent businessman);

M — franchisees managerial abilities (as a manager of franchise sales point);

$\bar{P}$ — average income of an independent businessman (working in the same region and with the same product as the franchise network). Independent businessman will get this income even without franchisors intellectual capital;

$\bar{\Pi}$ — average additional effect obtained by the franchisee thanks to franchisors intellectual property (franchisors brand value);

Obviously, $\bar{\Pi} = \bar{\pi} - \bar{P}$, where $\bar{\pi}$ — the average income of the franchisee.

The formula (1) clearly shows that:
– The franchisee income depends not only on franchisors brand value , but also on franchisees ability to use this brand potential;

– Franchisors are interested in selecting franchisees with high abilities as both independent and franchise managers (which is supported by empirical data

[2]);

– Franchisees with higher value of M (than average) may get a lower royalty rate (as their higher income is based mostly on their managerial abilities) which leads to non-linear range of royalty rates (this is also supported by empirical data);

– Franchisor may choose to invest either in franchisee selection or training (in order to increase m and M) or in his intellectual property (in order to increase brand value $\bar{\Pi}$).

His decision will depend on relative effectiveness of these investments: if

$$\frac{dm}{dr}\overline{P} + \frac{dM}{dr}\overline{\Pi} > m\overline{P} + M\frac{d\overline{\Pi}}{dr},$$

r — resources invested, then franchisor should invest into franchisee selection and training, otherwise he should invest in his brand value. Empirical data show that for mature networks franchisors generally invest into franchisee selection and training [3].

So the proposed model gives an adequate representation of the franchisee income from economical and mathematical points of view.

References

1. *R.D. Blair, F. Lafontaine.* The Economics of Franchising, Cambridge University Press, New York (2005).
2. *J.M. Ramirez-Hurtado, F.J. Rondan-Cataluna., F.M. Guerrero-Casas, J.M. Berbel-Pineda.* “Identifying the franchisee profiles franchisors prefer,” Journal of Business Economics and Management, **12**, No. 4, 567–588 (2011).
3. *T. Hossain, S. Wang.* “Franchisor’s Cumulative Franchising Experience and Its Impact on Franchising Management Strategies,” Journal of Marketing Channels, **15**, No. 1, 43–69 (2008).

Smoothing the frontier in the DEA models

Vladimir Krivonozhko[1], **Finn Førsund**[2], **Andrey Lychev**[3]

[1] *National University of Science and Technology "MISiS", Moscow, Russia;* `KrivonozhkoVE@mail.ru`

[2] *University of Oslo, Oslo, Norway;* `F.R.Forsund@econ.uio.no`

[3] *National University of Science and Technology "MISiS", Moscow, Russia;* `Lytchev@mail.ru`

Every mathematical model is just an approximation of the real-life processes and phenomena. In our previous papers [1, 2] we introduced the notion of terminal units. Moreover, we established relationships between different sets of units that may cause inadequacies in the DEA models (terminal units, different sets of anchor units, exterior units) [3-5]. It was also proved that only terminal units form necessary and sufficient set of units that were introduced in the DEA literature in order to smooth the efficient frontier.

In this paper, the approach is proposed for smoothing the frontier in the DEA models, which is based on using the set of terminal units as a starting point.

Our theoretical results are verified by computational experiments using real-life data sets and also illustrated by graphical examples.

The authors were supported by the Russian Foundation for Basic Research (project no. 14-07-00472).

References

1. *V.E. Krivonozhko, F.R. Førsund, A.V. Lychev.* "Terminal units in DEA: definition and determination," Journal of Productivity Analysis, **42**, 151–164 (2015).
2. *V.E. Krivonozhko, F.R. Førsund, A.V. Lychev.* "On comparison of different sets of units used for improving the frontier in DEA models," Annals of Operation Research, DOI: 10.1007/s10479-015-1875-8 (2015).
3. *M.-L. Bougnol, J.H. Dulá.* "Anchor points in DEA," European Journal of Operational Research, **192**, 668–676 (2009).
4. *E. Thanassoulis, M. Kortelainen, R. Allen.* "Improving envelopment in Data Envelopment Analysis under variable returns to scale," European Journal of Operational Research, **218**, 175–185 (2012).
5. *E. Thanassoulis, R. Allen.* "Simulating weight restrictions in data envelopment analysis by means of unobserved DMUs," Management Science, **44**, 586–594 (1998).

Numerical Simulation of a Surface Dielectric Barrier Discharge

V.M. Krivtsov[1,2], V.R. Soloviev[2]

[1] *Dorodnicyn Computing Center RAS, Moscow, Russia;* `krivtsov@ccas.ru`
[2] *Moscow Institute of Physics and Technology Dolgoprudnii, Russia;*

The barrier discharge is a discharge between two electrodes of which at least one is coated with dielectric to prevent the conduction current flowing from the discharge zone to its surface. This type of discharge develops either as a surface or as a volume one. A volume barrier discharge develops in a gas layer between the dielectric-coated electrodes. A surface barrier discharge (SDBD) between two electrodes having different widths and separated by a dielectric develops immediately along the dielectric surface. The electrode of smaller width, to which the voltage is applied and near whose edge the discharge develops, will be called a high-voltage, or working, electrode. The potential of the other electrode will be assumed to be zero.
Intense analysis of a boundary-layer and separation flow control using surface dielectric barrier discharge (SDBD) actuators continues during last ten years [1-3]. SDBD has a complex structure and evolves as a streamer and/or diffusive discharge depending on applied voltage polarity and its waveform. It consists of a set of tens microdischarges repeating each half cycle of alternating applied voltage. Numerical modeling of this bulk of microdischarges is extremely time-consuming problem. Because of this complexity, the main gap in theoretical and numerical predictions of SDBD aerodynamic performance is a lack of adequate physical and numerical modeling of its evolution in air.
In the report the analysis of various numerical approaches of modeling of this difficult phenomenon is given. Results of modeling and comparison with experimental data are given.

References

1. *E.Moreau* "Airflow control by non-thermal plasma actuators" J. Phys. D: Appl. Phys. 40:605-636. 2007
2. *Soloviev, V.R., Krivtsov V.M..* " Surface barrier discharge modeling for aerodynamic applications" Phys. D: Appl. Phys. 42 : 125208. 2009.
3. *T.C.Corke, M.L Post, D.M. Orlov* " Single dielectric barrier discharge plasma enhanced aerodynamics:physics, modeling and applications" Exp.Fluids 46: 1-26. 2009

Controlled systems of measure dynamics

Oleg Kuzenkov

Lobachevsky State University of Nizhni Novgorod, Nizhni Novgorod, Russia;
kuzenkov_o@mail.ru

The optimal control problem is considered in Banach space of measures. The study of measure dynamics has continued since the appearance of the basic works of L. Schwartz, who introduced the notions of the generalized function or distribution and considered differential equations in distributions or measures. These equations are often used for the description of various nature processes, for example, stochastic processes, processes of mathematical physics or quantum mechanics and others. Physical laws are often formulated concerning density of distribution (weight, charge, etc). To consider the dynamics of the distributed and concentrated objects, equations for measures are required. The problem of optimum control for such processes is of great importance.

In this work, the general principles of solving optimal control problem for a measure dynamics are presented. The general form of the equation of a probability measure dynamics is obtained. Necessary and sufficient conditions for the optimality of the probability measure dynamics are deduced. On the basis of these principles feedback control has been suggested for satisfying the state constraint in the form of equalities or inequality. Various applications of the theory to specific optimization problems in mathematical physics are considered. In particular, optimal control problems of heat conductivity with the state constraints as equality and ineqality are considered. Feedback is constructed by using bilinear control and integral transformation. Numerical solution is proposed on the basis of the method of moments. The efficiency of the proposed solution is confirmed by a numerical experiment.

The reported study was partially supported by the Russian Foundation for Basic Research (project no. 13-01-12452 ofi_m2).

References

1. *O.A. Kuzenkov, A.V. Novozhenin.* “Optimal control of measure dynamics,” Nonlinear Science and Numerical Simulation, **23**, No. 1–3, 159–171 (2015).

Exact penalty functions and convex extensions of functions in decomposition schemes in variables

Yuriy Laptin[1]

[1] *V.M.Glushkov Institute of Cybernetics of the NAS of Ukraine, Kyiv, Ukraine;* `laptin_yu_p@mail.ru`

The use of exact penalty functions in decomposition schemes in variables to solve block optimization problems allows to overcome some problems related with implicit description of the feasible region in the master problem. How to find proper penalty coefficients is discussed here. Let consider a problem

$$f^* = \min \{f_0(x) : x \in C\}, \tag{1}$$

where $C = \{x : \ f_i(x) \le 0, \ i = 1, ..., m, \ x \in R^n\}$, $f_i : R^n \to R$ is convex function with finite values for all values of variables , $i = 0, ..., m$.

Let $F_\lambda(x) = f_0(x) + \lambda h^+(x)$, where $h(x) = \max\{f_i(x), i = 1, ..., m\}$, $h^+(x) = \max\{0, h(x)\}$,

$$F^*_\lambda = \min\{F_\lambda(x) : x \in R^n\}. \tag{2}$$

$F_\lambda(x)$ is an exact penalty function, if the solutions of problems (1) and (2) coincide.

Lemma 1. *Let C be a closed set, values of penalty coefficients are fixed, $\varepsilon > 0$, and a sequence of points $x_k, k = 1, 2, ...$ converging to a solution $\tilde{x}$ of the problem (2) be given. Let a rule P establishing a correspondence between x_k and a point $z_k = P(x_k), z_k \in C, k = 1, 2, ...$ be given, and the following inequalities are fulfilled*

$$F_\lambda(x_k) \ge f_0(z_k) + \varepsilon \|x_k - z_k\|, \ \ if \ \ the \ point \ \ \ x_k \notin C \tag{3}$$

Then $\tilde{x} \in C$.

For a given point $x \notin C$ and the rule $P : R^n \to C$ let we denote $\lambda_P(x, \varepsilon) = \max(0, (f_0(z) + \varepsilon\|x - z\| - f_0(x))/h^+(x))$, where $z = P(x)$, $\lambda_P(\varepsilon) = \sup\{\lambda_P(x, \varepsilon), x \notin C\}$.

Let some converging algorithm A be used to solve the problem (2). For the exact penalty coefficients are not known in advance, their values will be specified (increased) in the course of the algorithm. Let λ_k be a value of the coefficient λ at the iteration k. For $k = 1$ the value $\lambda_1 > 0$ is given. The algorithm A uses the value λ_k to find the point x_k at the iteration k. If in the point x_k the inequality (3) is fulfilled at $\lambda = \lambda_k$, let put $\lambda_{k+1} = \lambda_k$, otherwise $\lambda_{k+1} = \lambda_P(x_k, \varepsilon) + R$, where $R > 0$ is a fixed given parameter.

Relations (3) are fulfilled, if $\lambda > \lambda_P(\varepsilon)$, and the number of corrections for the coefficients λ_k will be finite, if $\lambda_P(\varepsilon) < \infty$.

The use of penalty functions with too high values of penalty coefficients leads to problems related to rounding errors, worsening of the convergence of optimization algorithms. For this reason the value $\lambda_P(\varepsilon)$ is an important characteristics of the rule P.

For $x \notin C, y_0 \in C$ we denote $\pi_C(x, y_0)$ the point of intersection of a segment $[x, y_0]$ with a border of the set C.

Theorem 1. *Let a set C be bounded, a function f be Lipshitz continuous on C, and a point $y_0 \in C$, $h(y_0) < 0$, $P(x) = \pi_C(x, y_0)$ be given for $x \notin C$. Then $\lambda_P(\varepsilon) < \infty$.*

The rule $P(x) = \pi_C(x, y_0)$ gives rather efficient procedures to specify the penalty coefficient, but under unfortunate choice of the point y_0 the values of penalty coefficients may become rather large.

The ways to improve the rule P are considered. An analogous approach for finding penalty coefficients of the function $\Phi_\beta(x) = f_0(x) + \sum_{i=1}^m \beta_i f_i^+(x)$, and application in decomposition schemes in variables for block convex programming problems are also discussed. The proposed approaches do not require complex solutions of auxiliary problems. In the case when the functions of the initial problem are not defined on the whole space of variables, it is proposed to use convex extensions of functions.

References

1. *Laptin Yu.P.* “Decomposition in variables in some optimization problems,” Cybernetics and Systems Analysis, No. 1, 98–104 (2004).
2. *Laptin Yu.P., Lykhovyd A.P.* “The use of convex extensions of functions to solve nonlinear optimization problems,” Control Systems and Computers, No. 6, 25–31 (2010).
3. *Laptin Yu.P., Bardadym T.A.* “Some approaches to regularization nonlinear optimization problems,” Problems of control and informatics, No. 3, 57–68 (2011).
4. *Laptin Yu.P.* “Questions of construction of exact penalty functions,” Proc. St.-Petersb. Univ. Ser. 10: Applied mathematics, No. 4, 21–31 (2013).

The Solution of the Problem of Personal Computers Park Updating on the Basis of Modelling

Tatyana Lebedeva[1], **Tatyana Tyupikova**[2]

[1] *SUM, Moscow, Russia;* t-leb@ya.ru
[2] *JINR, Dubna, Russia;* tanya@jinr.ru

One of the directions of increase of efficiency of activity of the enterprise is the solution of the problem of effective management of its production assets which assumes investment of money into modernization of production and replacement of old assets by the new. There are various approaches to the solution of this problem of replacement of the equipment; the choice of approach is defined by concrete features of the technologies used in production. Relevance of this task in relation to updating of park of computer and office equipment is defined by its fast moral obsolescence that in big degree is connected with prompt improvement of technical characteristics.

For the solution of a problem of updating and replacement of computer and office equipment the approach based on work [1] is used. In this work elements of the theory of wear and replacement of the equipment are stated.

The developed mathematical model allowed to construct algorithm of definition of sequence of the replaced equipment and on its basis to receive computable estimates. On the basis of the considered model the automated information system program for the accounting of computers at the enterprise is realized in practice. The computer program allows to resolve different issues of the account in a complex; it included the accounting of the equipment, the accounting of demands of users and suppliers, the accounting of the equipment which is under repair and many other things the developed system switched on the block of support of making decision on replacement of equipment. The program is developed for the accounting of data on computers, accessories and office equipment, helps with carrying out inventory, allows to consider repair and service, to form budgets, to do demands to suppliers, to keep demands of users and to carry out some other important functions.The created information system is realized in the scientific organization; it can be applied at manufacturing enterprises.

The used approach is realized on a platform 1C:Enterprise 8.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-06-00389).

References

1. *A. Kaufmann, R. Faure.* Invitation a la recherche operationnelle. DUNOD, Paris (1963).

Application of Edgeworth-Pareto Hull approximation in block separable nonlinear multi-objective problems

Alexander Lotov[1], **Elena Zasukhina**[2]

[1] *Russian Academy of Sciences, Dorodnicyn Computing Centre, Moscow, Russia;* avlotov@gmail.com

[2] *Russian Academy of Sciences, Dorodnicyn Computing Centre, Moscow, Russia;* elenzet@bk.ru

Visualization of the Pareto frontier is used as a decision support in the case of multi-objective problems [1]. It helps the decision maker to study the Pareto frontier and the related objective tradeoffs. To implement such a technique in the case of more than two objectives, approximating the Edgeworth-Pareto Hull (EPH) of the feasible objective set is used. It helps to visualize the Pareto frontier on-line by displaying various collections of bi-objective slices of the EPH.

Herein, nonlinear multi-objective problems with the block separable structure are studied. Namely, a two-level system is considered, which consists of the lower level (subsystems) and of the upper level that is used for coordunating the decisions. It is assumed that the objectives are related to the variables of the upper level.

The EPH approximation method is based on the preliminary approximating the EPH's for the subsystems and on using the constructed approximations in the process of approximating the EPH for the whole problem [2]. The approximation technique was applied in the framework of the search for smart decisions of environmentally sound water management of the Volga River.

The authors were partially supported by the Russian Foundation for Basic Research (projects no. 13-01-00235 and no. 13-01-00779).

References

1. *A.V. Lotov, V.A. Bushenkov, G.K.Kamenev.* Interactive Decision Maps. Approximation and Visualization of Pareto Frontier, Kluwer, Boston (2004).
2. *A.V. Lotov.* "Decomposition of the Edgeworth-Pareto Hull approximation problem," Computational Mathematics and Mathematical Physics, **55**, No. 10 (2015).

Identification of linear dynamic systems on the harmonic signal

Elena Madaeva[1], **Arsalan Mizhidon**[2]

[1] *East Siberia State University of Technology and Management , Ulan-Ude, Russia;* elenamadaeva@gmail.com

[2] *East Siberia State University of Technology and Management , Ulan-Ude, Russia;* miarsdu@esstu.ru

One of the main stages implementing technology of mathematical modeling is the creation and identification of mathematical model the investigated object. Differentiate identification in the broadest sense - structural identification, in the narrow sense - parametric identification, assuming that the known structure and class of models describing real system.

In report are discuss an approach to the identification of linear stationary dynamical systems on results measurements of phase coordinates the system at the some interval time [1]. According to this approach, was developed a method of identification the linear systems on the input sinusoidal signal [2]. Suppose that the output variables of the system are solutions the certain stationary linear dynamic non-homogeneous Cauchy problem. Identification of the system following the approach is reduced to constructing and solving matrix linear algebraic equation. Constructing the equation is found by comparison of representation of solutions the Cauchy problem in the form of exponential matrix series and of results measurements of phase coordinates by the input sinusoidal signal.

For realization of the numerical experiments the software was made in the language Fortran. The results of numerical experiments showed that identification of system using approach allows on the exact solutions of the Cauchy problem reconstruct the system.

The authors were supported by the Russian Foundation for Basic Research and Republic of Buryatia (project no. 15-41-04020 r_siberia_a).

References

1. *A.D. Mizhidon, E.A. Madaeva* "About some approach to the identification of linear dynamic systems," Bulletin ESSUTM, **48**, No. 3, 5–12 (2014).
2. *A.D. Mizhidon, E.A. Madaeva* "Method of linear dynamic system identification by input sinusoidal action," Science Bulletin of NSTU, **58**, No. 1, 62–75 (2015).

Segmentation Methods of Speech Signal for Multimodal Speech Recognition

O. Mamyrbayev[1], Ye. Amirgaliev[1], M. Kalimoldayev[1], A.Keylan[2]

[1] *Institute of Information and Computational Technologies, Almaty, Kazakhstan;* `morkenj@mail.ru`

[1] *Institute of Information and Computational Technologies, Almaty, Kazakhstan;* `amir_ed@mail.ru`

[1] *Institute of Information and Computational Technologies, Almaty, Kazakhstan;* `mnk@ipic.kz`

[2] *Eurasian National University named after Gumilyov, Astana, Kazakhstan;* `keylan@live.jp`

Speech signal segmentation is one of the most important tasks in the field of computer science and information systems for processing and speech recognition. Segmentation of the speech signal is required for the feature extraction of the speaker's voice on certain segments of the speech signal and restore the shape of the vocal tract by acoustic-based, which can be used in speech synthesis of the input text and speech recognition [1].

In the studies, manual segmentation of speech can be used, but manual speech segmentation slows down and it is almost impossible to accurately reproduce the results of manual segmentation and it has many mistakes in speech recognition.

In information systems of speech recognition for speech signal segmentation is important:

1. the allocation of the basic elements (words, syllables, phonemes) of speech recognition
2. segmentation accuracy has a great impact on the optimal speech recognition

There are several basic types of automatic segmentation of the speech signal. One of these is speech segmentation, provided that the sequence of phonemes of the phrase, but the results are often unreliable, and the presence of transcription is possible only at the learning stage of lexical patterns [2].

The other type does not use a priori information of the speech, with the boundaries of speech segments are determined based on changes in the acoustic characteristics of the speech signal. It is desirable to use only the general characteristics of the speech signal with automatic segmentation , because usually at this stage there is no specific information about the content of the speech.

There is a "blind" segmentation method for simple segmentation of speech signal to pause and speech. This method is based on the magnitude and change rate of certain acoustic characteristics which is the ratio of the signal level transition through the zero (zero Cross Rate) and Spectral Transition Measure, but experiments show that it is not enough for a reliable segmentation of these values [3].

An income speech signal is recorded as a sequence of records of the y_i.

$$Y = y_0, y_1, \ldots, y_i, \ldots;$$

where $i = 0, 1, 2, \ldots$

The sequence of speech signal is divided into frames of 128 counts (respectively (128*1000)/11025 11ms). The frame size allows to pinpoint the boundaries between syllables. With the following formula we determine the average frame energy of a speech signal with length of 128 counts:

$$E_i = \frac{\sum_{j=i*128}^{i*128+127} y_i^2}{128}, where \;\; i = 0, 1, 2, \ldots \tag{1}$$

The value obtained by the formula (1.1) presents the average energy in a short span of time 11ms. Then we calculate the average energy of the short time of three adjacent plots by the formula:

$$E_i^* = \frac{E_i + E_{i+1}}{2}; where \;\; i = 0, 1, 2, \ldots \tag{2}$$

Thus, we calculate the average energy for frames 2*128=256. Frames are overlapping and shifting the adjacent intervals on 128 counts (Figure 1).

