Modeling and Optimization of the Production Cluster

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Agenda

- Clusters and clustering
- Three-sector model of a production cluster
- The problem of optimal equilibrium state of a cluster
- Parametric analysis of optimal equilibrium state
- Future work: the problem of an optimal cluster control

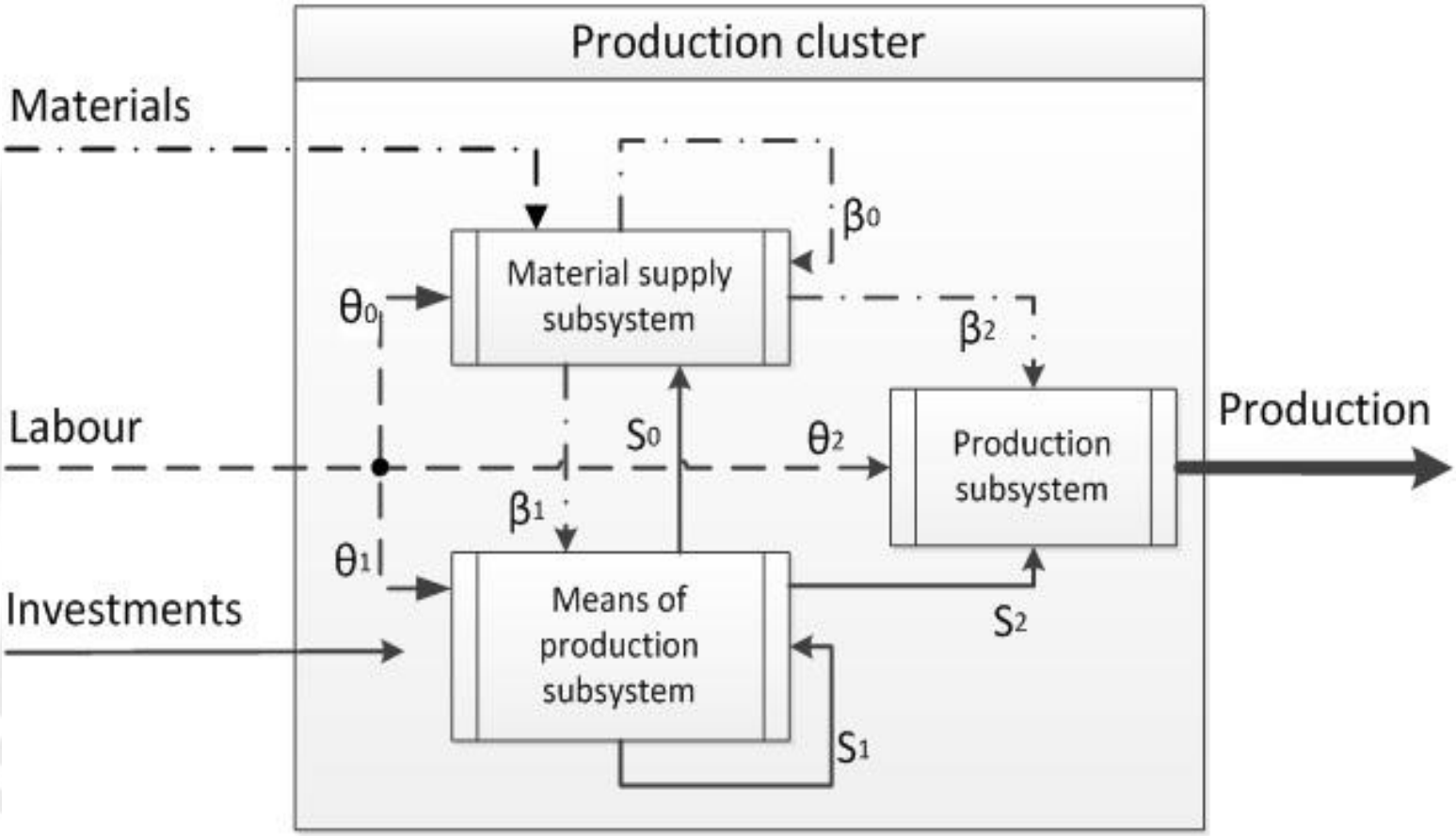
Clusters and clustering

- Clustering – one of the priorities in the global economy
- One way of grouping and consolidation of capitals
- Many governments stimulates development of clusters in various industries
- Decision-making processes in clusters can be supported by ekonomic models

