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Electric Charge of Dust Particles in a Plasma

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The problem of calculation of the electric charge of dust particles in a plasma is considered from different points of view. At first the charging of polarizable dust particles is studied within the orbital motion limited approach. Secondly, the plasma electrodynamics is applied to show that the electric charge of a dust particle is determined by the normal component of the dielectric displacement vector near the grain surface rather than the normal component of the electric field strength. And, finally, the chemical model, initially proposed for determination of partially ionized plasma composition, is demonstrated to be very productive in evaluating the electric charge of the dust component.

1 Introduction

In modern theoretical and experimental plasma physics an interest to the study of a dusty plasma is still heated up by its abundance in the near space [1, 2] and in the universe [3–5], as well as by its extensive exploitation in laboratories [6–8]. In particular, astrophysics explores the dusty plasma in stellar and planetary nebulae, supernova remnants, asteroids, planetary rings and comet tails. On Earth, a dusty plasma occurs at the lightning discharge, in craters of active volcanoes, at the fall of large meteorites, as well as in noctilucent clouds. From a technological point of view dust may play a negative role of plasma contaminant, as it is the case for plasma etching in microelectronics, or in installations designed for controlled nuclear fusion.

It has long been known that dust particles, immersed in a plasma, can form different structures with short- and long-range orders, which can be interpreted as liquid and crystalline phases, respectively [9–12]. In such systems, even phase transitions are observed and studied by different methods [13–15]. This straightforwardly testifies that a strong interaction does exist between the dust particles in the plasma, whose average energy can significantly exceed the thermal energy of their chaotic motion. Such nonideality in the system is a consequence of that, being placed in a plasma, the dust grains starts to intensively absorb electrons and acquire a high negative electric charge, which can reach tens of thousands of the elementary [16, 17]. Thus, it is clear that an ability to predict the electric charge of dust particles is crucial for comprehensive understanding of physical properties of dusty plasmas.

It is well known that the problem of theoretical calculation of the dust particle charge in a plasma is closely related to the theory of a so-called Langmuir probe, which is widely used for plasma diagnostics. The standard approach here is to use the orbital motion limited approximation [18], which assumes that the buffer plasma remains quasi-neutral and Maxwellian far away from the dust grain, and the mean free paths of plasma particles are much greater than the characteristic size of the sheath. This allows one to consider only ballistic trajectories of electrons and ions, and further use of the conservation laws of energy and angular momentum makes it possible to derive the corresponding absorption cross sections, and, hence, to determine the charge of the dust particle, or the current-voltage characteristics of a Langmuir probe.

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It is implied in the classical version of the orbital motion limited approximation that effective interaction energies between the plasma particles and the dust grain that include a centrifugal component are monotonic functions of the distance between them, which is not always accurate. Taking into account the shielding results in the appearance of the so-called absorption radius effect, which is caused by the onset of local maxima in the curve of the effective interaction energy [19, 20]. At the same time there is a need to treat the anisotropy of the ion flow near the dust grain, which is due to the accelerating field of the plasma sheath [21].

It is clear that a variety of physical conditions, under which a dusty plasma is encountered, may lead to a deviation of the velocity distribution functions of electrons and ions from the Maxwellian, which immediately affects the charge of dust particles themselves. Such deviations of the velocity distribution function are particularly frequent in space dusty plasmas and various astrophysical objects. Thus, the charge of the dust particle was studied for the Lorentzian spherical velocity distribution function [22], for the so-called bi-Maxwellian electron distribution function [23] as well as for the power distribution function [24, 25], obtained in the framework of non-extensive statistics, taking into account the long-range nature of the Coulomb interaction and the processes of the secondary electron emission [26].

A more consistent approach in the framework of the orbital motion limited theory was proposed in [27], where the Vlasov kinetic equation for a collisionless plasma was solved together with the Poisson equation. This allowed the authors to determine the so-called floating potential of the dust grain by imposing the equality of electron and ion fluxes on the dust surface, and thereby to calculate its electrical charge. Such a theoretical approach has a drawback that for sufficiently large dust particles ion concentration may turn out imaginary [28]. It took a further complication of the orbital motion limited theory, including an account for the acceleration of ions in an electric field of the sheath [29, 30].

With the growth of the plasma density the role of collisions, especially with the neutrals, increases dramatically so that the trajectories of electrons and ions in a plasma can no longer be regarded as ballistic. To treat interparticle collisions consistently it is necessary to solve the kinetic equation [31], which can be done both phenomenologically [32], and using computer simulations in the framework of the particles-in-cell method [33].

It is easy to imagine that the orbital motion limited approximation presumes that the electron and ion fluxes on the dust depend on their spatial distribution and the charge of the dust particle itself. In this sense, the equilibrium charge of the dust grain, usually derived from the equality of electron and ions fluxes, is completely determined by the parameters of the buffer plasma and independent of both the material the dust particle is made of and of elementary processes taking place on its surface. That is why the orbital motion limited approximation works rather well only for the dust particles whose dimensions are small compared to the Debye radius [34]. It is obvious that the above presented interpretation, in spite of its attractiveness, is unsatisfactory from the physical point of view, since it essentially exploits the idea that the surface of the dust particle is a perfect absorber of incoming electrons and ions [35]. To avoid such an unjustified assumption an attempt was made in [36, 37] to develop a true microscopic theory that takes into account the near-surface states of electrons and ions appearing as a result of the polarization of dust particles. Moreover, the electron emission from the surface of dust particles [38], which is determined by the work function of electrons, and the secondary electron emission [39] should be thoroughly included.

It should be noted that the electric charge and its dependence on the grain size can be measured in sophisticated experiments [40], which continue to develop at present [41, 42]. It is remarkable that the dust particles can themselves be used to diagnose the buffer plasma by their motion around a cylindrical Langmuir probe [43].