Figure 1. Splitting the speech signal into frames

The basic tone of the Kazakh language less than 256 / 11025 = 0.023 sec. which corresponds to the fundamental frequency of the /0.023 = $75.5Hz$. Therefore, the energy of the frame of 256 countdown concludes the energy of at least one period of the basic tone. Thus, from the sequence of speech signal $Y = y_0, y_1, , y_i, ;\ \ i = 0, 1, 2,$ we calculate the sequence of plots of the average energy in 192 counts $E^* = E_1^*, E_2^*, , E_i^*,$.

Each syllable has a syllable peak, where the signal energy reaches the highest values. Between the two syllabic peaks there is a point corresponding to the border that separates the syllables [4].

References

1. *W.P. McNeill, J.G. Kahn, D.L. Hillard, M. Ostendorf* "Parse Structure and Segmentation for Improving Speech Recognition," in: IEEE Spoken Language Technology Workshop, 2006. pp. 90–93.
2. *B. Petek, O. Andersen, P. Dalsgaard* "On the Robust Automatic Segmentation of Spontaneous Speech," in: Proceedings of ICSLP'96. 1996. pp. 913–916.
3. *D. Wang. L. Lu, H. J. Zhang* Speech segmentation without speech recognition, " International Conference on Multimedia and Expo (ICME 03). Vol. 1. 405-408. (2003).
4. *4. N.A.Baskakov* To the question of the classification of Turkic languages , " Proceedings of the Academy of Sciences of the USSR. The Department of literature and language. ., . 2. . . 121-134. (1952).

Optimal control and motion stability for technical systems described by differential equations with multiple-valued right parts

Olga Masina[1], **Olga Druzhinina**[2]

[1] *Yelets State University after I.A. Bunin, Yelets, Russia;* olga121@inbox.ru
[2] *Dorodnicyn Computing Center of RAS, Moscow, Russia;* ovdruzh@mail.ru

The problems of optimal control and motion stability of the technical systems modelled by the differential equations with multi-valued right parts play an important role when studying questions of qualitative analysis and optimization of dynamic systems [1–5]. Indicated problems are connected with decision-making in the conditions of uncertainty. Among them is the problem of freight movement by means of the aircraft when it is necessary to consider an uncertainty factor. Fundamental results in the area of optimization of dynamics of controlled systems are obtained in [6, 7] and in the other works. Existence and stability of differential inclusions are considered in [1, 2, 4, 8] and in the other works. Stability conditions and stabilization of the technical systems described by the differential inclusions containing control in the right parts are considered in [9, 10], where locally Lipschitz-continuous and regular Lyapunovs functions are used for solving of stability problem.

In the present work we consider model of controlled object which moves from the initial point in final with an achievement of the vertical direction. The specified system is described by the system of differential equations with multiple-valued right parts. The important aim of research is of stability of the nominal motion determined by criterion of optimality. The use of multi-valued right parts of the differential equations arises in view of resistance of the rarefied environment. We consider also the modifications of models described in [3, 5].

Stability analysis of differential equations with a multivalued is carried out by aid of results of works [1–4, 8–10]. The algorithm is developed for a control system on object motion to the purpose in three-dimensional space providing that the purpose moves randomly with a limited velocity on the plane. Algorithm of optimal control finding is offered. The set of programs is realized in the integrated mathematical environment is developed and series of computing experiments are carried out. This set of programs contains modules for data input, for a conclusion of results and for a graphic illustration of results. The program calculates draft force, coordinates and velocities of the control object.

The proposed algorithms and set of programs can be used for choice of suitable parameters of motion of the controlled object in the conditions of incomplete information.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-08-00710).

References

1. *A.A. Shestakov.* Generalized direct Lyapunov's method for systems with distributed parameters, URSS, Moscow (2007).
2. *Yu.N. Merenkov.* Stability-like properties of differential inclusions, fuzzy and stochastic differential equations, Russian University of Peoples' Friendship, Moscow (2000).
3. *O.N. Masina.* "Questions of motion control of transport systems", Transport: science, technique, control, No. 12, 10–12 (2006).
4. *O.V. Druzhinina, O.N. Masina.* Methods of stability research and controllability of fuzzy and stochastic dynamic systems, Dorodnicyn Computing Center of RAS, Moscow (2009).
5. *O.V. Druzhinina, O.N. Masina.* "Optimal control for technical systems modelled by differential inclusions", Proc. of V International conference on optimization methods and applications "Optimization and applications" held in Petrovac, Montenegro, 2014. Dorodnicyn Computing Center of RAS, Moscow, 2014, pp. 55–56.
6. *J. Varga.* Optimal control of the differential and functional equations, Translated from English, Moscow (1977).
7. *L.S. Pontryagin, V.G. Boltyansky, R.V. Gamkremidze, E.F. Mishchenko.* Mathematical theory of optimal processes, Science, Moscow (1961).
8. *O.N. Masina.* "On the existence of solutions of differential inclusions", Differential Equations, **44**, No. 6, 872–875 (2008).
9. *Bacciotti A.* "Stability and stabilization of discontinuous systems and nonsmooth Lyapynov functions", Rapporto Interno, **27**, 1–21 (1998).
10. *O.V. Druzhinina, O.N. Masina.* "Stability analysis and stabilization of discontinuous systems by the aid of generalized Lyapunov functions", Nonlinear World, **12**, No. 11, 10–20 (2014).

Numerical solution of shape optimization problem for elliptic elastic plates with hole

I.E. Mikhailov[1], **A.L. Skiba**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* mikh_igor@mail.ru
[2] *MIPT, Dolgoprudny, Moscow Region, Russia;* skiba_andrey@mail.ru

Control problems abroad for elliptic equations arise in various fields of mathematical physics [1], and in particular, in the theory of elasticity [2]. The mathematical formulation of such problems, called optimization problems form is to minimize in a bounded domain Ω of a functional $F(\Omega)$ on the solutions $u(x, y)$ of the boundary value problem for an elliptic equation.

In this part γ of the boundary $\partial\Omega$ is fixed, and the other part Γ (called free) can be changed in some class. Typically, the optimization problem is considered in the form of additional conditions, such as maintaining the area Ω. Existing approaches to the proof of existence of the solution of such problems under additional assumptions on the smoothness of the boundary Γ can be found in [3, 4], but they usually do not provide specific algorithms for finding optimal solutions.

In this report, we propose a numerical method for the solution of the following problem. As a region Ω we consider an ellipse with semi-long A and B, which is located inside the elliptic hole with semi-elliptical length a and b. Center of the outer ellipse may be shifted relative to the center hole by an amount d. Internal borders γ holes do not change. The outer boundary Γ can vary so that the area of the ellipse

$$S = \pi AB \tag{1}$$

is constant. Required to find the shape of the outer boundaries of the plate, the host deformation shift $u(x, y)$ under the action of a distributed load with density $\rho(x, y)$, which provides the minimum value of the functional

$$F(\Omega) = \iint\limits_{\Omega} \rho u \, dx \, dy, \tag{2}$$

characterizing the measure of the stiffness of the plate.

The dependence of $u(x, y)$ on the parameters A, d and conditions (1) shall be determined from the solution of the biharmonic equation

$$\Delta^2 u = \rho \tag{3}$$

in Ω with boundary conditions

$$u|_\gamma = \varphi(x, y); \quad u|_\Gamma = 0;$$

$$\left(\Delta u - \frac{1-\nu}{R}\frac{\partial u}{\partial n}\right)\bigg|_{\partial\Omega} = 0.$$

Here ν – Poisson's ratio, R – the radius of curvature of the boundary $\partial\Omega$ and n – normal to the boundary.

Prescribed boundary conditions equivalent hinging internal border at an altitude of $u = \varphi(x, y)$ relative hinged at the height of $u = 0$ external border.

A problem with a change of variables reduces to the problem with fixed boundaries and variable coefficients in the equation (3). Examples of calculations are presented.

This report is supported by Russian Foundation for Basic Research (project No. 15-01-07833).

References

1. *J.-L. Lions.* "Remark on the relationship between free surfaces and optimal control of distributed systems," in: Optimization Techniques. Part 1. Lectures Notes in Control and Information Science, **6**, Springer-Verlag, Berlin, 1978, pp. 28–40.
2. *N. V. Banichuk.* Shape Optimization of Elastic Bodies. Nauka, Moscow, 1980, 256 p.
3. *M. Goebel.* "Optimal control of coefficients in linear elliptic equations", Mat. operations gorsch. undstatistics ser optimiz., **12**, No. 4, 525–533 (1981).
4. *L.A. Muravey.* "The task of boundary control for elliptic equations," Vestnik of Moscow University. Ser.15. Computing Mathematic and Cybernetic, **3**, 7–13 (1998).

Estimation of functional optimal value for one control problem

Klara Mizhidon[1], **Arsalan Mizhidon**[2]

[1] *East Siberia State University of Technology and Management, Ulan-ude, Russia;* migka@mail.ru

[2] *East Siberia State University of Technology and Management , Ulan-Ude, Russia;* miarsdu@mail.ru

We propose an analytical and numerical method of estimation of the limit possibilities for one control system

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t), \quad x(t_0) = x^0, \quad t \geq t_0. \tag{1}$$

Here x is n-dimensional vector of state variables; u is r-dimensional control vector; A and B are constant matrices of dimensions $(n \times n)$ and $(n \times r)$ correspondingly; $f(t)$ is n-dimensional bounded continuous vector function of disturbances. Admissible controls $u(t)$ are piecewise continuous functions satisfying: $u'(t)R_j u(t) \leq 1, \quad (j = 1, 2, \ldots, p), \quad t \in [t_0, \infty)$. Let $u^*(t)$ be admissible control such that at the trajectories of the system (1) the minimum of functional

$$J(u(\cdot)) = \max_{t \geq t_0} \|x(t)\|^2 \tag{2}$$

is attained. It is required to estimate from below the optimal value of the functional (2). In other words, it is required to find $\bar{J}$ such that $\bar{J} \leq J(u^*(\cdot))$.

As the estimation of the functional limit (2) we take the optimal functional value of the problem:

$$\begin{cases} \dot{x} = Ax + Bu + f(t), \quad x(t_0) = x^0, \\ u'(t)R_j u(t) \leq 1, \quad (j = 1, 2, ..., p), \ t \in \ [t_0, t_1], \\ \|x(t)\|^2 \leq \|x(t_1)\|^2 \quad \forall t \leq t_1, \\ J_2(u(\cdot)) = \|x(t_1)\|^2 \to \min. \end{cases}$$

The authors were partially supported by the Russian Foundation for Basic Research and Republic of Buryatia (project no. 15-41-04020 r_siberia_a) and East Siberia State University of Technology and Management (Grant "Young scientists ESSTU").

Proximate time-optimal control of third-order integrators with phase constraints

Yury Morozov[1]

[1] *V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia;*
tot1983@inbox.ru

The equations describing the triple integrator are

$$\dot{x} = y,\ \dot{y} = z,\ \dot{z} = u, \tag{1}$$

where control law is defined by

$$u = -\mathrm{sign}(S(x, y, z)). \tag{2}$$

The system state X is defined as $X = (x, y, z)$, where x is the position, y is the velocity, z is the acceleration. For time-optimal control, the objective is to minimize the time required to transfer the system from an initial state $X_0 = (x_0, y_0, z_0) \in \Omega_0(x, y, z)$ to a final state $X_f = (0, 0, 0)$. Here $\Omega_0 \in R^3$ is attraction domain of X_f.

Time-Optimal control surface for the system given by (1)-(2) with no disturbances and no unmodeled dynamics is [1] as follows:

$$S_U^{opt} = x + \frac{z^3}{3U^2} + u_2\Big(\frac{yz}{U} + U^{-\frac{1}{2}}(u_2 y + \frac{z^2}{2U})^{\frac{3}{2}}\Big), \tag{3}$$

$$u_2 = y + \frac{z|z|}{2U} \tag{4}$$

We consider the same control problem, when the velocity and the acceleration are bounded($|y| < B$, $|z| < A$, $A, B > 0$). In this case, the proposed control law (2) with (3) does not guarantee the transfer the system from a sufficiently large neighbourhood of point X_f to point X_f. This occurs because velocity or acceleration reaches restrictions.

To solve this problem we can calculate new attraction domain $\Omega \in R^3$ of X_f for old control or construct new surface $S_U(x, y, z)$. we choose the second case.

If there is no restriction on the velocity ($B = +\infty$), the surface is easily modified to solve the problem. In this case, the surface is $S_U^{opt}(x, y, -A\ln(1 - z/A)\mathrm{sign}(z))$. We stretch the piece of the surface S_U^{opt} along the axis O_z.

If there is restriction on the velocity ($B < +\infty$) we can not move with a speed greater than a predetermined limit and can not influence the position x. So system (1) is equal

$$\dot{y} = z,\ \dot{z} = u(y, z), |y| < B, |z| < A \tag{5}$$

and the objective is to minimize the time required to transfer the system (5) from an initial state $(y_0, z_0) \in \Omega^*(y, z)$ to a final state $X_f^- = (-B, 0)$ or $X_f^+ = (+B, 0)$. The sign depends on the initial conditions. This problem can solve using the control law $u_y = -\mathrm{sign}(S_B(y, -A\ln(1 - z/A)\mathrm{sign}(z)))$, $S_B(y, \hat{z}) = y \pm B + \hat{z}|\hat{z}|/2$. May be show that, the attraction domain for this case is defined as $\Omega(y, z) = \{(y, z) : y \in [-B, B]; \min(-A, -\sqrt{(B - y)/2}) < z < \max(A, \sqrt{(B + y)/2})\}$.

So, The solution of the problem consists of two parts:

1. first, the system comes to the surface $S_B(y, z) = 0$,
2. second, the surface $S_U^{opt}(x, y, -A\ln(1 - z/A)\mathrm{sign}(z)) = 0$.

The attraction domain of point X_f is determined by the equation

$$\Omega(X) = \{(x, y, z) : x \in R; \\ y \in [-B, B]; \min(-A, -\sqrt{\frac{B - y}{2}}) < z < \max(A, \sqrt{\frac{B + y}{2}})\} \tag{6}$$

.

This work was supported by Russian Foundation for Basic Research, grant No. 13-01-00347 and the Department of Power Engineering, Mechanical Engineering, Mechanics, and Control Process of Russian Academy of Sciences (Program no. 1 "Scientific foundations of robotics and mechatronics").

References

1. *Lee, E. B., Markus, L.* Foundations of Optimal Control Theory. Wiley. 1967.

About market capitalization of low competitive company in the conditions of unstable demand

Nataliia Obrosova[1], **Alexander Shananin**[2]

[1] *MIPT, Dolgoprudnyi, CC RAS, Moscow, Russia;* nobrosova@ya.ru
[2] *MIPT, Dolgoprudnyi, CC RAS, Moscow, Russia;* alexshan@yandex.ru

Results of research of one model of production considering current assets deficit, instability of the sales channel of production and restriction of trade infrastructure (restriction from above on amount of one-timely implementable batch) are presented [1-4]. Motivation of research is attempt to analyze problems of functioning of low competitive macroeconomic structures in the conditions of unstable demand. The model allows to analyze production indicators dynamics as a result of the fluctuation in demand on production which can be caused, including, the competition to the import analogs. The model is formalized in the form of the Bellman equation for which a closed-form solution is found [2,3]. In terms of model the stochastic process of inventory change is described, its ergodicity is proved and final probability distribution is found. By analyzing the stochastic process expressions for the average production load and the average inventory are found [4]. The system of model equations related the model variables to official statistical parameters is derived. The model is identified using statistical data of the FIAT GROUP (FCA, Italy) and KAMAZ (Russia). By the method of a comparative statics the analysis of influence of a loan interest rate on a market capitalization of a company and the production loading level is carried out.

Changes of the interest rate on the credits and the fluctuations in demand both influence on market capitalization of the company and a position of the company in the market. The developed model allows to estimate dynamics of capitalization of the company proceeding from dynamics of revolving funds. Fig. 1 provides dynamics of the relation of market capitalization to cost of sales value for the FCA company (Fiat Chrysler Automobiles, Italy) for 2011-2014 received on the basis of statistical data (dashed line) and calculations for model (firm line) (on graphics the size of this indicator in relation to 2011 is showed).

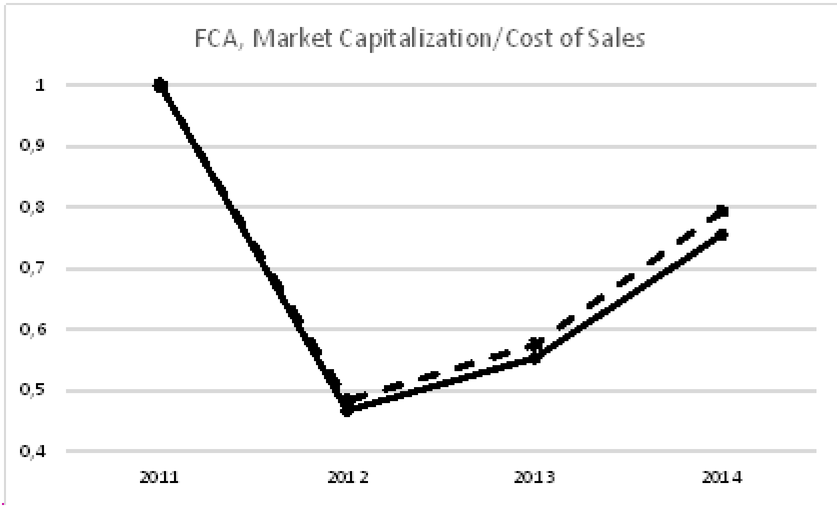

Fig. 1

The authors were supported by the Russian Foundation for Basic Research (project no. 14-07-00075).

References

1. *N.K. Obrosova, A.A. Shananin.* "Kvantoviy effect v modeli proisvodstva s uchetom deficit oborotnih sredstv i torgovoy infrastrukturi", Trudy Instituta Matematiki i Mekhaniki URO RAN, **16**, N5, 127-134 (2010).
2. *N.K. Obrosova, A.A. Shananin.* "A Production Model in the Conditions of Unstable Demand Taking into Account the Influence of Trading Infrastructure", Doklady Mathematics, **87**, N3, 1-4 (2013).
3. *N.K. Obrosova, A.A. Shananin.* "Study of the Bellman Equation in a Production Model with Unstable Demand", Computational Mathematics and Mathematical Physics, **54**, N9, 1411-1440 (2014).
4. *N.K. Obrosova, A.A. Shananin.* "Production Model in the Conditions of Unstable Demand Taking into Account the Influence of Trading Infrastructure: Ergodicity and Its Application", Computational Mathematics and Mathematical Physics, **55**, N4, 699-723 (2015).

A dynamic model of economy with social stratification

Nicholas Olenev[1,2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* nolenev@yahoo.com
[2] *People's Friendship University of Russia, Moscow, Russia;*

The paper presents a method of identification for a dynamic model of economic growth with social stratification. The model is a modification of a model presented in [1]. Here for describe an economic model for a stratum it is used the Uzawa-Lucas model [2-4] on the data of Russian economic statistics by application of high performance computations on multi-processors systems [5-7].

An optimal control problem for a typical household of a stratum dynasty can be formulated by the following way:

$$\int_0^\infty e^{-\rho t}(lnc + \phi ln\frac{n}{m} + \psi lnN)dt \longrightarrow \max_{u,c,n}, \tag{1}$$

$$\dot{h} = B(1-u)h, \tag{2}$$

$$\dot{N} = (n - M\mu(h))N, \tag{3}$$

$$\dot{k} = Ah_a^\gamma k^\alpha (uh)^{1-\alpha} - (n - M\mu(h) + \delta)k - c - qnh, \tag{4}$$

Here the control variables are a part of time for job $u(t)$, per capita consumption $c(t)$, and the number of children defined by fertility $n(t)$. State variables are the physical capital $k(t)$, the human capital $h(t)$, the size of dynasty $N(t)$, mortality $m(t) = M\mu(h)$, $\mu'(h) < 0$, $\mu(0) = \mu_+ > \mu(\infty) = \mu_- > 0$. Parameters are the discount rate $\rho > 0$, the utility weights ϕ and ψ, the intensity of schooling $B > 0$, the depreciation rate of physical capital $\delta > 0$, the technological level $A > 0$, the exponent $\gamma \geq 0$, the output elasticity of physical capital $0 < \alpha < 1$ in production, the cost of child rearing $q > 0$.

GDP for the whole economy, the population and the value the average human capital are determined as sum of all strata S.

$$Y_a = \sum_{i=1}^{S} y_i N_i, \tag{5}$$

$$N_a = \sum_{i=1}^{S} N_i, \tag{6}$$

$$h_a = \sum_{i=1}^{S} h_i N_i / N_a, \tag{7}$$

The author of the work was supported by the Russian Foundation of Basic Research (project no. 13-07-01020).

