Experimental Observation of Quasiperiodic Dynamics in Globally Coupled Oscillator Ensembles

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***Abstract*—We present the results of experiments with 20 electronic limit-cycle oscillators, globally coupled via a common load. We analyze collective dynamics of the ensemble in cases of linear and nonlinear phase-shifting unit in the global feedback loop. In the first case we observe the standard Kuramoto transition to collective synchrony. In the second case, we observe transition to a self-organized quasiperiodic state, predicted in [M. Rosenblum and A. Pikovsky, PRL, (2007)]. We demonstrate a good correspondence between our experimental results and previously developed theory. We also describe a simple measure which reveals the macroscopic incoherence-coherence transition in a finite–size ensemble.**

I. Introduction

Mean field approximation is widely used in description of oscillator networks with high degree of connectivity. Models of oscillator ensembles with mean field coupling, also known as global or all-to-all coupling, provide characterization of collective dynamics of oscillating objects of various nature, including fireflyes, spontaneously beating atrial cells, pedestrians on the footbridges, handclapping individuals in a large audience, Josephson junctions, lasers, electrochemical oscillators, spiking or bursting neurons, to name just a few. Analysis of collective behavior of such systems poses a number of problems which are highly nontrivial from the standpoint of nonlinear dynamics. Due to these reasons, this topic remains in the focus of interest within last three decades. Basic theory and further references can be found in the following books, book chapters, and review articles [1], [2], [3], [4], [5], [6], [7], [8], [9], [**?**].

A subject of recent interest are coherent though not synchronous states, also denoted as partial synchrony. Such regimes have been observed in networks of pulse coupled integrate-and-fire units [10], [11], [12] and in ensembles of Stuart-Landau and phase oscillators with global nonlinear coupling [13], [14]. The latter systems exhibit an interesting transition from synchrony to self-organized quasiperiodicity (SOQ). In the SOQ state the frequency of the mean field differs from the frequency of oscillators, i.e. the emergent collective mode and individual units are not locked. The primary goal of our current study is experimental verification of these results [15]. For this purpose, we performed experiments with electronic oscillators, globally coupled via a common feedback loop with a phase-shifting unit. The coupling is nonlinear in the sense that phase shift depends on the amplitude of the collective oscillation. We demonstrate, with increase of the strength of the global coupling, a transition from asynchronous state to collective synchrony and then to SOQ.

Before presenting our results, we briefly review the experimental studies of globally coupled systems. First of all, there is a number of observations of synchronous collective dynamics in systems, where the coupling is assumed to be of all-to-all type, although it is most likely not homogeneous. These observations include synchronous emission of optical or acoustical pulses by groups of insects [16], rhythmical hand clapping in opera houses [17], glycolytic oscillation in populations of yeast cells [18], [19], etc. A well-known example is pedestrian synchrony on the London Millennium Bridge; the experiments with the pedestrian groups of different size demonstrated that collective synchrony is a threshold phenomenon [20], in correspondence with the theoretical results [21], [22]. Next, we mention a brilliant demonstration of collective synchrony in a very simple experiments with metronomes, performed by B. Daniels within a framework of student research [23]. Numerous well-controlled experiments on globally coupled oscillators have been performed by J. Hudson, I. Kiss, and collaborators [24], [25], [26], [27]. Using an array of 64 electrochemical oscillators they have confirmed practically all theoretical predictions. In particular, they have demonstrated Kuramoto transition in ensembles of periodic and chaotic oscillators. Other laboratory experiments have been conducted with Josephson junctions [28], [29], photochemical oscillators [30], and vibrating motors on a common support [31].

II. Experimental setup

We performed experiments with 20 electronic generators, coupled via a global feedback loop, see Fig. 1. Coupling is organized via a common resistor R_c; a fraction of the voltage across this resistor is fed to the input of the phase-shifting unit. The output of this unit is fed back to all oscillators via resistors R_1. Scheme of

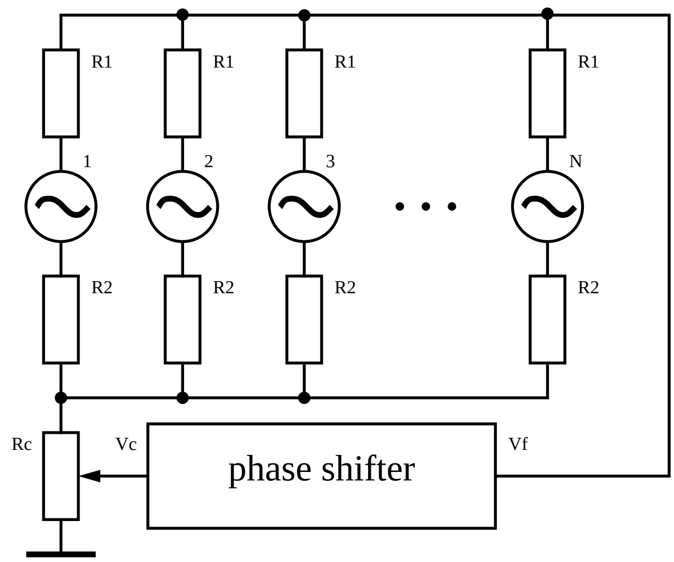

Fig. 1. Scheme of the globally coupled system. Individual generators are shown here by one symbol, there detailed scheme is given in Fig. 2, whereas the scheme of the phase-shifting unit is given in Fig. 3.