The sketch of the sequel in this paper is outlined as follows. In section 2 the orbital motion limited approximation is engaged to determine how the polarization phenomena influence the electric charge of a dust particle immersed in a buffer plasma. Section 3 is completely devoted to plasma electrodynamics as applied to the problem of the electric charge of a dust particle. In section 4 an attempt is undertaken to construct a chemical model of a dusty plasma in order to evaluate the electric charge of a dust component from the free energy minimization procedure. At the end conclusions are drawn and provisions for future works are stated.

2 Orbital motion limited approximation

For the sake of simplicity this section deals with the hydrogen buffer plasma with the electron number density n_e and the proton number density $n_p = n_e = n$, in which a spherical macroscopic particle of radius R and the electric charge $-Z_d e$ is placed Since the dust particle is solitary, the quasineutrality condition $n_e = n_p$ is imposed. The state of the electron component of the plasma is described by the density parameter $r_s = a/a_B$, where $a = (3/4\pi n)^{1/3}$ denotes the average distance between the electrons, $a_B = \hbar^2/m_e e^2$ stands for the first Bohr radius with $\hbar$ being the Planck constant and e being the elementary electric charge. Another dimensionless parameter relevant for description of the state of the buffer plasma is the so-called called coupling parameter given by $\Gamma = e^2/(ak_BT)$, where k_B is the Boltzmann constant and T designates the ambient temperature. It should be emphasized that the coupling parameter is common to represent the ratio of the average Coulomb interaction energy of the electrons to their average kinetic energy of thermal motion. To take into account the finite dimensions of the dust particle, the size parameter is introduced as $D = a/R$ to show how many times the average distance between the buffer plasma particles is larger or less than the radius of the dust grain. Note that to determine the electric charge of the dust particle in the classical case it is sufficient to only point out one dimensionless parameter $\Gamma_R = e^2/(Rk_BT) = D\Gamma$.

Further consideration of the charging process is carried out within the orbital motion limited approximation, in which trajectories of plasma particles, i.e. electrons and protons, are considered ballistic such that the interparticle collisions are completely ignored. To justify such an approach the mean free paths of plasma particles $\ell_{e(p)}$ should be much greater than the so-called Debye screening length $r_D = \sqrt{k_BT/4\pi(n_e + n_p)e^2}$, which, in its turn, should significantly exceed the dust grain radius R, i.e. $R \ll r_D \ll \ell_{e(p)}$,

It is believed in the classical treatment of dust particle charging that the material of the dust is a perfect absorber, i.e. all the plasma particles that reach the grain surface are inevitably taken up. This typically leads to that the grain charge is determined by the buffer plasma parameters and, thus, independent of the dust material. On the other hand it is known that a charged double-layer exists near the surface of solids, and whenever an attempt is undertaken to pull an electron out of a solid, the polarization phenomena come to play an essential role to cause an additional attraction. This results in that to extract an electron from the bulk of a solid it is necessary to perform some work, which is called a work function. The main idea of this section is to account for the polarization effects, which should ultimately lead to a true microscopic theory for the charge of the dust particle in a plasma.

Consider the interaction of a proton with a spherical dust particle, which is made of a conductive material. To take into account the polarization of the dust grain, the potential energy of the interaction is written with the aid of the charge image method as [44]:

$$U_{dp}(r) = -\frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)}. \tag{1}$$

Interaction potential (1) between the proton and the dust consists of two parts. The first part is determined by the dust particle charge and the plasma distribution around it, i.e. the sheath formation. It is that way the charging of dust particles was interpreted in the literature until very recently. It is visible from (1) that the screening of the electric field is completely dropped out to avoid explicit construction of the sheath theory that produces minor numerical corrections to the final result. The second part of the interaction potential is governed by the interaction with surface charges of the dust matter stemming from the polarization effects.

Let a dust particle absorb a proton with the fixed energy E and the impact parameter ρ. It is known [45] that this process is governed by the effective potential energy defined as

$$U_{dp}^{\text{eff}}(r, \rho, E) = -\frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)} + E\frac{\rho^2}{r^2}. \tag{2}$$

At the impact parameter $\rho = 0$, the effective potential energy of interaction between the proton and the dust particle is a monotonically increasing function of the distance, it is negative everywhere and tends to $-\infty$ when the proton approaches the surface of the dust particle. Thus, the proton with the impact parameter $\rho = 0$ is surely absorbed by the grain. At the fixed energy an increase in the impact parameter results in that a maximum appears in the curve of the effective potential energy whose height grows while increasing ρ. It is then evident that protons with small values of the impact parameter are absorbed by the dust particle, but at a certain value, $\rho = \rho_{dp}$, the height of the maximum of the effective potential energy turns equal to the total energy of the proton causing its rebound. Obviously, this value ρ_{dp} fully determines the absorption cross section as $\sigma_{dp} = \pi\rho_{dp}^2$. All above said is summarized in Figure 1, which shows that the protons with the energy $E/k_BT = 1$ and the impact parameters $\rho = 0$ and $\rho = 1$ are absorbed by the dust particle, and those with the impact parameter $\rho = 2$ are scattered. The black line in Figure 1 corresponds to the value $\rho = \rho_{dp} \approx 1.6$, which divides the whole region of the proton impact parameters into the absorption and rebound domains.

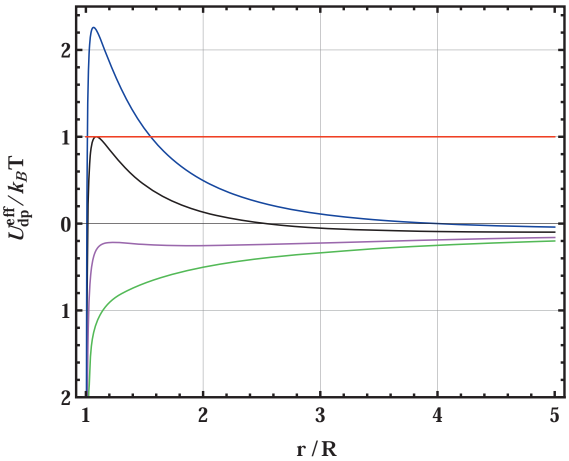

Fig. 1 The effective interaction energy between the proton and the dust particle at $E/k_BT = 1$ and $\Gamma_R = 0.1$. Green line: $\rho = 0$; magenta line: $\rho = 1$; black line: $\rho = \rho_{dp} \approx 1.6$ that corresponds to the critical value at which the proton begins to rebound from the dust grain; blue line: $\rho = 2$; red line: the total energy of the proton $E/k_BT = 1$.