References

1. *Olenev N.N.* Model ekonomicheskogo rosta s sotsialnoy stratifikatsiey // Pervaya rossiyskaya konferentsiya "Sotsiofizika i sotsioinzheneriya" (8-11 iyunya 2015g.) - Moskva: MGU im. M.V. Lomonosova, 2015. S.49. In Russian.
2. *Lucas R.* On the Mechanics of Economic Development. Journal of Monetary Economics, 1988, 22(1), pp. 3-42.
3. *Uzawa H.* Optimum Technical Change in an Aggregative Model of Economic Growth. International Economic Review, 1965, 6. pp. 18-31.
4. *Olenev N.* Identification of the Uzawa-Lucas Model for World Economy// International scientific conference "New chellenges of economic and business development - 2014". May 8-10, 2014. Riga, University of Latvia. Abstracts of reports. PP.87-88.
5. *Olenev N.* Parallel algorithms of global optimization in identification of an economic model// Proceedings of Scientific Seminar of CEDIMES-Russie occidentale and Dorodnicyn Computer Centre Modeling of social and economic systems. Moscow: CEDIMES-Russie occ., Vol. 1, No. 1, 2015. P.45-47.
6. *Olenev N.* A Ramsey type model with an endogenous production function for study of economic systems // 10th International Conference "Economic Integration, Competition and Cooperation" (April 22-24, 2015) . Book of Abstracts. Opatija-Croatia-Hotel Ambasador: University of Rijeka, 2015. P.54.
7. *Kamenev G.K., Olenev N.N.* Study of the Russian Economys Identification and Forecast Stability Using a Ramsey Type Model // Mathematical Models and Computer Simulations, Vol. 7, No. 2, 2015, P.179-189. ISSN: 2070-0482 (Print). 2070-0490 (Online).

Optimal investment problems in a dynamic model of economy with venture capital

Nicholas Olenev[1,2], **Vsevolod Ostapov**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* nolenev@yahoo.com
[2] *People's Friendship University of Russia, Moscow, Russia;*
vaostapov@gmail.com

The paper is denoted to the investigation of the schemes of investing in innovative projects. The work started to build a dynamic model of economy based on venture capital [1]. Seven economic agents are allocated: large producers, small producers, Government, venture capitalists, banking system, households, and trade intermediates. It is supposed that small enterprises exist by the venture capital. A dynamic model of a lifecycle during an investment period of such a firms is given [2].

The optimal control problem for a small firm can be formulated in the following way:

$$k(T) \longrightarrow \max_{y,w}, \tag{1}$$

$$\dot{m} = -\mu m + I,$$

$$\dot{k} = -\beta k + bI,$$

$$\Pi_y := (p_1 - se^{\mu t} v - ap)y = w + pbI, \tag{2}$$

$$(3)\ k(0) = bm(0) = \Lambda,$$

$$y \in [0, m],$$

$$W(T) := \int_0^T \frac{w}{p}\, dt + \gamma k(T) \geqslant \alpha\Lambda. \tag{3}$$

Where k – is a capital of a firm, m – its capacity (maximum output per unit of time), y – its real output, μ and β – rates of disposal for capacity and capital respectively, $\beta > \mu$. Parameters p and p_1 – prices for the products of large producers and innovative firms respectively. Λ is an amount of initial investments, I – own investments, w – payments for the initial investments. Investor's expected minimal rate of return is $\alpha > 1$, the interest of the investor in the capital of the firm is $\gamma \in [0, 1]$. T – is a duration of the investment period, parameters a and b are ratio of direct costs and incremental capital intensity, which shows an amount of a product consumed per creation a unit of capacity. v – nominal labor input, $\lambda = e^{\mu t} v$ – real labor input.

In this problem y and w are the control parameters. They are linked by the restriction (2), which defines the profit function Π_y of a firm. $W(T)$ - is a total payment for the investments, inequality (3) defines a specific scheme of payments. The optimal control problems, corresponding to different patterns of investment and payments are solved. In the first scheme it is assumed the withdrawal period of the capital during which the investor receives everything he is supposed to, in the second it is assumed that a part of the income the investor receives as a fixed share in the capital of the company at the end of the investment period.

In the given model the following statements are true.

Statement 1. *In the absence of bank credits it is optimal for small firms to make payments for the investments at the end of a period.*

Statement 2. *(1) gains its maximum on the same optimal control as a total discounted profit does, so it can be replaced by*

$$\int_0^T \Pi_y e^{-\delta t} dt \longrightarrow \max_{y,w}$$

.

The authors were supported by the Russian Foundation of Basic Research (project no. 13-07-01020).

References

1. *N.N. Olenev, V.A. Ostapov.* Toward a Dynamic Model for Economy with Venture Capital, CC RAS, Moscow (2014).
2. *V.A. Ostapov.* "Optimal investment problem," in: Modernization and innovative development of economic systems, PFUR, Moscow, 2014, pp. 344–362.

On the Local and Global Search in Polymatrix Games

Andrei Orlov[1]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* anor@icc.ru

Let us formulate for the sake of simplicity a 3-player polymatrix game (hexamatrix game) with mixed strategies as follows [1,2]:

$$\left.\begin{array}{c} F_1(x,y,z) \triangleq \langle x, A_1 y + A_2 z\rangle \uparrow \max\limits_x, \;\; x \in S_m, \\ F_2(x,y,z) \triangleq \langle y, B_1 x + B_2 z\rangle \uparrow \max\limits_y, \;\; y \in S_n, \\ F_3(x,y,z) \triangleq \langle z, C_1 x + C_2 y\rangle \uparrow \max\limits_z, \;\; z \in S_l, \end{array}\right\}$$

where $S_p = \{u = (u_1, \dots, u_p)^T \in I\!R^p \mid u_i \geq 0, \;\; \sum\limits_{i=1}^{p} u_i = 1\}, \;\; p = m, n, l.$

The triple $(x^*, y^*, z^*) \in S_m \times S_n \times S_l$ satisfying the inequalities

$$\left.\begin{array}{c} v_1^* = v_1(x^*, y^*, z^*) \triangleq F_1(x^*, y^*, z^*) \geq F_1(x, y^*, z^*) \quad \forall x \in S_m, \\ v_2^* = v_2(x^*, y^*, z^*) \triangleq F_2(x^*, y^*, z^*) \geq F_2(x^*, y, z^*) \quad \forall y \in S_n, \\ v_3^* = v_3(x^*, y^*, z^*) \triangleq F_3(x^*, y^*, z^*) \geq F_3(x^*, y^*, z) \quad \forall z \in S_l, \end{array}\right\}$$

be called a Nash equilibrium point in the game $\Gamma_3 = \Gamma(A, B, C)$ $(A = (A_1, A_2),\ B = (B_1, B_2),\ C = (C_1, C_2))$. Herewith, the strategies x^*, y^*, and z^* be called the equilibrium strategies. The numbers v_1^*, v_2^*, and v_3^* be called the payoffs of players 1, 2, and 3, respectively, at the equilibrium point (x^*, y^*, z^*).

Further consider the optimization problem $(\sigma \triangleq (x, y, z, \alpha, \beta, \gamma))$:

$$\left.\begin{array}{c} \Phi(\sigma) \triangleq \langle x, A_1 y + A_2 z\rangle + \langle y, B_1 x + B_2 z\rangle + \langle z, C_1 x + C_2 y\rangle \\ -\alpha - \beta - \gamma \uparrow \max\limits_\sigma, \qquad \sigma \in D \triangleq \{(x, y, z, \alpha, \beta, \gamma) \in I\!R^{m+n+l+3} \mid \\ \mid x \in S_m, \;\; y \in S_n, \;\; z \in S_l, \quad A_1 y + A_2 z \leq \alpha e_m, \\ B_1 x + B_2 z \leq \beta e_n, \quad C_1 x + C_2 y \leq \gamma e_l\}, \end{array}\right\} \qquad (\mathcal{P})$$

where $e_p = (1, 1, ..., 1) \in I\!R^p, p = m, n, l.$

Theorem 1. [2] *A point (x^*, y^*, z^*) is a Nash equilibrium point in the hexamatrix game $\Gamma(A, B, C) = \Gamma_3$ if and only if it is a part of a global solution $\sigma_* \triangleq (x^*, y^*, z^*, \alpha_*, \beta_*, \gamma_*) \in I\!R^{m+n+l+3}$ of Problem $(\mathcal{P})$. At the same time, the numbers α_*, β_*, and γ_* are the payoffs of the first, the second, and the third players, respectively, in the game Γ_3: $\alpha_* = v_1(x^*, y^*, z^*)$, $\beta_* = v_2(x^*, y^*, z^*)$, $\gamma_* = v_3(x^*, y^*, z^*)$. In addition, an optimal value $\mathcal{V}(\mathcal{P})$ of Problem $(\mathcal{P})$ is equal to zero: $\mathcal{V}(\mathcal{P}) = \Phi(\sigma_*) = 0$.*

In order to solve Problem ($\mathcal{P}$), we will use an approach based on Global Search Theory [3]. According to this theory the Global Search consists of two principal stages: 1) a local search, which takes into account the structure of the problem under scrutiny; 2) the procedures based on Global Optimality Conditions (GOC) [3], which allow to improve the point provided by the local search method, in other words, to escape a local pit.

The Local Search Method for problem with a bilinear structure applies the idea of consecutive solving partial linear programs with respect to different groups of variables: (x, β), (y, γ) and (z, α). The similar idea has previously demonstrated its efficiency in bimatrix games and bilinear programming problems [4]. In spite of coupled constraints in Problem ($\mathcal{P}$) the convergence theorem of 3-stage Local Search Method to a critical point was proved. Each critical point is a partly global solution to Problem ($\mathcal{P}$) with respect to any pair of variables.

The Global Search procedures (GSM) for Problem ($\mathcal{P}$) are based on Global Optimality Conditions for nonconvex problems with d.c. objective functions [3,4], because of the objective function of Problem ($\mathcal{P}$) can be represented as a difference of two convex functions with the help of the property of a scalar product.

The efficiency of methods developed for hexamatrix games is demonstrated by the results of computational solving of the test problems. Our future work will be direct to the elaborating of local and global search methods for polymatrix games with more than 3 players.

This research was supported by the Russian Science Foundation (project No. 15-11-20015).

References

1. *E.B. Yanovskaya* “Equilibrium points in polymatrix games,” (in Russian). Latv. Math. Collect. **8**, 381–384 (1968).
2. *A.S. Strekalovsky, R. Enkhbat.* “Polymatrix games and optimization problems,” Autom. Remote Control, **75**, No. 4, 632–645 (2014).
3. *A.S. Strekalovsky.* Elements of Nonconvex Optimization (in Russian), Nauka, Novosibirsk (2003).
4. *A.S. Strekalovsky, A.V. Orlov.* Bimatrix Games and Bilinear Programming (in Russian). FizMatLit, Moscow (2007).

Atmospheric general circulation and thermohaline ocean circulation coupled model numerical experiments realization

Valeriy Parkhomenko

Institution of Russian Academy of Sciences
Dorodnicyn Computing Centre of RAS,
Moscow, Russia; parhom@ccas.ru

The hydrodynamic three-dimensional global climatic model which consists of blocks of the atmospheric general circulation model, model of the thermohaline large-scale circulation of the ocean, model of the sea ice evolution is realized [1-3]. Before rather strongly aggregated heat-moisture-balance model of the atmosphere for temperature and humidity of a surface layer was used as model of the atmosphere [4]. The atmospheric general circulation model is significantly more detailed, and it also allows to describe more adequately processes in the atmosphere [5-7].

Functioning of coupled climatic model is considered in the mode of the seasonal cycle of solar radiation. Zero normal flow is required at all rigid surfaces. On the borders of the continents are also zero normal components of heat and salts fluxes. The ocean is forced by the surface friction wind stress. Heat and salts fluxes are equal to zero at the ocean bottom, and on the surface are determined by the interaction with the atmosphere. Thermodynamic sea ice model dynamic equations are solved for the fraction of the ocean surface covered by sea ice and the average height of sea ice. Growth and melting ice in the model depend only on the difference between the flow of heat from the atmosphere to sea ice and heat flow of ice into the ocean. A diagnostic equation is used for the ice surface temperature. All models are linked by exchange of momentum, heat and moisture. Real world configuration of continents and ocean depths distribution are used. Equations in spherical coordinates are solved by numerical finite-difference method. The depth of the ocean is represented as 8 levels logarithmic scale up to 5000 m. Initial state of the climate system is characterized by constant temperatures of the ocean, the atmosphere and ocean currents speeds of zero. Numerical experiments show that the model goes to the steady state for a period of about 2000 years.

Procedure of the coupled calculations organization of the ocean model and atmospheric general circulation model is considered. Synchronization of a number of parameters in both models is necessary for their collaboration. In this regard procedure of two-dimensional interpolation of the data defined on grids of ocean model and atmosphere model and back is developed [8]. Feature of this task is discrepancy of grids and configurations of continents in models. At the initial stage of initial data synchronization model is the ocean and atmospheric models for the time before the match day of the year. The next step in the model of the atmosphere is the time loop, total value in one day. After the completion of this phase all climate characteristics are averaged per day and are transferred to the calculated parameters in a model of the ocean. Further, in the ocean model is step at a time (one day) and passed the calculated parameters in the model atmosphere for the resumption of the account in the loop. Long-term calculations for more than 400 years for coupled model that showed its stable work are carried out. Results of calculations and comparison with observation data are discussed.

The distribution of mean global atmosphere temperature depending on time in the stationary mode demonstrated existence of interannual variability of atmosphere temperature. Distribution of a difference of ocean surface temperatures from observations and from thermohaline circulation ocean model for January is presented. Noticeable deviations of temperature are observed in the field of close Antarctica. Apparently, it is connected with inaccuracies at calculation of sea ice distribution in model. Geographical distribution of January ocean surface temperature at joint calculations shows in general zonal uniform structure of isolines with noticeable deviations from zonality near continents that will be coordinated with observation data. The calculated field of January surface atmosphere temperatures possesses strong variability over continents.

The author was supported by the Russian Foundation for Basic Research (projects no. 14-07-00037, 14-01-00308).

References

1. *Parkhomenko V.P.* Climate model with consideration World ocean deep circulation, //Vestnik MGTU im. Baumana. Issue Mathematical Modelling, p. 186-200 (2011) (in Russian)

2. *Marsh R., Edwards N.R., Shepherd J.G.* Development of a fast climate model (C-GOLDSTEIN) for Earth System Science. SOC, No.83. 54 p. (2002).

3. *Ghan S.J., Lingaas J.W., Schlesinger M.E., Mobley R.L., Gates W.L.* A documentation of the OSU two-level atmospheric general circulation model. Rep. No. 35, Climatic Research Institute. Oregon State University, Corvallis, 395 p. (1982).

4. *Parkhomenko V.P.* Numerical experiments on global hydrodynamic model to assess the sensitivity and climate sustainability. Vestnik MGTU im. Baumana. Issue Mathematical Modelling, p. 134-145 (2012) (in Russian)

5. *Parkhomenko V. P., Tran Van Lang.* Improved computing performance and load balancing of atmospheric general circulation model. Journal of Computer Science and Cybernetics. V. 29. N 2. P. 138 148. (2013).
6. *Arakawa A., Lamb V.* Computational design of the basic dynamical processes of the ucla general circulation model. Methods in Computational Physics. Academic Press. V. 17, p. 174-207. (1977).
7. *Belov P.N., Borisenkov E.P., Panin B.D.* Numerical methods of weather prognoses. L: Gydrometeoizdat. 1989. 375 p. (1989). (in Russian)
8. *Riabenky V.S.* Computational mathematics introduction. M: Fizmatlit. 296 p. (2000). (in Russian)

DEA models for the Internet Acquiring

Maxim Patrin[1], **Andrey Lychev**[2], **Vladimir Krivonozhko**[3]

[1] *Institute for Systems Analysis of Russian Academy of Sciences (ISA RAS), Moscow, Russia;* MaxPat@mail.ru

[2] *National University of Science and Technology "MISiS", Moscow, Russia;* Lytchev@mail.ru

[3] *National University of Science and Technology "MISiS", Moscow, Russia;* KrivonozhkoVE@mail.ru

The Internet Acquiring, or acceptance of payment cards via the Internet, is the service that is wide spread all over the world. Now it is impossible to imagine goods or service that cannot be got on the Internet, having made payment by any plastic card. Despite the difficult economic situation, the Internet Acquiring develops dynamically during the last years, the cumulative returns on the results of 2014 is more than 400 billion rubles.

The main players in the market of the Internet Acquiring among banks are the following: Alfa-Bank, VTB24, Rosbank (processing center UCS), Sberbank, Bank of Moscow, Promsvyazbank, Bank Russian Standard. More than 90% of all market of the Internet Acquiring in Russia fall to their share.

However, the Internet Acquiring, as any other business, assumes a certain set of risks that may include:

— financial losses in case of bankruptcy of the merchant and receiving claims from its clients acquiring goods and services which were not fully rendered to the merchant;

— financial losses because of roguish operations from a consequence of dishonesty or negligence of the merchant and its employees;

— financial losses from penalties of the international payment systems connected with sale of the merchant of illegal types of goods or services.

For reasons given above, it is very important for banks to evaluate the efficiency of merchants. In our paper, we propose to evaluate the activities of merchants with the help of DEA models. Our computational experiments showed that the proposed approach is very promising.

Synthesis of an optimal path following controller for a wheeled robot with constrained control resource

Alexander Pesterev[1]

[1] *Institute of Control Sciences RAS, Moscow, Russia;* a.pesterev@javad.com

Stabilization of motion of a wheeled robot governed by the nonlinear differential equations (the so-called *kinematic,* or *simple-car, model*)

$$\dot{x}_c = v\cos\theta,\ \dot{y}_c = v\sin\theta,\ \dot{\theta} = v\tan\delta/L, \tag{1}$$

along a straight target path is considered. In (1), x_c, y_c are coordinates of the target point located in the middle of the rear axle, θ is the orientation angle, L is the wheelbase distance, and δ is the turning angle of the front wheels. Motion of the robot is controlled by turning the front wheels, whereas the forward speed v is assumed to be an arbitrary (perhaps, unknown) function of time $v \equiv v(t)$. Owing to the equation $u = \tan\delta/L$ relating the angle δ with the curvature u of the trajectory described by the target point, we may consider u as the control. Further, without loss of generality, we may assume that the target path coincides with the x-axis of the coordinate frame. Then, y_c is the distance to the target path.

If $|\theta| < \pi/2$, we may take x_c to be the new independent variable instead of time. Denoting it as ξ, introducing the notation $z_1 = y_c$ and $z_2 = \tan\theta$, and replacing differentiation with respect to time in (1) by differentiation with respect to ξ (denoted by the prime), we obtain [1]

$$z_1' = z_2,\ z_2' = (1+z_2^2)^{3/2}u. \tag{2}$$

Closing system (1) by the feedback $u = -\sigma(z)/(1+z_2^2)^{3/2}$, where $\sigma(z) = c_1 z_1 + c_2 z_2$ is a linear function with positive coefficients, we obtain the linear system the zero solution of which is globally asymptotically stable.

Since the turning angle δ of the front wheels is constrained by an angle $\bar{\delta} < \pi/2$, the control is also constrained, $|u| \le \bar{u} = \tan\bar{\delta}/L$. To ensure the fulfillment of the control constraint, we apply saturation function $\mathrm{sat}_{\bar{u}}(\cdot)$ to the linearizing feedback,

$$u = -\mathrm{sat}_{\bar{u}}[\sigma(z)/(1+z_2^2)^{3/2}], \tag{3}$$

which results in a hybrid system that is linear in the region where $\sigma(z) \le \bar{u}(1+z_2^2)^{3/2}$ and nonlinear in the saturation region. It is proved in [1] that condition $|\theta| < \pi/2$ is fulfilled for any trajectory if it is fulfilled at the initial point and that system (2) closed by feedback (3) is asymptotically stable for any control resource $\bar{u}$ and any positive coefficients c_1 and c_2. The values of these coefficients, however, greatly affect performance of the stabilization process, which brings us at the problem of finding the coefficients that ensure the best (in some sense) performance of the closed-loop system.

The particular case of this problem where the feedback depends on only one coefficient (the case we arrive at by taking $\sigma(z) = \lambda^2 z_1 + 2\lambda z_2$, $\lambda > 0$) was studied in [2] (in this case, the corresponding linear system has one repeated pole). The desired value of the coefficient was defined to be the greatest value of λ for which the phase plane is divided into two half-planes such that any trajectory of the closed-loop system belongs to one of these half-planes (i.e., the phase portrait of the nonlinear system (2), (3) is topologically equivalent to that of the linear system with a stable node at the origin).

In this work, the general, two-parameter, case is studied. Like in the previous one-parameter case, we seek for values of the feedback coefficients that ensure the greatest exponential convergence while keeping the phase portrait of the closed-loop system topologically equivalent to that of the linear system. These requirements are shown to be satisfied for a one-parametric family of pairs of the coefficients. By selecting from this family two different pairs of the coefficients for the initial and final stages of the stabilization, one arrives at a hybrid robust control law that is optimal in terms of overshooting. The control reaches saturation ($u = \pm\bar{u}$) at the initial stage of motion until the system comes to an invariant set, where the control turns to be continuous and the system is linear.

References

1. *A. V. Pesterev.* "Synthesis of a stabilizing control for a wheeled robot following a curvilinear path," Automation and Remote Control, **73**, No. 7, 1134–1144 (2012).
2. *A. V. Pesterev.* "On optimal choice of the feedback coefficient in the path following problem for a wheeled robot with constrained control resource," Proc. of the V Int. Conf "Optimization and Applications," Petrovac, 146–147, (2014).

Illegal migration forecasting using system dynamics

Lev Petrov[1], **Alexey Stepanov**[2]

[1] *Plekhanov Russian University of Economics, Moscow, Russia;* lfp@mail.ru
[2] *Plekhanov Russian University of Economics, Moscow, Russia;* alexpro45@yandex.ru

The existing methodology of gathering statistical data about migration in Russian Federation consider only legal migrants, therefore, it is vital to develop a methodology for modelling of migration flows and population with regard to illegal migration.