an individual generator is given in Fig. 2; it represents a Wien bridge oscillator with a saturation of the amplitude. The saturation is ensured by the nonlinear circuit in the negative feedback loop of the operational amplifier; this circuit is built by diods $D_{1,2}$ and resitor R_7. With the help of the trimmer resistor R_6 amplitudes of all uncoupled oscillators were tuned to approximately same value $V \approx 1$ V. The scheme of the phase-shifting

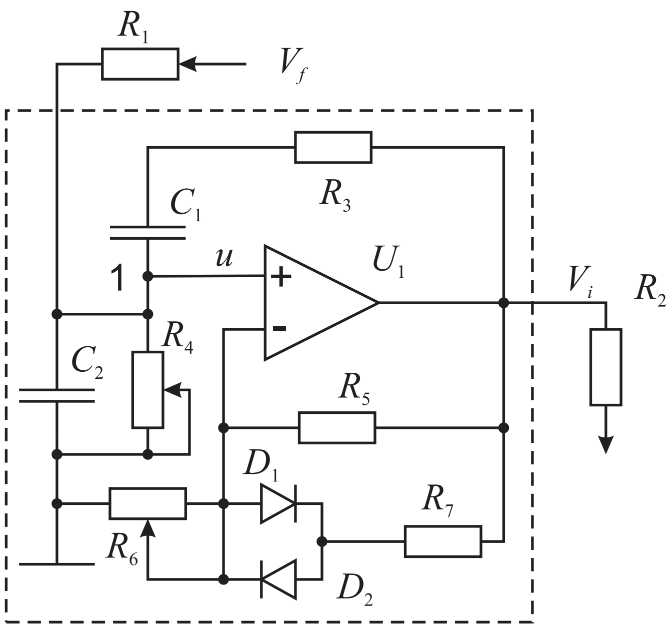

Fig. 2. Wien bridge oscillator. Here V_i is the output voltage of the i-th oscillator, V_f is the output voltage of the global feedback loop.

unit is depicted in Fig. 3. It consists of two identical linear subunits and one nonlinear. The linear subunit is a standard RC-circuit. The nonlinear part represents a high-pass first order filter, where nonlinear properties of diods provide a dependence of the phase shift between input and output on the amplitude of the input. We

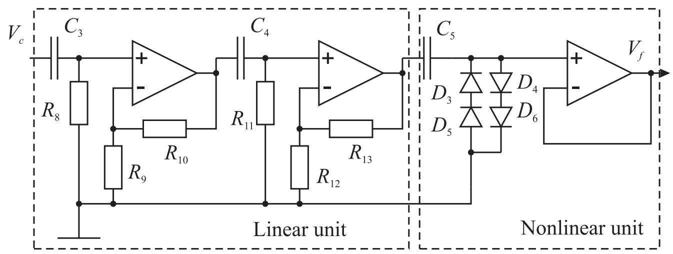

Fig. 3. Phase-shifting unit consists of two linear and one nonlinear shifting subunits.

performed 3 experiments: (i) Phase shifting unit excluded so that the signal from the common load was directly applied to the inputs of oscillators, i.e. $V_f = V_c$; (ii) only the linear phase shifting unit was included, and (iii) both linear and nonlinear units are used, as shown in Figs. 1,3. In each experiment we gradually changed the input to the feedback loop V_c from zero to its maximal value V_L and recorded the outputs of all oscillators, V_i, $i = 1, \ldots, N$, and the mean field voltage V_L. (It can be easily shown that $V_L = N^{-1}\sum_{j=1}^{N} V_j$, provided $R_2 \ll NR_c$.) In each recording we obtained 10^5 points per channel, with the sampling rate 65 kHz. For each value of the coupling strength $\varepsilon = V_c/V_L$ we performed 10 recordings.

III. Data analysis and results

For the presentation of results we have computed, for each value of the coupling strength ε, the following quantities:

1) instantaneous phases φ_i of all oscillators and instantaneous phase and amplitude A_{mf} of the mean field V_L were obtained with the help of the Hilbert transform;
2) frequencies f_i of all oscillators and frequency f_{mf} of the mean field were computed from the unwrapped phases for each recording and then averaged over 10 recordings;
3) the order parameter R was obtained by averaging the quantity $N^{-1}\sum_{j=1}^{N} e^{i\varphi_j}$ over time and over 10 measurements;
4) the minimal (over all 10 measurements) value A_{min} of the instantaneous mean field amplitude A_{mf}.