Thus, ρ_{dp} is obtained from the following equation

$$\max U^{\rm eff}_{dp}(r, \rho_{dp}, E)_{r \geq R} = E. \tag{3}$$

The numerical solution to (3) is found as follows. For a fixed value of the proton energy E it is necessary to find such $\rho = \rho_{dp}$ that the maximum of effective potential energy (2) should exactly be equal to the total energy E. Figure 2 shows the dependence of the absorption cross section $\sigma_{dp} = \pi\rho^2_{dp}$ on the energy of the incident proton at different values of the dust particle charge. It is seen that the proton absorption cross section grows when the dust charge increases.

The maximum in the effective interaction energy can be found from the relation $dU^{\rm eff}_{dp}(r, \rho, E)/dr = 0$ and since its position is very close to the dust particle surface, as evidenced by Figure 1, it is suitable to search for an approximate solution to equation (3) in the form of $r = R + \delta$ with $\delta \ll R$, which yields

$$\delta = \frac{R}{\sqrt{\frac{32E\rho^2}{e^2R} + 17 - 16Z_d}} \tag{4}$$

and for the absorption cross section of protons

$$\sigma_{dp} = \pi\rho^2_{dp} = \pi R^2 \left(1 + \frac{Z_d e^2}{RE} + \frac{e^2}{8RE}\left[\sqrt{1 + 16Z_d + 32\frac{RE}{e^2}} - 3\right]\right). \tag{5}$$

If the polarization effects are neglected, the following classical result is recovered [17]

$$\sigma^C_{dp} = \pi R^2 \left(1 + \frac{Z_d e^2}{RE}\right). \tag{6}$$

Figure 3 makes a comparison of the absorption cross section of protons, calculated from expression (3) with formulas (5) and (6) at $\Gamma_R = 0.1$ and $Z_d = 15$. Since polarization effects lead to an additional attraction of the proton by the dust particle, they are responsible for an increase in the corresponding absorption cross section. It is quite natural that formula (5) describes more accurately the behavior of the absorption cross section than formula (6), which is only valid for pure Coulomb interaction.

Consider the interaction of an electron with the same spherical dust particle. To account for the polarization of the dust grain, the potential energy of the interaction is written with the aid of the charge image method as [44]:

$$U_{de}(r) = \frac{Z_d e^2}{r} - \frac{e^2R^3}{2r^2(r^2 - R^2)}. \tag{7}$$

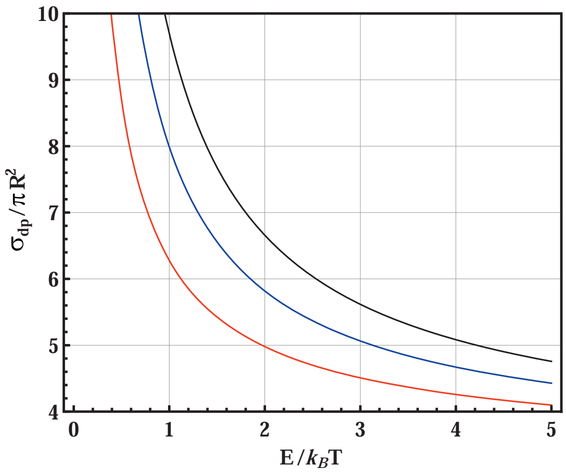

Fig. 2 The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $\Gamma_R = 0.1$. Red line: $Z_d = 5$; blue line: $Z_d = 10$; black line: $Z_d = 15$.

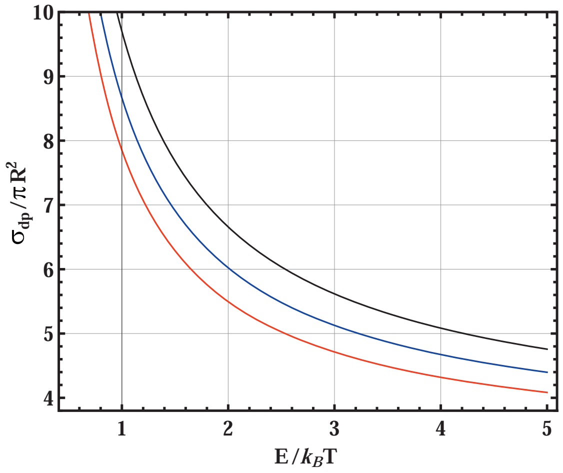

Fig. 3 The proton absorption cross section by the dust particle as a function of the energy of the incident proton at $\Gamma_R = 0.1$, $Z_d = 15$. Red line: formula (6); blue line: formula (5); black line: exact result from equation (3).

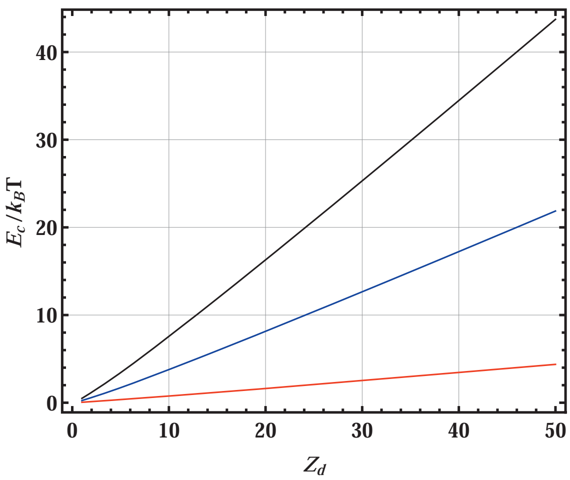

Fig. 4 The critical energy of electrons as a function of the dust particle charge. Red line: $\Gamma_R = 0.1$; blue line: $\Gamma_R = 0.5$; black line: $\Gamma_R = 1.0$.