In this regard, the task of creating mathematical models which are able to predict migration indicators in relation to different socio-economic factors is set. In this case, it is taken into account the impact of migration on the size and structure of the population. The implementation of these models in a dynamic modeling programing environment like PowerSim will allow to make forecasts on the basis of not only statistical data on migration, but also to analyze the flows of illegal migration on the basis of expert assessments in the retrospective period.

The change in the number of illegal migrants residing on the territory of the European part of Russia and Moscow region described by the differential equations (1), (2).

$$\frac{d(eIPop)}{dt} = eB * eIPop + ldeIMig - eD * eIPop - eIPop * ec, \qquad (1)$$

when $t = t_0, eIPop = eIPop_0$

$$\frac{d(mIPop)}{dt} = mB * mIPop + ldmIMig - mD * mIPop - mIPop * mc, \qquad (2)$$

when $t = t_0, mIPop = mIPop_0,$

where $eIPop$, $mIPop$ – the number of illegal migrants residing on the territory of the European part of Russia and the Moscow region; $erIPop_0$, $mIPop_0$ – initial conditions for the number of illegal migrants residing on the territory of the European part of Russia and the Moscow region; $ldeIMig$, $ldmIMig$ – flows of illegal migrants from Central Asia to the European part of the Russian Federation and the Moscow region; eB, mB – fertility rates in The European part of Russia and the Moscow region; eD, mD – mortality rates in the European part of the Russian Federation and the Moscow region; ec, mc – coefficients of legalization of migrants on the territory of the European part of Russia and the Moscow region.

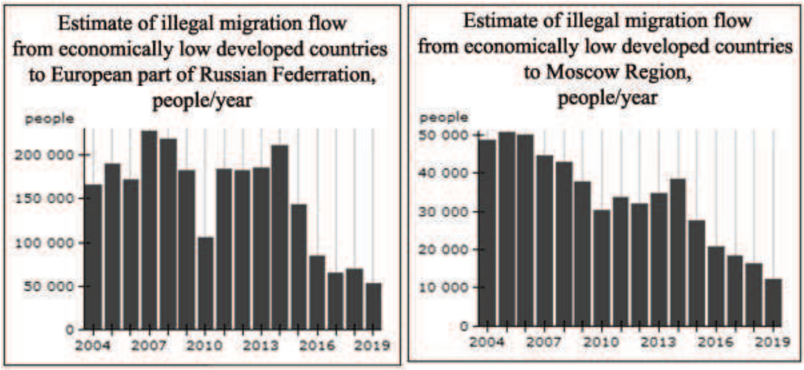

Fig. 1

In a more general version of modeling migration flows model parameters are selected based on officially published statistical data or expert estimates. It uses a built-in dynamic modeling PowerSimStudio genetic optimization algorithms. Multicriteria problem of minimization deviation of the calculated migration data and official statistics or expert estimates are solved.

References

1. *L.F. Petrov, A.V. Stepanov* Predictive modeling of migration flows. Environmental Economics No. 5, VINITI, Moscow, 2014 – p. 111-126, ISSN 1994-8336

Model of the Russian banking system in view of banks' demand for liquidity

Nikolay Pilnik[1], **Igor Pospelov**[2], **Stanislav Radionov**[3]

[1] *Higher School of Economics, Moscow, Russia;* u4d@yandex.ru
[2] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* pospeli@yandex.ru
[3] *Higher School of Economics, Moscow, Russia;* saradionov@edu.hse.ru

We present a relatively simple and plausible description of modern banking system applicable in the intertemporal equilibrium framework. This description cannot be replaced by simple relations based, for example, on money multipliers. The rational expectations model of banking system based on econometric analysis of banks' demand for liquidity depending on balances and turnovers of aggregated assets and liabilities is presented. Some econometric relations are used as restrictions while others are explained by the model. The model successfully reproduces the reaction of banking system on unexpected shocks such as large-scale sale of assets and world financial crisis.

The authors were supported by the Russian Scientific Fund, research project No 14-11-00432.

References

1. *M.Yu. Andreyev, V.P. Vrzheshch, N.P. Pilnik et al.* "Intertemporal Equilibrium Model of the Russian Economy, Based on Disaggregated Macroeconomic Balance," Trudy seminara im. I.G. Petrovskogo, **29**, 43–145 (2013).
2. *M.Yu. Andreyev, N.P. Pilnik, I.G. Pospelov.* "Intertemporal Equilibrium Model of the Russian Economy, Based on Disaggregated Macroeconomic Balance (in Russian)," Ekonomicheskiy zhurnal VShE, **13**, No.2, 143–171 (2009).
3. *J. Lewis.* "A Computable General Equilibrium (CGE) Model of Banking System Stability: Case of Jamaica," Journal of Business, Finance and Economics in Emerging Economies, **5**, No.2, 81–120 (2010).
4. *D.W. Diamond, P.H. Dybvig.* "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy, **91**, No.3, 401–419 (1983).
5. *J. Diaz-Gimenez, E.C. Prescott, T. Fitzgerald et al.* "Banking in Computable general Equilibrium Economies," Journal of Economic Dynamics and Control, **16**, No.3–4, 533–559 (1992).

P-th order maximum principle for nonregular optimal control problems

Agnieszka Prusińska[1], **Alexey A. Tret'yakov**[2]

[1] *Department of Mathematics and Physics, Siedlce University of Natural Sciences and Humanities, Siedlce, Poland* aprus@uph.edu.pl

[2] *Dorodnicyn Computing Centre of RAS, Moscow, Russia; System Research Institute, Polish Academy of Sciences, Warsaw, and University of Podlasie, Siedlce, Poland;* tret@uph.edu.pl

We consider the following nonregular optimal control problem

$$J(x,u) = \int_{t_1}^{t_2} f(t, x(t), u(t))dt \to \text{extr} \tag{1}$$

subject to

$$\begin{aligned} \dot{x} - \varphi(t, x(t), u(t)) &= 0, \\ M_1 x(t_1) + M_2 x(t_2) &= 0, \end{aligned} \tag{2}$$

where $f : R \times R^n \times R^r \to R$, $\varphi : R \times R^n \times R^r \to R^m$, M_1, M_2 are $l \times n$ matrices, $x \in \mathcal{C}_n^2[t_1, t_2]$, $u \in V \subseteq R^r$, V – some set and f, φ – sufficiently smooth mappings.

The system of equations (2) can be replaced by the following operator equation

$$G(x, u) = 0_Y, \tag{3}$$

where $G(x,u)(\cdot) = \dot{x}(\cdot) - \varphi(\cdot, x(\cdot), x(\cdot))$, $G : X \times V \to Y$, $X = \{x(\cdot) \in K\mathcal{C}_n^1[t_1, t_2] : M_1 x(t_1) + M_2 x(t_2) = 0\}$ and $Y = K\mathcal{C}_n[t_1, t_2]$. Denote by $L(t, x, \dot{x}, u) = \lambda(t)(\dot{x} - \varphi(t, x, u)) + \lambda_0 f(t, x, u)$, $\lambda(t) = (\lambda_1(t), \ldots, \lambda_n(t))^T$, $u \in K\mathcal{C}[t_1, t_2]$ piece wise continuous function and by $(x^*(t), u^*(t))$ optimal solution for (1)–(2). Let $T = [t_1, t_2]$, T' the set T without discontinuous points of optimal control $u^*(t)$. Denote by $H(t, x, u, \lambda) = \lambda\varphi(t, x, u) - \lambda_0 f(t, x, u)$ – Pontryagin function. Then maximum principle may be formulate as follows: there exists $\lambda_0^* \geq 0$ and $\lambda^*(t)$ such that

$$\max_{u \in V} (\lambda^* \varphi(t, x^*, u) - \lambda_0^* f(t, x^*, u)) = \lambda^* \varphi(t, x^*, u^*) - \lambda_0^* f(t, x^*, u) \tag{4}$$

for $t \in T'$. Moreover, if

$$\operatorname{Im} G'(x^*, u^*) = Y \tag{5}$$

then $\lambda_0^* = 1$.

But in singular (nonregular, degenerate) case when $\operatorname{Im} G'(x^*, u^*) \neq Y$ we cannot guarantee that $\lambda_0^* = 1$, it may be $\lambda_0^* = 0$!

Example. Consider the problem

$$J(x,u) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x_1^2 + x_2^2 + u^2 + u)dt \to \min \tag{6}$$

subject to

$$G(x,u) = \begin{pmatrix} \dot{x}_1 - x_2 + x_1^2 - \frac{1}{2}x_2^2 + u^2 \\ \dot{x}_2 + x_1 + x_1^2 - \frac{1}{2}x_2^2 + u^2 + ux_2 \end{pmatrix} \begin{matrix} = 0, \\ = 0, \end{matrix} \tag{7}$$

$x_1(-\frac{\pi}{2}) + x_1(\frac{\pi}{2}) = 0$, $x_2(-\frac{\pi}{2}) + x_2(\frac{\pi}{2}) = 0$, where $u \in V_\varepsilon = \{u \in R : \|u\| \leq \varepsilon\}$ and $\varepsilon > 0$ sufficiently small. Here $X = K\mathcal{C}_2^1[-\frac{\pi}{2}, \frac{\pi}{2}]$, $Y = K\mathcal{C}_2[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Optimal solution of (6)–(7) is $x^*(t) = 0$, $u^*(t) = 0$. However λ_0^* does not equal 1. Indeed, if $\lambda_0^* = 1$ then it must be

$$-u - u^2 + \lambda_1^*(t)u^2 + \lambda_2^*(t)u^2 \leq 0, \quad \forall_{u \in V_\varepsilon}$$

for sufficiently small ε and that is not true. For our example $\operatorname{Im} G'(x^*, u^*) \neq K\mathcal{C}_2[-\frac{\pi}{2}, \frac{\pi}{2}]$ i.e. regularity condition fails. But at the same time the mappings $G(x,u)$ are p - regular at the point (x^*, u^*) and we represent here p-th order maximum principle where coefficient $\lambda_0^* = 1$.

The authors were supported by the Russian Foundation for Basic Research (project no. 14-07-00805) and by the Leading Scientific Schools Grant no. 4640.2014.1

Discrete model of economy with delays

Vitaliy Pyankov[1]

[1] *South Ural state university, Chelyabinsk, Russia;*
`cold.karadras@gmail.com`;

Described model occurred from the ideas claimed in [1], but the previous form of mathematical description did not actually found itself useful. This model of economic system's core shows the very within of its functioning apart from the processes of its management and social relations emerging from production division problem.

The main idea is to divide industrial process functioning from its management. Neuman's model was took as a basis. Industrial process $j \in J$ of rate $x_{ij}(t)$ is characterised by a set of values $(a_{ij}, b_{ij}, c_{ij}, d_{ij}, \tau_{ij})$, where a_{ij} is a quantity of commodities produced $i \in I$ and b_{ij} – consumed current assets quantity per unit of production rate $x_{ij}(t)$, c_{ij} – unit of fixed assets contribution to the production rate, d_{ij} – fixed assets production rate loss in result of functioning within one step, τ_{ij} – number of steps from the moment t backwards, when for the current production $i \in I$ commodity was used.

Let $f_i(t)$ – be a commodity $i \in I$ quantity produced on step $t = 0, 1, \ldots, T$. Let also $\epsilon_j(I), \theta_j(I), \gamma_j(I)$ – subsets of a set of all commodities, which form process' j fixed and current assets and services, $p_{ij}(t)$ – produced commodity proportion of division, $\eta_j(t)$ – industrial capacity utilization.

Also suppose given industrial system is acting within a society with a particular set of labour specializations $k \in K$. Let $L_k(t)$ be a labour fund, e.g. labour amount of specialization $k \in K$ available on step (t), $q_{jk}(t)$ – labour fund $L_k(t)$ proportion of division between functioning industrial processes, r_{jk} – labour amount of specialization k needed by process j on a single step.

D e f i n i t i o n 1. A multiset $(a_{ij}, b_{ij}, c_{ij}, d_{ij}, \tau_{ij}, \epsilon_j(I), \theta_j(I), \gamma_j(I), r_{jk})$ $\forall i \in I, \forall j \in J, \forall k \in K$ is called a technological basis of the economic system.

Then the core of economic system of a given technological basis can be described as following:

$$f_i(t) = \sum_{j \in J} a_{ij} x_j(t), \forall i \in I;$$

$$x_j(t) = \min \begin{bmatrix} \eta_j(t-\tau_{ij}) \min\limits_{i \in \epsilon_j(I)} F_{ij}^{\epsilon}(t-\tau_{ij}); \\ \min\limits_{i \in \theta_j(I)} F_{ij}^{\theta}(t-\tau_{ij}); \\ \min\limits_{i \in \gamma_j(I)} b_{ij} p_{ij}(t-\tau_{ij}) f_j(t-\tau_{ij}); \\ \min\limits_{k \in \varphi_j(K)} \dfrac{1}{r_{jk}} q_{jk}(t-\tau_{jk}^L) L_k(t-\tau_{jk}^L); \end{bmatrix}, \forall j \in J;$$

$$F_{ij}^{\epsilon}(T) = \sum_{t=0}^{T} \left(c_{ij} p_{ij}(t) f_i(t) - d_{ij} x_j(t)\right), \forall j \in J, i \in \epsilon_j(I);$$

$$F_{ij}^{\theta}(T) = \sum_{t=0}^{T} \left(\frac{1}{b_{ij}} p_{ij}(t) f_i(t) - x_j(t)\right), \forall j \in J, i \in \theta_j(I);$$

$$\sum_{j \in J} p_{ij} \leq 1, \forall i \in I;$$

$$\sum_{j \in J} q_{jk} \leq 1, \forall k \in K.$$

This model can be used to test and compare different economic management ideas from the directive control with setting proportions $p_{ij}(t)$ and $q_{jk}(t)$ and capacity utilization $\eta_j(t)$ directly to the capitalistic management where these parameters are determined through quite a complex set of rules and equations.

References

1. *V.A. Pyankov, A.V. Panyukov.* “New mathematical approach to economic system development management”, V International Conference on Optimization Methods and Applications, 142–143 (2014).
2. *E.V. Shikin.* Mathematical methods and models in management, KDU, Moscow (2009).
3. *V.P. Petrov.* Cost. Money. Overproduction Economic crisis, Librokom, Moscow (2012).
4. *M.L. Khazin.* Zakat imperii dollara i konec ”Pax Americana”, Veche, Moscow (2003).

Application of H_∞–optimization for motion control of robotic platforms

L.A. Rybak[1], **D.I. Malyshev**[1], **A.A. Gunkin**[1]

[1] *Belgorod State Technological University of V.G. Shukhov, Belgorod, Russia;*
`rl_bgtu@intbel.ru`

The article describes the robotic platform designed to protect the technological objects, as well as a human-operator on the low-frequency effects from the base. At the same time put back robotic task at any random actions on the part of the mobile base is necessary to construct such a law controlling the drive mechanism, in which the object (platform) is fixed in an inertial coordinate system. Considered electromechanical actuator [1]. As the feedback sensors used accelerometers mounted on the object and the ground, and the relative movement of the sensor [2]. Drive including a two-phase stepper motor with active unsalient-pole rotor. Adopted as the reference angle position where the axis of the rotor pole coincides with the axis of phase 1. The equations of stress and torque are of the form

$$\frac{J}{p} \cdot \ddot{\theta} + M_H = p \cdot \psi_m \cdot (i_1 cos\theta - i_2 sin\theta)\,; \tag{1}$$

$$r_1 \cdot i_1 + L \cdot \dot{i}_1 + \psi_m \cdot \dot{\theta} \cdot \cos\theta = u_1; \tag{2}$$

$$r_2 \cdot i_2 + L \cdot \dot{i}_2 - \psi_m \cdot \dot{\theta} \cdot \sin\theta = u_2; \tag{3}$$

$$\ddot{\theta} = \omega; \tag{4}$$

where M_H — load torque, J — the moment of inertia of the engine, i_1, r_1, u_1 —the current, resisting, and tension of the 1st phase of the engine, i_2, r_2, u_2 — the 2nd phase of the engine, θ — the angle between the axis of the vector-poles and n. . , p — number of pole pairs, ψ_m — maximum flux linkage excited rotor, L — the coefficient of mutual induction, ω — the angular velocity.

Further, in the equations (1–4), passed the angle θ close to zero (in this case $\cos\theta \approx 1$, as well $\sin\theta \approx 0$), and then they are reduced to a form characteristic of the equations describing the system in the state space in vector-matrix form

$$\begin{cases} \mathbf{Y} = \mathbf{AX} + \mathbf{Bu} + \mathbf{DY} \\ \dot{v} = \mathbf{CX} \end{cases} \tag{5}$$

where $\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ — the state vector, $\mathbf{Y} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ — a vector of disturbances, $\mathbf{A} = \begin{bmatrix} -\frac{r_1}{L} & 0 & 0 & -\frac{\psi_m}{L} \\ 0 & -\frac{r_2}{L} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{J_{eq}} & 0 & 0 & 0 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{p}{J} \cdot M_H \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \frac{1}{L} \\ \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}$ — the coefficient matrix of the system, $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & 0 & 0 \end{bmatrix}^T$ — the vector of input signals, $\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & S_{eq} \end{bmatrix}^T$ — row vector of coefficients. Control is sought in the form $\mathbf{u} = K \cdot \mathbf{X}$, that minimizes [3] $J = sup_\omega \|y\|_2^2$

To do this in the second equation of the system (5) introduced control u used to limit the magnitude of the control, as otherwise, you can get arbitrarily small values J using sufficiently large $\mathbf{u}$.

As you know

$$J = \|H(s)\|_\infty^2 ,$$

where $H(S) = ((C_\Delta + B_{1\Delta} K)(sz - A_\Delta + BK))^{-1} D$ — the transfer function of a closed system with indignation ω to the output y, i.e., minimization of J is equivalent to H∞–optimization [4].

The inequality of the form

$$QA_\Delta^T + A_\Delta Q + QC_\Delta^T C_\Delta Q - B_\Delta S^{-1} B_\Delta^T + \frac{1}{y^2} D_\Delta D_\Delta^T \leq 0,$$

It has a positive-definite solution $Q > 0$, if such a decision is the Riccati equation, obtained by replacing the inequality to equality. According to this decision, restored the corresponding stabilizing controller

$$K = YQ^{-1} = -S^{-1} B_\Delta^T Q^{-1}.$$

As a result of the mathematical modeling are the resulting matrixes A, B

and C.

$$A = \begin{bmatrix} -13,3 & 0 & 0 & -0,0012 \\ 0 & -13,3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 8,9 \cdot 10^3 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2,38 \\ 2,38 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 2,936 \end{bmatrix}$$

And the following values of weighting coefficients: $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, r = 10^{-6}$. Thus, the synthesized optimal stabilizing control, the feedback coefficient matrix has the form

$$K = \begin{bmatrix} 0,0984 & 0 & 0,0001 & 0,0001 \end{bmatrix}$$

References

1. *Rybak L.A., Larin V.B. Sinev A.V.* "Synthesis of digital control states with invariant control of the active vibration isolation," Journal of Machinery Manufacture and Reliability, No. 2, 104 (1998).
2. *Pogonin A.A., Korop A.D.* "Improving the efficiency of the use of equipment for milling of titanic alloy," Herald of BSTU of V.G.Shukhov, No 3, 98–101 (2010).
3. *Besekersky V.A., Popov E.P.* Theory of automatic control systems, Nauka, Moscow (1972).
4. *Polyak B.T., Scherbakov P.S.* Robust stability and control, Nauka, Moscow (2002).

Application of splines for the hill diagram hydroturbine modeling

Alexander Salienko[1], **Yuriy Volkov**[2]

[1] *JSC Tyazhmash, Syzran, Russia;* sa_cae@yahoo.com
[2] *Sobolev Institute of Mathematics, Novosibirsk, Russia;* volkov@math.nsc.ru

The problem of constructing of a hill diagram for the hydro turbine wheels on the power test results of the model turbine is considered. The hill diagram is the basic document for selection of full-scale hydraulic turbine parameters (turbine wheel diameter, rotating frequency, etc.) that ensure the most efficient perfomance of the turbine at all modes of its operation in a particular hydropower station.

The basis of the proposed approach is the approximation methods for multidimensional functions at scattered data. The methods are modifications and generalizations of Hardys multiquadrics, thin plate and D^m-splines. These splines are an effective tool for the reconstruction of a function depending on three variables by its values known at irregularly (chaotically) located points of space. The definitions for interpolating and smoothing splines are given. The algorithmic problems related to their construction are also discussed. Note that such splines have much in common with the RBF-splines.

The software package for mathematical modeling of hill diagrams for the Fransis and Kaplan turbines was created. An example of modeling for real data on the basis of the program complex is given.

The second author was supported by the Russian Foundation for Basic Research (project no. 15-07-07530).

References

1. *Yu.S. Volkov, V.L. Miroshnichenko.* "Constructing a mathematical model of a universal characteristic for a radial-axial hydroturbine", Sibirskij Zhurnal Industrialnoj Matematiki, **1**, No. 1, 77–88 (1998). (in Russian)
2. *Yu.S. Volkov, V.L. Miroshnichenko, A.E. Salienko.* "Mathematical modeling of hill diagram for Kaplan turbine", Machine Learning and Data Analysis, **1**, No. 10, 1439–1450 (2014). (in Russian)

A problem of rythmical production

Vladimir Savelyev

Nizhny Novgorod State University, Nizhny Novgorod, Russia;
vpsavelyev@rambler.ru

1. Discrete version

Let an initial sequence $\{P_i\}$, $i = \overline{1,n}$, be the input of raw materials to the stock of the volume V and a smoothed sequence $\{x_i\}$, $i = \overline{1,n}$, be the output of raw materials to be manufactured.

