

*2<sup>nd</sup> INTERNATIONAL EURASIAN  
CONFERENCE ON  
MATHEMATICAL SCIENCES AND  
APPLICATIONS*

*PROCEEDING BOOK*

*26-29 AUGUST 2013  
SARAJEVO, BOSNIA and HERZEGOVINA*

## ***FOREWORD***

The 2<sup>nd</sup> International Eurasian Conference on Mathematical Sciences and Applications has been organized jointly by Sakarya University, Bilecik University, International University of Sarajevo, Kocaeli University and Turkish World Mathematical Society on 26-29 August 2013 in Sarajevo, the city of biggest cultural and economy center of Bosnia Herzegovina.

The first conference on these series was organized in Prishtine, Kosovo on 03-07 September 2012 and 351 scientists were attended the conference from 17 different countries, contributing 248 oral presentations and 36 posters.

The main aim of this conference is to bring mathematical society all over the world working in various areas of theoretical and applicational studies in mathematics. With this conference we have also aimed to promote the Bosnia-Herzegovina and show/announce the reality that the struggle of Aliya Izzetbegovic was right for his country and citizens.

The organizing scheme for the conference was formed by Organizing and Scientific committees. The Scientific committee, who are the best in their research areas, is formed from 17 different countries. 10 worldwide distinguished speakers are invited to the conference. We have taken a lot of applications from 30 countries and this gives us the opportunity to select the best works for the conference. We believe that this has raised the scientific level of the conference.

The first day of the conference is devoted to opening ceremony together with some presentations. The second day will comprise solely the talks. In the third day of the conference the presentation will be held till noon, following by a trip to Sarajevo. In the last day of the conference there will be an excursion to Mostar. Furthermore, in the first and second days' nights there will be Turkish and Bosnian culture nights with dinner.

We would like to thank to Sakarya University, Bilecik University, Kocaeli University and International University of Sarajevo for their invaluable supports. We would also like to thank to all contributors to the conference, especially to the invited speakers who share their scientific knowledge with us, to organizing and scientific committee for their great effort on evaluating the manuscripts. We do believe and hope that each contributor will get benefit from the conference.

We hope to see you in the third International Eurasian Conference on Mathematical Sciences and Applications.

Yours Sincerely,

***Prof.Dr. Murat TOSUN***

*Chairman of the Conference*

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*INVITED  
TALKS*



## On Biharmonic Legendre Curves in S-Space Forms and Generalized Sasakian Space Forms

Cihan Özgür<sup>1</sup>

**Abstract.** Let  $(M,g)$  and  $(N,h)$  be two Riemannian manifolds and  $f: (M,g) \rightarrow (N,h)$  a smooth map. If  $f$  is a critical point of the energy functional, then it is called harmonic [4].  $f$  is called a biharmonic map, if it is a critical point of the bienergy functional [5]. It is trivial that any harmonic map is biharmonic. If the map is non-harmonic biharmonic map, then we call it as proper biharmonic. A 1-dimensional integral submanifold of an S-space form [3] or a generalized Sasakian space form [1] is called a Legendre curve. In the present talk, we consider biharmonic Legendre curves in S-Space Forms and Generalized Sasakian Space Forms.

**Keywords.** Biharmonic curve, Legendre curve, S-space form, generalized Sasakian space form.

**AMS 2010.** 53C25, 53C40, 53A04

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## Small gaps between primes

Cem Yalçın Yıldırım<sup>1</sup>

**Abstract.** In the works by D. A. Goldston, J. Pintz and C. Y. Yıldırım the use of short divisor sums has led to strong results concerning the existence of small gaps between primes. The results depend on the information about the distribution of primes in arithmetic progressions, specially on the range where the estimate of the Bombieri-Vinogradov Theorem is taken to hold. Let  $p_n$  denote the  $n$ -th prime. We unconditionally obtain

$$\liminf_{n \rightarrow \infty} \frac{p_{n+1} - p_n}{\log p_n} = 0.$$

In fact, we have the stronger quantitative result

$$\liminf_{n \rightarrow \infty} \frac{p_{n+1} - p_n}{\sqrt{\log p_n} (\log \log p_n)^2} < \infty.$$

Furthermore for any fixed positive integer  $\nu$ , it is shown that

$$\liminf_{n \rightarrow \infty} \frac{p_{n+\nu} - p_n}{\log p_n} \leq e^{-\gamma} (\sqrt{\nu} - 1)^2,$$

along with a generalization for small differences between primes in arithmetic progressions

where the modulus of the progression can be taken to be as large as  $(\log \log p_n)^A$  with arbitrary

$A > 0$ . Assuming that the estimate of the Bombieri-Vinogradov Theorem holds with any level beyond the known level  $\frac{1}{2}$ , i.e. conditionally, the method establishes the existence of bounded gaps between consecutive primes. Another result is that given any arbitrarily small but fixed  $\eta > 0$ , a positive proportion of all gaps between consecutive primes are comprised of gaps which are smaller than  $\eta$  times the average gap. Again a variety of quantitative results, some unconditional and some conditional, are obtained.

The corresponding situation for  $E_2$  - numbers  $q_n$ , numbers which are the product of two distinct primes, have been studied by S. W. Graham and the three mentioned researchers with the unconditional result that

$$\liminf_{n \rightarrow \infty} (q_{n+r} - q_n) \leq C(r),$$

for certain constants  $C(r)$ , in particular  $C(1) = 6$ . The methods and results in this work also yielded stronger variants of the Erdős-Mirsky conjecture. For example, it is shown that there are infinitely many integers  $n$  which simultaneously satisfy  $d(n) = d(n+1) = 24$ ,  $\Omega(n) = \Omega(n+1) = 5$ ,  $\omega(n) = \omega(n+1) = 4$ , (here  $d(n), \Omega(n), \omega(n)$  denote respectively the number of positive integer divisors of  $n$ , the number of primes dividing  $n$  counted with multiplicity, and the number of distinct prime divisors of  $n$ ).

In my talk I shall try to give a presentation of the main ideas involved in these works.

**AMS 2000.** 11N05, 11N13.

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## On Generalized Sequence Spaces via Modulus Function and A- Statistical Convergence

Ekrem Savaş<sup>1</sup>

**Abstract.** Let  $w$  denote the set of all real and complex sequences  $x = (x_k)$ . By  $l_\infty$  and  $c$ , we denote the Banach spaces of bounded and convergent sequences  $x = (x_k)$  normed by  $\|x\| = \sup_n |x_n|$ , respectively. A linear functional  $L$  on  $l_\infty$  is said to be a Banach limit if it has the following properties:

1.  $L(x) \geq 0$  if  $x_n \geq 0$  (i.e.  $x_n \geq 0$  for all  $n$ ),
2.  $L(e) = 1$  where  $e = (1, 1, \dots)$ ,
3.  $L(Dx) = L(x)$ , where the shift operator  $D$  is defined by  $D(x_n) = \{x_n + 1\}$ .

Let  $B$  be the set of all Banach limits on  $l_\infty$ . A sequence  $x \in l_\infty$  is said to be almost convergent if all Banach limits of  $x$  coincide. Let  $\hat{c}$  denote the space of almost convergent sequences. Lorentz [2] has shown that

$$\hat{c} = \left\{ x \in l_\infty : \lim_m t_{m,n}(x) \text{ exist uniformly in } n \right\}$$

where

$$t_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \dots + x_{n+m}}{m+1}$$

By a lacunary  $\theta = (kr)$ ;  $r = 0, 1, 2, \dots$  where  $k_0 = 0$ , we shall mean an increasing sequence of non-negative integers with  $k_r - k_{r-1} \rightarrow \infty$  as  $r \rightarrow \infty$ . The intervals determined by  $\theta$  will be denoted by  $I_r = (k_{r-1}, k_r]$  and  $h_r = k_r - k_{r-1}$ . The ratio  $\frac{k_r}{k_{r+1}}$  will be denoted by

A modulus function  $f$  is a function from  $[0, \infty)$  to  $[0, \infty)$  such that

- (i)  $f(x) = 0$  if and only if  $x = 0$ ,
- (ii)  $f(x+y) \leq f(x) + f(y)$  for all  $x, y \geq 0$ ,
- (iii)  $f$  increasing,
- (iv)  $f$  is continuous from the right at zero.

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In the present paper, we introduce and study some properties of the following sequence space that is defined using the  $\varphi$ -function and generalized three parametric real matrix.

Let  $\varphi$  and  $f$  be given  $\varphi$ -function and modulus function, respectively and  $p = (p_k)$  be a sequence of positive real numbers. Moreover, let  $A = (A_i)$  be the generalized three parametric real matrix with  $A_i = (a_{n,k}(i))$  and a lacunary sequence  $\theta$  be given. Then we define the following sequence spaces,

$$N_{\theta}^0(\mathbf{A}, \varphi, f, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f \left( \left| \sum_{k=1}^{\infty} a_{nk}(i) \varphi(|x_k|) \right| \right)^{p_k} = 0, \text{ uniformly in } i \right\}.$$

If  $x \in N_{\theta}^0(\mathbf{A}, \varphi, f)$ , the sequence  $x$  is said to be lacunary strong  $(A, \varphi)$ -convergent to zero with respect to a modulus  $f$ . When  $\varphi(x) = x$  for all  $x$ , we obtain

$$N_{\theta}^0(\mathbf{A}, f, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f \left( \left| \sum_{k=1}^{\infty} a_{nk}(i) x_k \right| \right)^{p_k} = 0, \text{ uniformly in } i \right\}.$$

If we take  $f(x) = x$ , we write

$$N_{\theta}^0(\mathbf{A}, \varphi, p) = \left\{ x = (x_k) : \lim_r \frac{1}{h_r} \sum_{n \in I_r} f \left( \left| \sum_{k=1}^{\infty} a_{nk}(i) \varphi(|x_k|) \right| \right)^{p_k} = 0, \text{ uniformly in } i \right\}.$$

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## Hidden Attractors in Dynamical Systems: Fundamental Problems and Applied Models

Leonov G.A. Kuznetsov N.V.<sup>1</sup> and Seledzhi S.M.<sup>2</sup>

**Abstract.** An oscillation in dynamical system can be easily localized numerically if initial conditions from its open neighborhood lead to long-time behavior that approaches the oscillation. Thus, from a computational point of view in applied problems of nonlinear analysis of dynamical models, it is essential to regard attractors as *self-excited* and *hidden attractors* depending on simplicity of finding its basin of attraction in the phase space.

For a self-excited attractors its basin of attraction is connected with an unstable equilibrium: self-excited attractors can be localized numerically by *standard computational procedure*, in which after a transient process a trajectory, started from a point of unstable manifold in a neighborhood of equilibrium, reaches a state of oscillation therefore one can easily identify it. In contrast, for a *hidden attractor*, its basin of attraction does not intersect with small neighborhoods of equilibria.

This plenary lecture is devoted to affective analytical-numerical methods for localization of hidden oscillations in nonlinear dynamical systems and their application to well known fundamental problems (16th Hilbert problem, Aizerman conjecture and Kalman conjecture) and applied models (phase-locked loop, drilling system, and aircraft control system) [1-27].

**Keywords.** Hidden oscillation, hidden attractor, 16th Hilbert problem, limit cycles, Lyapunov focus values (Lyapunov quantities), Aizerman conjecture, Kalman conjecture, absolute stability, describing function method justification, phase-locked loop (PLL), drilling system, aircraft control, saturation, Chua circuits

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## Parseval Frame Wavelets

Hrvoje Šikić<sup>1</sup>

**Abstract.** Time-frequency analysis is fundamentally influenced by the Heisenberg uncertainty principle. Since the minimal time-frequency window is achieved for Gaussian functions, and it does not change by application of three basic transformations: translations, modulations, and dilations, it is natural that basic systems for the analysis are built using these facts. For example, Gabor systems are built from a single function via the use of translations and modulations. However, due to the well-known Balian-Low theorem, we know that Gabor systems which are also orthonormal bases have either the time or the frequency dispersion of infinite size, which is not suitable for applications. Interestingly enough, this particular problem does not occur in the realm of wavelets, systems built through the usage of translations and dilations. And, indeed, there are many very useful examples of orthonormal wavelets, interesting from both theoretical and practical point of view.

Orthonormal bases have many nice geometric properties, but from the applications point of view, there is at least one potentially negative feature. Consider signal analysis from the point of view of analysis-synthesis approach. If in the process some important coefficient is lost (say due to some technical conditions), then it can not be recovered from other members of the basis. Hence, it may be useful to apply some systems which have a redundancy property built in. Typical examples of such systems are frames, so in the last fifteen years we witness a strong development of the theory of frame wavelets, i.e., wavelet systems (based on translations and dilations) which are also frames. Of particular interest are so-called normalized tight frame wavelets, since they have a natural reproducing property as their defining feature. Recently such systems are named Parseval frame wavelets.

The development of Parseval frame wavelets relies on many ideas from the orthonormal case, like MRA structure, low-pass filters, characterizing equations, representation formulas, dimension function, connectivity properties, but they all exhibit much wider range of different properties than the orthonormal case. In addition they require different techniques. Furthermore, there are some completely new issues not to be observed in the orthonormal case. Wavelet systems can be considered as consisting of various levels of resolution (based on the fixed corresponding dilation order), and since in such an approach only translations run freely within a single resolution level, the theory of shift invariant spaces

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plays a key role in the analysis of Parseval frame wavelets. Consider for example the main (or zero) resolution level of such a system. Unlike in the orthonormal case, the properties of the generating system within the main resolution level can vary in numerous ways, without spoiling the frame property of the entire system. That does not mean, of course, that it will not influence the system. Quite to the contrary, the behavior of generating system within the main resolution level has an unexpectedly strong and diverse influence on the whole system. As an example of such a result, let us quote that Parseval frame wavelets are going to be semiorthogonal (property of interaction among various levels of resolution) if and only if the dimension function of the main resolution level is integer-valued (which is an inside property within a single resolution level). There is an entire family of various conditions that describe such relationships. One aim of this lecture is to present these results and the detailed structure of the entire set of Parseval frame wavelets, including some interesting open questions.

It is clear from the nature of results mentioned above, that our analysis focuses more and more on the main resolution level. This leads to various fascinating relationships between different basis properties of the generating system and some well-known notions and conditions from harmonic analysis, as well as from functional analysis. We plan to present some of these results and related open questions in the second part of the lecture. The entire lecture will be based on several results from articles listed below.

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## Geometric Modeling with Curves and Surfaces in Euclidean Spaces

Kadri Arslan<sup>1</sup>

**Abstract.** Mathematical and geometric concepts of curves and surfaces lead to the birth of Computer Aided Geometric Design (CAGD). In the present study we consider some examples of geometric modeling with curve and surfaces in Euclidean spaces.

**Keywords.** geometric modeling.

**AMS 2010.** 53A40

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## **Two Pillars of Probability Theory**

Nicholas Hugh Bingham

**Abstract.** Probability Theory is a vast and wonderful subject, full of interesting mathematics and full also of useful applications to the real world. We discuss here the two pillars on which all this rests.

The first pillar is known to the ‘man or woman in the street’, under the folklore name “Law of Averages”. If you keep tossing a fair coin, it will fall heads about half the time; similarly for biased coins. Mathematicians call such results Laws of Large Numbers (LLNs). Perhaps surprisingly, it was not until 1713 that this first found expression in a mathematical theorem : Bernoulli’s theorem. It was not until 1933 that this found its ultimate form: Kolmogorov’s strong law of large numbers.

If one tries to measure a physical constant to high precision, one replicates the relevant experiment a large number of times. The average of the results gives the best estimate of the true value; the spread in the results gives an error estimate. Hence such results are known to the ‘physicist in the street’ as the “Law of Errors”: errors are normally distributed about the mean. To mathematicians, this is the Central Limit Theorem (CLT). It goes back to de Moivre in 1733, and reached its modern form in the last century. It is precisely because of this that the normal (Gaussian) distribution dominates so much of Statistics.

There is lots more! The talk focuses on the two results above, with historical background, and their links with related results.

## **Analogues Of Some Tauberian Theorems For Bounded Variation**

Richard F. Patterson<sup>1</sup>

**Abstract.** In this paper definitions for “bounded variation”, “subsequences”, “Pringsheim limit points”, and “stretchings” of a double sequence are presented. Using these definitions and the notion of regularity for four dimensional matrices, the following two questions will be answered. First, if there exists a four dimensional regular matrix  $A$  such that  $Ay = \sum_{k,l=1,1}^{\infty,\infty} a_{m,n,k,l} y_{k,l}$  is of bounded variation (BV) for every subsequence  $y$  of  $x$ , does it necessarily follow that  $x \in BV$ ? Second, if there exists a four dimensional regular matrix  $A$  such that  $Ay \in BV$  for all stretchings  $y$  of  $x$ , does it necessarily follow that  $x \in BV$ ? Also some natural implications and variations of the two Tauberian questions above will be presented.

**AMS.** 40B05, 40C05.

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## **Fuzzy Logic Control in Perspective A 50-Year History**

Reza Langari<sup>1</sup>

**Abstract.** Fuzzy control, although officially introduced by E.H. Mamdani in 1974, traces its roots in L. A. Zadeh's original work on fuzzy set theory in 1965 but perhaps even earlier to Zadeh and Desoer's 1963 classic text on linear systems theory. In the past 50 years, fuzzy control has matured from a relatively simple use of *if, then* rules to a sophisticated combination of linear and nonlinear systems theoretic techniques that span the spectrum of sub-disciplines of control theory. These range from Lyapunov stability theory to the application of linear matrix inequalities in  $H_\infty$ -based optimization of multivariable fuzzy control systems to sum-of-squares optimal control of nonlinear systems. While the use of these sophisticated techniques have garnered a certain level of respectability for fuzzy control, one has also arguably lost the spirit in which fuzzy control was invented, namely reliance on the empirical knowledge of operation of a given system as opposed to formal construction of fuzzy logic controllers via system-theoretic techniques. In this presentation we will explore both sides of this argument and point to some potentially positive aspects of each approach as well as their limitations from both applied and theoretical perspectives. We conclude with a view toward the future of fuzzy control and discuss its capacity for innovation in areas that are not yet explored extensively by the community of researchers, namely the study of complex hierarchical-heterarchical systems that are commonly prevalent in engineering domains.