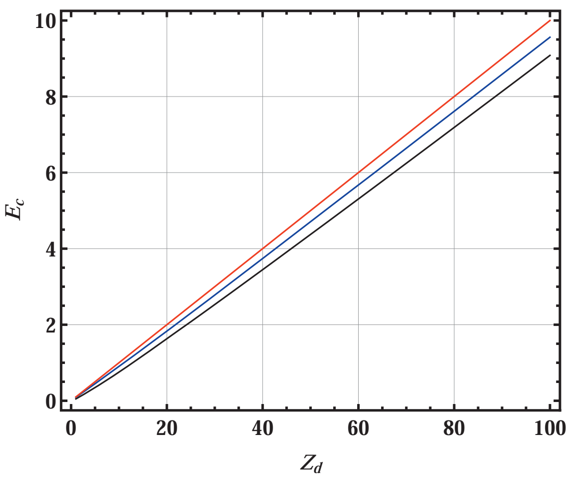

Fig. 5 The critical energy of electrons as a function of the dust particle charge at $\Gamma_R = 0.1$. Red line: formula (10); blue line: formula (9); black line: exact result.

There is a significant difference for the interaction of the electron with the dust particle as compared to its interaction with the proton. Due to the mutual repulsion the electron absorption is only possible when its energy reaches the critical value E_c determined as:

$$E_c = \max U_{de}(r). \tag{8}$$

Under the assumption that the rebound of the electron occurs close to the dust particle surface, series expansion allows one to roughly solve equation (8) as

$$E_c^a = \frac{e^2}{R}\left(Z_d + \frac{5}{8} - \frac{1}{8}\sqrt{17 + 16Z_d}\right). \tag{9}$$

Note that in case of the pure Coulomb interaction between the electron and the dust grain the critical energy is exactly found from the energy conservation law as follows [17]

$$E_c^C = \frac{Z_d e^2}{R}. \tag{10}$$

Figures 4 displays the dependence of the critical energy on the dust particle charge and the coupling parameter of the buffer plasma to show that both dependencies are almost linear whereas Figure 5 provides a comparison of exact expression (8) with approximate formulas (9) and (10). It can be seen that equation (9) better describes the exact data obtained from formula (8) than expression (10) which completely ignores the polarization of the dust particle. At the same time, it is rather obvious that the polarization phenomena are responsible for an additional attraction of electrons, and thus the value of the critical energy is reduced as compared to the expression for pure Coulomb interaction (10).

Let a dust particle absorb an electron with the fixed energy E and the impact parameter ρ. Again, this process is governed by the effective potential energy defined as [45]:

$$U_{de}^{\text{eff}}(r, \rho, E) = \frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)} + E\frac{\rho^2}{r^2}. \tag{11}$$

The analysis implemented above for the absorption of the proton is simply repeated for the absorption of the electron to find ρ_{de} from the equation

$$\max U_{de}^{\text{eff}}(r, \rho_{de}, E)_{r \geq R} = E \tag{12}$$

and, then, to obtain an approximate solution in the form

$$\sigma_{de} = \pi\rho_{de}^2 = \pi R^2 \left(1 - \frac{Z_d e^2}{RE} - \frac{e^2}{8RE}\left[\sqrt{1 - 16Z_d + 32\frac{RE}{e^2}} + 3\right]\right) \tag{13}$$

that recovers the classical result for the Coulomb interaction potential [17]

$$\sigma_{de}^C = \pi R^2 \left(1 - \frac{Z_d e^2}{RE}\right). \tag{14}$$

It is known that the proton flux on the surface of the dust particle is obtained from the relevant absorption cross section by integrating over the velocity distribution function as:

$$J_p = n_p \int v \sigma_{dp} f_p(v) d\mathbf{v}, \tag{15}$$

where $f_p(v) = (2\pi v_{Tp}^2)^{-3/2} \exp(-v^2/2v_{Tp}^2)$ is the Maxwell distribution with the thermal velocity $v_{Tp} = \sqrt{k_B T/m_p}$ and m_p being the proton mass.

Substituting expression (5) into (15) gives rise to the following analytical approximation for the proton flux on the dust grain surface

$$J_p^a = \sqrt{\frac{8\pi k_B T}{m_p}} nR^2 \left(1 + \frac{e^2}{Rk_B T}\left[Z_d + \frac{\sqrt{1 + 16Z_d}}{8} - \frac{3}{8}\right] + \right.$$
$$\left. + \sqrt{\frac{\pi e^2}{8Rk_B T}} \exp\left(\frac{e^2(1 + 16Z_d)}{32Rk_B T}\right) \operatorname{erfc}\left(\sqrt{\frac{e^2(1 + 16Z_d)}{32Rk_B T}}\right)\right), \tag{16}$$

where the error function and its complementary counterpart are mathematically defined as $\operatorname{erf}(z) = 1 - \operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int\limits_0^z \exp(-t^2) dt$.

In case of the pure Coulomb interaction, expression (16) reduces to the classical result of the form [17]

$$J_p^C = \sqrt{\frac{8\pi k_B T}{m_p}} nR^2 \left(1 + \frac{Z_d e^2}{R k_B T}\right). \tag{17}$$

Figure 6 portrays the dependence of the proton flux on the particle surface as a function of its charge to demonstrate that it grows due to reciprocal attraction when the charge of the dust particle increases. It is rather interesting to observe that the corresponding relationships are quite linear. In Figure 7 a comparison is made of the proton flux on the dust particle, calculated from expression (15), with formulas (16) and (17) for a fixed value of the charge number $Z_d = 15$. The polarization effects lead to an additional attraction of electrons by the dust particle and are responsible for an increase in the corresponding flux. Figure 7 reveals that analytical formula (16) neater follows the behavior of the proton flux than formula (17), which is valid for the case of the pure Coulomb interaction.