The problem of optimal smoothing (the optimization of streamlining manufacturing operations) can be stated and solved [1] as a problem of convex programming: find a vector $X^0 = (x_1^0, x_2^0, \ldots, x_n^0) \in D$ minimizing the function $F(X) = \sum_{i=1}^n f(x_i)$, where $f(x_i)$ is a continuous convex function, and the feasible set

$$D = \left\{ X \in R^n : \ A_j \le \sum_{i=1}^{j} x_i \le B_j, \ j = \overline{1, n-1}, \ \sum_{i=1}^{n} x_i = P \right\}$$

is a special set of R^n with lower (A_j) and upper (B_j) constraints.

Statement 1. *A vector $X^0 = (x_1^0, x_2^0, \ldots, x_n^0) \in D$ minimizes the function $F(X)$ iff every pair of its components x_k^0 and x_j^0 ($k > j$) verifies one of the following conditions:*

A) $x_k^0 = x_j^0$;

B) $x_k^0 > x_j^0$, *and there exists such a number $m \in \{j, j+1, \ldots k-1\}$ that $\sum_{i=1}^{i=m} x_i^0 = B_m$;*

C) $x_k^0 < x_j^0$, *and there exists such a number $l \in \{j, j+1, \ldots k-1\}$ that $\sum_{i=1}^{i=l} x_i^0 = A_l$.*

Statement 2. *Let $\mu_i = B_i - i\frac{B_n}{n}$ and $\nu_i = i\frac{B_n}{n} - A_n$, $i = \overline{1, n-1}$.*

If $\mu_i \ge 0$, $\nu_i \ge 0$, $i = \overline{1, n-1}$, then all the components of optimal vector are equal to $\frac{B_n}{n}$;

if $\mu_k = \min \mu_i < 0$, then the components of optimal vector satisfy the condition $\sum_{i=1}^{i=k} x_i^0 = B_k$;

if $\nu_l = \min \nu_i < 0$, then the components of optimal vector satisfy the condition $\sum_{i=1}^{i=l} x_i^0 = A_l$.

2. Continuous version

Let $p(t)$, $0 \leq t \leq T$, be continuous nonnegative function, representing the speed of delivery of raw materials to the stock of the volume V, $u(t)$, $0 \leq t \leq T$, be piecewise continuous function representing the speed of manufacturing of raw materials, then the quantity $x(t)$ of raw materials in the stock will satisfy the following equation

$$x(t) = x(0) + \int_0^t p(\tau)d\tau - \int_0^t u(\tau)d\tau, \quad 0 \leq t \leq T. \tag{1}$$

The integral amount of raw materials $P(t) = x(0) + \int_0^t p(\tau)d\tau$ and the integral amount of manufactured raw materials $v(t) = \int_0^t u(\tau)d\tau$ have to satisfy the restrictions

$$A(t) \leq v(t) = \int_0^t u(\tau)d\tau \leq B(t), \quad 0 \leq t \leq T, \quad \int_0^T u(\tau)d\tau = P. \tag{2}$$

A piecewise continuous function $u^0(\tau)$, $0 \leq \tau \leq T$, satisfying the inequality (2) and minimizing the functional $F[u] = \int_0^T u^2(\tau)d\tau$ is called *optimal control.*

Statement 3. *Let $u^0(\tau)$, $0 \leq \tau \leq T$, be the optimal control.*

a) If $u^0(\bar{t} - 0) < u^0(\bar{t} + 0)$, then $v^0(\bar{t}) = \int_0^{\bar{t}} u^0(\tau)d\tau = B(\bar{t})$;

b) if $u^0(\bar{t} - 0) > u^0(\bar{t} + 0)$, then $v^0(\bar{t}) = \int_0^{\bar{t}} u^0(\tau)d\tau = A(\bar{t})$.

Statement 4. *Let $\mu(t) = \frac{B(t)-ct}{\sqrt{(1+c^2)(t^2+B^2(t))}}$, $\nu(t) = \frac{ct-A(t)}{\sqrt{(1+c^2)(t^2+A^2(t))}}$, $0 \leq t \leq T$, where $c = P/T$.*

If $\mu(t) \geq 0$, $\nu(t) \geq 0$, then $u^0(\tau) \equiv c$, $0 \leq t \leq T$, will be optimal control;

if $\mu(\bar{t}) = \min \mu(t) < 0$, then $v^0(\bar{t}) = B(\bar{t})$;

if $\nu(\bar{t}) = \min \nu(t) < 0$, then $v^0(\bar{t}) = A(\bar{t})$.

References

1. *V.P. Savelyev.* "Optimization of work balancing and minimizing changes of production intensity," Vestnik NNGU, No. 4, 115–119 (2007). (in Russian)

Differentiation Functors and its Application in the Optimization Control Theory

Simon Serovajsky[1]

[1] *Al-Farabi Kazakh national university, Almaty, Kazakhstan*
serovajskys@mail.ru

Optimization control problems are analyzed as a rule by necessary conditions of optimality. It can be the stationary condition, Euler equation, the variational inequality, the maximum principle, etc. These conditions include the first derivative of the state functional. So the optimal control theory has serious relations with the differentiation theory.

The differentiation is an operation of the local linearization. It supposes that the nonlinear phenomenon has become weakly apparent in a small enough set. Then the regular enough nonlinear object can be approximated by a linear one. Note that the derivative is used for the definition of the tangent. If the local structure of the nonlinear object is analyzed by means of its linear approximation, we use the differentiation. It is true for the optimization control theory too. The differentiation relates with the local structure of the object. Two functions (functionals, operators) that are equal in a neighbourhood of a point have the same derivative in this point. So the derivative characterizes the local structure of the class of objects, but not a concrete object. These objects are equivalent in some way. This equivalence class is the germ of functions (functionals, operators) in this point. Then the differentiation relates with the germs theory naturally enough.

Thus the differentiation transforms the germ of operators to a linear operator, which is its derivative in the given point. This map can be interpreted as a functor. It transforms the category, which has germs of operators as morphisms, to the category, which has linear operators as morphisms. So our problem allows an interpretation in the categories theory We consider classic and extended operator derivatives.

Inverse problems for two dimensional parabolic equations with infinite horizon

Ilyas Shakenov[1]

[1] *Al-Farabi Kazakh National University, Almaty, Kazakhstan*
ilias.shakenov@gmail.com

Practical problems often lead to difficulty that we call inverse problems. For example, if you need to know the temperature of soil at the depth of several meters while it is possible to measure the temperature only on the surface. In this type of problems there is a lot of information (over-determination) on one side of the boundary but no any data at the other side. We consider one of such problems with the following mathematical problem definition.

We have a two-dimensional heat conduction equation and initial boundary problem. The right boundary state function is unknown and has a meaning of heat flow. To determine this value we can use additional information at the left side. To solve the problem numerically we cannot use infinite time interval, so we replace the original problem with its finite approximation, where the final time becomes larger and larger.

We convert this problem to optimization by standard method. We use a familiar method to solve the problem by constructing an iterative process. At first we find Gato derivative of the functional. Then we have constructed a computational algorithm, we can do a lot of experiments accompanied with graphs and tables.

Additionally we give an outline of the future works to improve and develop this research, which we believe has a great applicability and concerns about new unknown effects in inverse problem theory.

Numerical solution of the some parametric inverse problem of atmospheric optics by Monte Carlo methods

Kanat Shakenov[1]

[1] *Al-Farabi Kazakh National University, Almaty, Kazakhstan*
shakenov2000@mail.ru

In the paper parametric inverse problems of atmospheric optics are considered. To solve these problems we applied algorithm "the method of dependent tests for transport theory problems" of Monte Carlo methods. The problems reduced to linear system of equations for parameters and solved by optimizations methods. The numerical solution of the optical depth of the extinction specified. The approximation error is no more than 5-10 percent, which is quite satisfactory for Monte Carlo methods.

Estimation of derivatives of I_k by Monte Carlo methods with respect to parameter τ (is optical depth) to estimate this parameter with the methods mentioned in the proceeding.

This parameter $\tau(\mathbf{r}_n, \mathbf{r}_k, \lambda) = \int \sigma(\mathbf{r}_n + \omega_k l, \lambda) dl$ is called "the optical depth", where $(\omega_k)l = (\frac{(\mathbf{r}_k - \mathbf{r}_n)}{|\mathbf{r}_k - \mathbf{r}_n|})l$ is called "the optical length from $\mathbf{r}_n$ to $\mathbf{r}_k$", ω_k is unit length vector.

Lattices of tringulations of linear polyhedral cones and their f-vectors

Valery Shevchenko

Nizhny Novgorod State University, Nizhny Novgorod, Russia;
shevchenko@vmk.unn.ru

The problem of connections of the structures of polyhedral and symplicial complexes is discussed. This problem naturally appeared in control theory and optimization.

Consider a polyhedral cone K, i.e. the set of all solutions to a homogenous system of linear inequalities over the field of rational numbers

$$\sum_{j=1}^{d} a_{ij}x_j \geq 0 \qquad (i = 1, \dots, m). \tag{1}$$

The set $K_I = \{x = (x_1, \dots, x_d) \in K : \sum_{j=1}^{d} a_{ij}x_j = 0,\ i \in I\}$, where $I \subseteq \{1, \dots, m\}$, is called I-face of K. If $K_I \neq K$ then K_I is called the proper face of K. The set of all proper faces is called the boundary of K. It is known that the set of all faces of K ordered by inclusion is the lattice of K denoted by $\Gamma(K)$ (with maximal face K).

Let $B = (b_{ik})$ be $(d \times n)$-matrix with columns b_j $(j = 1, \dots, n)$ and $\Gamma(B(J)) = \Delta(J)$, where $J \subseteq \{1, \dots, n\}$.

Denote by $B^{\angle} = \left\{\sum_{j=1}^{n} b_j y_j : y_j \geq 0\right\}$ the set of all nonnegative linear combinations of its columns and suppose that $B^{\angle}$ is coincide with the set of solutions to system (1). Analogously, let $A_{\angle}$ be the set of all nonnegative linear combinations of rows of A. To determine the structure of $\Delta(J)$ it's enought to know non-zero elements of matrix $(c_{ij}) = C = AB$. Let $\Gamma(C) = (\gamma_{ij})$, where $\gamma_{ij} = 1$ if $c_{ij} = 0$ and $\gamma_{ij} = 0$ otherwise. We call a $\{0, 1\}^{m \times n}$ matrix d-realizable iff there exist matrices $A \in \mathbf{Z}^{m \times d}$ and $B \in \mathbf{Z}^{d \times n}$ such that $AB = C$. For $k = 0, \dots, d$ we denote by $\Delta_k = \bigcup_{\tau=1}^{t} \Delta_k(S_\tau)$ the set of k-dimensional faces of simplicial complex Δ. Assume that $f_k(\Delta) = |\Delta_k|$, $f(\Delta) = (f_0(\Delta), \dots, f_d(\Delta))$ and $f(\lambda, \Delta) = 1 + \sum_{k=1}^{d} f_{k-1}(\Delta)\lambda^k$. Represent the polynomial $f(\lambda, \Delta)$ as $f(\lambda, \Delta) = \sum_{k \in \mathbf{Z}_+} \gamma_k(\Delta)\lambda^k(1 + \lambda)^{d-k}$. The integer sequence $\gamma = (\gamma_0, \gamma_1, \dots)$ is called (d, n)*-realized* if $\gamma_k = \gamma_k(\Delta)$ for $k = 0, \dots, d$ and $\gamma_k = 0$ for $k > d$.

The *triangulation of K with knots from B* is the set $T(B) = \{S_1, \dots, S_t\}$ such that S_τ $(\tau = 1, \dots, t)$ satisfy the following conditions:

1) $S_\tau \subseteq \{1, \dots, n\}$, 2) $|S_\tau| = r = \operatorname{rank} B(S_\tau)$, 3) $B^{\angle} = \bigcup\limits_{\tau=1}^{t} B^{\angle}(S_\tau)$,

4) $B^{\angle}(S_\tau) \cap B^{\angle}(S_\sigma) = B^{\angle}(S_\tau \cap S_\sigma)$.

A criterion for the realizability of f-vector of the triangulation was given last year [2].

Since the strutures of $A_{\angle}$ and $B^{\angle}$ are anti-isomorhic. This allows to determ f-vector of regular triangulations. The main tool here is eigen-values of $A \cdot A^T$. Using γ_k instead of f_k allows us to estimate the power of n as function of d. This is illustrated by the following table.

rank	$H_{d,n}$	$G^s_{d,n}$
2	$0 \le \gamma_2 \le \gamma_1 \le n-3$	$g = (1, \gamma_1 - \gamma_2)$; $0 \le g_1 = \gamma_1 - \gamma_2 \le n-3$
3	$0 \le \gamma_3 \le \gamma_1 \le n-4$ $\gamma_3 \le \gamma_2 \le (\gamma_1)^{\langle 1 \rangle}$	$g = (1, \gamma_1 - \gamma_3)$; $0 \le g_1 = \gamma_1 - \gamma_3 \le n-3$
4	$0 \le \gamma_4 \le \gamma_3 \le \gamma_2 \le (\gamma_1)^{\langle 1 \rangle}$ $\gamma_4 \le \gamma_1 \le n-5$ $\gamma_2 - \gamma_3 \le (\gamma_1 - \gamma_4)^{\langle 1 \rangle}$	$g = (1, g_1, g_2); 0 \le g_1 \le n-5$ $0 \le g_2 \le g_1^{\langle 1 \rangle}$ $h = (1, 1+g_1, 1+g_1+g_2, 1+g_1, 1)$
5	$0 \le \gamma_5 \le \gamma_4 \le \gamma_3 \le \gamma_2^{\langle 2 \rangle}$ $0 \le \gamma_2 \le \gamma_1^{\langle 1 \rangle}$ $\gamma_5 \le \gamma_1$; $\gamma_4 \le \gamma_2$ $(\gamma_2 - \gamma_4) \le (\gamma_1 - \gamma_5)^{\langle 1 \rangle}$ $\gamma_3 - \gamma_4 \le (\gamma_2 - \gamma_5)^{\langle 2 \rangle}$	$g = (1, g_1, g_2)$ $0 \le g_1 \le n-6$ $0 \le g_2 \le g_1^{\langle 1 \rangle}$ $h = (1, 1+g_1, 1+g_1+g_2,$ $1+g_1+g_2, 1+g_1, 1)$

References

1. *V.N. Shevchenko.* “Triangulations of convex polyhedra and realizablity of their f-vectors,” in: Russian conference ”Discrete optimization and operation research”: Proceedings of the conference (Altay, June 27 – July 3 2010). Novosibisk, Institute of Mathematics, 2010, pp. 75–81.
2. *V. Shevchenko.* “Triangulations of polyhedra and convex cones and realization of their f-vectors,” in: V International Conference on Optimization Methods and Applications (OPTIMA-2014), Moscow, 2014, pp. 176–177.
3. *V.N. Shevchenko.* “Triangulations of convex polyhedra and their Boolean functions,” in: Mathematical Problems of Cybernetics, vol. 16, Fizmatlit, Moscow, 2007, pp. 43–56.
4. *P. McMullen.* “The numbers of faces of simplicial polytopes,” Israel J. Math., **9**, 559–570 (1971).

A Method of Meeting Paths for Piecewise Linear Exchange Model

V.I. Shmyrev[1]

[1] *Sobolev Institute of Mathematics, Siberian Branch, Russian Academy of Science, Novosibirsk;* shvi@math.nsc.ru

The exchange model with piecewise linear separable concave utility functions of participants is considered. The consideration extends an original approach to the equilibrium problem in a linear exchange model and its variations. The conceptual base of this approach is the scheme of polyhedral complementarity. It has no analogs and made it possible to obtain the finite algorithms for some variations of the exchange model [1]. The consideration is based on the new notion of *consumption structure*. Two sets of the price vectors can be associated for each structure : a preference zone as a set of prices by which the participants prefer the connections of the structure, and a balance zone as a set of prices by which the budget conditions and balances of goods are possible when the connections of the structure are respected, but the participants preferences are ignored. In this way a point-to-set mapping is obtained.The fixed points of this mapping , and only they, give the equilibrium price vectors of the model. The mapping has some monotonicity property and should be characterized as a decreasing mapping. For the fixed point searching in the case of linear exchange model and production-exchange model [2] the method of meeting paths was proposed. The presenting consideration deals with its generalization on the piecewise linear exchange model.

The author was supported by the Russian Foundation for Basic Research (project no. 12-01-00667-a)

REFERENCES

1. Shmyrev V. I. *Polyhedral complementarity and equilibrium problem in linear exchange models* - Far East Journal of Applied Mathematics, vol. 82, Number 2, 2013, pp.67-85.

2. Shmyrev V. I. *A method of meeting paths for the linear production-exchange model* - J. Appl. Indust. Math.,vol. 6, Number 4,2012, pp. 490–500.

On the symmetries of the b-matching polytope

R.Yu. Simanchev[1,2], **I.V. Urazova**[2]

[1] *Omsk Scientific Center, Omsk, Russia;* osiman@rambler.ru
[2] *Omsk state university, Omsk, Russia;* urazovainn@mail.ru

Let G is usual graph without loops and multiple edges with the vertex set V and edge set E, $b = (b_u, u \in V)$ – vector with integer positive components. Subgraph $H \subseteq G$ is called b-matching if $d_H(u) \leq b_u$ for all vertices $u \in V$. The b-matching is called a matching if $b_u = 1$ for all $u \in V$. Let R^E is the space of column vectors that axes are in one-to-one correspondence with an edge set E. For each b-matching H define its incidence vector as $(0,1)$-vector $x^H \in R^E$ with the components $x_e^H = 1$, if $e \in EH$ and $x_e^H = 0$ otherwise. The b-matching polytope is the convex hull all b-matching incidence vector, i.e

$$P(G,b) = conv\{x^H \in R^E | \text{ for all } H \subset G \text{ that is } b\text{-matching}\}.$$

We consider the set of symmetries of the polytope $P(G,b)$. We define the symmetry of $P(G,b)$ as the affine nondegenerate transformation $\varphi : R^E \to R^E$ such that $\varphi(P(G,b)) \equiv \{\varphi(x) | x \in P(G,b)\} = P(G,b)$. It is clear that the set of symmetries is a group with respect to composition of transformations.

The interest to the symmetries of the polytopes is due to the following. Every symmetry translates face of the polytope to the face of the same dimension. In particular this fact allows to "duplicate" the inequalities that generate facets of a polytope (see [1]). In [2] the algorithm for reducing the dimension of the independence system problem that is based on symmetries of polytope is proposed. Another important useage of the symmetries of the polytope is the possibility of "adjustment" of the linear objective function on a polytope. Let $\varphi(x) = Ax + h$ (A is a square non-singular matrix of order $|E|$, $h \in R^E$) is the symmetry of $P(G,b)$. If a vector $\bar{x}$ is optimal solution of the problem $max\{c^T Ax | x \in P(G,b)\}$ then a vector $x^* = A\bar{x} + h$ is optimal solution of the problem $max\{c^T x | x \in P(G,b)\}$.

In this report the properties of the group of symmetries of $P(G,b)$ are discussed. The modification of the graph G is obtained. The automorphism group of this modification is isomorphic to the group of symmetries of the b-matching polytope. A special case of this result is isomorphic of the group of all linear symmetry of matching polytope and automorphism group of the original graph G (see [3]).

References

1. *G. Bolotashvili, E. Girlich, M. Kovalev.* New facets of the linear ordering polytope. Preprint Otto-von-Guericke-University Magdeburg, N 15 (1995).
2. *O.V. Chervyakov.* “Affine symmetries of the polytope of an independence system with unit shift”, Diskretn. and assayed. Oper., Ser. 1, 6: 2, p.82–96 (1999).
3. *R.Yu. Simanchev.* “Linear Symmetries of a Matching Polytope and Automorphisms of the Graph”, Bulletin of Omsk University, N.1. p.18–20 (1996).

On an algorithm for optimal correction of inconsistent problems of convex programming

Vladimir Skarin[1]

[1] *Krasovsky Institute of Mathematics and Mechanics UB RAS, Ekaterinburg, Russia;* skavd@imm.uran.ru

Consider the convex programming (CP) problem

$$\min\{f_0(x) \mid x \in X\}, \tag{1}$$

where $X = \{x \mid f(x) \leq 0\}$, $f(x) = [f_1(x), \dots, f_m(x)]$, $f_i(x)$ are convex functions defined on $\mathbb{R}^n$ $(i = 0, 1, \dots, m)$. The CP problems with contradictory constraints ($X = \varnothing$) often arise in mathematical modeling of complex real-life systems. The correction of similar improper [1] problems is understood as a transformation of an inconsistent model into a model that belongs to a family of feasible problems.

Let $X_\xi = \{x \mid f(x) \leq \xi\}$, $E = \{\xi \in \mathbb{R}^m_+ \mid X_\xi \neq \varnothing\}$ and $\bar{\xi} = \arg\min\{\|\xi\| \mid \xi \in E\}$. Along with (1) we consider the problem

$$\min\{f_0(x) \mid x \in X_{\bar{\xi}}\}. \tag{2}$$

If $X \neq \varnothing$, then $\bar{\xi} = 0$, and problem (2) coincides with problem (1). Otherwise, (2) is an example of possible optimal correction for improper problem (1).