Three-sector model of a production cluster – sectors and resources flows

- Sectors:
  - Material supply (*i=0*)
  - Production means (*i=1*)
  - Production (*i=2*)
- Flows:
  - Materials
  - Labour
  - Investments (capital)

Three-sector model of a production cluster – the structure



Three-sector model of a production cluster – the math model

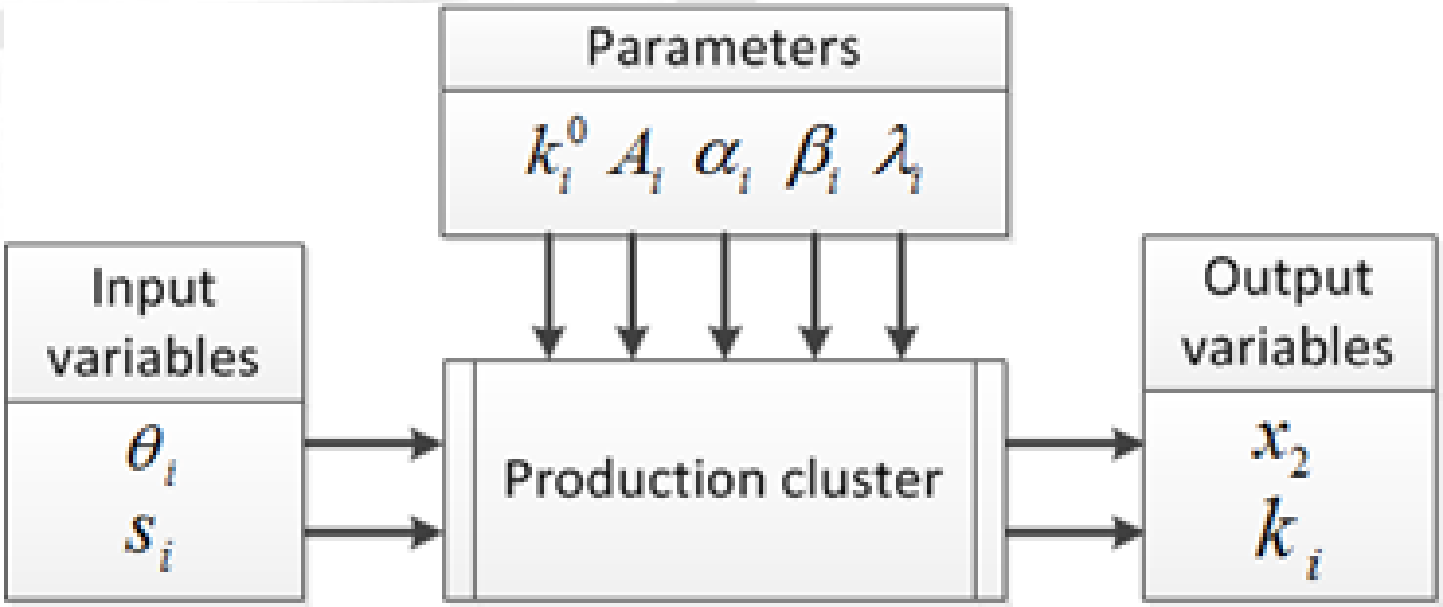
$$\frac{dk_i}{dt} = -\lambda_i k_i + \frac{s_i}{\theta_i} x_1,\ \lambda_i > 0,\ k_i(0) = k_i^0,\ i = 0,1,2.$$