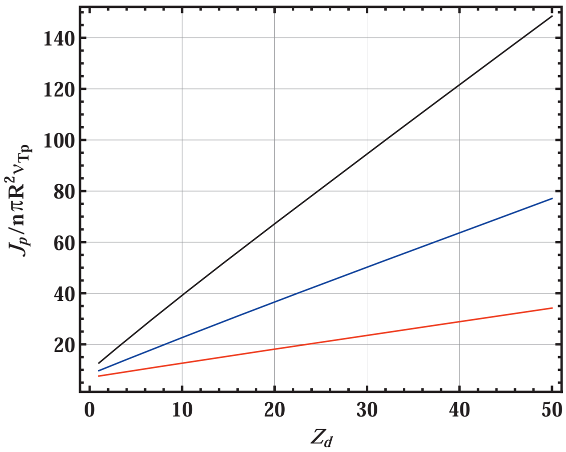

Fig. 6 The normalized proton flux on the surface of the dust particle as a function of the dust charge number Z_d. Red line: $\Gamma_R = 0.1$; blue line: $\Gamma_R = 0.25$; black line: $\Gamma_R = 0.5$.

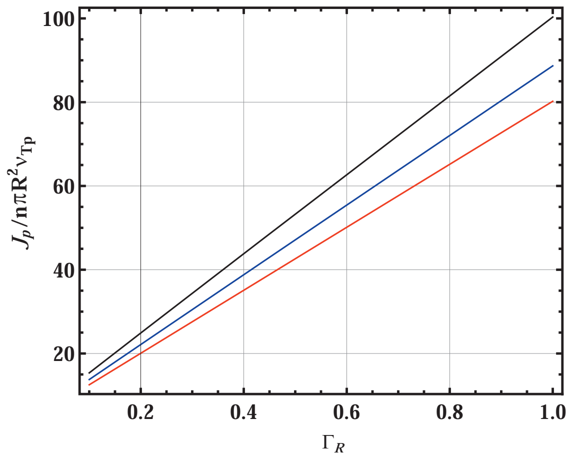

Fig. 7 The normalized proton flux on the surface of the dust particle as a function of the coupling parameter Γ_R at $Z_d = 15$. Red line: formula (17); blue line: formula (16); black line: exact result from formula (15).

Quite an analogous procedure provides the following approximate expression for the electron flux on the dust grain surface

$$\begin{aligned} J_e^a = \sqrt{\frac{8\pi k_B T}{m_e}} nR^2 \Bigg[\Bigg(1 - \frac{e^2}{8 R k_B T} \left(\sqrt{17 + 16 Z_d} - 2\right) \exp\Bigg(- \frac{e^2}{R k_B T} \Bigg(Z_d + \frac{5}{8} \\ - \frac{\sqrt{17 + 16 Z_d}}{8} \Bigg)\Bigg)\Bigg) + \sqrt{\frac{\pi e^2}{8 R k_B T}} \left(1 + \exp\left(- \frac{e^2 (21 + 16 Z_d - 4\sqrt{17 + 16 Z_d})}{32 R k_B T} \right) \right. \\ \left. \sqrt{\frac{e^2 (21 + 16 Z_d - 4\sqrt{17 + 16 Z_d})}{8\pi R k_B T}} + \mathrm{erf}\left(- \sqrt{\frac{e^2 (21 + 16 Z_d - 4\sqrt{17 + 16 Z_d})}{32 R k_B T}} \right) \right) \\ \exp\left(- \frac{e^2 (16 Z_d - 1)}{32 R k_B T} \right) \Bigg] \end{aligned} \tag{18}$$

that, in the classical case of Coulomb interaction, is simplified to [17]

$$J_e^C = \sqrt{\frac{8\pi k_B T}{m_e}} nR^2 \exp\left(- \frac{Z_d e^2}{R k_B T} \right). \tag{19}$$

It is known that the charge of the dust particle is found by equating the electron and proton fluxes on its surface as

$$J_e = J_p, \tag{20}$$

which, with the aid of expressions (16) and (18), can approximately be written in the simple analytical form

$$\sqrt{\frac{m_e}{m_p}}\left(1+\frac{e^2}{Rk_BT}\left[Z_d+\frac{\sqrt{1+16Z_d}}{8}-\frac{3}{8}\right]+\sqrt{\frac{\pi e^2}{8Rk_BT}}\exp\left(\frac{e^2(1+16Z_d)}{32Rk_BT}\right)\right.$$
$$\left.\operatorname{erfc}\left(\sqrt{\frac{e^2(1+16Z_d)}{32Rk_BT}}\right)\right)=\left[\left(1-\frac{e^2}{8Rk_BT}\left(\sqrt{17+16Z_d}-2\right)\right.\right.$$
$$\left.\left.\exp\left(-\frac{e^2}{Rk_BT}\left(Z_d+\frac{5}{8}-\frac{\sqrt{17+16Z_d}}{8}\right)\right)\right)+\sqrt{\frac{\pi e^2}{8Rk_BT}}(1+\right.$$
$$\exp\left(-\frac{e^2(21+16Z_d-4\sqrt{17+16Z_d})}{32Rk_BT}\right)\sqrt{\frac{e^2(21+16Z_d-4\sqrt{17+16Z_d})}{8\pi Rk_BT}}+$$
$$\left.+\operatorname{erf}\left(-\sqrt{\frac{e^2(21+16Z_d-4\sqrt{17+16Z_d})}{32Rk_BT}}\right)\right)\exp\left(-\frac{e^2(16Z_d-1)}{32Rk_BT}\right)\Bigg] \quad (21)$$

that asserts the following classical result for Coulomb interaction [17]

$$\sqrt{\frac{m_e}{m_p}}\left(1+\frac{Z_de^2}{Rk_BT}\right)=\exp\left(-\frac{Z_de^2}{Rk_BT}\right). \quad (22)$$

Figure 8 is drawn to make a comparison of the dust particle charge, calculated from expression (20), with formulas (21) and (22) as a function of the coupling parameter. Since the polarization effects lead to a stronger increase in the electron flux than the proton flux on the dust particle surface, this results in the growth of the grain charge. Solutions to equations (21) and (22) better describe the behavior of the grain charge at low values of the coupling parameter since the polarization plays less significant role in this case. It is rather natural that formula (21) treats more accurately the behavior of the dust particle charge than formula (22).