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## **Quaternionic and Clifford analysis in applications**

Wolfgang Sproessig<sup>1</sup>

**Abstract.** Quaternionic and Clifford analysis (higherdimensional function theories) have in recent years become increasingly important tools in the analysis of partial differential equations and their application in mathematical physics and engineering. The lecture reflects the main ideas in the field and covers quaternions, Clifford numbers, Clifford (quaternion) operator calculus, boundary value and problems numerical implementations. It is intended to make the audience familiar with basic ideas in order to adapt these methods to their own work. Examples are taken from fluid dynamics, elasticity theory and Maxwell equations.

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# *ALGEBRA*



## A New Study on Generalized Intuitionistic Fuzzy Subhyperalgebras of Boolean Hyperalgebras

B. A. Ersoy<sup>1</sup>, S. Onar<sup>1</sup>, K. Hila<sup>2</sup> and B. Davvaz<sup>3</sup>

**Abstract.** This paper deals with a special hyperalgebra, called Boolean hyperalgebra which is redefined in it. We introduce the concepts of generalized intuitionistic fuzzy subhyperalgebras and generalized intuitionistic fuzzy hyperideals of Boolean hyperalgebras. A necessary and sufficient condition for a intuitionistic fuzzy subset of Boolean hyperalgebra to be a generalized intuitionistic fuzzy subhyperalgebra (hyperideal) is proved. Images and inverse-images of generalized intuitionistic fuzzy subhyperalgebra (hyperideal) under Boolean hyperalgebra homomorphism are studied.

**Keywords.** Boolean hyperalgebra, generalized intuitionistic fuzzy subhyperalgebra, generalized intuitionistic fuzzy hyperideal, generalized intuitionistic fuzzy quotient Boolean hyperalgebra

**AMS 2010.** 03E72, 08A72.

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## Primitive Sets of $F_n/R$ Lie Algebras

Cennet Eskal<sup>1</sup> and Naime Ekici<sup>2</sup>

**Abstract.** Let  $F_n$  be the free Lie algebra freely generated by a set  $\{x_1, x_2, \dots, x_n\}$  and let  $R$  be a verbal ideal of  $F_n$ . We prove that if  $W$  is a primitive subset of  $F_n/R$  which all of elements do not involve  $x_n$  then  $W$  is primitive in  $F_{n-1}/\widehat{R}$ , where  $\widehat{R} = R \cap F_{n-1}$ .

**Keywords.** Free Lie algebras, Solvable, nilpotent (super)algebras.

**AMS 2010.** 17B01, 17B30.

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## Crossed Modules of Commutative Algebras and Whiskered R-Algebroids

Çağrı Ataseven<sup>1</sup> and Erdal Ulualan<sup>2</sup>

**Abstract.** We define whiskered R-categories and investigate the relationship between crossed modules of commutative algebras and whiskered R-algebroids.

**Keywords.** Crossed Module, R-category, Monoidal category

**AMS 2010.** 18G50, 18G55

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## Primitive Elements and Preimage of Primitive Sets of Free Lie Algebras

Dilek Ersalan<sup>1</sup>, Zerrin Esmerligil<sup>2</sup> and Naime Ekici<sup>3</sup>

**Abstract.** Let  $F$  and  $L$  be free Lie algebras of finite rank  $n$  and  $m$  respectively and  $\varphi$  be a homomorphism from  $F$  to  $L$ . We prove that the preimage of a primitive set of  $L$  contains a primitive set of  $F$ . As a consequence of this result we obtain that an element  $h$  of a subalgebra  $H$  of  $F$  is primitive in  $H$  if it is primitive in  $F$ . Also we show that in a free Lie algebra of the form  $F/\gamma_{m+1}(R)$  if the ideal  $\langle g \rangle_{id}$  of this algebra contains a primitive element  $h$  then  $h$  and  $g$  are conjugate by means of an inner automorphism.

**Keywords.** Primitive element, Free Lie algebra.

**AMS 2010.** 17B01, 17B40.

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## A Study on Multiplication Lattice Module

Emel Aslankarayiğit<sup>1</sup>, Fethi Çallialp<sup>2</sup> and Ünsal Tekir<sup>3</sup>

**Abstract.** In this paper we determine multiplicative lattice module with principal element. Also we define small element in lattice module and hollow lattice module. Then we characterize small element and hollow lattice module. Consequently, we obtain the following results.

**Theorem:** Let  $L$  be a  $PG$  –lattice and  $M$  be a faithful multiplication  $PG$  –lattice  $L$  –module with  $1_M$  compact. Then  $M$  is hollow  $L$  –module if and only if  $L$  is hollow  $L$  –module.

**Theorem:** Let  $L$  be a  $PG$  –lattice and  $M$  be a faithful multiplication  $PG$  –lattice  $L$  –module with  $1_M$  compact. Then  $N$  is small if and only if there exists a small element  $a \in L$  such that  $N = a1_M$ .

**Keywords.** Multiplication lattices, lattice modules, principal element, small element and hollow lattice module.

**AMS 2010.** 16F10, 16F05.

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## Primitive Decompositions of Elements of a Free Metabelian Lie algebra of Rank Two

Ela Aydın<sup>1</sup> and Naime Ekici<sup>2</sup>

**Abstract.** We give a primitive decomposition of any element of a free metabelian Lie algebra and we determine the primitive length of an element.

**Keywords.** Primitive element, primitive decomposition, free metabelian Lie algebra.

**AMS 2010.**17B01.

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## On Multiplication and Comultiplication Lattice Modules

Fethi Çallıalp<sup>1</sup>

**Abstract.** This study concerns with the further investigation of multiplication and comultiplication lattice modules, especially faithful and principally generated comultiplication lattice modules over a multiplicative lattice. We obtain the following important result.

**Theorem :** Let  $L$  be a  $PG$  –lattice and  $M$  be a faithful multiplication  $PG$  –lattice  $L$  –module with  $1_M$  compact. The following statements are equivalent:

- (1)  $N = a1_M$  is a pure elements of  $M$ .
- (2)  $a = (N;_L 1_M)$  is a pure elements of  $L$ .
- (3)  $a = (N;_L 1_M)$  is a multiplication and idempotent element of  $L$ .
- (4)  $N = a1_M$  is a multiplication and idempotent element of  $M$

**Keywords.** Multiplication lattice module, Comultiplication lattice module, idempotent element, multiplication element and pure element.

**AMS 2010.** 16F10, 16F05.

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## Idempotent Generators of Skew Cyclic Codes over $F_q$

Fatmanur Gursoy<sup>1</sup>, Irfan Siap<sup>2</sup> and Bahattin Yıldız<sup>3</sup>

**Abstract.** D. Boucher et al. [1] introduced cyclic codes over skew polynomial rings. In their study they defined skew cyclic codes by using the noncommutative polynomial ring  $F_q[x; \theta]$ . This polynomial ring is known as a skew polynomial ring where the addition is defined as the usual addition on polynomials and the multiplication is defined by the rule  $ax^i * bx^j = a\theta^i(b)x^{i+j}$ ,  $(a, b \in F_q)$  where  $\theta$  is an automorphism over the finite field with  $q$  elements ( $F_q$ ) [2]. In the commutative case ( $F_q[x]$ ), if  $(n, q) = 1$  then every cyclic code of length  $n$  over  $F_q$  has a unique idempotent generator. However in the noncommutative case, polynomial factorization is not unique, and as a result it is difficult to determine the number of skew cyclic codes and their idempotent generators. In this paper, we study idempotent generators of skew cyclic codes over  $F_q$ . Under some restrictions, we show that skew cyclic codes have idempotent generators. Moreover, we derive a formula for the number of skew cyclic codes with length  $n$  over  $F_q$  in the case  $(n, m) = 1$ , where  $m$  is the order of the automorphism  $\theta$ .

**Keywords.** Skew Polynomial Rings, Skew Cyclic Codes, Codes over Rings.

**AMS 2010.** 94B05, 94B15.

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## A Note on Modules Over Commutative Rings

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**Abstract.** In this paper we study expansion of submodules and  $\delta$  – primary submodules of modules over commutative rings. Particularly, we investigate  $\delta$  – primary submodules of multiplication modules over commutative rings. Next the following important results are obtained:

**Theorem:** Let  $R$  be a ring,  $M$  be a multiplication  $R$  –module and  $N$  be a submodule of  $M$  such that  $N \neq M$ . Let  $\delta$  be a quotient and multiplication preserving expansion.  $N$  is a  $\delta$ -primary if and only if for any two submodules  $N_1$  and  $N_2$ , if  $N_1N_2 \subseteq N$  and  $N_1 \not\subseteq N$ , then  $N_2 \subseteq \delta(N)$ .

**Proposition:** Let  $M$  and  $M^*$  be multiplication  $R$  –modules and  $f : M \rightarrow M^*$  be a surjective module homomorphism. Let  $\delta$  be a global-homomorphism and quotient preserving. Then a submodule  $N$  of  $M$  that contains  $\ker(f)$  is  $\delta$  –primary if and only if  $f(N)$  is  $\delta$ -primary.

**Keywords.** Expansion of submodules,  $\delta$  –primary, Multiplication modules.

**AMS 2010.** 16D80, 16D10.

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## Braided Crossed Modules of Associative Algebras and Related Structures

Hasan Atik<sup>1</sup> and Erdal Ulualan<sup>2</sup>

**Abstract.** We give the Notion of braiding for a crossed module over non-commutative associative algebras and investigate the relations between this structures and some models.

**Keywords.** Braided Crossed Module, Simplicial Algebras, Moore Complex

**AMS 2010.** 18G50, 18G55

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## **Ordinal Length**

Hans Schoutens

**Abstract.** Commutative algebra is a generalization of linear algebra, in which a field gets replaced by a ring, and a vector space by a module; length then generalizes the notion of dimension. However, anything of infinite length gets thrown onto the same "does-not-apply" pile. I will argue that using a tool from logic, to wit, ordinals, we can nonetheless distinguish between them by means of ordinal length. This invariant tells us quite a lot about the module. Looking at a few examples, one sees that one can no longer expect additivity (which itself is a generalization of the rank-nullity theorem), but a vestige remains, namely semi-additivity, which often suffices to calculate the ordinal length. I will give a brief overview of the theory and some applications.

## Secret Sharing Scheme via Fractional Repetition Codes for $\rho > 2$

İbrahim Özbek<sup>1</sup> and İrfan Şiap<sup>2</sup>

**Abstract.** In this work we will show the relationship between Fractional Repetition Codes for  $\rho > 2$  to Secret Sharing Scheme [1]. In the previous study, we have constructed Secret Sharing Scheme via Fractional Repetition Codes for  $\rho = 2$  [2]. In this work we will generalize this construction by using design theory [3]. Furthermore, this construction can be resistant up to  $\rho - 1$  error participant.

**Keywords.** Secret Sharing Scheme, Fractional Repetition Codes, Error Participant.

**AMS 2010.** 53A40, 20M15.

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## Homotopy of 2-Crossed Module Morphisms on Commutative Algebras

Kadir Emir<sup>1</sup> and İ. İlker Akça<sup>2</sup>

**Abstract.** 2-crossed modules on groups were introduced by Conduché D. in [2], and Grandjeán A.R., Vale M.L. were defined algebra case of the notion in [3].

Homotopy of crossed complex morphisms on groupoids introduced by Brown R. and Galensiski in [1]. Also, Martins J.F. and Gohla B. examined the homotopy of 2-crossed module morphisms on groups in [4].

In this work, we will define homotopy of 2-crossed module morphisms on commutative algebras and construct a groupoid structure by 2-crossed module morphisms and their homotopies.

**Keywords.** 2-Crossed Module, Quadratic Derivation, Homotopy, Groupoid.

**AMS 2010.** 18D05, 55U35, 13N15.

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## A Topology for Cat-algebras

Koray Yılmaz<sup>1</sup>

**Abstract.** In this work, we established a topology on categorical algebras in terms of Barr-Wells lemma [3] by using an idempotent filter.

**Keywords.** Crossed module, categorical algebra , filter.

**AMS 2010.** 18G50, 18G55.

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## On Newer Concepts of Flat Modules

Muhammad Rashid Kamal Ansari<sup>1</sup>, Asma Zaffar<sup>1</sup>, Shuja Muhammad Qureshi<sup>1</sup> and  
Muhammad Zakaullah Khan<sup>1</sup>

**Abstract.** Flat modules which are a generalization of projective modules have given rise to many new classes of modules. For example module embedded in a flat module (*EF* modules) [1], [2]. Such modules demonstrate special generalized features which in some aspects resemble torsion free modules. There are other classes of modules and rings which arise from flat modules such as modules over *IF* rings and modules over a ring whose injective hulls are flat [4]. Flat modules are initially defined with the help of exact sequences of tensor product of modules. That is, a module  $M \in \text{mod-}A$ , the category of all right modules over a ring  $A$  is said to be flat if for all exact sequences  $\alpha : 0 \rightarrow P \rightarrow Q \rightarrow R \rightarrow 0$  in  $A\text{-mod}$ , the category of all left modules over the ring  $A$ , the sequence  $M \otimes_A \alpha$  is two sided exact that is the sequence  $0 \rightarrow M \otimes_A P \rightarrow M \otimes_A Q \rightarrow M \otimes_A R \rightarrow 0$  is exact. However, recently newer definitions of flat modules have appeared in literature. For example some authors use cotorsion modules to define flat modules [3], [5].

Generalizing these definitions, in this study it is considered that if  $A$  is an  $I$ -ring, that is a ring embedded in a ring  $I$ , then  $M$  in  $A\text{-mod}$  is said to be  $I(n)$ -cotorsion ( $I$ -cotorsion of order  $n$ ) if  $\text{Ext}(I, M) = 0$  for all  $n \geq 0$ . These concepts can be used to define  $I(n)$ -flat modules and  $I$ -flat modules. If  $A$  is an  $I$ -ring, then  $N$  in  $A\text{-mod}$  is said to be  $I(n)$ -flat ( $I$ -flat of order  $n$ ) if  $\text{Ext}_A^n(N, M) = 0$  for every  $I(n)$ -cotorsion module. This definition generalizes the definition of flat modules by Bazzoni and Salce. After discussing these developments in detail some applications regarding *EF* modules [5] and module approximations [6], [7] are also presented.

**Key Words.** 16D40.

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## Some Properties of Self-similar Groups in the Sense of IFS

Mustafa Saltan<sup>1</sup> and Bünyamin Demir<sup>2</sup>

**Abstract.** Self-similarity property of the fractal sets is defined on the group structure by Ş. Koçak. We give some properties of self-similar groups in the sense of iterated function system (IFS). We also show the relations between profinite groups and strong self-similar groups in the sense of IFS.

**Keywords.** Iterated function system, self-similar set, fractal, profinite group.

**AMS 2010.** 28A80, 47H10, 20E18.

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## Type II codes obtained via a chain ring and Quantum codes

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**Abstract.** In this study we deal with the Gray images of the codes over a special chain ring. By Gray map in [1], we also derive a family of optimal codes for suitable length and look at the special class of self-dual codes called Type II over the binary field in case where special values as the Gray images of the codes over the given chain ring. Finally, by the Gray map, we present a class of quantum codes over  $F_2$ ,  $F_3$  and  $F_4$  as applications.

**Keywords.** Gray map, Optimal codes, Type II codes, Quantum codes.

**AMS 2010.**68P30, 81P70

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## Some Identities in the Twisted Group Algebra of Symmetric Groups

Milena Sošić<sup>1</sup>

**Abstract.** In this presentation we will examine a natural action of the symmetric group  $S_n$  on the polynomial ring  $R_n$  in  $n^2$  commuting variables  $X_{ab}$  and also describe a twisted group algebra (defined by this action of  $S_n$  on  $R_n$ ), which we denote by  $\mathcal{A}(S_n)$ .

In the twisted group algebra  $\mathcal{A}(S_n)$  we will present some identities and give some factorizations of certain canonical elements in  $\mathcal{A}(S_n)$ , which can be decomposed into the product of simpler elements of  $\mathcal{A}(S_n)$ . These identities can be used to simplify some computations in [2], [3].

**Keywords:** symmetric group, polynomial ring, group algebra, twisted group algebra.

**Subject Classification:** 05E15.

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## Some Properties on the Disjunctive Product of a Special Graphs

Nihat Akgüneş<sup>1</sup>

**Abstract.** In [1], it has been recently defined a new graph  $\Gamma(S_M)$  on monogenic semigroups  $S_M$  (with zero) having elements  $\{0, x, x^2, x^3, \dots, x^n\}$ . The vertices are the non-zero elements  $x, x^2, x^3, \dots, x^n$  and, for  $1 \leq i, j \leq n$ , any two distinct vertices  $x^i$  and  $x^j$  are adjacent if  $x^i \cdot x^j = 0$  in  $S_M$ .

Our main aim in this study is to extend these studies over  $\Gamma(S_M)$  to the disjunctive product. In detail, we will investigate the diameter, radius, girth, maximum and minimum degree, chromatic number, clique number and domination number for the lexicographic product of any two (not necessarily different) graphs  $\Gamma(S_{M_1})$  and  $\Gamma(S_{M_2})$ .

**Keywords.** Disjunctive product, Domination number.

**AMS 2010.** 05C12, 15A42.

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## Symmetric Derivation On Kähler Modules

Necati Olgun<sup>1</sup>

**Abstract.** Let  $k$  be an algebraically closed field of characteristic zero,  $R$  an affine  $k$ -algebra and let  $\Omega^{(q)}(R/k)$  denote its universal finite Kähler module of differentials over  $k$ . In this paper, generalized symmetric derivation is introduced on Kähler modules. We then give some interesting results by using this definition and related to Kähler modules and symmetric derivations introduced by H. Osborn [4].

**Keywords.** Kähler module, symmetric derivation, regular ring.

**AMS 2010.** 13N05, 13D05.

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### **$k$ - Primitivity and Images of Primitive Elements**

Nazar Şahin Ögürlü<sup>1</sup> and Naime Ekici<sup>2</sup>

**Abstract.** Let  $F_n$  be a free Lie algebra of finite rank  $n$ ,  $n \geq 2$ . We give another proof of the following criterion which is proven by Mikhalev and Yu, using the idea of  $k$  - primitivity: An endomorphism of  $F_n$  preserving primitivity of elements is an automorphism.