We note that typically synchronization transition in a globally coupled system is traced by plotting R vs. ε. This approach is efficient in the limit $N \to \infty$, where $R = 0$ in the incoherent state. However, since in our case $N = 20$, the finite-size fluctuations of the mean field in this state are quite large (they are known to scale as $1/\sqrt{N}$) and therefore R is not small either. We find that the distinction between incoherent (fluctuating mean field) and coherent (oscillatory mean field) states can be better revealed by A_{min}.

In the first and second experiments (no phase shifting unit and linear unit, respectively), we observed standard Kuramoto transitions to collective synchrony, characterized by a monotonic dependence of R and A_{min} on ε. In the third, main, experiment, we observed a non-monotonic dependence of R and A_{min} on ε (Fig. 4). We have found, that with increase of ε, 10 oscillators formed a cluster at $\varepsilon \approx 0.12$, while other 10 remained asynchronous. Next, the frequency locked oscillators leaved the cluster one by one. Finally, SOQ state appeared at $\varepsilon \approx 0.72$. In order to show that this is indeed a transition to SOQ but not simply a breakup of synchrony, we have plotted the Hilbert transform of the mean field vs. the mean field itself (not shown). We have seen that in the asynchronous state the pattern is typical for a narrow-band random process, with the amplitude dropping practically to zero, whereas in the SOQ state the mean field is clearly oscillatory and its phase and frequency are well-defined.

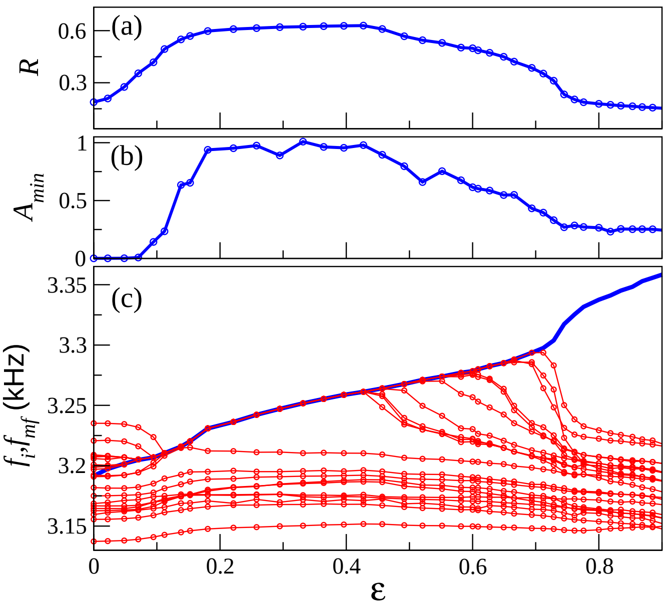

Fig. 4. Results of the experiment with the linear and nonlinear phase-shifting unit. Order parameter R (a) and minimal mean field amplitude A_{min} (b, blue circles). At $\varepsilon \approx 0.12$ we observe the transition to a partially synchronous state, where the fastest oscillators lock to each other and to the mean field. Between $\varepsilon \approx 0.43$ and the $\varepsilon \approx 0.72$ oscillators leave the cluster and for $\varepsilon > 0.72$ the SOQ state is observed: mean field is faster than all oscillators. Although the values of the order parameter in the asynchronous ($\varepsilon < 0.12$) and SOQ states are almost the same, these states are qualitatively different and can be easily distinguished by the quantity A_{min}.

IV. Summary

In summary, we have experimentally demonstrated a state where oscillators are synchronized neither with each other nor with the mean field, but the amplitude of the latter is, nevertheless, non-zero. This peculiar coherent state is possible because phases of oscillators, though not locked, are coordinated in a way that their distribution is non-uniform. Our results well correspond to analytical results for phase oscillators [13], [14], [32]. The SOQ regime we observe emerges when the system is brought, due to the phase shift, close to the point where attractive interaction becomes repulsive. Thus, we expect SOQ to be observed in other physical systems where the global coupling is characterized by an amplitude-dependent phase shift or time delay. For example, these dynamics appear in systems where the global coupling contains linear and cubic terms [14], cf. [**?**]. Moreover, numerical observations, e.g., reported in [33], indicate that SOQ states can appear in linearly coupled ensembles of strongly nonlinear oscillators. Analysis of such systems is a topic for future studies, both theoretical and experimental. Finally, we notice that we have used a simple and easily computable measure A_{min} and have shown that it reliably reveals macroscopic incoherence-coherence transition. We suggest that this quantity can be efficiently used in other experimental studies, as well as in numerical studies of finite size ensembles.

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