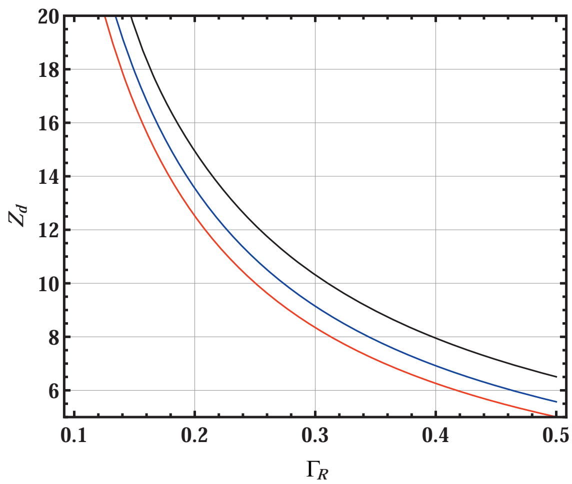

Fig. 8 The electric charge of the dust particle as a function of the coupling parameter Γ_R. Red line: solution to equation (22); blue line: solution to equation (21); black line: exact result from (20).

3 Plasma electrodynamics

It is widely believed in the literature that the dust charge is determined by the normal component of the electric field strength at the particle surface. This inference is usually made from the following equation for the electric field strength $\mathbf{E}$ as applied to the cylinrical volume of the cross section S shown in Figure 9:

$$\oint \mathbf{E}\cdot d\mathbf{S}=4\pi(\sigma S+\rho_{pl}V), \quad (23)$$

where σ stands for the surface charge density on the dust particle and ρ_{pl} refers to the plasma charge density.

Without any loss of generality the dust grain material can be assumed to be conductive so that the electric field strength under the grain surface turns zero and, then, applying equation (23) to the infinitesimally thin cylinder, $V \to 0$, finally yields

$$E_n = 4\pi\sigma. \tag{24}$$

Boundary condition (24) is incorrect from the viewpoint of plasma electrodynamics because the cylinder in equation (23) cannot be taken infinitesimally thin, otherwise one has to inevitably turn to consideration of the microscopic electric field which rapidly fluctuates over time in contrast to the macroscopically averaged electric field entering equation (23).

To correctly derive the boundary condition one has to use the following explicit equation for the dielectric displacement vector $\mathbf{D}$ which stems from the plasma electrodynamics:

$$\oint \mathbf{D} \cdot d\mathbf{S} = 4\pi\sigma. \tag{25}$$

When applied to a rather small cylinder still containing enough number of plasma particles to treat the electric field macroscopically, equation (25) gives rise to the correct boundary condition

$$D_n = 4\pi\sigma \tag{26}$$

for the dielectric displacement vector in a plasma near the dust surface.

Boundary condition (25) differs significantly from (24) because the displacement vector $\mathbf{D}$ is expressed in terms of the electric field strength $\mathbf{E}$ in the static case of plasma electrodynamics via the integral relation

$$\mathbf{D}(\mathbf{r}) = \int \epsilon(\mathbf{r} - \mathbf{r}_1)\mathbf{E}(\mathbf{r}_1)d\mathbf{r}_1, \tag{27}$$

where $\epsilon(\mathbf{r})$ stands for the plasma dielectric function defined in the configurational space.

It is, thus, rather clear how to accurately work out the problem of the electric charge of the dust grain. One has to consider a spatially finite plasma with boundary condition (26) and spatially varying plasma parameters, i.e. to construct an exact theory of the plasma sheath. This is quite a complicated problem to solve analytically and all further simplified consideration in this section is aimed at establishing what impact expression (26) has on the interaction between plasma particles and the dust grain.

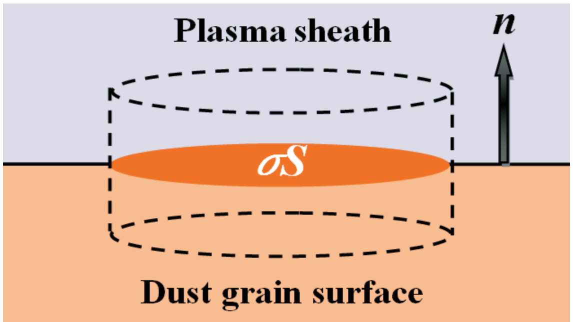

Fig. 9 To the derivation of a boundary condition between the plasma and the dust grain surface

It is well known that in a spatially infinite plasma the Fourier transform of the screened interaction potential $\tilde{\Phi}_{ab}(\mathbf{k})$ between the particles of species a and b is expressed in terms of the Fourier transform of the true microscopic interaction potential $\tilde{\varphi}_{ab}(\mathbf{k})$ and the plasma static dielectric function $\varepsilon(\mathbf{k})$ as:

$$\tilde{\Phi}_{ab}(\mathbf{k}) = \varepsilon(\mathbf{k})^{-1}\tilde{\varphi}_{ab}(\mathbf{k}), \tag{28}$$

in which the static dielectric function can be taken in the form of the random phase approximation as

$$\varepsilon(\mathbf{k}) = 1 + \frac{k_D^2}{k^2}, \tag{29}$$

where $k_D = 1/r_D$ denotes the wavenumber inversed to the Debye screening radius r_D.