**Keywords.** Free Lie algebras, primitive elements, primitive systems.

**AMS 2010.** 17B01, 17B40.

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## Hecke Groups and Numbers of the Form $n = x^2 + Ny^2$

Nihal Yılmaz Özgür<sup>1</sup>

**Abstract.** Let  $\lambda$  be a fixed positive real number. Hecke groups  $H(\lambda)$  are the discrete subgroups of  $\text{PSL}(2, \mathbf{R})$  (the group of orientation preserving isometries of the upper half plane  $\mathbf{U}$ ) generated by two linear fractional transformations

$$R(z) = -\frac{1}{z} \text{ and } T(z) = z + \lambda.$$

Hecke showed that  $H(\lambda)$  is discrete only for  $\lambda \geq 2$  or  $\lambda = \lambda_q = 2\cos(\pi/q)$ ,  $q \geq 3$  is an integer [3].

Let  $n$  be a given natural number. We introduce an algorithm that computes the integers  $x$  and  $y$  satisfying the equation  $n = x^2 + Ny^2$  by using the group structure of the Hecke groups  $H(\sqrt{N})$ ,  $N \geq 2$ .

**Keywords.** Modular group, Hecke groups, representations of integers.

**AMS 2010.** 11D84, 11F06, 20H10.

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## Some Results on Weak Hyper Residuated Lattices

R. A. Borzooei<sup>1</sup>, S. Niazian<sup>2</sup>

**Abstract.** In this paper, we introduced the notion of weak hyper residuated lattice which is a generalization of residuated lattice and we state and prove some related results. Moreover, we define the concepts of deductive systems, (positive) implicative and fantastic deductive systems and then state, we get some properties and we show the relations among them

**Keywords.** Weak hyper residuated lattice, (positive) implicative and fantastic deductive systems.

**AMS 2010.** 03G10, 06B99, 06B75.

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## On $(m;n)$ -ary Hypermodules

Reza Ameri<sup>1</sup>

**Abstract.** The authors in [1] introduced prime and primary hyperideals in Krasner  $(m; n)$ -ary hyperrings. In this paper, we state this concept on a canonical  $(m; n)$ -ary hypermodule, and we investigate the connection between of them. Also, maximal subhypermodule, Jacobson radical of canonical  $(m; n)$ -ary hypermodules are defined and several properties in this respect are investigated.

**Keywords.** Krasner  $(m, n)$ -ary hyperring,  $(m, n)$ -ary hypermodule,  $n$ -ary prime and  $n$ -ary primary subhypermodule.

**AMS 2010.** 20N20, 20N15.

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## The Trace Formula for $GL(2)$ over a Global Field

Rukiye Öztürk<sup>1</sup>, Ali Aydoğdu<sup>2</sup>, Engin Özkan<sup>3</sup>, Yuval Z.Flicker<sup>4</sup>

**Abstract.** We write out the trace formula explicitly for the group  $GL(2)$  over a global field of any characteristic. We give a detailed and complete proof. Then we write it in an invariant form. This is necessary for applications of counting special automorphic representations. We sketched some basic applications.

**Keywords.** trace formula, global fields, invariant distributions, Eisenstein series, intertwining operators, automorphic representations,  $GL(2)$ .

**AMS 2010.** 11F70,11F72,11M36, 22E55, 22E57.

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## Exponential Permutable Semigroups

Rushadije Ramani Halili<sup>1</sup> and Eip Rufati<sup>2</sup>

**Abstract.** We say that a semigroup  $S$  is a permutable semigroup if for every congruences  $\alpha$  and  $\beta$  of  $S$ ,  $\alpha \circ \beta = \beta \circ \alpha$ . A semigroup  $S$  is called a permutative semigroup if there is a positive integer  $n$  and a non-identity permutation  $\sigma$  of  $\{1, 2, \dots, n\}$  such that  $S$  satisfies the identity  $x_1 x_2 \dots x_n = x_{\sigma(1)} x_{\sigma(2)} \dots x_{\sigma(n)}$ .

In this paper we show that every permutable semigroup satisfying a permutation identity  $x_1 x_2 \dots x_n = x_{\sigma(1)} x_{\sigma(2)} \dots x_{\sigma(n)}$ ,  $\sigma(1) \neq 1, \sigma(n) \neq n$  is exponential. We also prove that every strictly permutative permutable semigroup is exponential. A semigroup is said to be exponential semigroup if it satisfies the identities  $(xy)^n = x^n y^n$  for all integers  $n > 0$ . Every medial semigroup is exponential.

**Key words.** exponential semigroups, permutative, permutable semigroups.

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**On the Permanents and Determinants of Some Matrices with Applications to the  
Recursive Sequence Order-k**

Tugba Sarişahin<sup>1</sup> and Ayse Nalli<sup>2</sup>

**Abstract.** In this paper, we derive a family of identities on the arbitrary subscripted recursive sequence order-k which are more general than Fibonacci, Tribonacci, Tetranacci,... etc. and their sums. Furthermore we construct matrices whose determinants and permanents form any linear subsequence of the recursive sequence order-k which are more general than that given in literature [J.Feng, More Identities On The Tribonacci Numbers, *Ars Combinatoria*,100(2011), 73-78 and E. Kilic,Tribonacci Sequence With Certain Indices And Their Sums, *Ars Combinatoria*, 86 (2008), 13-22].

**Keywords.** Recursive Sequence Order-k, Permanent, Determinant

**AMS 2010.** 53A40, 20M15.

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## Strongly 0-Dimensional Rings

Ünsal Tekir<sup>1</sup>, C. Jayaram<sup>2</sup>, Kürşat Hakan Oral<sup>3</sup>

**Abstract.** In this study, a commutative ring  $R$  with identity is called strongly 0-dimensional ring if whenever a prime ideal  $P$  of  $R$ , contains the intersection of any family of ideals, then  $P$  contains one of the ideals of family. Next the following important results are obtained.

Theorem: The following statements on  $R$  are equivalent:

- (i)  $R/\sqrt{0}$  is Noetherian regular ring;
- (ii)  $R$  is a zero dimensional compactly packed ring;
- (iii)  $R$  has Noetherian spectrum and  $\dim R = 0$ ;
- (iv)  $R$  is a zero dimensional  $Q$  – ring.

**Keywords.** Compactly packed rings, Regular rings,  $Q$  – ring, Strongly prime ideals, Strongly 0-dimensional rings.

**AMS 2010.** 13A15, 13A99.

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## Complete Semigroups of Binary Relations Defined by Semilattices of the Class $Z$ – Elementary $X$ – Semilattice of Unions

Yasha Diasamidze<sup>1</sup>, Shota Makharadze<sup>2</sup>, Neşet Aydın<sup>3</sup> and Ali Erdoğan<sup>4</sup>

**Abstract.** Let  $X$  be a nonempty set and  $D$  be an  $X$  – semilattice of unions that is a nonempty set of subsets of  $X$  closed under unions.  $D$  is said to be  $Z$ -elementary if  $D$  satisfies the following conditions:

- a)  $D$  is not a chain;
- b) every subchain of the semilattice  $D$  is finite;
- c) the set  $D_Z = \{T \in D \mid Z \subseteq T\}$  be a chain with smallest element  $Z$  ;
- d) the condition  $T \cup T' = Z$  is hold for any incomparable elements  $T$  and  $T'$  of  $D$  .

In this paper we investigate idempotent elements of  $Z$ -elementary  $X$ -semilattice of unions  $D$ . In case  $X$  is finite we derived formulas for the number of idempotent elements of the semigroup  $B_X(D)$ .

**Keywords.** semilattice, semigroup, binary relation.

**AMS 2010.** 05C38, 15A15, 05A15, 15A18.

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## Partial Metrics L-Valued Equalities and Relations Between Them

Zekiye Çiloğlu<sup>1</sup>

**Abstract.** In this work, I introduced GL-monoids and dual GL-monoids, changing the range of equivalence relations and metrics in the classical sense with GL-monoids and dual GL-monoids. This interchanging gave the definitions of L-Valued Equalities and V- partial Metrics. Also, I mentioned the categorical structure of them. Consequently, I generalized the studies which are carried out for V-partial metrics and L-valued Equalities to V-quasi Metrics and L-preordered Equalities, which are defined by losing symmetry axioms in the definition of L-valued Equalities and V-partial metrics.

**Keywords.** L-valued Equality, L-preordered Equality, V-pseudop Metric Spaces, V-pseudopq Metric Spaces.

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## The Orbit Problem of a Finitely Generated Lie Algebra

Zeynep Özkurt<sup>1</sup> and Naime Ekici<sup>2</sup>

**Abstract.** Let  $F_n$  be a free Lie Algebra of finite rank  $n \geq 2$ . We consider the orbit problem which is in the following form: Given an element  $u \in F_n$  a finitely generated subalgebra  $H$  of  $F_n$  does  $H$  contain  $\phi(u)$  for some automorphism  $\phi$  of  $F_n$ . Our main results state that this problem is decidable in the rank  $n$  free Lie algebra.

**Keywords.** Free Lie Algebra, Automorphisms, Orbit.

**AMS 2010.** 17B01,17B40.

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## Direct Limit of Parafree Lie Algebras

Zehra Velioglu<sup>1</sup> and Naime Ekici<sup>2</sup>

**Abstract.** We consider direct limits of parafree Lie algebras and we prove that the direct limit of any directed system of parafree Lie algebras exists in the variety of parafree Lie algebras.

**Keywords.** Parafree Lie algebras, Free Lie algebras, Direct Limit.

**AMS 2010.** 17B99.

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# *ANALYSIS*



## Generalized Durrmeyer Operators and Voronovskaya Type Results

Ali Aral<sup>1</sup> and Tuncer Acar<sup>2</sup>

**Abstract.** The concern of this paper is to introduce a new generalized Durrmeyer type operators which including some well-known operators in approximation theory by linear positive operators, inspiring from the Ibragimov-Gadjiev operators [Gadjiev, A.D., Ibragimov, I.I., On a sequence of linear positive operators, Soviet Math. Dokl. 11 (1970), 1092-1095.]. By this way, we not only combine most of well-known Durrmeyer operators but also present new type Durrmeyer operators. After fundamental concepts of approximation theory and construction of new Durrmeyer operators in first and second sections, we obtain local approximation result and Voronovskaya type results not only asymptotic formula but also in quantitative mean using the at least concave majorant of modulus of continuity. Hence, we obtain the degree of pointwise convergence in Voronovskaya formula. Moreover we furnish various applications to some classical Durrmeyer type operators

**Keywords.** Durrmeyer type operators, Voronovskaya theorem, Modulus of continuity.

**AMS 2010.** 41A36, 41A25

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## On $A$ -statistical Convergence with Speed

Ants Aasma<sup>1</sup>

**Abstract.** In [4] Kolk introduced the concept of  $A$ -statistical convergence for a non-negative regular matrix  $A$  and studied matrix transforms of  $A$ -statistically convergent sequences. The speed (or the rate) of convergence of statistically and  $A$ -statistically convergent sequences were introduced in various ways (see, for example, [1]-[2]). We choose another way to define the speed of convergence of  $A$ -statistically convergent sequences, applying the notion of convergence of sequences with speed (where the speed was defined by a monotonically increasing positive sequence  $\lambda$ ), introduced by Kangro in [3]. Also we investigate the matrix transforms of  $A$ -statistically convergent sequences with speed. Let  $X, Y$  be sequence spaces and  $(X, Y)$  the set of all matrices mapping  $X$  into  $Y$ . We describe the class of matrices  $(st_A^\lambda \cap X, Y)$ , where  $st_A^\lambda$  is the set of all  $A$ -statistically convergent sequences with speed  $\lambda$ . We show that in the special case, if  $\lambda$  is bounded, from our results follow some results of Kolk [4].

**Keywords.** Statistical convergence,  $A$ -density,  $A$ -statistical convergence with speed, convergence with speed, matrix transformations, regular matrices.

**AMS 2010.** 40A35, 40G15, 40C05.

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## Almost Statistical Convergence of Order $\beta$ of Sequences of Fuzzy Numbers

Abdulkadir Karakaş<sup>1</sup>, Yavuz Altın<sup>2</sup> and Hıfısı Altınok<sup>3</sup>

**Abstract.** In this article we introduce the concepts of almost  $\lambda$ - statistical convergence of order  $\beta$  and strongly almost  $\lambda_p$ - summability of order  $\beta$  for sequences of fuzzy numbers. Also, we establish some relations between the almost  $\lambda$ - statistical convergence of order  $\beta$  and strongly almost  $\lambda_p$ - summability of order  $\beta$  and we define  $\hat{w}_\lambda^\beta(\mathbf{F}, f, p)$ , where  $f$  is a modulus function and give some inclusion relations.

**Keywords.** Fuzzy number, Statistical convergence, Cesàro summability, Modulus function .

**AMS 2010.** 03E72, 40A05, 40D25

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## Fractional Order Derivative and Relationship between Derivative and Complex Functions

Ali Karci<sup>1</sup> and Ahmet Karadoğan<sup>2</sup>

**Abstract.** The concept of fractional order derivative can be found in wide range of many different areas. Due to this case, there are many methods about fractional order derivative (FOD). The most of them are Euler, Riemann-Liouville and Caputo which are fractional order derivatives as mentioned in the literature. However, they are not sound and complete for constant and identity functions. This case means that they are curve fitting or curve approximation methods.

FOD concept was defined in [10]. In this study, the concept of FOD and its relationships with complex functions was handled. Some knowledge about FOD is as follow.

**Definition 1:**  $f(x):R \rightarrow R$  is a function,  $\alpha \in R$  and the fractional order derivative can be considered as follows.

$$f^{(\alpha)}(x) = \lim_{h \rightarrow 0} \frac{f^\alpha(x+h) - f^\alpha(x)}{(x+h)^\alpha - x^\alpha}$$

In the case of very small value of  $h$ , the limit in the Definition 1 concluded in indefinite limit.

$$f^{(\alpha)}(x) = \lim_{h \rightarrow 0} \frac{f^\alpha(x+h) - f^\alpha(x)}{(x+h)^\alpha - x^\alpha} = \frac{0}{0}$$

In this case, the method used for indefinite limit (such as L'Hospital method) can be used, and the fractional order derivative can be redefined as follows.

**Definition 2:** Assume that  $f(x):R \rightarrow R$  is a function,  $\alpha \in R$  and  $L(.)$  be a L'Hospital process. The fractional order derivative of  $f(x)$  is

$$f^{(\alpha)}(x) = \lim_{h \rightarrow 0} L \left( \frac{f^\alpha(x+h) - f^\alpha(x)}{(x+h)^\alpha - x^\alpha} \right) = \lim_{h \rightarrow 0} \frac{\frac{d(f^\alpha(x+h) - f^\alpha(x))}{dh}}{\frac{d((x+h)^\alpha - x^\alpha)}{dh}}$$

The fractional order derivative definition can be demonstrated that it obtained same results as classical derivative definition for  $\alpha=1$ .

**Theorem .** Assume that  $f(x)$  is a function such as  $f:R \rightarrow R$  and  $\alpha \in R$ . then  $f^{(\alpha)}(x)$  is a function of complex variables.

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**Proof.** Assume that  $\alpha = \frac{\beta}{\delta}$  and  $\delta \neq 0$ . If  $f(x) \geq 0$ , the results obtained in Theorem 1,

Theorem 2, Theorem 3 and Theorem 4 are the cases. The fractional order derivative of  $f(x)$  is

$$f^{(\alpha)} = \frac{f'(x)f^{\alpha-1}(x)}{x^{\alpha-1}} = f'(x)^{\delta} \sqrt[\delta]{\frac{f^{\beta-\delta}(x)}{x^{\beta-\delta}}} = f'(x)^{\delta} \sqrt[\delta]{\left(\frac{f(x)}{x}\right)^{\beta-\delta}}$$

If the fractional derivative is a function of complex variables, then  $f^{(\alpha)}(x) = g(x) + ih(x)$  where  $i$  is the  $i = \sqrt{-1}$ .

If  $f(x) < 0$ , there will be two cases:

**Case 1:** Assume that  $\delta$  is odd.

$$\text{If } \left(\frac{f(x)}{x}\right)^{\beta-\delta} \geq 0 \text{ or } \left(\frac{f(x)}{x}\right)^{\beta-\delta} < 0,$$

then the obtained function  $f^{(\alpha)}(x)$  is a real function and  $h(x) = 0$  for both cases. Since the multiplication of any negative number in odd steps yields a negative number.

**Case 2:** Assume that  $\delta$  is even.

$$\text{If } \left(\frac{f(x)}{x}\right)^{\beta-\delta} \geq 0, \text{ then } h(x) = 0 \text{ and } f^{(\alpha)}(x) \text{ is a real function.}$$

$$\text{If } \left(\frac{f(x)}{x}\right)^{\beta-\delta} < 0,$$

then the multiplication of any number in even steps yields a positive number for real numbers. However, it yields a negative result for complex numbers, so,  $h(x) \neq 0$ . This means that  $f^{(\alpha)}(x)$  is a complex function. In fact,  $f^{(\alpha)}(x)$  is a complex function for both cases. The  $h(x) = 0$  for some situations.

**Keywords.** Fractional Order Derivative

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## Taylor's theorem for functions defined on Atanassov IF-sets

Alžbeta Michalíková<sup>1</sup>

**Abstract.** If sets were first time defined in the paper [1] and they represent the natural extension of the theory of fuzzy sets. In the paper [5] there was proved that Lagrange theorem holds for functions defined on IF-sets. Therefore it is natural question if it is possible to prove also Taylor's theorem for functions defined on IF-sets. To prove this theorem we defined polynomial function and Taylor's formula for functions defined on IF-sets. Then the Taylor's theorem is proved.