For the solving of the problem (2) we apply the residual method [2] for the regularization of ill-posed models. This method consists in solving of a sequence of problems

$$\min\{\|x\|^2 \mid x \in X \cap M_\delta\}, \tag{3}$$

where $M_\delta = \{x \mid f_0(x) \leq \delta\}$, $\delta \in \mathbb{R}^1$. If the problem (1) is solvable and f^* is the optimal value of (1), then the problem (3) has a unique optimal point x^*_δ for any $\delta \geq f^*$ and sequence x^*_δ converges to the normal solution of the problem (1) as $\delta \to f^*$.

If the set X is empty, then we replace the restrictions of the problem (3) by a penalty function, for examples by the quadratic penalty function

$$F_\delta(x, r) = \|x\|^2 + \rho\|f^+(x)\|^2 + \rho_0(f_0(x) - \delta)^{+^2}, \quad r = [\rho, \rho^0] > 0.$$

The function $F_\delta(x, r)$ is strongly convex with respect to x in $\mathbb{R}^m$; hence problem $\min\limits_x F_\delta(x, r)$ has a unique solution for any $r \in \mathbb{R}^2$, $r > 0$ and $\delta \in \mathbb{R}^1$, including the case $X = \varnothing$ in contrast to problem (3). Therefore, the function $F_\delta(x, r)$ can by used for the analysis of improper CP problems.

The practical application of the method (3) for the approximation of improper CP problems is connected with the necessity to implement the condition $\delta \to \bar{f}$, where $\bar{f}$ is the optimal value of the problem (2). We present here an algorithm to find a sequence of δ_k that converges to the solution of (2) as $k \to \infty$.

Let $\varepsilon_k > 0$, $\varepsilon_k > \varepsilon_{k+1}$, $\lim\limits_{k\to\infty} \varepsilon_k = 0$ be given and bounds δ_0, C be known, where $\delta_0 < \bar{f} - \varepsilon_0$, $C \geq \|\bar{x}_0\|^2$, $\bar{x}_0 = \arg\min\{f_0(x) \mid x \in X_{\bar{\xi}}\}$. Assuming $\delta_k < \bar{f} - \varepsilon_k$, let us find

$$\delta_{k+1} = \delta_k + (f_0(\tilde{x}_k) - \delta_k - \varepsilon_k)^+, \qquad k = 0, 1, \ldots, \tag{4}$$

where $\tilde{x}_k = \arg\min\limits_x \widetilde{F}_k(x)$, $\widetilde{F}_k(x) = \widetilde{F}_{\delta_k}(x, r_k, \varepsilon_k) = \|x\|^2 + \rho_k \|f^+(x)\|^2 + \rho_k^0 (f_0(x) - \delta_k - \varepsilon_k)^+$, $r_k = [\rho_k, \rho_k^0] > 0$, $\rho_k^0(\varepsilon_k - \varepsilon_{k+1}) > C$.

Theorem. *Suppose that problem* (2) *is solvable and parameters* ε_k, ρ_k, ρ_k^0 *are chosen such that* $\varepsilon_k \to 0$, $\rho_k \to \infty$, $\rho_k^0 \to \infty$, $\rho_k^0 \rho_k^{-1} \to \infty$ *as* $k \to \infty$. *Then,* $\lim\limits_{k\to\infty} \delta_k = \bar{f}$ *in method* (4) *and any limit point of the sequence* $\{\tilde{x}_k\}$ *solves the problem* (2). *If* $\lim\limits_{k\to\infty} (\rho_k^0)^2 \rho_k^{-1} = 0$ *holds, then* $\lim \tilde{x}_k = \bar{x}_0$.

The author was supported by the Russian Scientific Foundation, grant no. 14-11-00109.

References

1. *I.I. Eremin, V.D. Mazurov, and N.N. Astaf'ev.* Improper problems of linear and convex programming, Nauka, Moscow (1983) (in Russian).
2. *V.D. Skarin.* On the application of the residual method for the correction of inconsistent problems of convex programming. Trudy Inst. Math. Mekh. UrO RAN (2014), v. 20, no. 2, pp. 268–276 (in Russian).

Sufficient optimality condition based on the increment formula

Vladimir Srochko[1], **Vladimir Antonik**[2]

[1] *Irkutsk State University, Irkutsk, Russia;* srochko@math.isu.ru
[2] *Irkutsk State University, Irkutsk, Russia;* antonik-vg@bk.ru

A typical optimal control problem is considered:

$$\Phi(u) = \varphi(x(t_1)) + \int_T F(x(t), u(t), t)dt \to \min,\ u \in V, \tag{P}$$

$$\dot{x} = f(x, u, t),\ x(t_0) = x^0,$$

$$V = \{\, u(\cdot) \in \mathrm{PC}(T) :\ u(t) \in U,\ t \in T \,\}.$$

We assume that $\varphi(x)$ is a convex, continuously differentiable function.

Let $u, v \in V$. Nonstandard formula for the functional increment in the problem (P) is presented:

$$\Delta_v \Phi(u) = -\int_T \Delta_{v(t)} H(\psi(t, u), x(t, v), u(t), t)dt + \eta.$$

Here $H(\psi, x, u, t)$ is the Pontryagin function, $\psi(t, u)$, $x(t, v)$ are solutions of conjugate and phase systems with respect to corresponding controls $u(t)$, $v(t)$, and η is a residue with respect to phase increment $\Delta x(t) = = x(t, v) - x(t, u)$.

Theorem. *Let $u \in V$ and following conditions are true:*
1)

$$u(t) = \arg\max_{w \in U} H(\psi(t, u), x(t, v), w, t)\ \forall t \in T,\ v \in V;$$

2) the function $g(x, t) = H(\psi(t, u), x, u(t), t)$ is concave with respect to x.

Then the control $u(t)$ is optimal in the problem (P).

Procedures and examples of effective application of this result are given.

Merit functions for d.c. problems

Alexander S. Strekalovskiy[1]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* strekal@icc.ru

Consider the problem

$$(\mathcal{P}): \qquad \left.\begin{array}{c} f_0(x) := g_0(x) - h_0(x) \downarrow \min\limits_x, \quad , x \in S, \\ f_i(x) := g_i(x) - h_i(x), \quad i \in I = \{1, \ldots, m\}, \end{array}\right\} \tag{1}$$

where the functions $g_i(\cdot)$, $h_i(\cdot)$, $i \in \{0\} \cup I$ are convex on $I\!R^n$, and the set S is convex in $I\!R^n$ [1–6]. Along with Problem $(\mathcal{P})$ we address two following auxiliary problems

$$(\mathcal{P}_\eta): \qquad F_\eta(x) \downarrow \min_x, \quad x \in S, \tag{2}$$

$$(\mathcal{L}): \qquad \mathcal{L}(x, \lambda) \downarrow \min_x, \quad x \in S, \tag{3}$$

where

$$F_\eta(x) = \max\{f_0(x) - \eta, f_i(x), i \in I\}, \tag{4}$$

$$\mathcal{L}(x, \lambda) = f_0(x) + \sum_{i=1}^{m} \lambda_i f_i(x). \tag{5}$$

For problems $(\mathcal{P}_\eta)$–(2) and $(\mathcal{L})$–(3) we establish, first, the relations with Problem $(\mathcal{P})$–(1).

For that purpose we used the following results [2–4].

Proposition 1. *Suppose that a feasible point z is a solution to Problem $(\mathcal{P})$, $z \in Sol(\mathcal{P})$. Then, the point z is a solution to the following auxiliary problem $(\mathcal{P}_\eta)$ with $\eta = \zeta \triangleq f_0(z)$.*

Proposition 2. *If the pair $(z, \lambda) \in S \times \mathbb{R}^m_+$ is a saddle point of the Lagrange function $\mathcal{L}(x, \mu)$ on the set $S \times \mathbb{R}^m_+$, then the point z is a global solution to Problem $(\mathcal{P})$.*

Proposition 3. *Suppose $z \in D$, z is a KKT-point, but it is not a global solution to Problem $(\mathcal{P})$. Then, there does not exist a Lagrange multiplier $\lambda \in \mathcal{M}(z)$ such that $(z, \lambda) \in Sdl(\mathcal{L})$.*

Second, we develop new apparatus of Global Optimality Conditions (GOC) [6–8], which is connected with the classical Optimization Theory and the previous corresponding results for canonical nonconvex optimization problems as convex maximization, revers-convex optimization, d.c. minimization and optimization with d.s. constraints.

These new tools allow us not only to escape critical points in examples considered, but to estimate the adequacy of Problems $(\mathcal{P}_\eta)$ and $(\mathcal{L})$ with respect to the original problem $(\mathcal{P})$–(1).

The authors were supported by the Russian Science Foundation (project no. 15-11-20015).

References

1. *J. Nocedal, S.J. Wright* Numerical Optimization, Springer, New York (2006).
2. *J.-B. Hiriart-Urruty* "Generalized Differentiability, Duality and optimization for Problems dealing with Difference of Convex Functions," in: Convexity and Duality in Optimization. Lecture Notes in Economics and Mathem. Systems, **256**, Berlin, Springer-Verlag, 1985, pp. 37–69.
3. *J.B. Hiriart-Urruty,C. Lemaréchal* Convex Analysis and Minimization Algorithms, Springer Verlag, Berlin (1993).
4. *J.-B. Hiriart-Urruty* Optimization et Analyse Convex, Presses Universitaires de France, Paris (1998).
5. *J.-F. Bonnans, J.C. Gilbert, C. Lemaréchal, C.A. Sagastizábal* Numerical Optimization: Theoretical and Practical Aspects, 2nd edn. Springer-Verlag, Berlin, Heidelberg (2006).
6. *A.S. Strekalovsky.* Elements of Nonconvex Optimization, Nauka, Novosibirsk (2003).
7. *A.S. Strekalovsky.* "On local search in d.c. optimization problems," Applied Mathematics and Computation, **255**, 73–83 (2015).
8. *A.S. Strekalovsky* "On Solving Optimization Problems with Hidden Nonconvex Structures," in: T.M. Rassias, C.A. Floudas, S. Butenko (Eds.). Optimization in Science and Engineering, Springer, New York, 2014, pp. 465–502.

The models of optimal economic development with risk criteria

Nikolai Tikhomirov, Damir Ilyasov

Plekhanov Russian University of Economics, Moscow, Russia;
nik.tikhomirov.46@mail.ru

In economic research, especially at the macro level, as the main purpose of the social development is often considered the maximization of consumption. This meant that the growth of consumption, the main cause of which is the expansion of productive accumulation, causes increasing of the protection of the population from the negative effects of environment. The validity of this assumption is confirmed by the results of comparing the average duration and standard of living in the countries of the world community and in each country over a sufficiently long period of its development. Life expectancy grows approximately logarithmically with incising standards of living, measured by per capita expenditure on food, health care, education, culture, improvement of sanitary and hygienic conditions of life and other areas determining the structure of personal consumption.

However, in modern conditions social development also is characterized by increasing the risks of new man-made and socio-political hazards for the health and human life. These include, in particular, the risks of industrial accidents, man-made and natural disasters, unlawful acts, including terrorist acts, etc. These risks are generally classified as external. To reduce it society is forced to spend some funds that could be directed to increasing accumulation or consumption. Thus, the distribution of national income should be divided on the three components according to the following relationship:

$$y(t) = u(t) + c(t) + z(t) \tag{1}$$

where t - the index of the year; $y(t)$ - national income produced in year t; $u(t)$ accumulation in year t; $c(t)$ - personal consumption in year t (standard of living); $z(t)$ - costs on protective measures against external risks in year t.

In such a situation as a purpose of social process instead of the maximization of consumption can be considered the minimization of the overall risk of life $R_0(c(t), z(t))$ or the maximization of life expectancy $T_{av}(c(t), z(t))$ depending on the personal consumption and costs on social safety.

To describe the indicators $R_0(c(t), z(t))$ and $T_{av}(c(t), z(t))$ can be used the following expressions:

$$R_0(c(t), z(t)) = R_1(c(t)) + R_2(z(t)) \tag{2}$$

$$T_{av}(c(t), z(t)) = \frac{T_{max}}{R_1(c(t)) + R_2(z(t))} \quad (3)$$

where T_{max} maximum average optimal life expectancy (eg, 100 years); $R_1(c(t))$ - average individual risk of death in year t for internal reasons (sickness, accidents in home, etc.), delivered in dependence on the level of personal consumption, for example, according to the following expression:

$$R_1(c(t)) = \alpha_0 e^{-\alpha_1 z(t)} \quad (4)$$

$R_2(z(t))$ average risk of death due to external reasons depending on the costs on security:

$$R_2(z(t)) = \beta_0 e^{-\beta_1 z(t)} \quad (5)$$

The coefficients α_0, α_1, β_0, β_1 can be evaluated according to the official statistics of the levels of these processes in the countries of the international community, or in a particular country for a number of years.

In view of the above expressions enlarged (single-sector) model of optimal socio-economic development in a range (0, t_k) can be represented by the following differential equation:

$$y(t) = B\frac{\delta y(t)}{\delta t} + c(t) + z(t) \quad (6)$$

where B - the capital intensity of national income, with criteria such as the following:

$$min\sum_t R_0(c(t), z(t)) \quad (7)$$

under constraints, reflecting no increase of risks $R_0(t) \geq R_0(t+1)$, consumption growth $C(t+1) \geq C(t)$ and nonnegative arguments y, u, c, z.

Note that the problem (6), (7) corresponds to the task of optimal operating with a free boundary and a fixed time tk. Its solution can be found using the Pontryagin maximum principle with Lagrangian numerical methods.

This paper is preapered under the grant of Plekhanov RUE "The methodology of comparative analysis of modes of reproduction in regions of Russian Federation in conditions of turbulent economy "

The approach to the assessment of health loss due to premature mortality in Russia

Tatiana Tikhomirova, Valeriya Gordeeva

Plekhanov Russian University of Economics, Moscow, Russia;
t_tikhomirova@mail.ru

The increase of life expectancy is one key priorities stipulated he concept of long-term socioeconomic development of the Russian Federation for the period up to 2020. Nowadays the current life expectancy in Russia is lower than in most developed countries. For instance, in 2013 in Western Europe this indicator was 80,54 years, in comparison to 70,76 years in our country.

It hardly can be explained by shortage of funding, because in a number of countries with a similar level of investments allocated to health care, the life expectancy is 4,88 years longer than in Russia. The increase of life expectancy along with recourse deficit requires prioritization of the measurements dedicated to health care modernization. The optimum recourse allocation is impossible without adequate indicators of health losses resulted from different causes of death.

The optimum recourse allocation means the goal-oriented recourse distribution between the regions of the Russian Federation in accordance with the level of health loss from specific causes of illness or death in these areas. Recently global burden of disease indicators have been widely applied to get estimates of health losses. However, the calculations of global burden of disease requires huge amounts of information on age-specific morbidity and mortality rates, that makes its implementation in Russia quite complex [2]. As a result, indicators of the global burden of disease are not widely used in the Russian Federation, despite the fact that in 2007 the method of DALE calculation was adapted for our country in order to analyze the regional health system effectiveness.

An alternative method for determination of health losses can be a comparative analysis of the age distributions on mortality from a particular disease.

The comparison of the age distribution on mortality in a particular area with the "standard" distribution determines the loss, expressed as years of life lost due to premature mortality. In order to test the method, there were calculated estimates of health loss due to cardiovascular diseases and cancer. The reduction in mortality due to these causes is especially important for Russia, as these diseases annually cause more than 70% from the total number of deaths.

As a result of analysis of age-specific mortality rates in 40 European countries, there were identified the countries with the lowest death risk.

In the comparison of mortality risks at different ages were constructed mathematical relationships risk of death by age in Russia and some European countries. To determine the standard distribution of deaths were classified 40 countries in Europe in terms of mortality from cardiovascular diseases and cancer and identified the countries in which the lowest level of risk [1]. Comparing the levels of risk in lead European countries and Russian Federation, it was determined that the losses from premature mortality from cardiovascular diseases and cancer in our country can be expressed by formulas (1) and (2) respectively:

Based on the results of simulation, we can conclude that the average loss in the form of premature mortality is 10.34 years from cardiovascular diseases and 2.98 years from cancer. That means that today the problem of premature death from cardiovascular diseases has more areas for development than mortality from cancer due to the fact that the gap in the level of losses is greater.

$$\mathcal{L}_{cardio}(x_{cardio}) = 2,39 * ln x_{cardio} + 3,99 \tag{1}$$

$$\mathcal{L}_{cancer}(x_{cancer}) = 20,02 * x_{cancer}^{0,19} - 17,321 * x_{cancer}^{0,2047} \tag{2}$$

L_{cardio}, L_{cancer} - loss from premature mortality from cardiovascular diseases and cancer respectively.

x_{cardio}, x_{cancer} - risk of death from cardiovascular diseases and cancer respectively.

Based on the results of simulation, we can conclude that the average loss in the form of premature mortality is 10.34 years from cardiovascular diseases and 2.98 years from cancer. That means that today the problem of premature death from cardiovascular diseases has more areas for development than mortality from cancer due to the fact that the gap in the level of losses is greater. The propose approach to the determination of losses due to premature mortality can be used to justify the investment attractiveness of projects aimed at fundamental reforms in health care. The fundamental reforms include programs that will reduce the mortality risk from a particular cause.

This work was supported by the grant provided by "Russian Foundation for Humanities" (RFH), project number 15-02-00412a "Risk assessment and risk management related to loss of life and health of the population in emergency situations with radiation leakage".

References

1. *Tikhomirov N.P., Tikhomirova T.M., Ushmaev O.S.* Metody Ekonometriki i mnogomernogo statisticheskogo analiza: Uchebnik M.: Ekonomika, 2011. 647 s.
2. *Tikhomirova T.M.* Metody analiza sostojanija i poter' zdorov'ja naselenija v regionah Rossii: monografija Moskva: FGBOU VPO "REU im. G. V. Plekhanova", 2012 353 s.

On complexity of one variety of the packing problem

Oleksandr Trofymchuk[1], **Liudmila Ushakova**[1], **Vladimir Vasyanin**[1]

Institute of telecommunications and global information space NAS of Ukraine, Kiev, Ukraine; itelua@kv.ukrtel.net

We consider one of a variety of packing problem, in which the elements of the rows and columns of a square matrix, given by natural numbers a certain way unite (merges, packing) in blocks fixed size. A practical formulations of problems, in which used packing of elements of a matrix of demands (commodity), subject to distribution in a network, are provided in [1, 2].

Given matrix $A = ||a_{ij}||_{n\times n}$ and number ω, which is define size of block (container). Values a_{ij}, $\omega \in \mathrm{N}$, $i, j = \overline{1, n}$, N – the set of natural numbers with zero. Let $a_{ii} = 0$, $i = \overline{1, n}$, and for $a_{ij} \neq 0$, may be performed $a_{ij} < \omega$, $a_{ij} \geq \omega$, $i, j = \overline{1, n}$. There are matrix $X = ||x_{ij}||_{n\times n}$, which will be transformed, initially $X = A$. The transformation of a matrix is that, of its elements (decision variable) $x_{ik}, x_{kj}, x_{ij} \neq 0$, $k \neq i \neq j$, may be iteratively performed the following operations:

$$x_{ik} \leftarrow x_{ik} + x_{ij}, x_{kj} \leftarrow x_{kj} + x_{ij}, c_{ij} \leftarrow k, y_k \leftarrow y_k + x_{ij}, x_{ij} \leftarrow 0. \tag{1}$$

In (1) are accepted the notation: k – column index, through which converts (merger) the x_{ij}; c_{ij} – elements of help matrix of the merge $C = ||c_{ij}||_{n\times n}$; y_k – elements of vector of sum values of a merged elements $Y = ||y_k||$, $k = \overline{1, n}$.

It is required a minimize function:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\lceil x_{ij}/\omega\rceil, \tag{2}$$

where $\lceil\cdot\rceil$ – ceiling function, $\lceil\cdot\rceil : x \to \lceil x\rceil$, $\lceil x\rceil = \min\{n \in \mathrm{N}\ |n \geq x\}$. Since for $\forall\ i = \overline{1, n}$ the must be performed equality $\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} a_{ij} = y_i$, $\sum_{j=1}^{n} x_{ji} - \sum_{j=1}^{n} a_{ji} = y_i$, write the balance conditions in the form

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} a_{ij} = \sum_{j=1}^{n} x_{ji} - \sum_{j=1}^{n} a_{ji}, i = \overline{1, n}. \tag{3}$$

Constraints on the variables values x_{ij} and y_i are defined as follows:

$$x_{ij} \leq \lceil a_{ij}/\omega\rceil\omega, \forall i, j = \overline{1, n}, \tag{4}$$

$$y_i = \sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} a_{ij} \leq h_i, i = \overline{1, n}. \tag{5}$$

In formulas accepted notation: $x_{ij} = x_{ij} + \sum_{rs\in\Omega_{ij}} x_{rs}$, if x_{ij} has not merges with the other elements, $x_{ij} = 0$ otherwise; Ω_{ij} – the set of index rs elements $x_{rs} \in \{x^*_{ij}\}$, which are have been merged with the element x_{ij}. The set Ω_{ij} may be empty, and is determined from the help matrix C by using algorithms given in [3]; $h_i \in$ N – defined bound; $i, j = \overline{1, n}$.