Using the convolution theorem it is convenient in the sequel to rewrite relation (28) as

$$\Phi_{ab}(\mathbf{r}) = \int \varepsilon^{-1}(\mathbf{r} - \mathbf{r}_1)\varphi_{ab}(\mathbf{r}_1)d\mathbf{r}_1 \tag{30}$$

where the kernel is found from equation (29) as

$$\varepsilon^{-1}(\mathbf{r}) = \delta(\mathbf{r}) - \frac{k_D^2}{2\pi r}\exp(-k_D r). \tag{31}$$

To practically apply formulas (30) and (31), initially worked out for an infinite plasma, to a spatially finite plasma of interest it is proposed herein to treat dust grains as point-like charges by counting all distances r from the dust grain surface so that the interaction micropotentials acquire the form:

$$\varphi_{ab}(r) = \frac{Q_{ab}}{r + R_{ab}} \tag{32}$$

where the following notation $Q_{ed} = -Q_{pd} = Z_d e^2$, $R_{ed} = R_{pd} = R$ and $Q_{dd} = Z_d^2 e^2$, $R_{dd} = 2R$ is used for the interaction between the plasma particles and the dust grain and for the intergrain interaction, respectively.

On substituting expressions (31) and (32) into (30), one ultimately gets

$$\Phi_{ab}(r) = \varphi_{ab}(r) - \frac{Q_{ab}}{r}\left(1 - \exp(-rk_D) - \frac{R_{ab}k_D}{2}B_{ab}(r)\right), \tag{33}$$

where

$$\begin{aligned} B_{ab}(r) = {} & \exp((R_{ab}+r)k_D)\mathrm{Ei}((R_{ab}+r)k_D) - \exp(k_D(R_{ab}-r))\mathrm{Ei}(k_D R_{ab}) \\ & + \exp\left(-(R_{ab}+r)k_D\right)\left[\mathrm{Ei}(-R_{ab}k_D) - \mathrm{Ei}(-(R_{ab}+r)k_D)\right] \end{aligned} \tag{34}$$

with the exponential integral function defined as $\mathrm{Ei}(x) = \int_x^\infty \exp(-t)/t\,dt$.

Note that it is ordinarily assumed in the literature that the screening effects start from the dust grain surface such that the Debye-like theory gives rise to the following interaction potential

$$\Phi_{ab}^D(r) = \frac{Q_{ab}}{(r + R_{ab})(1 + k_D R_{ab})}\exp\left(-k_D r\right). \tag{35}$$

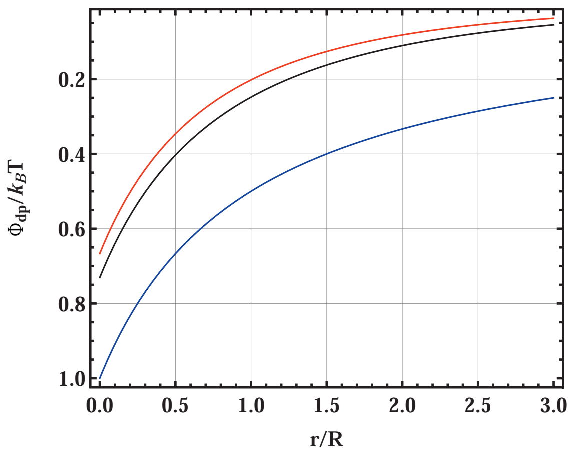

Fig. 10 The interaction potential between the dust particle and the proton at $Z_d = 10$, $\Gamma = 0.1$ and $\kappa = 0.5$. Blue line: micropotential (32); red line: Debye-like potential (35); black line: formula (33).

In Figure 10 a comparison is made between expressions (32), (33) and (35) for the proton-grain interaction at $Z_d = 10$, $\Gamma = e^2/(Rk_BT) = 0.1$ and $\kappa = k_D R = 0.5$. It is seen that micropotential (32) lies high above Debye-like potential (35) and the proposed potential (33) that has a gap, say $\delta = \Phi_{ab}(r \to 0) - \varphi_{ab}(r \to 0)$, at the origin, $r \to 0$, caused by engaging of the plasma electrodynamics. It appears from expression (33) and (34) that the gap δ has an exact value of

$$\delta = -k_D Q_{ab} \left[1 - k_D R_{ab} \mathrm{Ei}(k_D R_{ab}) \exp(k_D R_{ab})\right]. \tag{36}$$

It is worth noting that the onset of the gap is a straightforward consequence of boundary condition (25) and has an immediate influence on the the dust grain charge in a plasma which will be in focus of a forthcoming paper.

4 Chemical model of dusty plasmas

As it is clear from the consideration above the only tool at hand to analytically calculate the electric charge is the orbital motion limited approximation. The vital question posed in this section is whether the power of thermodynamics can be employed to determine the electric charge of dust particles in a plasma.

To give a positive answer to this question consider a situation that is quite different from those treated in the previous sections in that the dust is assumed to be an ordinary plasma component. The original idea is to use the chemical model of a plasma which is commonly applied to evaluate the composition of a partially ionized plasma. It is important for the chemical picture to properly define each particle specie which is done hereinafter as follows. Assume that the plasma medium of volume V contains N_p number of protons, N_e number of electrons and N_d number of dust particles, then, the free energy F of the system is written as [46, 47]:

$$\frac{F}{k_B T} = N_e \left[\ln\left(\frac{N_e \lambda_e^3}{V}\right) - 1\right] + N_p \left[\ln\left(\frac{N_p \lambda_p^3}{V}\right) - 1\right] + N_d \left[\ln\left(\frac{N_d \lambda_d^3}{V\Sigma}\right) - 1\right] - \frac{k_B T V}{12 r_D^3}, \tag{37}$$

where $\lambda_a = (2\pi\hbar^2/m_a k_B T)^{1/2}$ denotes the thermal wavelength of particles of specie a and $r_D^2 = k_B T/4\pi(n_e + n_p + n_d Z_d^2)e^2$ designates the Debye screening radius with n_p, n_e, n_d being the corresponding number densities. Note that the first three terms in (37) correspond to an ideal system of noninteracting particles whereas the last term comes from the plasma contribution in the simplest Debye approximation.