**Keywords.** IF-set, polynomial function, Taylor's formula, Taylor's theorem.

**AMS 2010.** 26E50, 03E72.

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## **Existence of Three Symmetric Positive Solutions for a Second-Order Multi-Point Boundary Value Problem on Time Scales**

Aycan Sinanoğlu<sup>1</sup>, İlkay Yaslan Karaca<sup>2</sup>, Fatma Tokmak<sup>3</sup> and Tuğba Şenlik<sup>4</sup>

**Abstract.** In this article, we establish symmetric positive solutions of second-order multi-point boundary value problem on time scales. The ideas involve Bai and Ge's fixed point theorem. As an application, we give an example to demonstrate our main result.

**Keywords.** boundary value problem, fixed point theorem, symmetric positive solution, time scales.

**AMS 2010.** 34B10, 34B18, 39A10.

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## Cone Two Normed Spaces

Ayse Sonmez<sup>1</sup>, Huseyin Cakalli<sup>2</sup> and Cigdem Gunduz Aras<sup>3</sup>

**Abstract.** A pair  $(X, d)$  is a cone two metric space where  $d : X \times X \times X \rightarrow E$  is called a cone two metric on  $X$  if it satisfies: for two distinct elements  $x$  and  $y$  in  $X$  there is an element  $z$  in  $X$  such that  $d(x, y, z) \neq 0$ ;  $d(x, y, z) = 0$  when two of the three elements are equal;  $d(x, y, z) = d(x, z, y) = d(y, z, x)$  and  $d(x, y, z) \leq d(x, y, w) + d(x, w, z) + d(w, y, z)$  for all  $x, y, z$  and  $w$  in  $X$ .

Let  $X$  be a real vector space of dimension  $d$ , where  $2 \leq d < \infty$  and  $E$  be a real Banach space.

A pair  $(X, \|\cdot, \cdot\|)$  is a cone two normed space where the mapping  $\|\cdot, \cdot\| : X \rightarrow E$  satisfies:

$$\forall x, y \in X, \forall \alpha \in R$$

$$\|x, y\| = 0 \Leftrightarrow x \text{ and } y \text{ are linearly dependent,}$$

$$\|x, y\| = \|y, x\|,$$

$$\|\alpha x, y\| = |\alpha| \|x, y\|, \alpha \in R,$$

$$\|x, y + z\| \leq \|x, y\| + \|x, z\|$$

In this paper, we investigate cone two metric spaces, cone two normed spaces and prove interesting theorems.

**Keywords.** Two normed space, two metric space.

**AMS 2010.** 40J05, 40G05, 46S20.

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## On Some Subspaces and F-Dual of Fk-Spaces

Asuman Ulu<sup>1</sup> and Bilal Altay<sup>2</sup>

**Abstract.** The concept of sectional convergence (AK), weak sectional convergence (SAK) and functional sectional convergence (FAK) in FK-spaces were investigated by Zeller [6]. Convergent and boundedness sections were studied by Garling [3,4] in more general topological sequence spaces.

In this study, we shall examine the properties of some distinguished subspaces and f-dual of sequence spaces along the lines of previous investigations by Buntinas [2], Grosse-Erdmann [5], Boos and Leiger [1] and many others on related sectional properties.

**Keywords.** FK-spaces; f-dual; AK, AB, SAK, FAK and AD property, .

**AMS 2010.** 46A45, 40C05.

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## Buck-Pollard Property for (C,1,1) Method

Cihan Orhan<sup>1</sup> and Emre Taş<sup>2</sup>

**Abstract.** In the present study we consider the Buck-Pollard property for double sequences. We show a double bounded sequence is (C,1,1) summable if and only if almost all of its subsequences are (C,1,1) summable.

**Keywords.** Double sequences, Pringshiem convergence, the Buck-Pollard Property, double subsequences.

**AMS 2010.** 40B05, 40C05.

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## Approximation by Complex $q$ -Baskakov Operators in Compact Disks

Dilek Söylemez<sup>1</sup>, Gülen Başcanbaz-Tunca<sup>2</sup> and Ali Aral<sup>3</sup>

**Abstract.** In this work, we assume that  $f$  is analytic on a disk  $|z| < R, R > 1$ , and has exponential growth in a compact disk with all derivatives are bounded in  $[0, \infty)$  by the same constant. For such  $f$  we consider complex form of  $q$ -Baskakov operators  $W_n^q(f)(z)$  for  $q > 1$  and obtain quantitative estimates for simultaneous approximation, Voronovskaja-type result and degree of simultaneous approximation in the compact disks.  $q$ -derivative plays essential role in the work.

**Keywords.**  $q$ -Baskakov operator, complex approximation, Voronovskaja-type result, exact degree of approximation

**AMS 2010.** 30E10, 41A36.

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## **Of Non-Linear Modelling Steel I Beams Strengthened with Hm-Cfrp**

Elif Ağcakoca<sup>1</sup>, Yusuf Sümer<sup>2</sup> and Zeynep Yaman<sup>3</sup>

**Abstract.** There are two options to solve problems of steel bridges, which are suffering from loss in cross section due to corrosion or those whose cross sections are insufficient as a result of increasing traffic loadings: either to change the beam with a new one or to strengthen the beam. New technology materials such as Fiber Reinforced Polymers (FRP) are preferred in strengthening processes as these materials are resistive against corrosion, thin and light, i.e., they do not cause a significant increase in the weight of the system, and good for preventing the fatigue cracks. Laboratory testing is limited and took long times considering the member sizes, establishment of the experimental apparatus, support conditions, loading etc. and it is time consuming. Composite behavior of three different materials such as steel, epoxy and fiber reinforced polymers can be provided by non-linear finite element analysis. Therefore such elements can be modeled numerically and further parametric studies can be carried out more easily.

This study discussed the stages of numerical modeling and aims to compare tested laboratory studies with the nonlinear finite element models. ABAQUS finite elements program was used in the study and non-linear analyses were conducted in terms of material and geometry. It is shown that a good agreement exist both load deflection graphics and failure modes of test beams.

**Keywords.** HM-CFRP, Finite elements, ABAQUS

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## Convergence Properties of Ibragimov-Gadjiev-Durrmeyer Operators

Emre Deniz<sup>1</sup> and Ali Aral<sup>2</sup>

**Abstract.** The purpose of the present paper is to study the local and global direct approximation properties of the Durrmeyer type generalization of Ibragimov-Gadjiev operators defined in [1] which, we show that, are an approximation process in polynomial weighted space of continuous functions on the interval  $[0, \infty)$ . The results obtained in this study consist of Korovkin type theorem which enables us to approximate a function uniformly by new Durrmeyer operators, and estimates for approximation error of the operators in terms of weighted modulus of smoothness. The all afore mentioned results are obtained for the functions belongs to weighted space with polynomial weighted norm by new operators acts on functions defined on the non compact interval  $[0, \infty)$ . We finally discuss another weighted approximation formula not including any Korovkin type theorem but has a direct proof.

**Keywords.** Durrmeyer operators, Ibragimov-Gadjiev operators, Korovkin theorem, rate of convergence, modulus of continuity, weighted approximation.

**AMS 2010.** 41A36, 41A25.

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## Levelwise $\lambda$ – Statistical Cluster and Limit Points of a Sequence of Fuzzy Numbers

Fatma Berna Benli<sup>1</sup>

**Abstract.** Steinhaus [1] and Fast, [2] defined statistical convergence of a sequence. Then, Buck [3] generalized statistical convergence. Bounded and convergent sequences of fuzzy numbers was introduced by Matloka [4]. Nuray and Savas [5] extended and also discussed the concepts of statistically convergent and statistically Cauchy sequences of fuzzy numbers. Then, Tuncer and Benli [6] have introduced  $\lambda$ -statistically Cauchy sequences of fuzzy numbers. Also Aytar and Pehlivan [7] defined levelwise statistical cluster and limit points of a sequence of fuzzy numbers.

In this paper we will introduce the concept of levelwise  $\lambda$  – statistical cluster and limit points of a sequence of fuzzy numbers.

**Keywords.** Fuzzy number,  $\lambda$  – statistical convergence,  $\lambda$  – statistical cluster points,  $\lambda$  – statistical limit points, levelwise  $\lambda$  – statistical convergence.

**AMS 2010.** 03E72, 40A35.

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## Relations Between $\lambda$ -Statistical Limit Inferior and Limit Superior for Sequence of Fuzzy Numbers

Fatma Berna Benli<sup>1</sup>

**Abstract.** Fast ([1]) defined statistical convergence of a sequence. Matloka [2] introduced bounded and convergent sequences of fuzzy numbers. Nuray and Savas [3] extended the concepts of statistically convergent and statistically Cauchy sequences of fuzzy numbers. Then, Tuncer and Benli [4] have introduced  $\lambda$ -statistically Cauchy sequences of fuzzy numbers. Also sup and inf notions have been given only for bounded sets of fuzzy numbers ([5],[6]). Then in [7] Aytar, Mammadov and Pehlivan have introduced the concepts of statistical limit inferior and limit superior for statistically bounded sequences of fuzzy numbers. Recently, Benli [8] extended  $\lambda$ -statistical limit inferior and limit superior for  $\lambda$ -statistically bounded sequences of fuzzy numbers.

In this paper we will discuss some relations between  $\lambda$ -statistical limit inferior and limit superior for  $\lambda$ -statistically bounded sequences of fuzzy numbers.

**Keywords.** Fuzzy number,  $\lambda$ -statistical limit inferior,  $\lambda$ -statistical limit superior.

**AMS 2010.** 03E72, 40A35.

**Acknowledgement.** This work is supported by the Scientific Research Center at the Erciyes University with the project code FBA-12-4137.

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## Convergence and Stability Results for Some New Kirk Type Hybrid Fixed Point Iterative Algorithms

Faik Gürsoy<sup>1</sup> and Vatan Karakaya<sup>2</sup>

**Abstract.** In this presentation we introduce Kirk multistep-SP and Kirk-S iterative algorithms and we prove some convergence and stability results for these iterative algorithms. Since these iterative algorithms are more general than some other iterative algorithms in the existing literature, our results generalize and unify some other results in the literature.

**Keywords.** Kirk multistep-SP and Kirk-S iterative algorithms, Convergence, Stability.

**AMS 2010.** 53A40, 20M15.

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## **The Existence of Symmetric Positive Solutions for a Nonlinear Multi-Point Boundary Value Problem on Time Scales**

Fatma Tokmak<sup>1</sup>, Ilkay Yaslan Karaca<sup>2</sup>, Tugba Senlik<sup>3</sup>, Aycan Sinanoglu<sup>4</sup>

**Abstract.** In this paper, we study the existence of symmetric solutions for the nonlinear multi-point boundary value problem on time scales. By applying fixed-point index theorem, the existence of at least two positive solutions is obtained. An example is given to illustrate our main result.

**Keywords.** boundary value problem, fixed point theorem, symmetric positive solution, time scales.

**AMS 2010.** 34B10, 34B18, 39A10.

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## Approximation Properties of Ibragimov-Gadjiev Durrmeyer Operators on $L_p[0, \infty)$

Gülsüm Ulusoy<sup>1</sup> and Ali Aral<sup>2</sup>

**Abstract.** As a continuation of recent paper given in [1],  $L_p$  approximation behaviour of generalized Durrmeyer operators, called Ibragimov-Gadjiev Durrmeyer operators, on  $L_p[0, \infty)$  spaces for  $1 \leq p \leq \infty$  is investigated. In particular, rate of convergence and weighted norm convergence are presented for new Durrmeyer operators. After we show that Ibragimov-Gadjiev Durrmeyer operators are an approximation process on  $L_p[0, \infty)$  spaces for  $1 \leq p \leq \infty$ , rate convergence of these operators is obtained by Ditzian-Totik modulus of smoothness and its equivalence to appropriate K-functionals. Furthermore, weighted norm convergence which has the proof based on Korovkin type theorem on  $L_p[0, \infty)$  spaces for  $1 \leq p \leq \infty$  is given.

**Keywords.** Durrmeyer type generalization of Ibragimov-Gadjiev operators, modulus of continuity, weighted approximation.

**AMS 2010.** 41A36, 41A25.

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## Generalized Statistical Boundedness in Sequences of Fuzzy Numbers

Hıfı Altınok<sup>1</sup> and Damla Yağdıran<sup>2</sup>

**Abstract.** In this study, we generalize the concept of statistical boundedness for sequences of fuzzy numbers and define  $\Delta^m$ -statistical boundedness and examine some properties of it.

**Keywords.** Fuzzy numbers, Statistical convergence, Statistical boundedness, Difference sequence

**AMS 2010.** 40A05, 40C05, 46A45, 03E72

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## A Study on Sets of Filter Cluster Functions

Hüseyin Albayrak<sup>1</sup> and Serpil Pehlivan<sup>2</sup>

**Abstract.** In [1], we generalized the concepts of pointwise convergence, uniform convergence and  $\alpha$ -convergence for sequences of functions on metric spaces by using the filters on  $\mathbb{N}$ . Then, in [2], we defined the concepts of limit function,  $\mathcal{F}$ -limit function and  $\mathcal{F}$ -cluster function respectively for each of these three types of convergence, where  $\mathcal{F}$  is a filter on  $\mathbb{N}$ . In this work, we investigate some topological properties of the sets of  $\mathcal{F}$ -pointwise cluster functions and  $\mathcal{F}$ -uniform cluster functions by using pointwise and uniform convergence topologies.

**Keywords.**  $\mathcal{F}$ -uniform cluster function,  $\mathcal{F}$ -pointwise cluster function.

**AMS 2010.** 40A35, 40A30, 54A20.

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## Approximation in Generalized Lebesgue Spaces $L^{p(x)}$

Hatice Aslan<sup>1</sup> and Ali Guven<sup>2</sup>

**Abstract.** In this study, analogues of results of B. Szal ([4]) are obtained in generalized Lebesgue spaces  $L^{p(x)}$ .

**Keywords.** Fourier series, Matrix transformation, generalized Lebesgue space.

**2010 AMS.** 41A25, 42A10, 46E30.

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### $\lambda$ -Statistically Ward Continuity

Huseyin Cakalli<sup>1</sup>, Ayse Sonmez<sup>2</sup> and Cigdem Gunduz Aras<sup>3</sup>

**Abstract.** A sequence  $(\alpha_k)$  of points in  $\mathbf{R}$  is called to be  $\lambda$ -statistically quasi-Cauchy if  $(\Delta\alpha_n)$  is  $\lambda$ -statistically convergent to 0, i.e.

$$\lim_{n \rightarrow \infty} \frac{1}{\lambda_n} |\{k \in I_n : |\alpha_k - \alpha_{k+1}| \geq \epsilon\}| = 0$$

where  $I_n = [n - \lambda_n + 1, n]$  and  $\lambda = (\lambda_n)$  be a non-decreasing sequence of positive numbers tending to  $\infty$  such that  $\lambda_{n+1} \leq \lambda_n + 1$ ,  $\lambda_1 = 1$ . The main object of this paper is to introduce, and investigate  $\lambda$ -statistical quasi-Cauchy sequences. It turns out that the set of uniformly continuous functions coincides with the set of  $\lambda$ -statistically ward continuous functions on a  $\lambda$ -statistically ward compact subset of  $\mathbf{R}$ .

**Keywords.** Continuity, sequences, compactness.

**AMS 2010.** 40A05, 26A15, 40A30.

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## Up and Down Continuities

Huseyin Cakalli<sup>1</sup>

**Abstract.** A function  $f$  is up continuous if it preserves upward half Cauchy sequences, is down continuous if it preserves downward half Cauchy sequences, and a subset  $E$  of the set of real numbers,  $\mathbb{R}$ , is up half compact if any sequence of points in  $E$  has an upward half Cauchy subsequence, is down half compact if any sequence of points in  $E$  has a downward half Cauchy subsequence. We investigate up continuity, down continuity, up half compactness, down half compactness and prove related theorems.

**Keywords.** Continuity, sequences, compactness.

**AMS 2010.** 40A05, 26A15, 40A30.

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### More on the Borwein-Ditor Theorem

Harry I. Miller<sup>1</sup> and Leila Miller-Van Wieren<sup>2</sup>

**Abstract.** D. Borwein and S.Z. Ditor have proved the following theorem, answering a question of P. Erdos.

**Theorem** (Borwein, Ditor 1978)

(1) If  $A$  is a measurable set in  $R$  with  $m(A) > 0$  and  $(d_n)$  is a sequence of reals converging to 0, then for almost all  $x \in A$ ,  $x + d_n \in A$  for infinitely many  $n$ .

(2) There exists a measurable set  $A$  in  $R$  with  $m(A) > 0$ , and a (decreasing) sequence  $(d_n)$  converging to 0, such that, for each  $x + d_n$  is not in  $A$  for infinitely many  $n$ .

Here we prove some new related results:

**Theorem 1.** Suppose  $f : R \times R \rightarrow R$  is continuous and satisfies:

(1) there exists  $e \in R$  such that  $f(x, e) = x$  for all  $x \in R$ ,

(2) there exist  $x_1, y_1 \in R_2$ ,  $x_0 \in R$ ,  $x_1 > x_0$ ,  $y_1 > e$ , such that the partial derivatives  $f_x, f_y$  exist and are continuous on  $T$  the closed rectangle with corners  $(x_0, e), (x_0, y_1), (x_1, e)$  and  $(x_1, y_1)$ ,

(3) there exist  $a, b \in R$ , with  $a, b > 0, b > 1$  and  $a < f_x, f_y < b$  on  $T$ .

If  $E \subset [x_0, x_1]$  is an arbitrary nowhere dense set, then there exists  $(e_n)$ , monotonically converging to  $e$  such that  $f(x, e_n)$  is not in  $E$  infinitely often for each  $x$  in  $R$ .

**Theorem 2.** Suppose  $f : R \times R \rightarrow R$  is continuous and satisfies:

(1) there exists  $e \in R$  such that  $f(x, e) = x$  for all  $x \in R$ ,

(2) there exist  $x_1, y_1 \in R_2$ ,  $x_0 \in R$ ,  $x_1 > x_0$ ,  $y_1 > e$ , such that the partial derivatives  $f_x, f_y$  exist and are continuous on  $T$  the closed rectangle with corners  $(x_0, e), (x_0, y_1), (x_1, e)$ , and  $(x_1, y_1)$ ,

(3) there exist  $a, b \in R$ , with  $a, b > 0, b > 1$  and  $a < f_x, f_y < b$  on  $T$ .