The problem (2)-(5) is NP-hard. The proof is based on a polynomial transformation of the known [4] NP-complete minimum-cost integer multicommodity flow problem (MC IMCF) to a problem (2)-(5).

Theorem. *Problem MC IMCF polynomially transformed to the problem (2)-(5).*

The key idea the proof consist in the formulation of the problem (2)-(5) in the form of MC IMCF and individual problem MC IMCF on a complete oriented network, when the indexes of all commodity are replaced on the numbers of nodes of origin s and destination t, and these coincide with numbers arcs in the complete network (i.e. for each commodity st exist the arc ij and $s = i, t = j$). In the individual problem MC IMCF also entered additional conditions on decision variables and the arc capacity, which ensure correctness of operations (1).

References

1. *A.N. Trofimchyk, V.A. Vasyanin.* "A Technique to Solve Packing Problem for the Management of Perspective Development of Communication Network Nodes", Cybernetics and Systems Analysis, **50**, No. 4, 560-569 (2014).
2. *V.A. Vasyanin.* "Problem of Distribution and Routing of Transport Blocks with Mixed Attachments and Its Decomposition", Journal of Automation and Information Sciences, **47**, No. 2, 56-69 (2015).
3. *V.A. Vasyanin.* "A help matrix of the merge flows in optimization problem of packings on multicommodity networks", System Research and Information Technologies, No 3, 42-49 (2014). (In Russian)
4. *S. Even, A. Itai, A. Shamir.* "On the complexity of timetable and multicommodity flow problems", SIAM J: Comput., **5**, 691-703 (1976).

A parallel subgradient method for maximizing a Lagrangian dual function in the p-median problem

Anton Ushakov[1], **Igor Vasilyev**[1]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* {aushakov,vil}@icc.ru

Nondifferentiable optimization problems arise in many fields of applications and especially when dealing with integer programs. One of the most successful approach to solving integer programs is the so-called Lagrangian relaxation method or Lagrangian heuristic [2]. The main issue when applying the method is that in order to find a lower bound of the objective function one should solve the nondifferentiable Lagrangian dual problem, which is to maximize a concave piecewise-linear Lagrangian dual function. For that purpose one often uses subgradient algorithms that are extremely efficient in the case when a special heuristic step-rule is used.

In this abstract we propose a parallel heuristic subgradient algorithm for maximizing the Lagrangian dual function derived from the well-known relaxation of the p-median problem. Recall that the p-median problem is one of the basic problems in combinatorial optimization and location theory. It consists in locating p facilities that can be placed at a set $I = \{1, \ldots, m\}$ of potential sites in order to satisfy the demand of a set $J = \{1, \ldots, n\}$ of clients such that the sum of distances from each client to the nearest facility is minimal. Note that the distance between client $j \in J$ and potential site $i \in I$ is denoted as $d_{ij} > 0$. This combinatorial optimization problem can easily be formulated as an integer program by introducing two sets of binary variables. For each $i \in I$ let y_i takes the value 1 if the facility at site i is open, and 0 otherwise, and for each pair $i \in I$, $j \in J$ let x_{ij} is equal to 1 if client j is served by facility at i, and 0 otherwise. Thus the p-median problem is

$$
\begin{aligned}
&\min_{(x,y)} \sum_{i\in I}\sum_{j\in J} d_{ij}x_{ij} && \\
&\sum_{i\in I} x_{ij} = 1 && j \in J,\\
&x_{ij} \le y_i && i \in I, j \in J,\\
&\sum_{i\in I} y_i = p, && \\
&y_i \in \{0,1\} && i \in I,\\
&x_{ij} \in \{0,1\} && i \in I, j \in J.
\end{aligned}
$$

One of the most popular type of Lagrangian relaxation for this problem is obtained by relaxing the first constraints. In order to get the best lower bound of the objective value one has to maximize the following Lagrangian dual function

$$\mathcal{L}(\lambda) = \sum_{i \in T(\lambda)} \rho_i(\lambda) + \sum_{i \in I} \lambda_i,$$

where $\rho_i(\lambda) \triangleq \sum_{j \in J} \min\{0, d_{ij} - \lambda_j\}$ and $T(\lambda)$ is a set of first p location sites $i \in I$ with minimum values of $\rho_i(\lambda)$.

To maximize the function $\mathcal{L}(\lambda)$ we develop a parallel version of the subgradient algorithm [1], that follows the Master-Slave parallel scheme and implemented by means of the message passing interface and OpenMP. We present some computational results for large-scale problem instances of the p-median problem obtained by running the parallel algorithm on the computer cluster "Akademik Matrosov" of the Irkutsk Supercomputing Center.

The authors were supported by the Russian Science Foundation (project no.15-11-20015).

References

1. *P. Avella, M. Boccia, S. Salerno, I. Vasilyev.* "An aggregation heuristic for large scale p-median problem," Comput. Oper. Res., **39**, No. 7, 1625–1632 (2012).
2. *J. E. Beasley* "Lagrangean heuristics for location problems," Eur. J. Oper. Res., **65**, No. 3, 383–399 (1993).

The Development of the Logistic Transport System for Solving the Problem of Homogeneous Goods Delivery

Aida Valeeva[1], **Ruslan Valeev**[2], **Yuliya Goncharova**[3], **Ivan Koshcheev**[4]

[1] *USATU, Ufa, Russia;* aida_val2004@mail.ru
[2] *USATU, Ufa, Russia;* ruslan__valeev@inbox.ru
[3] *USATU, Ufa, Russia;* yuliagonch@mail.ru
[4] *USATU, Ufa, Russia;* ivan.koscheev.ufa@gmail.com

In this article the logistic transport system for solving the problem of homogeneous goods delivery (PHPD, Problem homogeneous product delivery), produced by the petrochemical company, to different customers, including the following subproblems:

1. Inventory management.
2. Warehousing.
3. Creation of the rational routes for the delivery of the product, produced by the petrochemical company, by the car vehicles to the different customers subject to the following conditions: 3.1. There are one or several depots for the vehicles. 3.2. Every route starts and ends at the depot. 3.3. A fleet of vehicles is heterogeneous. 3.4. The time windows are considered. The service of each customer must start within the associated time window. Moreover, in case of early arrival at the location of customer, the vehicle generally is allowed to wait until the service may start. 3.5. Delivery period of days is considered, where days is the number of days, during which delivery of the containers with reagent must be carried out. 3.6. The possibility that customers return some commodities is contemplated. 3.7. The split delivery is considered: each customer can be visited more than one vehicle. 3.8. The mass of containers with product, loading in vehicle, doesnt exceed the capacity of the vehicle. 3.9. The penalty of the route, which doesnt correspond to the rational packing of containers with product in vehicles during the delivery of the containers with product to the different customers on this route. 3.10. The demand of customers in reagent must be satisfied.
4. Packing of the containers with finished products in vehicles.

The aim of solving the problem of inventory management is to choose an effective strategy to minimize maintenance costs, execution of order, taking into account the cost associated with pending deliveries. To this end, inventory control system is proposed, including different models (deterministic, probabilistic, simulation), which are used depending on the initial information. The population based ant colony optimization algorithm for solving the problem of creation of the rational routes for the delivery of the containers with produced product by the car vehicles to the different customers subject to the mentioned above constraints and evolutionary algorithm (1+1)-EA3D for solving the packing problem of the containers with produced product in the vehicles with packing plan subject to the technological constraints are developed. Some numerical experiments on random generated data, taking into account such restrictions as vehicle capacity, time windows, split delivery, delivery period, multi depot and so on, and tested examples, taken from International OR-library tests (http://people.brunel.ac.uk/ mastjjb/jeb/info.html), are resulted.

The results of the population based ant colony optimization algorithm were compared with the results of the genetic algorithm [1] and of the tabu search algorithm [1]. The proposed algorithm showed the best values of the objective function on four test examples. The results of the evolutionary algorithms were compared with the results obtained in [2, 3]. The proposed algorithms showed the best results for two examples. The developed algorithms for solving PHPD were combined into the logistic transport system as an optimization kernel of the system.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-07-00579).

References

1. *Reimann M.* Ant Based Optimization in Good Transportation. // PhD Thesis. University of Vienna. Vienna, Austria. – 2002. – P. 149.
2. *Lim A., Rodrigues B., etc.* 3D Container Packing Heuristics. // Applied Intelligence. – 2005. – V. 22. N. 2. – P. 125-134.
3. *Nepomuceno N., etc.* Tackling the Container Loading Problem: A Hybrid Approach Based on Integer Linear Programming and Genetic Algorithms. // Comp. Sc. – 2007. – V. 4446. – P. 154-165.

Optimization of Network Structure for Tree-type Market

Alexander Vasin[1], **Polina Kartunova**[2]

[1] *Lomonosov Moscow State University, Moscow, Russia;* `foravas@yandex.ru`
[2] *Lomonosov Moscow State University, Moscow, Russia;*

We consider a model of a perfect competitive market for a homogeneous good with a network structure without cycles. Local markets are connected by transmitting lines with limited capacities and given cost functions for capacity increments. Let a graph with a set of nodes N and a set of edges $L \subseteq N \times N$ correspond to the market. Each node i is characterized by demand and supply functions $D_i(p), S_i(p)$ related to consumption utility and production cost functions:

$$U_i(q) = \int_0^q D_i^{-1}(v)dv, S_i(p) = Arg\max_v (pv - c_i(v))$$

Each edge $(i,j) \in L$ is described by construction cost function $E_{ij}(q_{ij})$ including fixed cost c_f^{ij} and convex variable cost $c_v^{ij}(|q_{ij}|)$. Under given transmission capasities and production volumes, the total social welfare is determined as follows:

$$\overline{W} = \sum_{i \in N} \left[U_i \left(v_i + \sum_{l \in \sigma^{-1}(i)} q_{li} - q_{i\sigma(i)} \right) - c_i(v_i) - E_{i\sigma(i)}(q_{i\sigma(i)}) \right],$$

where $\sigma(i)$ is the proceding node for node i in the network tree. We consider the total welfare optimization problem and provide a method that determines optimal investments for tree-type networks.

Proposition 1. *Under any fixed flows of the good between the lockal markets* $(q_{i\sigma(i)}, i \in N)$ *the optimal production volume at node* i *is* $v_i = S(\widetilde{p}_i)$, *where* $\widetilde{p}_i$ *meets equation* $\Delta S_i(\widetilde{p}_i) = -\sum\limits_{l \in \sigma^{-1}(i)} q_{li} + q_{i\sigma(i)}$.

In order to find optimal transmission capacities, we introduce the following notations:

$W^i(p_i)$ — maximal total welfare at node i and all subsequent nodes under a given price at node i and optimal flows between the nodes;

$\Delta S_i(p_i) = S_i(p_i) - D_i(p_i)$ — net supply-demand balance at node i;

$\Delta \overline{S^J}(p_j) = \Delta S_j(p_j) + \sum_{\{i | i \in \sigma^{-1}(j)\}} q_{ij}(p_j)$ — the same value with account of optimal flows of the good in the submarket corresponding to the node j.

A method for determination of optimal transmission capacities is as follows.

Stage 1. Consider the set N_1 of final nodes (such that $\sigma^{-1}(i) = \varnothing$). For every node i from this set the optimal welfare under price p_i is $W^i(p_i) = U_i\left(D_i(p_i)\right) - c_i(S_i(p_i))$.

Stage k. Consider the set N_k of k-level nodes (such that $\sigma^{-1}(i) \subseteq N_1 \cup \ldots \cup N_{k-1}$). Let p_j be a fixed price at node $j \in N_k$. At the previous stages, for every node i from the sets $N_1, \ldots, N_{k-1}$ we determined the optimal value $W^i(p_i)$ of the total welfare, functions $\Delta\overline{S^i}(p_i)$ and the optimal flows $q_{l\sigma(l)}(p_{\sigma(l)})$ in the submarket corresponding to the node depending on the price p_i. For every node $i \in \sigma^{-1}(j)$ let $q^*_{ij}(p_j), p^*_i(p_j)$ denote the solution of the system

$$\begin{cases} \Delta\overline{S^l}(p_i) = q_{ij}, \\ p_j - p_i = E'_{ij}(q_{ij}). \end{cases}$$

Consider also the price p^0_i proceeding from equation $\Delta\overline{S^l}(p_i) = 0$.

Proposition 2. *For every $i \in \sigma^{-1}(j)$, the optimal price $p_i(p_j)$ and the optimal flow $q_{ij}(p_j)$ that maximize the total social welfare meet the following system:*

$$(p_i, q_{ij})(p_j) = \begin{cases} (p^*_i, q^*_{ij})(p_j) \; if \; W^i\left(p^*_i(p_j)\right) + p_j q^*_{ij} - E_{ij}\left(q^*_{ij}(p_j)\right) \geqslant W^i(p^0_i), \\ (p^0_i, 0) \; otherwise. \end{cases}$$

The proofs base on the Welfare Theorem for the competitive market (Arrow and Debreu, 1956).

The research was supported by RFBR, project No. 14/01/91163 GFEN a.

References

1. *K. Arrow and G. Debreu* The existence of an equilibrium for a competitive economy, Econometrica22, 265-290, 1954.

A Numerical Method for Solving Convex Optimal Control Problems

Maxim Yanulevich[1]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* max@irk.ru

Consider the following linear control system [2]:

$$\dot{x}(t) = A(t)x(t) + b(u(t),t) \quad \overset{\circ}{\forall} t \in T = [t_0, t_1], \quad x(t_0) = x_0, \tag{1}$$

$$u(\cdot) \in \mathcal{U} = \{\, u(\cdot) \in L^r_\infty(T) \mid u(t) \in U \quad \overset{\circ}{\forall} t \in T \,\}, \tag{2}$$

where $A(\cdot)$ is $(n \times n)$-matrix function with continuous elements $t \mapsto a_{ij}(t)$, $i,j = 1,2,\ldots,n$ on $T := [t_0, t_1]$, and U is a compact set. Assume also that vector function $(u,t) \mapsto b(u,t)$ is continuous with respect to variables $u \in I\!R^r$ and $t \in T$. Further, let us denote absolutely continuous solution $x(\cdot,u)$ of system (1)–(2).

We study the following convex optimal control (OC) problem form the numerical solution point of view:

$$(\mathcal{CP})\colon\ I(u) = \varphi_1(x(t_1,u)) + \int_T \varphi(x(t,u),t)\,dt \downarrow \min_u, \quad u(\cdot) \in \mathcal{U}, \tag{3}$$

where functions $x \mapsto \varphi_1(x)\colon I\!R^n \to I\!R$ and $(x,t) \mapsto \varphi(x,t)$ are convex functions.

Note that problem $(\mathcal{CP})$ is auxiliary problem for solving the following nonconvex optimal control problem (see [3,4]):

$$(\mathcal{P})\colon \quad J(u) = F_1(x(t_1,u)) + \int_T F(x(t,u),t)\,dt \downarrow \min_u, \quad u(\cdot) \in \mathcal{U}, \tag{4}$$

with d.c. functions $x \mapsto F_1(x)\colon I\!R^n \to I\!R$ and $(x,t) \mapsto F(x,t)$ (A.D. Alexandrov's functions, see [3]), which can be represented as a difference of two convex functions with respect to variable x (for all $t \in T$):

$$F_1(x) = g_1(x) - h_1(x), \quad F(x,t) = g(x,t) - h(x,t) \quad \forall x \in \Omega \subset I\!R^n, \quad t \in T.$$

Here $x \mapsto g_1(x)$, $x \mapsto h_1(x)$, $x \mapsto g(x,t)$, and $x \mapsto h(x,t)$ are convex functions with respect to variable x for all $t \in T$. Note that in auxiliary problem (CP) functions $x \mapsto \varphi_1(x)$ and $(x,t) \mapsto \varphi(x,t)$ are partially linearized as follows ($x \in I\!R^n$, $t \in T$):

$$\varphi_1(x) = g_1(x) - \langle \nabla h_1(y(t_1)), x \rangle, \quad \varphi(x,t) = g(x,t) - \langle \nabla h(y(t),t), x(t) \rangle,$$

where $y(t)$, $t \in T$ is a given vector function. Because of nonconvexity of problem $(\mathcal{P})$, which is created by objective functional $J(\cdot)$, it might possess a number processes, satisfying the Pontryagin maximum principle (PMP) [1], which are rather far from a global solution.

On the basis of the global optimality conditions [3] we previously proposed the method global search method for problem $(\mathcal{P})$ [4,5], which combines a local search search procedure and the procedure for improving process, satisfying PMP [1]. In the same time, in these procedures it is necessary to solve some convex (linearized) problems of the form $(\mathcal{CP})$ on each iteration of the developed global search method.

In order to improve efficiency and velocity numerical solving of auxiliary convex problems, we propose the new method for solving problem $(\mathcal{CP})$ that uses the Pontryagin maximum principle and ideas of mathematical programming method. Numerical experiment is performed on series of test problems with quadratic objective functional.

The work is supported by the Russian Science Foundation (project no. 15-11-20015).

References

1. *L.S. Pontryagin, V.G. Boltyanskii, R.V. Gamkrelidze, E.F. Mishchenko.* The Mathematical Theory of Optimal Processes, Interscience, New York (1976).
2. *R. Gabasov, F.M. Kirillova.* Optimization of Linear Systems, Plenum Press, New York (1979).
3. *A.S. Strekalovsky.* "Global Optimality Conditions for Optimal Control Problems with Functions of A.D.Alexandrov," Journal of Optimization Theory and Applications, **159**, No. 6, 297–321 (2013).
4. *A.S. Strekalovsky, M.V. Yanulevich.* "Global Search in a Nonconvex Optimal Control Problem," Journal of Computer and Systems Sciences International, **52**, No. 6, 893–908 (2013).
5. *A.S. Strekalovsky, M.V. Yanulevich.* "Global search in the optimal control problem with a terminal objective functional represented as a difference of two convex functions," Comput. Math. Math. Phys. **48**, No. 7, 1119–1132 (2008).

The study procedure of solvability of the system of interval algebraic equations for the object control problem with inaccurate data in parameters

N.R. Yunicheva[1]

[1] *Institute of Information and Computational Technologies of MES of the RK, Almaty, Kazakhstan;* naduni@mail.ru

In the proposed paper, the interval arithmetic is used with nonstandard subtraction and division, as well as centered form of intervals representation [1,2].

In applied interval analysis the system of linear algebraic equations is called solvable if it has a solution, and valid if it has a nonnegative solution [3].

For the interval linear systems, there are additional definitions: the term "weak" is linked to the performance of the specified property for "some" of the system from a given set, and the term "strong" is linked with the implementation of the specified property for all systems from this family. The introduction of strong and weak properties has its own reasons.

For example, we want to find out whether the some point system of algebraic equations is solvable , but we do not know the exact data about the system (data obtained from some measurements, subjects to rounding errors, etc.). Instead, we know only that it satisfies the conditions. Then, our system is solvable unless it is known that the system is strongly solvable. Conversely, the system is unsolvable, unless it is known that the system is weakly solvable. In case of feasibility is the same reasoning.

In this article, to facilitate computational difficulties in the handling of interval values, the problem of parametric synthesis control of object with inaccurate data in parameters is reduced to the solution of the algebraic system of interval equations, therefore there is a need to study the solvability of the resulting system.

The property of strong solvability of interval equations system is examined:

$$\{P \cdot K \,|P \in \mathbf{P}\} \subseteq H.$$

Let us call a vector $K \in R^n$, a strong solution of the system if it satisfies the point system $\mathbf{P} \cdot K = \mathbf{H}$, for any $P \in \mathbf{P}$ and $H \in \mathbf{H}$.

The proof of this assertion is held by the scheme proposed in [4]. The result shows that the system of interval linear equations, which reduced to the problem of parametric synthesis of control is solvable (in this case, it has a strong solvability).

References

1. *Sharyi S. P.* Finite-dimensional interval analysis. - ICT SB RAS. - Novosibirsk: XYZ, 2015. - p.605.
2. *G. Alefeld, G. Mayer* "Interval analysis: theory and applications," in: Journal of Computational Applied Mathematics. -V.121. 2001. pp. 421-464.
3. *L. Zholen, M. Kifer, O Deidre, E. Walter* Applied interval analysis. M.: Institute of computer science. 2007. - p.467
4. *M.Fiedler ,J.Nedoma , J.Ramik , J.Rohn , K. Zimmerman* Speech segmentation without speech recognition, Linear optimization problems with inexact data. M.: Institute of computer researches. 2008. 288 p.

Solving one location problem of rectangles in the plane with weighted points

Gennady Zabudsky[1], **Tatiana Keyner**[2]

[1] *Sobolev Institute of Mathematics SB RAS, Omsk, Russia;*
zabudsky@ofim.oscsbras.ru
[2] *Omsk State University n.a. F.M. Dostoevskiy, Omsk, Russia;*
volka89@yandex.ru

Let $P = \{p_1, ..., p_m\}$ be a set of m weighed points within an axis-paralel rectangle R. Denote the weights of the points by $\{w_1, ..., w_m\}$, respectively. Let $Q_1, ..., Q_n$ be another axis-paralel rectangles which are smaller than R (both in width and in height). The goal is to place $Q_1, ..., Q_n$ within R without intersections among themselves such that the maximal sum of the weights of the points of P that are contained in $Q_i, i = 1, \ldots, n$ is minimal. One application of this problem aries in the context of obnoxious facilities location [1].