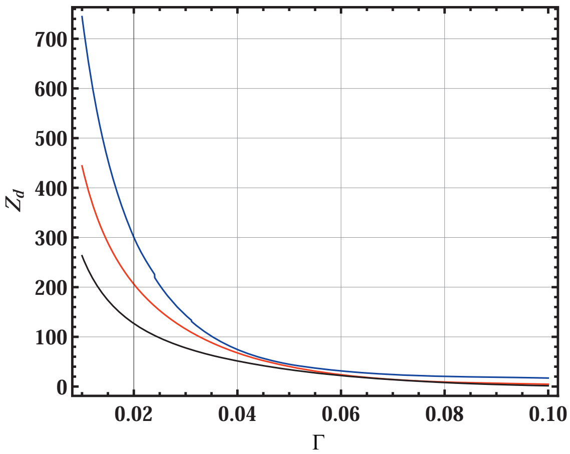

Fig. 11 Electric charge Z_d of the dust grain as a function of the coupling parameter Γ at $r_s = 200$. Blue line: $\kappa = 10^{-5}$; red line: $\kappa = 10^{-4}$; black line: $\kappa = 10^{-3}$.

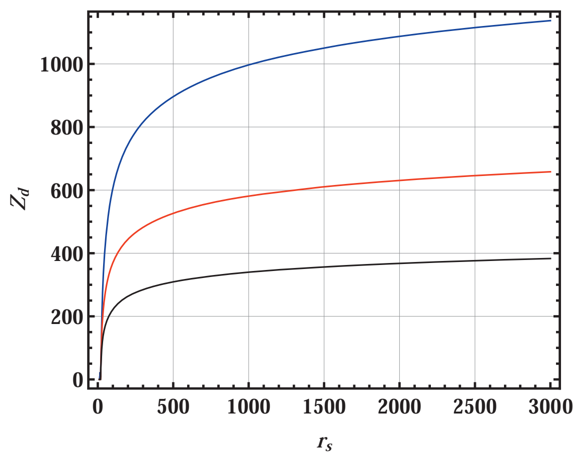

Fig. 12 Electric charge Z_d of the dust grain as a function of the density parameter r_s at $\Gamma = 0.01$. Blue line: $\kappa = 10^{-5}$; red line: $\kappa = 10^{-4}$; black line: $\kappa = 10^{-3}$.

The only factor yet left undefined is the partition function Σ which can be handled as follows. A dust grain is a solid state body which is a potential well for electrons characterized by the so-called work function A. If each dust particle have absorbed Z_d number of electrons, then, the partition function is obtained as

$$\Sigma = \exp(Z_d A). \tag{38}$$

The last restriction to be imposed is an overall neutrality of the system invoked by the relation

$$n_e + Z_d n_d = n_p. \tag{39}$$

From the practical point of view evaluation of the dust grain charge is entirely based on the minimization of free energy (37) under the condition that the number density of the protons n_p is kept constant. Numerical results are presented in Figures 11 and 12 to show the dependence of the dust grain charge on the coupling parameter $\Gamma = e^2/(ak_BT)$ and the density parameter $r_s = am_ee^2/\hbar$ where $a = (3/4\pi n_p)^{1/3}$. It is seen that when the parameter $\kappa = n_d/n_p$ grows the charge number of the dust grains Z_d decreases because the number density of dust particles effectively increases. It is rather self explanatory that the electric charge of dust particles vanishes when the coupling parameter grows since the decrease in plasma temperature results in lower plasma particles mobilities. At the same time decreasing the proton number density or increasing the density parameter give rise to the growth of the dust particle charge. Quite an analogous behavior was established in [48].

Of course, the approach developed above has its own merits and demerits. One of the apparent advantages of the chemical model is that it is absolutely insensitive to the details such as, for example, absorption cross sections of plasma particles and, on the other hand, success strongly depends on the choice of the free energy. Further improvement can readily be achieved by developing a self-consistent model to correctly take into account interparticle interactions as it could be done in the pair correlation approximation [49].

5 Conclusions

This paper has been solely concentrated on the problem of the calculation of the electric charge of dust particles immersed in a buffer hydrogen plasma.

Consideration has started from the orbital motion limited approximation, which implies the collisionless ballistic trajectories of plasma particles in an electric field of the charged dust grain. It has been demonstrated that the polarization effects lead to a substantial modification of the calculation technique to find that the proton and electron fluxes on the grain surface strongly depend on its charge and the coupling parameter of the buffer plasma. In particular, the proton flux grows linearly with increasing the grain charge and the coupling parameter, which is explained by their mutual attraction. The opposite pattern is observed for the the electron flux since the electrons are repelled by the negatively charged dust particle. The influence of polarization effects on the grain charge has been studied to show that it increases when the coupling parameter grows which is prescribed to the studied behavior of the electron and proton fluxes on the dust grain surface.

The plasma electrodynamics has provided an important insight that the charge of the dust grain is determined by the normal component of the dielectric displacement vector near the grain surface. A simple model has been put forward to derive the screened interaction potential between the constituents of the dusty plasma to discover that a gap at the grain surface has occurred in comparison with the microscopic potential which must have a significant impact on the grain charge.

Finally, the chemical model of dusty plasmas has been developed for the first time ever as an efficient analogue of the chemical picture of a partially ionized plasmas. The free energy of the system has taken into account the plasma contribution in the simplest form of the Debye approximation and its minimization together with the total system neutrality has allowed us to evaluate the electric charge of dust particles treated as a distinct specie.

There are so many possible ways to improve the approaches proposed above. For example, one can develop a more sophisticated theory of a charged double layer at the solid surface to mimic generation of a work function. Another problem is to create a consistent theory for the grain charge in the framework of plasma electrodynamics. And, of course, the chemical model of dusty plasmas can be significantly modified to take into account various effects that may play a tangible role.

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