Suppose  $A \subset [x_0, x_1]$  is such that  $[x_0, x_1] \setminus A$  is of first category, and  $(e_n)$ , converges to  $e$ .

Then there exists  $x$  in  $A$ ,  $m$  in  $N$ , such that  $f(x, e) \in A$  for  $n > m$ .

**Keywords.** Nowhere dense, measurable, category

**AMS 2010.** 28A05, 26A21.

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## Partial Sums of Generalized Bessel Functions

Halit Orhan<sup>1</sup> and Nihat Yağmur<sup>2</sup>

**Abstract.** Let  $(g_{p,b,c})_n(z) = z + \sum_{m=1}^n b_m z^{m+1}$  be the sequence of partial sums of generalized and normalized Bessel functions  $g_{p,b,c}(z) = z + \sum_{m=1}^{\infty} b_m z^{m+1}$  where  $b_m = \frac{(-c/4)^m}{m!(\kappa)_m}$  and  $\kappa = p + (b+1)/2 \neq 0, -1, -2, \dots$ . The purpose of the present paper is to determine lower

bounds for  $\operatorname{Re} \left\{ \frac{g_{p,b,c}(z)}{(g_{p,b,c})_n(z)} \right\}$ ,  $\operatorname{Re} \left\{ \frac{(g_{p,b,c})_n(z)}{g_{p,b,c}(z)} \right\}$ ,  $\operatorname{Re} \left\{ \frac{g'_{p,b,c}(z)}{(g_{p,b,c})'_n(z)} \right\}$  and  $\operatorname{Re} \left\{ \frac{(g_{p,b,c})'_n(z)}{g'_{p,b,c}(z)} \right\}$

Further we give lower bounds for  $\operatorname{Re} \left\{ \frac{A[g_{p,b,c}](z)}{A[(g_{p,b,c})_n](z)} \right\}$  and  $\operatorname{Re} \left\{ \frac{A[(g_{p,b,c})_n](z)}{A[g_{p,b,c}](z)} \right\}$  where

$A[g_{p,b,c}](z)$  is the Alexander transform of  $g_{p,b,c}$ .

**Keywords.** Partial sums, Analytic functions, Generalized Bessel functions, Bessel, modified Bessel and spherical Bessel functions.

**AMS 2010.** 30C45, 33C10.

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## On Double Hausdorff Summability Method

Hamdullah Şevli<sup>1</sup> and Rabia Savaş<sup>2</sup>

**Abstract.** In [1], Das proved that every conservative Hausdorff matrix is absolutely  $k$ th power conservative. Savaş and Rhoades [2] proved the result of Das for double Hausdorff summability. In this presentation we will consider the double Endl- Jakimovski (E-J) generalization and we will prove the corresponding result of [3] for double E-J generalized Hausdorff matrices.

**Keywords.** Hausdorff matrices, double series, absolute summability.

**AMS 2010.** 40F05, 40G05.

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## Fixed Point Problems Of Picard-Mann Hybrid Method Of Continuous Functions on an Arbitrary Interval

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**Abstract.** In this paper, we consider the iteration method called "Picard-Mann Hybrid method " for finding a fixed point of continuous functions on an arbitrary interval. We give a necessary and sufficient condition for the convergence of this iteration of continuous functions on an arbitrary interval. Also, we compare the convergence speed of Picard-Mann Hybrid method with the other iterations and prove that it converges faster than the others. Moreover, we support our proofs by numeric examples.

**Keywords.** Fixed Point, Continuous function, Arbitrary interval, Convergence Theorem

**MSC 2010.** 26A18, 47H10, 54C05.

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## On the Existence of the Solutions for Some Nonlinear Volterra Integral Equations

İsmet Özdemir<sup>1</sup>, Ümit Çakan<sup>2</sup> and Bekir İlhan<sup>3</sup>

**Abstract.** We present a theorem which gives sufficient conditions for existence of at least one solution for some nonlinear functional integral equations in the space of continuous functions on the interval  $[0, a]$ . To do this, we will use Darbo's fixed-point theorem associated with the measure of noncompactness. We give also an example satisfying the conditions of our main theorem but not satisfying the conditions described by Maleknejad et al. in [5].

**Key words.** Nonlinear integral equations, Measure of noncompactness, Darbo fixed point theorem.

**AMS 2010.** 45M99, 47H09.

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## Reciprocal Continuity and Common Fixed Points of Non-Self Mappings

Javid Ali<sup>1</sup> and M. Imdad<sup>2</sup>

**Abstract.** We extend the notions of reciprocal continuity and Cq-commutativity to non-self setting besides observing equivalence between compatibility and  $\varphi$ -compatibility, and utilize the same to obtain some results on coincidence and common fixed points for two pairs of non-self mappings in metrically convex metric spaces. As an application of our main result, we also prove a common fixed point theorem in Banach spaces besides furnishing several illustrative examples.

**Keywords.** Fixed point, metrically convex metric space and reciprocal continuity.

**AMS 2010.** 47H10, 54H25.

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## Time Scale Integral Inequalities for Superquadratic Functions

Josipa Barić<sup>1</sup>, Rabia Bibi<sup>2</sup>, Martin Bohner<sup>3</sup> and Josip Pečarić<sup>4</sup>

**Abstract.** Two different methods (based on [1] and [2]) of proving Jensen's inequality on time scales for superquadratic functions are demonstrated. Some refinements of classical inequalities on time scales are obtained using properties of superquadratic functions and some known results for isotonic linear functionals.

**Keywords.** Time scales, superquadratic functions, Jensen's inequality, Holder's inequality, Minkowski's inequality, Jessen-Mercer's inequality, Slater's inequality, Hermite-Hadamard's inequality.

**AMS 2010.** 26D15, 26D20, 26D99, 34N05.

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## On E.A Like Property and Fixed Point Results in Dislocated and Dislocated Quasi- Metric Spaces

Kastriot Zoto<sup>1</sup>

**Abstract.** In this paper we establish several fixed points theorems for weakly compatible selfmappings on a dislocated and dislocated quasi-metric space, which satisfy E. A Like and common E. A. Like property, satisfying a liner type of contraction.

**Keywords:** dislocated metric, dislocated quasi-metric, common E.A Like property, fixed point.

**AMS. Subject classification:** 47H10, 54H25, 55M20.

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## On Exact Oscillatory Property of Sturm-Liouville Equation with Alternating-sign Potential

L. Kussainova<sup>1</sup>

**Abstract.** Let us consider equations

$$-y'' - u(x)y = 0 \quad (x > 0), \quad (1)$$

$$-y'' + v(x)y - u(x)y = 0 \quad (x > 0) \quad (2)$$

with continuous non-negative functions  $v$  and  $u$  defined on  $I = [0, \infty)$ . For the equation (1) the following oscillation condition is well known:

$$\limsup_{x \rightarrow \infty} x \int_x^\infty u(t) dt > 1. \quad (3)$$

See [1]. In [2] for (1) another oscillation condition was proved:

$$\liminf_{x \rightarrow \infty} A(x | u) > 1,$$

where

$$A(x | u) = x^{-1} \int_x^\infty t^2 u(t) dt + x \int_x^\infty u(t) dt.$$

We are interested in oscillatory properties of equation (2) at infinity. Let  $H$  be a set of all positive continuous functions  $h(x)$  ( $x > 0$ ) such that

$$\liminf_{x \rightarrow \infty} x^{-1} h(x) \geq 1.$$

**Theorem.** Let for one of  $h \in H$  the condition

$$\limsup_{x \rightarrow \infty} x \int_x^{x+h(x)} (u - v) dt > 1 + \limsup_{x \rightarrow \infty} A(x | v) \quad (4)$$

holds. Then equation (1) is oscillatory.

**Remark.** Then condition (4) in case  $v = 0$  is equal to (3) for all  $u$  such that

$$\limsup_{x \rightarrow \infty} x \int_{h(x)}^\infty u(t) dt = 0, \text{ where function } h \text{ is constructed by } u \text{ only.}$$

**Keywords.** Oscillatory Property, Ordinary Differential Equation.

**AMS 2010.** 53A40, 20M15.

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## On Composition Identities for the Caputo Fractional Derivatives with Applications to Opial-type Inequalities

Maja Andrić<sup>1</sup>, Josip Pečarić<sup>2</sup> and Ivan Perić<sup>3</sup>

**Abstract.** Using the Laplace transform we prove that some conditions in composition identities for the left-sided and right-sided Caputo fractional derivatives can be relaxed. As an application, an Opial-type inequality involving the Caputo fractional derivatives is presented.

**Keywords.** Caputo fractional derivative, Opial-type inequality, Laplace transform.

**AMS 2010.** 26A33, 26D15.

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## Some New Sequence Spaces Derived From The Spaces of Bounded, Convergent and Null Sequences

Murat Candan<sup>1</sup>

**Abstract.** In this article, we introduce the new paranormed sequence spaces  $Z(p, A_\lambda)$  consisting of all sequences whose generalized weighted mean  $A_\lambda$  transforms are in the linear space  $Z(p)$ , where  $Z(p)$  was defined by Maddox [Quart. J. Math. Oxford [1], 18 (1967), 345--355] and  $Z$  denotes one of the classical sequence spaces  $l_\infty$ ,  $c$  or  $c_0$ . Meanwhile, we have also presented the Schauder basis of  $c_0(p, A_\lambda)$  and  $c(p, A_\lambda)$  and computed its  $\beta$ - and  $\gamma$ -duals. In addition to this, the fact that sequence space  $\{c_0\}_{A_\lambda}$  has  $AD$  property is shown and then the  $f$ -dual of the space  $\{c_0\}_{A_\lambda}$  presented. In conclusion, we characterize the classes of matrix mappings from the sequence spaces  $Z(p, A_\lambda)$  to the sequence space  $\mu$  and from the sequence space  $\mu$  to the sequence spaces  $Z(p, A_\lambda)$ .

**Keywords.** Paranormed sequence space, alpha-, beta- and gamma-duals and matrix mappings.

**AMS 2010.** 46A45, 40C05.

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## A New Sequence Space Isomorphic to the Space $l(p)$ and Compact Operators

Murat Candan<sup>1</sup>

**Abstract.** As a major issue in this work, we present the paranormed sequence space  $l(u, v, p; \tilde{B})$  consisting of all sequences whose  $\tilde{R}$ -transforms are in the linear space  $l(p)$  introduced by Maddox [Quart. J. Math. Oxford [1], 18 (1967), 345--355], where  $\tilde{B} = B(\tilde{r}, \tilde{s})$  denotes double sequential band matrix provided that  $(r_n)_{n=0}^{\infty}$  and  $(s_n)_{n=0}^{\infty}$  are given convergent sequences of positive real numbers. For this purpose, we have used the generalized weighted mean  $G$  and double sequential band matrix  $\tilde{B}$ . Meanwhile, we have also presented the basis of this space and computed its  $\alpha$ - ,  $\beta$ - and  $\gamma$ -duals. Then, we have characterized the classes of matrix mappings from  $l(u, v, p; \tilde{B})$  to  $l_{\infty}$  ,  $c$  and  $c_0$ . In conclusion, in order to characterize some classes of compact operators given by matrices on the space  $l_p(u, v, \tilde{B})$  ( $1 \leq p < \infty$ ), we have applied the Hausdorff measure of noncompactness.

**Keywords.** Paranormed sequence space,  $\tilde{B} = B(r_n, s_n)$  double sequential band matrix, Weighted mean,  $\alpha$ - ,  $\beta$ - and  $\gamma$ - duals, Matrix mappings, Hausdorff measure of noncompactness, Compact operators

**AMS 2010.** 46A45, 40C05.

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## Univalence Criteria and Quasiconformal Extensions

Murat Çağlar<sup>1</sup> and Halit Orhan<sup>2</sup>

**Abstract.** In the present paper, we obtain a more general conditions for univalence of analytic functions in the open unit disk  $U$ . Also, we obtain a refinement to a quasiconformal extension criterion of the main result.

**Keywords.** Analytic function, univalence condition, Loewner or subordination chain, quasiconformal extension.

**AMS 2010.** 30C45, 30C55.

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## Approximate Fixed Point Property in Intuitionistic Fuzzy Normed Space

Müzeyyen Ertürk<sup>1</sup> and Vatan Karakaya<sup>2</sup>

**Abstract.** In this presentation, we study approximate fixed point property and investigate approximate fixed point property of some operators in intuitionistic fuzzy normed space.

**Keywords.** Approximate fixed point property, intuitionistic fuzzy normed space.

**AMS 2010.** 47H10; 46S40.

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## On Pointwise Lacunary Statistical Convergence of Order $\alpha$ of Sequences of Function

Mikail Et<sup>1</sup> and Hacer Şengül<sup>2</sup>

**Abstract.** In this study, we introduce the notion of pointwise lacunary statistical convergence of order  $\alpha$  of sequences of real-valued functions. Also some relations between  $\alpha$   $S_\theta^\alpha(f)$ -statistical convergence and strong  $w_p^\alpha(f, \theta)$ -summability are given. Also some relations between the spaces  $S_\theta^\alpha(f)$  and  $w_p^\alpha(f, \theta)$  are examined.

**Key words.** Statistical convergence, sequences of functions, Cesàro summability.

**AMS 2010.** 40A05, 40C05, 46A45.

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## Some properties of the sequence space $B\widehat{V}_\theta(f, p, q, s)$

Mahmut Işık<sup>1</sup>

**Abstract.** In this paper we define the sequence space  $B\widehat{V}_\theta(f, p, q, s)$  on a seminormed complex linear space, by using a modulus function. We give various properties and some inclusion relations on this space.

**Keywords.** Modulus function, sequence spaces

**AMS 2010.** 40A05, 40C05, 40D05

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## The New Almost Sequence Space

Murat Kirişci<sup>1</sup>

**Abstract.** Hahn [6] defined the space  $h$  and gave some general properties. Goes and Goes[5] studied the functional analytic properties of this space. The study of Hahn sequence space was initiated by Rao[11] with certain specific purpose of Banach space theory. Rao and Srinivasalu[12] introduced a new class of sequence space called semi replete space. Rao and Subramanian[13] defined the semi Hahn space and proved that the intersection of all semi Hahn spaces is Hahn space. Balasubramanian and Pandiarani[1] defined the new sequence space  $h(F)$  called Hahn sequence space of fuzzy numbers and proved that beta and gamma duals of  $h(F)$  is the Cesaro space of the set of all fuzzy bounded sequences. Matrix domain of Hahn sequence space  $h(C)$  determined by Cesaro mean order one gave and some inclusion relations and some topological properties investigated by Kirisci[7]. Dual spaces of the space  $h(C)$  are computed and matrix transformations are characterized in [7]. Also Kirisci[8] defined the  $p$ -Hahn sequence space.

In this paper, we investigate new sequence space with the concept of almost convergence. The main purpose of this paper is to introduce to the new almost sequence space and to determine the beta and gamma duals of this space. Furthermore, some classes of matrix mappings on/in the new space are characterized.

**Keywords.** Hahn sequence space, matrix domain, dual spaces, matrix transformations.

**AMS 2010.** 46A45, 46A35.

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## Common Fixed Point Theorems for a Sequence of Mappings with Different Type of Continuity

Mahpeyker Öztürk<sup>1</sup>

**Abstract.** In the present paper, we establish some common fixed point theorems for a sequence of mappings  $\{S_n\}_{n=0}^{\infty}$  and a self mapping  $T$  by using different type of continuity. Some results which are extension and generalization of literature also have been given.

**Keywords.** Reciprocal Continuity, Fixed Point, Semi-Compatibility, Partially Order.

**AMS 2010.** 47H10, 54H25.

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## Extension of Generalized Kannan Type Contraction in Complete Partially Ordered Metric Space

Mahpeyker Öztürk<sup>1</sup> and Işıl Arda Kösal<sup>2</sup>

**Abstract.** The purpose of this paper to extend the generalised Kannan type contraction and to present some fixed point theorems in complete partially ordered metric space. Also as application of the theorems we give a unique solution for first-order ordinary differential equation.

**Keywords.** Kannan type mapping, partially ordered set, fixed point.

**AMS 2010.** 47H10, 54H25.

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## Inverse Problems for Sturm-Liouville Differential Operators with Constant Delay

Milenko Pikula<sup>1</sup> and Vladimir Vladićić<sup>2</sup>,

**Abstract.** Inverse problems in the spectral theory of operators, especially differential operators, have been studied since the 1930s until now.

Inverse problems classical Sturm-Liouville operators is fully solved. In the problems with delay the main methods classical problem (transformation operator method, method of spectral mappings and others) do not give solutions. For these reasons inverse problems second order differential operators with delay is not solved until today.

In this work paper we study non-self-adjoint second order differential operators with constant delay  $L$  generated with

$$-y''(x) + q(x)y(x - \tau) = \lambda y(x) , \quad x \in (0, \pi) \quad (1)$$

$$y(x) = 0, \quad x \in [0, \tau] \quad (2)$$

$$y(\pi) = 0 \quad (3)$$

where  $\lambda$  is spectral parameter,  $\tau \in (0, \pi)$  is delay,  $q(x)$  is real-valued function,  $q \in L^2(0, \pi)$ .

We establish properties of the spectral characteristics and investigate the inverse problem of recovering operators from their spectra. We construct potential and delay for this operators.

More precisely, it was shown that the spectrum of the operator  $L$  is countable. We construct the characteristic function  $F$  of the operator  $L$ .

Let  $(\lambda_n)_{n=1}^{\infty}$  is the spectrum of the operators  $L$ . Using the properties of the function  $F$  we find Fourier's coefficients of  $q(x)$  and  $\tau$  from  $(\lambda_n)_{n=1}^{\infty}$ . It is shown that the delay is uniquely determined. It also shows that the series  $(\lambda_n)_{n=1}^{\infty}$  correspond to two functions  $q(x)$  and  $q(\pi - x)$ .