$$\theta_0 + \theta_1 + \theta_2 = 1, 0 < \theta_i < 1,$$

$$s_0 + s_1 + s_2 = 1, 0 < s_i < 1,$$

$$(1-\beta_0)x_0 = \beta_1 x_1 + \beta_2 x_2, \beta_i > 0.$$

Three-sector model of a production cluster – input and output parameters



The problem of optimal equilibrium state of a cluster – the problem

- Fixed (equilibrium) state = value of sectors' ratios are constant
- Optimal fixed state = max of cluster **Production**
- Optimal fixed state = solution of the problem:

$$x_2 = \theta_2 A_2 k_2^{\alpha_2} \xrightarrow[\theta_i, s_i]{} \max$$

under conditions:

$$(1-\beta_0)\theta_0 A_0 k_0^{\alpha_0} = \beta_1 \theta_1 A_1 k_1^{\alpha_1} + \beta_2 \theta_2 A_2 k_2^{\alpha_2}, \beta_i > 0,$$

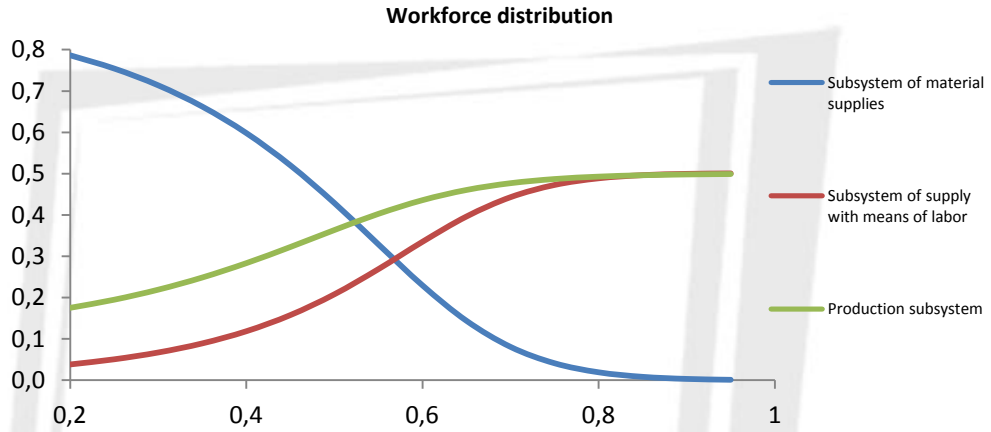
The problem of optimal equilibrium state of a cluster – the solution

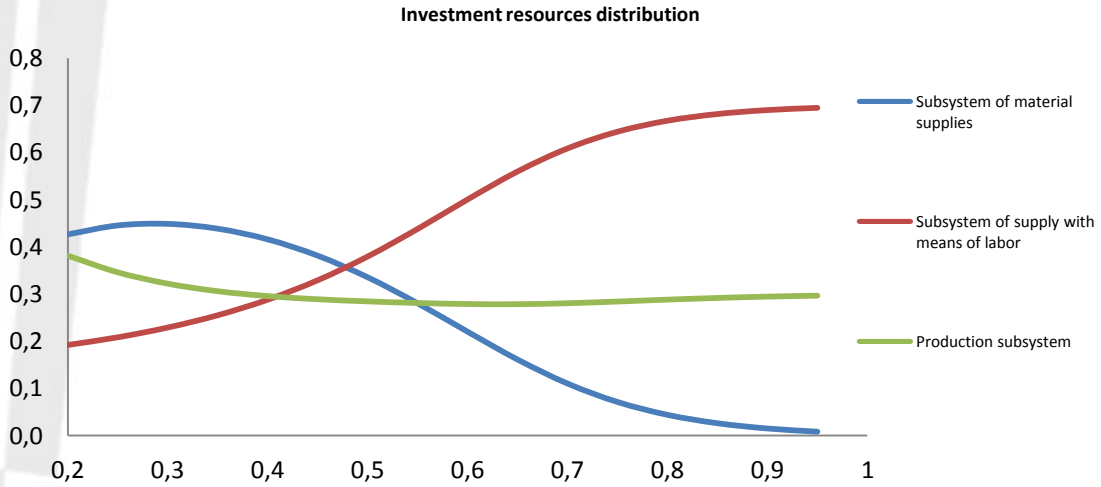
- Method: Lagrange multipliers
- Results: system of algebraic equations
- Solution: equations system resolve using Maple software package