In the report the problem with rectilinear metric and when $n = 2$ is considered. It is proved that for search of the optimum of the problem sufficient to consider a discrete subset of admissible solutions. The algorithm for constructing the subset is developed. Results of computing experiment in comparison of efficiency of the algorithm and the solving of the problem by means of mixed-integer linear programming model and package IBM ILOG CPLEX are represented.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-01-00862).

References

1. *G.G. Zabudsky, T.I. Keyner, Koval T.I.* “Minimax and maximin problems to locate obnoxious facilities on a plane,” Proceedings of the National Conference ”Statistics. Modeling. Optimization”, South Ural State University, Chelyabinsk, 2011, pp. 117–120 (in Russian).

On average number of feasible solutions of a class of the generalized set packing problem

Lidia Zaozerskaya[1]

[1] *Omsk Branch of Sobolev Institute of Mathematics SB RAS, Omsk, Russia;*
`zaozer1ofim@gmail.com`

Consider the generalized set packing problem (GSPP):

$$\max\{cx \mid Ax \leq b,\ x \in \{0,\ 1\}^n\}.$$

Here $A = ||a_{ij}||$ is a boolean $m \times n$ matrix; $c = (c_1, ..., c_n)$; $b = (b_1, ..., b_m)$; $x = (x_1, ..., x_n)^{\mathrm{T}}$ is a vector of the boolean variables. All $c_j \geq 1$, $b_i \geq 1$ are integer.

We investigate the class $\mathcal{G}(n, p, B)$ of the GSPP, where all the elements of the matrix A are independent random variables, notably $P\{a_{ij} = 1\} = p$, $P\{a_{ij} = 0\} = 1 - p$, where $p \in (0,\ 1)$, and $b_i = B$, $i = 1, ..., m$. Denote by $\mathbf{E}|D(n, p, B)|$ the mathematical expectation of the cardinality of feasible solutions set for the problems from the class $\mathcal{G}(n, p, B)$. For every integer $k \geq 1$ and fixed $n \geq 2k + 1$, p and $B \leq k$ we have obtained some condition on the parameter m, at which $\mathbf{E}|D(n, p, B)|$ does not exceed value $O(n^{k+1})$. Earlier similar results were obtained for a class of the set packing problem ($B = 1$) [1].

Using the approach from [1] and the presented result the polynomial upper bounds on the average number of iterations for some known integer linear programming algorithms for solving of problems from class $\mathcal{G}(n, p, B)$ may be find.

The authors were supported by the Russian Foundation for Basic Research (project no. 13-01-00862).

References

1. *A.A. Kolokolov, L.A. Zaozerskaya.* "On the Approach of Obtaining the Upper Bounds on the Average Number of Iterations of Some Integer Programming Algorithms", in: Proceedings of II International Conference "Optimization and applications" (OPTIMA-2011), Budva, Montenegro, September 25 – October 2, 2012, pp. 137–140.

The technology for solving the boundary value problems for systems of functional-differential equations of point type

T.S. Zarodnyuk[1], **A.L. Beklaryan**[2], **E.A. Finkelshtein**[3], **F.A. Belousov**[4]

[1] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* tz@icc.ru
[2] *National Research University Higher School of Economics, Moscow, Russia;* abeklaryan@hse.ru
[3] *Matrosov Institute for System Dynamics and Control Theory of SB RAS, Irkutsk, Russia;* evgeniya.finkelstein@gmail.com
[4] *Central Economics and Mathematics Institute of RAS, Moscow, Russia;* abeklaryan@hse.ru

A class of nonlinear functional-differential equations, including equations with deviating argument of various types with time-lag and advance, as well as combine both of these elements (see. e.g., [1, 2]) is considered.

The proposed technology for solving boundary value problems is based on the Ritz method and spline collocation approaches. To solve the problem the system trajectories are discretized on the grid with a constant step and it is formulated the generalized residual functional, including both weighted residuals of the original differential equation and residuals of boundary conditions. To evaluate the derivatives of the system trajectories we use a technique of "spline-differentiation", based on two methods of spline approximation: using cubic splines and using a special type of splines, which second derivatives at the edges are controlled by the optimized parameters.

To solve the finite-dimensional optimization problems, in general non-convex and ravine, it is implemented a set of algorithms of local optimization (BFGS quasi-Newton method, two versions of the Powell method, the Barzilai-Borwein method, variant of trust regions method, stochastic search techniques in subspaces of dimension 3, 4 and 5) and global optimization algorithms (random multistart, curvilinear search technique, the tunneling method, "parabola" method and others).

The proposed technology includes the algorithm of increase the accuracy of approximations by increasing the number of points of the sampling grid (see. e.g., [3]), the algorithms of the functional derivative estimation by finite differences schemes with accuracy degrees from one to six, methodology for improving the accuracy of the spline-differentiation.

The report examines a small collection of test problems, created by the traditional techniques ([4]), and the results of computational experiments are carried out for all the problems generated from the collection.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-37-20265).

References

1. *L.A. Beklaryan.* “The optimal control problem for systems with deviating argument and its relationship with finitely generated group of homeomorphisms of R, generated by the deflection function,” DAN SSSR, Vol. 317, **16**, 1289–1294 (1991).
2. *A.L. Beklaryan.* “On the existence of solutions of the first boundary value problem for elliptic equations on unbounded domains,” International Journal of Pure and Applied Mathematics, Vol. 88, **4**, 499–522 (2013).
3. *Gornov A.Y.* Computing technologies for solving optimal control problems, Nauka, Novosibirsk (2009).
4. *Gornov A.Y., Zarodnyuk T.S., Madzhara T.I., Daneyeva A.V., Veyalko I.A.* “A collection of test multiextremal optimal control problems,” Optimization, Simulation and Control, Springer Optimization and Its Applications, **76**, 257–274 (2013).

On some variant of simplex-like algorithm for solving linear semi-definite programming problem

Vitaly Zhadan[1,2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* zhadan@ccas.ru
[2] *Moscow Institute of Physics and Technology (State University), Moscow, Russia*

Consider the linear semi-definite programming problem

$$\min\ C \bullet X, \qquad A_i \bullet X = b^i, \quad 1 \le i \le m, \quad X \succeq 0, \tag{1}$$

where C, X and A_i, $1 \le i \le m$, are symmetric matrices of order n, the inequality $X \succeq 0$ indicates that X must be a semi-definite matrix. The operator $\bullet$ denotes the Frobenius inner product between two matrices.

Many methods have been proposed for solving (1), and overwhelming majority of them belongs to the class of interior point techniques. Nevertheless, there are some simplex-like algorithms (see, for example, [1]). In this paper we also consider some variant of simplex-like algorithm for solving (1). The algorithm is based on some approach for solving the system of optimality conditions for (1)

$$X \bullet V = 0, \quad A_i \bullet X = b^i, \quad V = C - \sum_{i=1}^{m} u^i A_i,$$

where $X \succeq 0, V \succeq 0,\ 1 \le i \le m$. These conditions can be rewritten in vector form, using direct sums of columns of all matrices.

Let $\mathcal{F}_{\mathcal{P}}$ be the feasible set in (1), and let $\mathcal{E}(\mathcal{F}_{\mathcal{P}})$ be the subset of extreme points of $\mathcal{F}_{\mathcal{P}}$. Starting from the extreme point X_0, algorithm generates the sequence of points $\{X_k\} \subset \mathcal{E}(\mathcal{F}_{\mathcal{P}})$, which converges to the solution of (1). Two cases are considered separately. The first one is the case where the number of equations m is a "triangle" number. The second one is the case where this congruence is not fulfilled.

The author was supported by the Russian Foundation for Basic Research (project no. 15-01-08259), and by the Leading Scientific Schools Grant no. 4640.2014.1

References

1. *J.B.Lasserre* "Linear programming with positive semi-definite matrices," MPE, **2**, 499 – 522 (1996).

On the Effective Gradient Calculation With Help of Fast Automatic Differentiation Technique

Vladimir Zubov[1], **Alla Albu**[2]

[1] *Dorodnicyn Computing Centre of RAS, Moscow, Russia; Moscow Institute of Physics and Technology (State University), Moscow, Russia;* zubov@ccas.ru

[2] *Dorodnicyn Computing Centre of RAS, Moscow, Russia;* alla.albu@mail.ru

In recent years problems of optimal control of thermal processes, in which the substance under study undergoes phase transitions, have become increasingly important. A key feature of these problems is that they involve a moving interface between two phases (liquid and solid). The law of motion of the interface is unknown in advance and is to be determined. It is on this interface that heat release or absorption associated with phase transitions occurs. The mathematical model of such problems is based on a nonstationary two-phase initial-boundary value problem of the Stefan type.

Problems of this class are usually solved numerically using gradient methods to minimize the cost functional. It is extremely important to use the exact value of the gradient of the functional. One approach that can determine the exact value of the gradient is the generalized Fast Automatic Differentiation Technique.

This paper evaluates the effectiveness of the Fast Automatic Differentiation Technique to compute the gradient of the cost function in optimal control problems of thermal processes with phase transitions.

The evaluation is made on an example of optimal control of the melting process. This problem is formulated as follows: it is required to melt a given portion of a metal sample with minimal input of heat. This problem is analyzed here in a one-dimentional (radially symmetric) time-dependent setting. The heat source is located along the axis of symmetry. We analyze the case of a distributed source. The time distribution of the heat input by the heat source is chosen as the control.

In the paper we formulate and justify the statement that the time required to calculate the components of the gradient of the cost function using the above method doesnt exceed the time required to calculate two values of this function.

This work was supported by the Russian Science Foundation, project no. 14-11-00782.

Integrity constraint of incomplete data

Vladimir Zykin

Omsk State Technical University, Omsk, Russia; vszykin@mail.ru

The problem of finding of the irredundant set of the integrity constraints is considered. These restrictions allow regulating business rules on the enterprise where used relational database. The theoretical basis of the restrictions are based on inclusions dependencies that this work got a generalization that allows to use undefined value [1]. For these dependencies presents a set of axioms, the reliability and completeness of axioms are presented and proved.

The aim of this work is to study the properties of the reference integrity constraints. The system of axioms was developed within creating new mathematical formalism and using known algorithms of relationships on the database schemes. This system allows using undefined values. The its soundness and completeness was proved.

D e f i n i t i o n 1. The tuple $t_a[X]$ from relation $R_a[X]$ corresponds to $t_b[X]$ from relation $R_a[X]$ via sequence of members X ($t_a[X] \preccurlyeq t_b[X]$), if $t_b[A_j] \neq Null \Rightarrow t_a[A_j] = t_b[A_j]$ or $t_a[A_j] = Null$; if $t_b[A_j] = Null \Rightarrow t_a[A_j] = Null$, where A_j – single attribute from set X.

D e f i n i t i o n 2. Inclusion dependency $\sigma = R_a[X] \subsetneq R_b[X]$ from main relation $R_b[X]$ to dependent relation $R_a[X]$ via attributes X is exists, if for each tuple $t_a[X]$ from $R_a[X]$ it is exists tuple $t_b[X]$ from relation $R_b[X]$.

In general case the axiom system for inclusion dependencies with potential undefined values is presented as follow:

1) IND1 (reflexivity): $X \subseteq [R] \Rightarrow R[X] \subsetneq R[X]$
2) IND2 (projection): $R[Y] \subsetneq S[Y]$, $X \subseteq Y \Rightarrow R[X] \subsetneq S[X]$
3) IND3 (transitivity): $R[X] \subsetneq S[X]$ and $S[X] \subsetneq T[X] \Rightarrow R[X] \subsetneq T[X]$.

The completeness and soundness of the axioms system are proved.

The author was supported by the RFBR (project no. 15-41-04436).

References

1. *A. Cuzzocrea, A. Nucita.* "Enhancing accuracy and expressive power of range query answers over incomplete spatial databases via a novel reasoning approach," Data & Knowledge Engineering, **70**, No. 8, 702–716 (2011).

Complementarity problem in non-linear two-stage problem of stochastic programming

Anna Zykina[1], **Olga Kaneva**[2]

[1] *Omsk State Technical University, Omsk, Russia;* avzykina@mail.ru
[2] *Omsk State Technical University, Omsk, Russia;* okaneva@yandex.ru

Let consider the non-linear programming problem of the following type:

$$\min_x \varphi(x) \tag{1}$$

under conditions

$$g(x) \leq b, \tag{2}$$

$$X = \{x \in R^n |\ g^{(1)}(x) \leq b^{(1)},\ x \geq 0\}. \tag{3}$$

Where $\varphi(x)$ – a scalar function of the vector x; $g(x) = (g_1(x), ..., g_m(x))$, $g^1(x) = (g_1^1(x), ..., g_{m_1}^1(x))$ and $g_i(x), g_i^1(x)$ – scalar functions of the vector x; $b = (b_1, ..., b_m)$ and $b^1 = (b_1^1, ..., b_m^1)$.

Let ω – a random variable, which determines the nature, it is member of the probability space, $\varphi(x) = (x, \omega)$, $g_i(x) = g_i(x, \omega)$, $i = 1, ..., m$ – random functions, $b = b(\omega)$ – random vector, and restrictions (3) are determinate. Discrepancy $[g(\omega, x) - b(\omega)]^+$, which may arise in conditions (2) we will compensate by the correction vector $y = y(\omega, x)$, it is calculated via following equations:

$$g(\omega, x) - b(\omega) \leq B(\omega)y,\ y \geq 0, \tag{4}$$

$$y(g(\omega, x) - b(\omega)) = yB(\omega)y. \tag{5}$$

Penalty for implementation of the compensation plan y we will give as follow function $\psi(\omega, x, y)$. In this case we get the following formulation of non-linear two-stage problem of stochastic programming:

$$\min_x M_\omega\{\varphi(\omega, x) + \min_y \psi(\omega, x, y)\} \tag{6}$$

under the conditions (4), (5), (3).

If the inequalities (4) to understand coordinatewise as associated inequality, than condition (5) ia an analog of the classical complementary slackness conditions: in each pair of coupled inequalities there is at least one equality. This approach can be interpreted as an extension of the classical concept of dual problem to the problem of stochastic programming. For the problem of mathematical programming in a deterministic setting, these issues were discussed in the works [1,2]. In paper [3] as an example, a model of production planning is constructed, in which the foreign market value of resources coincides with internal objectively determined resource estimates. In paper [4] penalty function for the implementation of the plan compensation y is presented in the form

$$\psi(\omega, x, y) = y(g(\omega, x) - b(\omega)).$$

The difficulties associated with analysis of two-stage problems in general is determined by the need to choose the best of the preliminary plan of the original problem x, which would guarantee the existence of residual compensation for all implementations of parameters of uncertainty ω. Construction of complementarity (4), (5) for the second stage in a new production of non-linear two-stage problem of stochastic programming problems (6), (4), (5), (3), which is presented in paper, is ensures the solvability of the problem for the positive semidefinite matrix $B = B(\omega)$, if positive definition of matrix $B = B(\omega)$ than it is a unique solution $y = y(\omega, x)$ in all implementations of ω and x.

The authors were supported by the Russian Foundation for Basic Research (project no. 15-41-04436).

References

1. *A.V. Zykina.* "Generalized duality for problems of mathematical programming," Omsk scientific Bulletin, **44**, No. 8, 59–62 2006).
2. *A.V. Zykina.* "Return additionality: formulation, methods, applications, " Scientific Bulletin of NSTU, **23**, No. 2, 91–104 (2006).
3. *A.V. Zykina.* "Two-objective stochastic programming for portfolio securities," Journal of Computational Mathematics and Mathematical Physics, **48**, No. 11, 1968–1978 (2008).
4. *A.V. Zykina, O.N. Kaneva.* "The problem of additionality for modeling decision making under conditions of incomplete information, Applied mathematics and fundamental Informatics, No. 2, 176–181 (2015).

Author index

Abdullaev Vagif 20
Abgaryan Karine 12, 14
Abramov Alexander P. 16
Afanasyev Alexander 18
Aida-zade Kamil 20, 22, 24, 26
Albu Alla 192
Alexeyeva Alexandra 89
Amirgaliev Ye. 121
Amirkhanova Gulshat 89
Andreeva Elena 28
Andrianov Alexander 30
Anikin Anton 29, 30
Antipin Anatoly 32
Antonik Vladimir 168
Asadova Jamilya 22
Ashrafova Yegana 24
Astrakov Sergey 34
Atink Dmitriy 91

Baklanov Artem 35
Balashov Maxim 36
Beklaryan A.L. 189
Belokrys-Fedotov A.I. 38
Belousov F.A. 189
Berdyshev A.S. 40
Bolodurina Irina 28
Brezhneva Olga 42

Chentsov Alexander 44
Chernov Alexey 73
Chernova Iuliia 46

Denisov Dmitry 48
Dikusar Vasily 50, 52
Druzhinina Olga 54, 124
Dvurechensky Pavel 56, 58, 73

Elkin V.I. 60
Emelyanov Stanislav 18
Eremeev Anton 61
Erzin Adil 63
Evtushenko Yuri G. 65, 66

Finkelshtein E.A. 189
Finkelstein Evgeniya 29
Fomin Andrey 67
Førsund Finn 113

Ganin Bogdan V. 68
Garanzha V.A. 38, 71
Gasnikov Alexander 56, 58, 73
Gasnikova Evgenia 56
Gimadi Edward 75–77
Golikov Alexander I. 65
Golshteyn Evgeny 79
Goncharova Yuliya 179
Gordeeva Valeriya 173
Gornov Alexander 30, 70
Gruzdeva Tatiana 81
Guliyev Samir 26
Gunkin A.A. 152

Ilyasov Damir 171
Ivanova Ulia 28

Jaćimović Milojica 83
Jaćimović Vladimir 84
Jenaliyev M.T. 85

Kalashnikov Vyacheslav 87
Kalashnykova Nataliya 87
Kalimoldayev M.N. 85, 89, 121
Kaneva Olga 91, 194
Kaporin Igor 92
Kartunova Polina 181
Kaznacheeva Marina 58
Kel'manov Alexander . 93, 94, 96, 98, 99

Keylan A. 121
Keyner Tatiana 187
Khachaturov Ruben.V. 100
Khachaturov V.R. 102
Khachay Daniel 44
Khachay Mikhail 44, 77, 104
Khamidullin Sergey 94
Khandeev Vladimir 94, 96
Khoroshilova Elena 105
Klochkova Liudmila 107
Kobylkin Konstantin 109
Konatar Nikola 84
Koshcheev Ivan 179
Kotliarov Ivan 111
Kovalenko Julia 61
Krivonozhko Vladimir 113, 142
Krivtsov V.M. 114
Kudryavtseva L.N. 38, 71
Kurochkina Anna 76
Kuzenkov Oleg 115

Laptin Yuriy 116
Leal-Coronado Mariel 87
Lebedeva Tatyana 118
Lotov Alexander 119
Lychev Andrey 113, 142

Madaeva Elena 120
Malkov Ustav 79
Malyshev D.I. 152
Mamyrbayev O. 121
Masina Olga 124
Mijajlović Nevena 83
Mikhailov I.E. 126
Mizhidon Arsalan 120, 128
Mizhidon Klara 128
Mladenovic Nenad 63
Morozov Yury 129
Motkova Anna 98

Nekrasova Olga 48
Neznakhina Katherine 104

Obrosova Nataliia 131
Olenev Nicholas 50, 133, 135
Orlov Andrei 81, 137
Orlov Yuri 67
Ostapov Vsevolod 135

Parkhomenko Valeriy 139
Patrin Maxim 142
Pesterev Alexander 143
Petrov Lev 145
Petrova Natalia 54
Pilnik Nikolay 147
Plotnikov Roman 63
Pospelov Igor 147
Posypkin Mikhail 67
Prusińska Agnieszka 148
Putilina Elena 18
Pyankov Vitaliy 150
Pyatkin Artem 99

Radionov Stanislav 147
Reviznikov D.L. 12
Rybak L.A. 152
Rykov Ivan 77

Salienko Alexander 155
Savelyev Vladimir 156
Serovajsky Simon 158
Shakenov Ilyas 159
Shakenov Kanat 160
Shananin Alexander 131
Shchennikova Elena 54
Shevchenko Valery 161
Shmyrev V.I. 163
Sigal Izrael 67
Simanchev R.Yu. 164
Skarin Vladimir 166
Skiba A.L. 126
Sokolov Nikolay 79
Soloviev V.R. 114
Srochko Vladimir 168
Stepanov Alexey 145

Stetsyuk Petro 73
Strekalovskiy Alexander S. 169
Suvorikova Alexandra 73

Tikhomirov Nikolai 171
Tikhomirova Tatiana 173
Tishkin Vladimir 107
Tret'yakov Alexey A. 42, 148
Trofymchuk Oleksandr 175
Tsidulko Oxana 77
Tyupikova Tatyana 118

Urazova I.V. 164
Ushakov Anton 177
Ushakova Liudmila 175
Uvarov Sergey 14

Valeev Ruslan 179
Valeeva Aida 179
Vasilyev Igor 177
Vasin Alexander 181
Vasyanin Vladimir 175
Volkov Yuriy 155
Volodina Olga 14

Yanulevich Maxim 183
Yunicheva N.R. 185

Zabudsky Gennady 187
Zaozerskaya Lidia 188
Zarodnyuk T.S. 189
Zasukhin Sergey 52
Zasukhina Elena 52, 119
Zhadan Vitaly 191
Zlotov A.V. 102
Zubov Vladimir 66, 192
Zykin Vladimir 193
Zykina Anna 194