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## A-Distributional Convergence in Topological Spaces

Mehmet Ünver<sup>1</sup>

**Abstract.** (This talk is based on a joint research with M.K.Khan and C.Orhan) In this talk we present a new concept of A-distributional convergence in an arbitrary Hausdorff topological space which is equivalent to A-statistical convergence for a degenerate distribution function. We investigate A-distributional convergence as a summability method in an arbitrary Hausdorff topological space. We also study the summability of spliced sequences, in particular, for metric spaces and give the Bochner integral representation of A-limits of the spliced sequences for Banach spaces.

**Keywords.** A-distributional convergence, A-statistical convergence , Hausdorff spaces, Matrix summability.

**AMS 2010.** 54A20 , 40J05, 40A05, 40G15, 11B05.

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## Monotonicity Properties of $q$ -Digamma and $q$ -Trigamma Functions

Necdet Batır<sup>1</sup>

**Abstract.** We extend monotonicity properties of some functions involving  $q$ -digamma and  $q$ -trigamma functions proved in [8] to complete monotonicity

**Keywords.**  $q$ -digamma function,  $q$ -psi function,  $q$ -gamma function,  $q$ -extensions.

**AMS 2010.** 33B15, 26D15.

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## Asymptotic Properties of Rapidly Varying Functions

Nebojša Elez<sup>1</sup> and Vladimir Vladićić<sup>2</sup>

**Abstract.** A measurable function  $f : [a, \infty) \rightarrow (0, \infty)$  ( $a > 0$ ) is called rapidly varying in the sense of de Haan with the index of variability  $+\infty$  if it satisfies the following condition

$$\lim_{x \rightarrow \infty} \frac{f(\lambda x)}{f(x)} = +\infty$$

for every  $\lambda > 1$ . This functional class is denoted by  $R_\infty$ .

In this paper we present the asymptotic properties of rapidly varying function analogous to classical inequalities such as Jensen's inequality and Chebyshev's inequality.

Specifically, we give conditions for  $f, g \in R_\infty$  under which holds

1.  $f(x_1) + f(x_2) + \dots + f(x_n) = o(f(x_1 + x_2 + \dots + x_n)), (n \rightarrow \infty)$

2.  $f\left(\frac{1}{x} \int_a^x \varphi(t) dt\right) = o\left(\frac{1}{x} \int_a^x f(\varphi(t)) dt\right), (x \rightarrow \infty)$

3.  $\frac{1}{x} \int_a^x f(t) dt \int_a^x g(t) dt = o\left(\int_a^x f(t) g(t) dt\right), (x \rightarrow \infty)$

where  $o$  is the Landau symbol.

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## On Some Improvements of Hölder's Inequality

Neda Lovričević<sup>1</sup>, Mario Krnić<sup>2</sup> and Josip Pečarić<sup>3</sup>

**Abstract.** Hölder's inequality, expressed in terms of positive linear functionals acting on the linear space of real-valued functions is analysed in view of superadditivity and monotonicity properties of the Jensen-type functionals, on the set of nonnegative (weighted) functions. Obtained non-weighted bounds for these functionals resulted in new refinements and converses of Hölder's inequality.

**Keywords.** Hölder's inequality, Jensen's inequality, Jessen's and McShane's functionals.

**AMS 2010.** 26D15, 26A51.

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## Continuity of the modulus of noncompact convexity

Nermin Okičić<sup>1</sup> and Amra Rekić-Vuković<sup>2</sup>

**Abstract.** Normal structure plays an essential role in some problems of metric fixed point theory. In 1960s Kirk proved that every (singlevalued) nonexpansive mapping  $T: C \rightarrow C$ , defined from convex closed bounded subset  $C$  of reflexive Banach space with normal structure, has a fixed point. That was a starting point for many researchers to try to exploit connection between geometric theory of Banach spaces and fixed point theory. So they considered geometrical properties (uniform smoothness, Opial property, nearly uniform convexity, etc.) that could imply normal structure. One of the tools that provides classification of Banach spaces considering their geometrical properties is modulus of convexity. Using the concept of Kuratowski measure of noncompactness, K. Goebel and T. Sekowski defined modulus of noncompact convexity by which they proved some facts about geometrical structure of Banach spaces. 1980s Huff started studying nearly uniform convexity and proved that space is nearly uniform convex if and only if it is a reflexive and has Kadec-Klee property.

Banas J., has proved that modulus  $\Delta_{X,\chi}(\epsilon)$ , where  $\chi$  is Hausdorff measure of noncompactness and  $X$  is reflexive Banach space, is continuous function on the interval  $[0,1)$ , [3].

**Theorem.** Let  $X$  be a Banach space that has Radon-Nikodym property and  $\phi$  strictly minimalizable measure of noncompactness. Modulus of noncompact convexity  $\Delta_{X,\phi}(\epsilon)$  is continuous function on the interval  $[0, \phi(\overline{B_X})]$ .

**Keywords.** Measure of noncompactness, modulus of noncompact convexity, normal structure, fixed point

**AMS 2010.** 46B20, 46B22, 47H09, 47H10.

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## Operators Ideal of Generalized Modular Spaces of Cesaro Type Defined by Weighted Means

Necip Şimşek<sup>1</sup>, Vatan Karakaya<sup>2</sup> and Harun Polat<sup>3</sup>

**Abstract.** We study the ideal of all bounded linear operators between any arbitrary Banach spaces whose sequence of approximation numbers belong to the sequence space by using generalized weighted means defined by [4] and generalized by [6].

**Keywords:** Approximation numbers; operator ideal; generalized weighted mean.

**AMS 2010:** 46B20, 46B45, 47B06.

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## On Some Lacunary Ideal Convergent Sequence Spaces in $n$ -normed Spaces

Selma Altundağ<sup>1</sup> and Ömer Faruk Çelik<sup>2</sup>

**Abstract.** In this paper, we define new generalized sequence spaces using ideal convergence and a sequence of modulus function in  $n$ -normed spaces. Further we obtain some inclusion relations involving these sequence spaces.

**Keywords.** Sequence spaces,  $n$ -normed spaces,  $I$ -convergence.

**AMS 2010.** 40A05, 40A35.

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## About the Zeroes of the Solutions of One Areolar Equation of Second Order

Slagjana Brsakoska<sup>1</sup>

**Abstract.** In the paper analysis for the solutions and the zeroes of the solutions of one areolar equation of second order is given (with coefficient – a complex constant or an analytic function), for their nature, oscillatory properties and number if they exist. Here analogy with the properties of the zeroes of the solutions of an ordinary differential equation of second order analogue to the equation analyzed in the paper is used. The results are formulated in theorems and conclusions and corresponding examples are given.

**Keywords.** areolar differential equation of II order, analytic function, zeroes of a solution, oscillatory properties.

**AMS 2010.** 34M45, 35Q74.

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## On Convergence of the Family of Nonlinear Integral Operators

Sevgi Esen Almalı<sup>1</sup>

**Abstract.** We investigate the problem of pointwise convergence of the family of nonlinear integral operators

$$L(f, x, \lambda) = \int_a^b \sum_{m=1}^{\infty} f^m(x) K_{\lambda, m}(x, t)$$

where  $\lambda$  is a real parameter,  $K_{\lambda, m}(x, t)$  is non-negative kernels and  $f$  is the function in  $L_1(a, b)$ . We consider two cases where  $(a, b)$  is a finite interval and when is the whole real axis..

**Keywords.** Approximation, nonlinear integral operators, lebesgue point.

**AMS 2010.** 41A25.

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## Some Estimations on the Fejér Kernel Functions With Respect to the Walsh-Paley System

Samra Pirić<sup>1</sup> and Nacima Memić<sup>2</sup>

**Abstract.** Let  $I = [0,1]$  be the unit interval. Set  $I_n(x) = \{y \in I: y_0 = x_0, \dots, y_{n-1} = x_{n-1}\}$  for  $x \in I, I_n = I_n(0)$  for  $n \in N$  and  $I_0(x) = I$ . We consider the Walsh-Paley system [4]  $(\chi_n, n \in N)$ , that is,  $\chi_n(x) = \prod_{k=0}^{\infty} (-1)^{n_k x_k}$ ,  $n \in N, x \in N, x \in I$ , and the Fejér kernel function  $K_n^\chi = \frac{1}{n} \sum_{k=1}^n D_k^\chi$ , where  $D_n^\chi = \sum_{k=0}^{n-1} \chi_k$ ,  $K_0^\chi = D_0^\chi = 0$ , and  $n \in N$  ( $n_k, x_k$  are the  $k$ -th coordinates of  $n, x$  respectively). Using the previous notations, we prove results that provide better estimate for the maximal function of the Walsh-Fejér kernels than the estimate obtained in [2].

**Lemma 0.1.**  $n K_n^\chi = \sum_{j=1}^{N-1} D_{2^{A_j}} \left( \sum_{i=j+1}^N 2^{A_i} \right) \chi_{2^{A_1}} \chi_{2^{A_2}} \dots \chi_{2^{A_{j-1}}} + \sum_{j=1}^N (\chi_{2^{A_1}} \chi_{2^{A_2}} \dots \chi_{2^{A_{j-1}}}) 2^{A_j} K_{2^{A_j}}.$

**Theorem 0.2.** The integral of the maximal function of Fejér means can be estimated by

$$\int_{I-I_k} \sup_{|n| \geq A} |K_n^\chi(x)| dx \leq C \frac{A-k}{2^{A-k}}, \quad A \geq k.$$

**Keywords.** Walsh-Paley system, Fejér kernel function, Dyadic group.

**AMS 2010.** 42 C10.

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## A Study on the Jost Solution and the Spectral Properties of the Matrix-Valued Difference Operators

Şerifenur Cebesoy<sup>1</sup> and Yelda Aygar<sup>2</sup>

**Abstract.** In this paper, we find polynomial type Jost solution of the selfadjoint matrix-valued difference equation of second order. Then we investigate analytical properties and asymptotic behaviour of the Jost solution. Using the Weyl compact perturbation theorem we prove that, the selfadjoint operator  $L$  generated by the matrix-valued difference expression of second order has the continuous spectrum filling the segment  $[-2,2]$ . We also study the eigenvalues of  $L$  and prove that it has a finite number of simple real eigenvalues.

**Keywords.** Difference Equations, Spectral analysis, Eigenvalues, Continuous Spectrum, Jost Function

**AMS 2010.** 39A05, 34L05

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## Weighted Statistical Convergence Through A Lacunary Sequence In Locally Solid Riesz Space

Şükran Konca<sup>1</sup> and Metin Başarır<sup>2</sup>

**Abstract.** The purpose of this paper is to introduce the concepts of weighted statistical  $\tau$  – convergence, weighted lacunary  $\tau$  – convergence, weighted lacunary statistical  $\tau$  – bounded in the framework of locally solid Riesz space and prove some topological results related to these notions.

**Keywords.** Locally solid Riesz space; statistical topological convergence, weighted lacunary statistical  $\tau$  – convergence; Nörlund-type mean.

**AMS 2010.** 46A40; 40A35.40G15; 46A45.

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## ( $\sigma, f$ )-Asymptotically Lacunary Equivalent Sequences

Tunay Bilgin<sup>1</sup>

**Abstract.** We introduce the strong  $(\sigma, f)$ -asymptotically equivalent and strong  $(\sigma, f)$ -asymptotically lacunary equivalent sequences which are some combinations of the definitions for asymptotically equivalent, statistically limit, modulus function,  $\sigma$ -convergence and lacunary sequences. Then we use these definitions to prove strong  $(\sigma, f)$ -asymptotically equivalent and strong  $(\sigma, f)$ -asymptotically lacunary equivalent analogues of Connor's results in [3], Fridy and Orhan's results in [9,10] and Das and Patel's results in [5].

**Keywords.** Asymptotically equivalence, Lacunary sequence, Modulus function,  $\sigma$ -convergence, Statistically limit

**AMS 2010.** 40A99, 40A05.

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## On the Solvability of a System of Multi-Point Second Order Boundary Value Problem

Tugba Senlik<sup>1</sup>, Ilkay Yaslan Karaca<sup>2</sup>, Aycan Sinanoglu<sup>3</sup>, Fatma Tokmak<sup>4</sup>

**Abstract.** In this paper, by using Leray-Schauder fixed point theorem, we obtain existence of symmetric solution for multi-point second order boundary value problem. We give an example to demonstrate our result.

**Keywords.** boundary value problem, fixed point method, existence of solution.

**AMS 2010.** 34B10, 34B18, 39A10.

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## Inclusion Results on Statistical Cluster Points

Tuğba Yurdakadim<sup>1</sup> and Emre Taş<sup>2</sup>

**Abstract.** In this note, we study the concepts of statistical cluster point and statistical core of a sequence for  $A_\lambda$ -methods defined by deleting some rows from a nonnegative regular matrix  $A$ . We also relate  $A_\lambda$ -statistical convergence to  $A_\mu$ -statistical convergence. Also by giving a consistency theorem for  $A$ -statistical convergence, we consider conditions for which the equality  $st_A\text{-core}\{x\}=st_B\text{-core}\{x\}$  holds.

**Keywords.**  $A$ -density,  $A$ -statistical cluster point,  $A$ -statistical core of a sequence.

**AMS 2010.** 40A05, 26A03, 11B05.

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## n-Tuplet Fixed Point Theorems in Partially Ordered Metric Space

Vatan Karakaya<sup>1</sup> and Müzeyyen Ertürk<sup>2</sup>

**Abstract.** In this work, we study existence and uniquenesses of fixed points of operator  $F : X^n \rightarrow X$  where  $n$  is an arbitrary positive integer and  $X$  is a partially ordered complete metric space.

**Keywords.** fixed point theorems, partially ordered metric space, mixed g-monotone.

**AMS 2010.** 47H10; 54H25; 54E50

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## Single Layer Potentials on Surfaces with Small Lipschitz constant

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**Abstract.** This talk considers to the equation

$$\int_S \frac{U(Q)}{|P-Q|^{N-1}} dS(Q) = F(P), \quad P \in S,$$

where the surface  $S$  is the graph of a Lipschitz function  $\varphi$  on  $\mathbf{R}^N$ , which has a small Lipschitz constant. The integral in the left-hand side is the single layer potential corresponding to the Laplacian in  $\mathbf{R}^{N+1}$ . Let  $\Lambda(r)$  be a Lipschitz constant of  $\varphi$  on the ball centered at the origin with radius  $2r$ . Our analysis is carried out in local  $L^p$ -spaces and local Sobolev spaces, where  $1 < p < \infty$ , and results are presented in terms of  $\Lambda$ . Estimates of solutions to the equation are provided, which can be used to obtain knowledge about the behaviour of the solutions near a point on the surface. The estimates are given in terms of seminorms. Solutions are also shown to be unique if they are subject to certain growth conditions. Local estimates are provided and some applications are supplied.

**Keywords.** Singular integrals, Lipschitz surfaces, local estimates.

**AMS 2010.** 45E99, 45M05.

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## Some New Identities on the Bernoulli Polynomials $B_{n,q}(x, y)$ and Euler Polynomials

$$E_{n,q}(x, y)$$

Veli Kurt<sup>1</sup>

**Abstract.** The main purpose of this paper is to introduce and investigate a new class  $q$ -Bernoulli polynomials  $B^{(\alpha)}_{n,q}(x, y)$  order  $\alpha$  and  $q$ -Euler polynomials  $E^{(\alpha)}_{n,q}(x, y)$  order  $\alpha$ . We give some identities for the  $q$ -Bernoulli polynomials  $B_{n,q}(x, y)$  and  $q$ -Euler polynomials  $E_{n,q}(x, y)$ . Furthermore, we prove some relations between the 2-variable Bernoulli polynomials  $B^{(\alpha)}_{n,q}(x, y)$  order  $\alpha$  and Euler polynomials  $E^{(\alpha)}_{n,q}(x, y)$  order  $\alpha$ .

**Keywords.** Bernoulli Polynomials, Euler polynomials, Generating Function.

**AMS 2010.** 11B68, 11B73.

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## Metric-affine vs Spectral Theoretic Characterization of the Massless Dirac Operator

Vedad Pasic<sup>1</sup> and Elvis Barakovic<sup>2</sup>

**Abstract.** A classical pp-wave is a 4 - dimensional Lorentzian spacetime which admits a nonvanishing parallel spinor field. This definition was previously generalized to metric compatible spacetimes with torsion and used to construct new explicit vacuum solutions of quadratic metric-affine gravity.

In this work, we begin by examining the physical interpretation of the pp-wave solutions of metric - affine gravity by comparing them to pp-wave type solutions of the classical massless Dirac and Einstein - Weyl models of gravity. We then proceed to consider an elliptic self-adjoint first order differential operator acting on pairs of complex-valued half-densities over a connected compact 3-dimensional manifold without boundary. We then address the question: is this operator a massless Dirac operator on half-densities? Several problems arise and we compare the spectral analytical results with previous work done with particular emphasis on the work to be done on spectral asymmetry on the 3-torus and the 3-sphere.

**Keywords.** pp-waves, metric-affine gravity, massless Dirac operator , Einstein - Weyl, spectral theory, asymptotic distribution of eigenvalues.

**AMS 2010.** Primary 35P20; Secondary 35J46, 35R01, 83C15, 83C35.

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## Some Properties of the Sequence Space $ces(f, p, \Delta)$

Yavuz Altın<sup>1</sup>, Birgül Torgut<sup>2</sup> and Rifat Çolak<sup>3</sup>

**Abstract.** In this paper we define the sequence space  $ces(f, p, \Delta)$  using a modulus function  $f$ . We give various properties and some inclusion relations on this space

**Keywords.** Cesàro summability, Modulus function .

**AMS 2010.** 46A45, 40A05

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## The Notion of Exhaustiveness in Asymmetric Metric Spaces

Zeynep Hande Toyganözü<sup>1</sup> and Serpil Pehlivan<sup>2</sup>

**Abstract.** We introduce here the notion of exhaustiveness which is closely connected to the notion of equicontinuity in asymmetric metric spaces. We give the relation between equicontinuity and exhaustiveness in such spaces and some theorems and results about it. We show that if asymmetric condition is dropped, the limit of a sequence of exhaustive function might be discontinuous even if the limit is unique.