Parametric analysis of the optimal equilibrium state

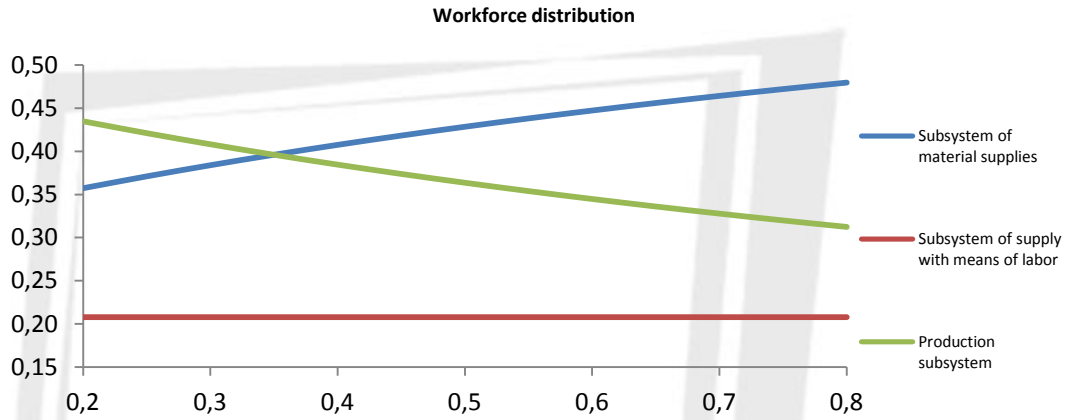
- The influence of parameters on the optimal equilibrium state of the production cluster
- Calculated and presented for the following parameters:
  - Funds elasticity inthe material supply subsystem
  - Direct material costs in the production subsystem
- An example initial paremeters have been used

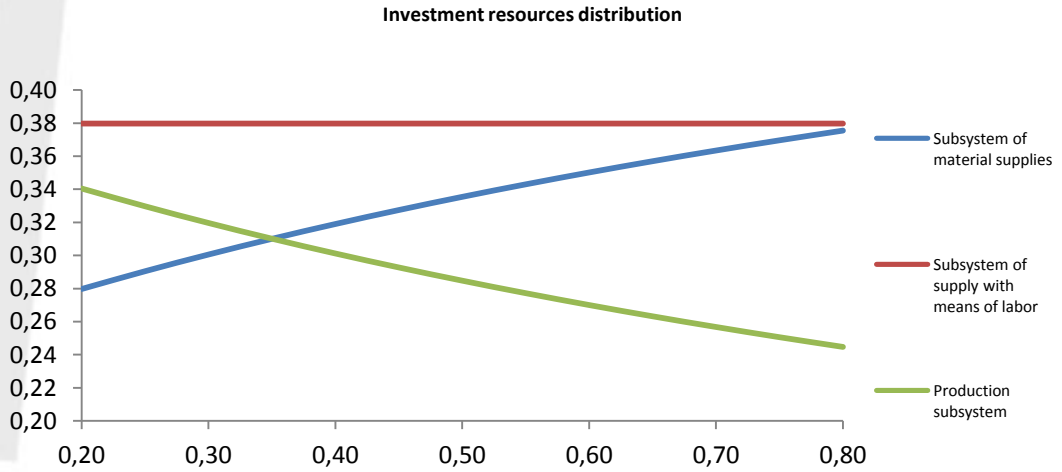
Effect of changes in the elasticity parameter in materials subsystem on labour and investment resources





Effect of changes in material costs in production subsystem on labour and investment resources





The problem of an optimal cluster control

- Future work
- The problem to determine the steady state is an initial step in solving the problem of optimal control with fixed ends of trajectories for economic model of production cluster
- The problem definiction:

$$J(\theta_i, s_i) = \int_0^T e^{-\delta t} \theta_2 A_2 k_2^{\alpha_2} dt \to \max$$

THANK YOU

Questions?