**Keywords.** Forward exhaustiveness, backward exhaustiveness, asymmetric metric.

**AMS 2010.** 40A30.

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## Selfadjoint Extensions for a First Order Differential Operator

Z. I. Ismailov<sup>1</sup>, M. Sertbaş<sup>2</sup> and E. Otkun Çevik<sup>3</sup>

**Abstract.** In this work, all selfadjoint extensions of the minimal operator generated by linear singular multipoint symmetric differential-operator expression  $l = (l_-, l_1, \dots, l_n, l_+)$ ,  $l_{\mp} = i \frac{d}{dt} + A_{\mp}$ ,  $l_k = i \frac{d}{dt} + A_k$ , where the coefficients  $A_{\mp}$ ,  $A_k$  are selfadjoint operators in separable Hilbert spaces  $H_{\mp}$ ,  $H_k$ ,  $k = 1, \dots, n$ ,  $n \in \mathbb{N}$  respectively, are researched in the direct sum of Hilbert spaces of vector-functions

$$L_2(H_-, (-\infty, a)) \oplus L_2(H_1, (a_1, b_1)) \oplus \dots \oplus L_2(H_n, (a_n, b_n)) \oplus L_2(H_+, (b, +\infty)),$$

$-\infty < a < a_1 < b_1 < \dots < a_n < b_n < b < +\infty$ . Also, the structure of the spectrum of these extensions is investigated.

**Keywords.** Selfadjoint extension, Multipoint differential operator, Spectrum.

**AMS 2010.** 47A10, 47A20.

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*APPLIED*  
*MATHEMATICS*



## Shallow Water Waves (KdV) Equation by Painleve's Property

Attia A.H Mostafa<sup>1</sup>

**Abstract.** The Korteweg-de Vries (KdV) equation which is the third order nonlinear PDE has been of interest for two centuries. In this paper we study this kind of equation by Painleve's equation and through this study, we find that KdV equation satisfies Painleve's property, but we could not find a solution directly, so we transformed the KdV equation to the like-KdV equation, therefore, we were able to find four exact solutions to the original KdV equation.

**Keywords.** Kortewege-de Vrise equation · Painleve's property · Transformed KdV equation Resonance points · Exact solutions

**AMS 2000.** 35Q53 · 37K10

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## **A Collocation Method for Linear Fredholm-Volterra Integro-Differential Equations in the Most General Form by Means of Generalized Bernstein Polynomials**

Aysegul Akyuz-Dascioglu<sup>1</sup> and Nese Isler Acar<sup>2</sup>

**Abstract.** In this study, a collocation method based on Bernstein polynomials defined on the interval  $[a, b]$  is introduced for the approximate solutions of higher order linear Fredholm–Volterra integro-differential equations in the most general form under the mixed conditions. The convergence of the proposed method and error analysis are given. To illustrate the applicability and efficiency of the proposed method, some examples are presented.

**Keywords.** Bernstein approximation, collocation method, linear Fredholm-Volterra integro-differential equations.

**MSC 2010.** 45J05, 65L60.

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## A New Operational Matrix Algorithm

Ayşe Betül Koç<sup>1</sup>, Musa Çakmak<sup>2</sup> and Aydin Kurnaz<sup>3</sup>

**Abstract.** In this study, we present a matrix method with new operational matrices for various initial and boundary value problems. By using this method, approximate solutions of the problems are obtained in form of the special polynomials. Also, numerical examples are given to illustrate the effectiveness and advantages of this approach.

**Keywords.** Polynomial approximation, operational matrices, matrix method

**AMS 2010.** 65D25.

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## Numerical Solution of the Diffusion Equation with B-Spline Functions

Ahmet Boz<sup>1</sup>

**Abstract.** We solved linear Diffusion equation using restrictive Taylor approximation. We use the restrictive Taylor approximation to approximate the exponential matrix  $\exp(xA)$ . The effect of the cubic B-Splines in the restrictive Taylor approximation method is sought. The advantage of the method is that has the exact value at the certain point. We will use a new technique for solution of the Diffusion equation. Numerical comparison of results of algorithms and some other published numerical results are done.

**Keywords.** Diffusion equation, Finite Element, Restrictive Taylor Approximation

**AMS 2010.** 35K57, 65L60,41A58.

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## **Solutions of Travelling Salesman Problem Using Genetic Algorithm and Atom Algorithm**

Ayşe Erdoğan Yıldırım<sup>1</sup>, Ali Karacı<sup>2</sup>

**Abstract.** Travelling Salesman Problem (TSP) is an optimization problem that aims navigating given a list of city in the shortest possible route and visits each city exactly once. When number of cities increases, solution of TSP with mathematical methods becomes almost impossible. Therefore it is better to use heuristic methods to solve the problem [1].

In this study, for the solution of TSP, Genetic and Atom Algorithms which are based on heuristic techniques were used and their performances were compared. Genetic Algorithm which is an evolutionary algorithm is inspired by biological changes and it uses operators such as natural selection, reproduction, crossover and mutation. Population evolution is occurred by applying these operators iteratively and as the other heuristic techniques, it gives exact or approximate solutions [2,3].

Another algorithm was applied for TSP is Atom Algorithm. This is a new meta-heuristic algorithm based on process of compound formation. To form a compound, there are two methods called as Ionic Bond and Covalent Bond. Ionic Bond is formed by removing some electrons from some elements and attaching new electrons to these elements. In the other case, Covalent Bond is based the joint use of one electron by two elements. Change of atom set appears by applying these two operators iteratively. Atom algorithm differs from the other algorithms are in this field by taking into account the effect of parameter values on the solution. Covalent Bond operator is applied according to effect of electrons [4].

In the applications of Genetic and Atom Algorithms for TSP, to avoid repetition of city, candidate solution sets were generated by codification in form of permutation and to prevent degradation of this form when operators were applied, some of the techniques were used [5].

As a result, it was observed that for the solution of TSP, Atom Algorithm gives better results than Genetic Algorithm. That means, the salesman completes the tour by travelling less distance in Atom Algorithm. On the other hand, the results of Atom Algorithm had better stability. But Atom algorithm has a disadvantage in the aspect of time. It works slower than Genetic Algorithm.

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**Keywords.** Meta-Heuristic Approaches, Atom Algorithm, Genetic Algorithm, Travelling Salesman Problem.

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## Strongly damped wave equation with exponential nonlinearities

Azer Hanmehmetli (Khanmamedov)<sup>1</sup>

**Abstract.** In this work, we study the initial boundary value problem for the two dimensional strongly damped wave equation with exponentially growing source and damping terms. We first show the well-posedness of this problem and then prove the existence of the global attractor in  $H_0^1(\Omega) \cap L^\infty(\Omega) \times L^2(\Omega)$ .

**Keywords.** 35L05, 35B41

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## On Finite Spectrum of Sturm-Liouville Problem

Abdullah Kablan<sup>1</sup>

**Abstract.** For a regular two point Sturm-Liouville problems exactly one of the following four cases: There are no eigenvalues in  $\mathbb{C}$  or every complex number is an eigenvalue or there are exactly  $n$  eigenvalues in  $\mathbb{C}$  for some  $n \in \mathbb{N} = \{1, 2, 3, \dots\}$  or there are an infinite but countable number of eigenvalues in  $\mathbb{C}$  and these have no infinite accumulation point in  $\mathbb{C}$ .

In this study we construct Sturm-Liouville problem with exactly  $n$  eigenvalues for each non-negative integer  $n$ . We also show these eigenvalues can be arbitrarily distributed throughout the complex plane  $\mathbb{C}$ , [1], [2].

**Keywords.** Sturm–Liouville problems, eigenvalues, finite spectrum.

**AMS 2010.** 34B24, 47B25.

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## **Taylor-Collocation Method for The Numerical Solution of The Classical Boussinesq Equation Using Extended Cubic B-spline Basis**

A. Murat Aksoy<sup>1</sup> and Idris Dağ<sup>2</sup>

**Abstract.** In this study, we construct a Taylor Collocation method for the numerical solution of the Classical Boussinesq equation. We use suitable initial and boundary conditions. Taylor series expansion is used for time discretization. The cubic B-spline collocation method is applied to spatial discretization. Test problems concerning the traveling waves, interaction of two colliding waves are studied to evaluate the method. The  $L_2$  and  $L_\infty$  error norms are calculated to improve the accuracy of the numerical results.

**Keywords.** Extended Cubic B-Spline, Taylor Collocation, Classical Boussinesq.

**AMS 2010.** 41A15, 65L60.

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## **Analytical Solutions to Orthotropic Variable Thickness Disk Problems**

Ahmet N. Eraslan<sup>1</sup>, Yasemin Kaya<sup>2</sup> and Ekin Varlı<sup>3</sup>

**Abstract.** Analytical solutions to nonisothermal orthotropic variable thickness rotating and stationary disk problems are obtained. The fact that the modulus of elasticity and Poisson's ratio are different in  $r$  – and  $\theta$  – directions in an orthotropic disk brings an additional difficulty in the analytical treatment of the problem. Considering a two–parametric parabolic thickness profile and using the equation of equilibrium, the compatibility relation, and the stress-strain relations, the governing differential equation describing the elastic/thermoelastic response of the disk is obtained in terms of a predefined stress function. The governing differential equation turns into a familiar hypergeometric type by performing an appropriate transformation. The analytical solution is then obtained in terms of hypergeometric functions. In nonisothermal cases, the temperature distribution in the disk is computed numerically by the use of an accurate model which takes into account the convective heat transfer resulting from rotation. The solutions are specifically derived for annular disks subjected to free–free, pressurized–free and free–pressurized boundary conditions. The results of the calculations are presented in graphical form.

**Keywords.** Orthotropic Disk, Variable Thickness, Rotating Disk, Convection, Elasticity, Thermoelasticity.

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## **A Computational Model for Partially-Plastic Stress Analysis of Orthotropic Variable Thickness Disks Subjected to External Pressure**

Ahmet N. Eraslan<sup>1</sup>, Yasemin Kaya<sup>2</sup> and Busra Ciftci<sup>3</sup>

**Abstract.** A computational model is developed to predict elastoplastic stresses in orthotropic variable thickness disks with free-pressurized boundary conditions. The modulus of elasticity and Poisson's ratio of an orthotropic disk are different in  $r$  – and  $\theta$  – directions and the thickness of the disk vary radially in accordance with a two-parametric parabolic thickness profile. A general derivation for the governing differential equation is carried out by taking into account the rotation and existence of a radial temperature gradient. In addition to basic deformation equations of the disk geometry, Hill's quadratic yield condition with its associated flow rule and a Swift type nonlinear hardening law are used. Therefrom, a single nonlinear differential equation describing the partially plastic response of an orthotropic variable thickness disk is obtained. The numerical solution of this differential equation is obtained by a nonlinear shooting technique. The resulting IVP system is integrated using Runge-Kutta-Fehlberg four-fifth order method to achieve high order accuracy. It is observed that plastic deformation commences at the inner surface of the disk and as the applied pressure is further increased, another plastic region develops at the outer surface. These plastic regions propagate into the disk with increasing pressures. The effects of elastic and plastic orthotropy parameters on the propagation of plastic regions are investigated and the results are presented in graphical form.

**Keywords.** Orthotropic Disk, Variable Thickness, Elastoplasticity, Hill's Quadratic Yield Criterion, Shooting Method

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## Qualitative Analysis of Solutions of Certain Nonlinear Problems

Alma Omerspahić<sup>1</sup>

**Abstract.** The paper present some results on the existence and behavior of solutions of some nonlinear differential equations. The obtained results contain an answer to the question on approximation as well as stability of solutions whose existence is established. The errors of the approximation are defined by the functions that can be sufficiently small. The qualitative analysis theory of differential equations and topological retraction method are used.

**Keywords.** nonlinear differential equations, behavior of solutions, approximation of solutions

**AMS 2010.** 34C05

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## Heat Transfer Modeling in Non-Homogeneous Media with Adomian Decomposition

### Method

Ali Sahin<sup>1</sup> and Yasemin Karagöz<sup>2</sup>

**Abstract.** In this study, the one dimensional reaction diffusion equation in composite media that is named as Functionally Graded Materials (FGMs) is examined by using Adomian Decomposition Method (ADM). The method can be applied to linear and non-linear ordinary or partial differential equations, integral equations and their systems [1-4]. Encountered problem of this study is the reaction diffusion equation with variable coefficients which represents the material properties in spatial coordinates. In the solution process, the conductivity of the material is defined as a function of space variable and a non-homogeneity parameter is represented by an exponential function. Then, in terms of different case of the reaction parameter,  $p(x,t)$ , a model is set up for the solution via ADM. Finally, examples are given for the different states of the non-homogeneity parameter of the material and results are plotted.

**Keywords.** ADM, Non-homogeneous Materials, Reaction Diffusion Equation.

**AMS 2010.** 58J35, 35K05, 35K20.

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## Diffraction of Sound Waves from a Lined Duct with Lined Afterbody

Ayşe Tiryakioğlu<sup>1</sup> and Ahmet Demir<sup>2</sup>

**Abstract.** In this study, diffraction of sound waves from a coaxial duct is investigated using Wiener-Hopf technique. The outer duct wall is lined from inside and liner impedance is denoted by  $Z_1$ . The inner duct wall is lined from outside and liner impedance is denoted by  $Z_2$  (after body).

This boundary-value problem related to the diffraction problem has been reduced to Wiener-Hopf equation via Fourier transform. The scattered field was found analytically by solving the Wiener-Hopf equation. The integral encountered in finding the scattered field is evaluated via saddle point technique. At the end of the analysis, numerical results illustrating the effects of the acoustic absorbent lining on the coaxial duct on the sound spread are presented.

**Keywords.** Wiener-Hopf technique, fourier transform, saddle point technique.

**AMS 2010.** 78A45, 34B30, 97M50.

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## A Numerical Approach for Solving Fredholm-Integro Differential Difference Equations

Burcu Gürbüz<sup>1</sup> and Mehmet Sezer<sup>2</sup>

**Abstract.** This paper presents a numerical method for the approximate solution of Fredholm-integro differential difference equations with variable coefficients

$$\sum_{h=0}^J \sum_{k=0}^m P_{hk}(x) y^{(k)}(\alpha_{hk}x + \beta_{hk}) = g(x) + \lambda \int_a^b K_f(x,t) y(t) dt, \quad a \leq x, t \leq b < \infty$$

which is under the mixed conditions

$$\sum_{k=0}^{m-1} [a_{jk} y^{(k)}(0) + b_{jk} y^{(k)}(b)] = \lambda_j, \quad j = 0, 1, 2, \dots, m-1$$

the approximate solution of Fredholm-integro differential difference equation is obtained by truncated Laguerre series in the form

$$y(x) = \sum_{n=0}^N a_n L_n(x), \quad L_n(x) = \sum_{r=0}^n \frac{(-1)^r}{r!} \binom{n}{r} x^r$$

where  $P_{hk}(x)$ ,  $K(x,t)$  and  $g(x)$  functions are defined on the interval  $a \leq x, t \leq b < \infty$ ,  $\lambda$ ,  $a_{jk}$ ,  $b_{jk}$  and  $\lambda_j$  are constants,  $a_n$   $n = 0, 1, 2, \dots, N$  is unknown Laguerre coefficients and  $N$  is any positive integers. The aim of this article is to present an efficient numerical procedure for solving a class of the Fredholm-integro differential difference equations. Our method transforms Fredholm-integro differential difference equations and the given conditions into matrix equation which corresponds to a system of linear algebraic equation. The reliability and efficiency of the proposed scheme are demonstrated by some numerical experiments and performed on the computer. Moreover, we develop the residual error analysis for our method.

**Keywords.** Fredholm integral equations, differential- difference equations, Laguerre polynomials, collocation method, residual error analysis.

**AMS 2010.** 45B05, 33E30, 33C45, 65M75, 65G99.

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## Solitary Waves of the MRLW Equation Using Fully Implicit Finite Difference Method

Bilge İnan<sup>1</sup> and Ahmet Refik Bahadır<sup>2</sup>

**Abstract.** In the present paper, the modified regularized long wave (MRLW) equation is solved numerically by a fully implicit finite difference method. Numerical results for different particular cases of the problem are presented. Comparisons are made with published numeric and analytic solutions. The present method performs well in terms of efficiency and simplicity.

**Keywords.** Modified regularized long wave equation, Finite differences, Fully implicit finite difference method, Solitary waves

**AMS 2010.** 65M06, 74J35

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## The Quenching Behavior of a Parabolic System With Singular Boundary Conditions

Burhan Selçuk<sup>1</sup> and Nuri Özalp<sup>2</sup>

**Abstract.** In this paper, we study the quenching problem for the following parabolic system:

$$u_t = u_{xx}, 0 < x < 1, 0 < t < T,$$

$$v_t = v_{xx}, 0 < x < 1, 0 < t < T,$$

with boundary conditions

$$u_x(0, t) = 0, u_x(1, t) = (1 - v(1, t))^{-q_1}, 0 < t < T,$$

$$v_x(0, t) = 0, v_x(1, t) = (1 - u(1, t))^{-q_2}, 0 < t < T,$$

and initial conditions

$$u(x, 0) = u_0(x) < 1, v(x, 0) = v_0(x) < 1, 0 \leq x \leq 1,$$

where  $q_1, q_2$  are positive constants and  $u_0(x), v_0(x)$  are positive smooth functions satisfying the compatibility conditions

$$u'_0(0) = v'_0(0) = 0, u'_0(1) = (1 - v_0(1))^{-q_1}, v'_0(1) = (1 - u_0(1))^{-q_2}.$$

Firstly, we prove finite-time quenching for the solution. Further, we show that quenching occurs on the boundary under certain conditions. Furthermore, we show that the time derivative blows up at quenching time. Finally, we get a quenching criterion by using a comparison lemma and we also get a quenching rate.

**Keywords.** Parabolic system, singular boundary condition, quenching, quenching point, quenching time, maximum principles

**AMS 2010.** 35K55, 35K60, 35B35, 35Q60.

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## **Diffraction of Sound Waves from a Semi-Infinite Circular Cylindrical Pipe with an Acoustically Absorbing External Surface**

Burhan Tiryakioğlu<sup>1</sup> and Ahmet Demir<sup>2</sup>

**Abstract.** In this study, diffraction of sound waves from a semi-infinite circular cylindrical pipe is investigated using Wiener-Hopf technique. The cylinder of radius  $\rho = R_2$  have been assumed to rigid from inside and lined from outside.

This boundary-value problem related to the diffraction problem has been reduced to Wiener-Hopf equation via Fourier transform. The scattered field was found analytically by solving the Wiener-Hopf equation. The integral encountered in finding the scattered field is evaluated via saddle point technique. At the end of the analysis, numerical results illustrating the effects of the acoustic absorbent lining on the outer surface of the cylinder on the sound spread are presented.

**Keywords.** Wiener-Hopf technique, fourier transform, saddle point technique.

**AMS 2010.** 78A45, 34B30, 97M50.

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## Iterative Procedure for Common Quadratic Lyapunov Function

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**Abstract.** In this study, the problem of finding a common Lyapunov function for a large family of stable systems is considered. Gradient algorithms which give deterministic convergence for finite system families are proposed to this problem. Finally some examples are presented by using proposed algorithm.

**Keywords.** Common Lyapunov function, gradient algorithms.

**AMS 2010.** 37B25

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## **Fuzzy Minimal Covering Location Problem with Different Conorms**

Darko Drakulić<sup>1</sup>, Miroslav Marić<sup>2</sup> and Aleksandar Takači<sup>3</sup>

**Abstract.** Covering location problems represents a very popular subset of location problems and it has been well studied in the past. Objective of the covering location problem is to find the best positions for various facilities. Some objects need to service as more as possible locations (like shops, schools, antennas, light sources, emergency units etc.) but some objects need to cover as less as possible locations (like landfills, pollution plants, chemical waste incinerators etc.). There are two different types of covering location problem for described cases - Maximal covering location problem (MCLP) with the aim to cover maximal number of locations with fixed number of facilities and Minimal covering location problem (MinCLP) with the aim to cover minimal number of locations with fixed number of facilities. The aim of this paper is to present new methods for better modelling of real life minimal covering location problem using fuzzy sets for problem's conditions and different conorms for computation of the coverage degree.

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## An Numerical Analysis with Low and High-Reynolds Numbers in Vascular Flows

Dilek Pandir<sup>1</sup> and Yusuf Pandir<sup>2</sup>

**Abstract.** While much of the hemodynamics in a healthy human body has low Reynolds number, resulting in laminar flow, relatively high Reynolds number flow is observed at some specific locations, which can cause transition to turbulence (The term “turbulence” refers to the motion of a fluid having local velocities and pressures that fluctuate randomly). For example, the peak Reynolds number  $P$  in the human vessel has been measured to be approximately 4000 [1-3]. The two-dimensional, unsteady, incompressible Navier-Stokes equation was solved with the assumptions of rigid vessel walls and constant viscosity (Newtonian fluid). Laminar to turbulent flow transition in the mammalian vessel is generally characterized by Reynolds number [4].

**Keywords.** Reynolds number, vascular flows, stream lines.

**AMS 2010.** 35Q30, 76B10.

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## Spectral Analysis of Fractional Hydrogen Atom Equation

Erdal Baş<sup>1</sup> and Funda Metin<sup>2</sup>

**Abstract.** In this study, spectral analysis of fractional Sturm Liouville problem defined on  $(0,1]$  having the singularity of type  $-\frac{2}{r} + \frac{l(l+1)}{r^2}$  at zero and research the fundamental properties of the eigenfunctions and eigenvalues for the operator. We show that the eigenvalues and eigenfunctions of the problem are real and orthogonal, respectively. Furthermore, we give some important theorems and lemmas for fractional hydrogen atom equation.

**Keywords.** Sturm-Liouville, Fractional, Hydrogen atom, Singular, Spectral.

**AMS 2010.** 26A33, 34A08

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## Numerical Solution For Fuzzy Heat Equation

Emine Can<sup>1</sup> and Aylin Bayrak<sup>2</sup>

**Abstract.** In this work, we study numerically for fuzzy heat equation under generalized differentiability in the form [4]

$$\tilde{u}(x, t) = \tilde{u}_{xx} + \tilde{p}(t)\tilde{u}(x, t) + \tilde{f}(x, t) \quad 0 < x < L, \quad 0 < t < T$$

with initial and boundary conditions

$$\tilde{u}(x, 0) = \tilde{\varphi}(x) \quad 0 < x < L$$

$$\tilde{u}(0, t) = \tilde{g}_1(t) \quad 0 < t < T$$

$$\tilde{u}(1, t) = \tilde{g}_2(t) \quad 0 < t < T$$

where  $\tilde{p}(t)$ ,  $\tilde{\varphi}(x)$ ,  $\tilde{g}_1(t)$ ,  $\tilde{g}_2(t)$ ,  $\tilde{f}(x, t)$  and  $\tilde{u}(x, t)$  are fuzzy functions [1-3]. An algorithm is presented, and the finite difference method is used for solving obtained problem. Some numerical examples are given with graphical representations for fuzzy heat equation.

**Keywords.** Heat Equation, Fuzzy Numbers, Generalized Differentiability, Fuzzy-valued Function

**AMS 2010.** 53A40, 20M15.

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**Positive Periodic Solutions in Shifts  $\delta_{\pm}$  for a Nonlinear First Order Functional  
Dynamic Equation on Time Scales**

Erbil Çetin<sup>1</sup>

**Abstract.** Let  $\mathbb{T} \subset \mathbb{R}$  be a periodic time scale in shifts  $\delta_{\pm}$  with period  $P \in [t_0, \infty)_{\mathbb{T}}$ . We consider the existence of positive periodic solutions in shifts  $\delta_{\pm}$  for the nonlinear functional dynamic equation of the form

$$x^{\Delta}(t) = -a(t)x^{\sigma}(t) + \lambda b(t)f(t, x(h(t))), \quad t \in \mathbb{T}$$

using cone the theory techniques. We extend and unify periodic differential, difference, h-difference and q-difference equations and more by a new periodicity concept on time scales.

**Keywords.** periodic time scale; periodic solution; shift operator; time scale.

**AMS No.** 39A12; 34C25; 34N05; 34K13; 35B10.

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## **Collocation Spline Methods for Semilinear Reaction-Diffusion Problem on Shishkin Mesh**

Enes Duvnjakovic<sup>1</sup> and Samir Karasuljic<sup>2</sup>

**Abstract.** The paper discusses discretization of semilinear reaction-diffusion problem. The theorem of existence and unique numerical solution generated by the difference scheme has been suggested and proved. Moreover, the paper provides the uniform convergence theorem for specific difference schemes obtained from the class. The numerical results that support these results have been listed.

**Keywords.** Singular perturbation, difference scheme, Shishkin mesh.

**AMS 2010.** 65L10, 65L50, 65L60.

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## Taylor Collocation Approach for Delayed Lotka-Volterra Predator-Prey System

Elçin Gökmen<sup>1</sup> and Mehmet Sezer<sup>2</sup>

**Abstract.** In this study, a numerical approach is proposed to obtain approximate solutions of the system of nonlinear delay differential equations defining Lotka Volterra prey-predator model. By using Taylor polynomials and collocation points, this method transforms the population model into a matrix equation. The matrix equation corresponds to a system of nonlinear equations with the unknown Taylor coefficients. Numerical examples are given to demonstrate the validity and applicability of the presented technique. The method is easy to implement and produces accurate results. All numerical computations have been performed on the computer algebraic system Maple 9.

**Keywords.** Lotka Volterra prey-predator model, System of nonlinear delay-differential equations, Taylor polynomials and series, Collocation points.

**AMS 2010.** 53A40, 20M15.

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**A Note on the Numerical Approach for the Reaction-Diffusion Problem to Model the  
Tumor Growth Dynamics in 1D**

Ersin Özüğurlu<sup>1</sup> and Nuri Kuruğlu<sup>2</sup>

**Abstract.** The equation modeling the evolution of the glioma (the most malignant form of brain tumor quantified in terms of net rates of proliferation and invasion) is solved numerically. This model is described by the reaction-diffusion differential equation. We use finite difference methods in combination with Newton's method, are proposed for solving the governing equations. The solution of traveling wave equation is considered as a special case.

**Keywords.** Glioma growth, non-linear partial differential equation, finite difference methods, brain tumor growth, proliferation, reaction--diffusion problem.

**AMS 2010.** 35K55; 35R35; 65M06; 65M12; 76B45; 76D99.

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## A Uniqueness Theorem for Singular Dirac Operator

Etibar Panakhov<sup>1</sup> and Murat Şat<sup>2</sup>

**Abstract.** In this paper, using Mochizuki and Trooshin's method [1], we discuss the inverse problem for the singular Dirac operator. Using spectral data of a kind, it is shown that the potential function can be uniquely determined by a set of values of eigenfunctions at some internal point and one spectrum. Interior inverse problem for Sturm-Liouville and Dirac operators were studied [2,3].

**Keywords.** Eigenfunctions, spectrums, internal point.

**AMS 2010.** 34B24, 45C05.

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**Stability Analysis of a Mathematical Model: Interaction of the Brain Tumor  
Glioblastoma Multiforme (GBM) and the Immune System (IS)**

Fatma Bozkurt<sup>1</sup>

**Abstract.** In this paper, fractional-order is introduced into the interaction model between GBM and IS. GBM (Glioblastoma Multiforme) is a brain tumor, that has a monoclonal origin and produces after reaching a specific density another tumor with different growth rate and treatment susceptibilities. The IS cells are also divided into two populations, namely, the macrophages and the activated macrophages. Hence, this model show two conversions, the conversion from sensitive tumor cells to resistant tumor cells and the conversion from macrophages to active macrophages, and an interaction between the tumor cells and the macrophages. In this work, the local and global stability analysis of the constructed system was analyzed. Furthermore, the damped oscillation of the tumor cell were investigated. Data about the drug treatment of GBM and the theoretical results will give a discussion at the end of the paper.

**Keywords.** difference equations; local stability; global stability; damped oscillation; boundedness

**AMS 2010.** 39A10, 92B10, 92D10.

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## A Fractional-Order Early Brain Tumor Growth Model with Density

Fatma Bozkurt<sup>1</sup> and Thabet Abdeljawad<sup>2</sup>

**Abstract.** Investigating the growth of tumor is an active research area and has lots of interests in biology, medicine and mathematics. In this work a fractional order logistic equation is considered such as

$$D^\alpha x(t) = x(t) \left[ \left( r(1 - \beta x(t)) \right) + \gamma x(t) \right] \quad (\text{A})$$

that was previously constructed as a logistic equation with piecewise constant arguments to investigate the behavior of a brain tumor growth [1]. The parameter  $r$  and  $\beta$  denote positive real number,  $\gamma$  is a negative number and  $0 < \alpha \leq 1$ . Eq. (A) explain a brain tumor growth, where  $\gamma$  is embed to show the drug effect on the tumor. Eq.(A) is considered taking into account the tumor density. For this, we have constructed two models of a brain tumor growth; one is Eq.(A) and the other one explain an early brain tumor growth by incorporating Allee function at time  $t$ . In this study we show the stability, existence and the uniqueness of both models. Simulations give a detailed description of the behavior of solutions in (A) with and without Allee effect.

**Keywords.** Fractional order differential equation, stability, existence, uniqueness, Allee effect

**AMS 2010.** 34A08, 34A12, 34D20.

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## **Stability Analysis of a Mathematical Model of the HIV Epidemic with Piecewise Constant Arguments**

Fatma Bozkurt<sup>1</sup> and Fatma Peker<sup>2</sup>

**Abstract.** In this work, a nonlinear mathematical model of differential equations with piecewise constant arguments is proposed. This model is analyzed by using the theory of both differential and difference equations to show the spread of HIV/AIDS in a homogeneous population. The population of the model is divided into three subclasses, which are the HIV negative class, the HIV positive class that don't know they are infected and the HIV positive class that know they are infected. As an application of the model we took the spread of HIV/AIDS in India into consideration.

**Keywords.** logistic differential equations; difference equations; piecewise constant arguments; stability

**AMS 2010.** 34D20, 34D23.

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## An Iterative Scheme for the Solution of Singularly Perturbed Delay Differential Equation

Fevzi Erdogan<sup>1</sup> and Onur Saldır<sup>2</sup>

**Abstract.** This study deals with the numerical solution of singularly perturbed initial value problems for a quasilinear first-order delay differential equation. A quasilinearization technique for the appropriate delay difference problem theoretically and experimentally is analyzed. The method is shown to uniformly convergent with respect to the perturbation parameter. The parameter uniform convergence is confirmed by numerical computations.

**Keywords.** Delay Differential equation, Singularly perturbed, Quasilinearization technique.

**AMS 2010.** 65L12, 65L20, 34K70.

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## A Finite Difference Analysis for Singularly Perturbed Differential Difference Equations with Delay and Advance

Fevzi Erdogan<sup>1</sup> and Onur Saldır<sup>2</sup>

**Abstract.** We present an uniform finite difference method for numerical solution of boundary-value problems for singularly perturbed differential-difference equations with delay and advance. An fitted difference scheme is constructed in an equidistant mesh, which gives first order uniform convergence in the discrete maximum norm. The method is shown to uniformly convergent with respect to the perturbation parameter. A numerical experiment illustrate in practice the result of convergence proved theoretically.

**Keywords.** Differential-difference equation, Singularly perturbed, Finite difference scheme.

**AMS 2010.** 65L12, 65L20, 34K70.

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## On Inertia Problem for Two Parameter Affine Polynomial Families

Gökhan Çelebi<sup>1</sup>, Taner Büyükköroğlu<sup>2</sup> and Vakıf Dzhafarov<sup>3</sup>

**Abstract.** In this study we investigate the root invariant regions in the parameter space of affine polynomial families. To determine the root regions we use well-known segment lemma and value set concept from stability theory of linear systems [1-3] and  $D$ -decomposition approach from [4].

**Keywords.** Hurwitz Stability, Segment Lemma,  $D$ -decomposition.

**AMS 2010.** 93D20.

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## Numerical Solution of an Optimal Control Problem Governed by Two Dimensional Schrodinger Equation

Gabil Yagubov<sup>1</sup> and Fatma Toyoğlu<sup>2</sup>

**Abstract.** In this study the finite difference method is applied to an optimal control problem controlled by two functions which are in the coefficients of two-dimensional Schrödinger equation. Convergence of the finite difference approximation according to the functional is proved. We have used an implicit method for solving the two-dimensional Schrödinger equation. Although the implicit scheme which is obtained from solving a system of linear equations always numerically stable and convergent when it is time independent, the equation in this paper is numerically stable with time-step condition.

**Keywords.** Optimal control, Schrödinger Operator.

**AMS 2010.** 49J20, 35J10.

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## An Estimated Upper Bound for Chebyshev Polynomial Approximation of Linear Partial Differential Equations with Mixed Conditions

Gamze Yüksel<sup>1</sup>, Osman Rasit Işık<sup>2</sup> and Mehmet Sezer<sup>3</sup>

**Abstract.** In this paper, the Chebyshev collocation method is applied to linear second order partial differential equations (PDEs) and an estimated upper bound is given for the absolute errors. The method has applied before to solving of ordinary differential [1], integral and integro-differential equations [2-4], nonlinear differential equations [5-7], partial differential equations [8-9]. In this study, different from similar collocation methods given an upper bound for the absolute errors. Finally, the effectiveness of the method is illustrated in several numerical experiments such as Laplace and Poisson equations.

**Keywords.** Partial differential equations; Chebyshev collocation method, Chebyshev polynomial solutions, Error analysis of collocation methods, Chebyshev matrix method.

**AMS 2010.** 35G16, 65N35, 65N15.

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## Density of States in GaAs-Al<sub>x</sub>Ga<sub>1-x</sub> Asymmetric Quantum Well

H.Akbaş<sup>1</sup> and C.Dane<sup>2</sup>

**Abstract.** The density of states in GaAs-Al<sub>x</sub>Ga<sub>1-x</sub>As single asymmetric quantum wells are calculated numerically using the variational method within the effective mass approximation. As a general feature, we observed that the density of states depend on the barrier height ratio  $V_R/V_L$ .

**Keywords.** Density of states, asymmetric quantum well, binding energy

**AMS 2010.** 53A40, 20M15.

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## **Investigation of the Elastic Bending Deformations of the Metacarpus Bone by Analytical and Experimental Methods**

Ibrahim Kutay Yilmazcoban<sup>1</sup>, Ahmet Cagatay Cilingir<sup>2</sup> and Nursel Kiratli Yilmazcoban<sup>3</sup>

**Abstract.** To understand the skeleton system which is the basement of the human body structure, it is important to determine the tasks that are held by skeleton. The initial task of this system is: supporting the limbs, organs, muscles and tissues etc. In macro perspective, the main part of the skeleton system is the bones and the joints (articulations). If the behavior of the bones will be understood, one of the most important problems of the whole skeleton system will be evaluated. To determine the supporting task of the system knowledge of biology, anatomy, statics and strength of materials can be used together. All of this knowledge comes together under the biomechanics title. Recent 20 years, too many biomechanics applications were figured out about the structure and the mechanical behaviors of different kind of bones. Especially long bones of the limbs are subjected to compression and bending loads in general. In this study, the mechanical behaviors like stresses (pressures), strains, displacements and deformations for the critical regions of the specimens taken from medial parts of the sheep metacarpus bones in elastic bending conditions were determined by analytical methods with the help of basic mechanics and the processed 10 bone bending experimental results are calculated well with the generated methods to create graphics which are used for the most of the engineering applications. However, the useful analytical methods for suitable calculations are presented for the different kind of stresses for a variety of the bone structure. With this study it is determined that, generated analytical methods are eligible enough for the calculation of the values of the needed expressions and preparations of the graphical backgrounds.

**Keywords.** Bone, three point bending, stress, strain, deformation, displacement.

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## A New Crank Nicholson Type Difference Scheme For Time Fractional Heat Equations

Ibrahim Karatay<sup>1</sup>, Nurdane Kale<sup>2</sup>, Serife R. Bayramoglu<sup>3</sup>

**Abstract.** In this paper, we consider the numerical solution of a time-fractional heat equation with Caputo derivative. We propose an approximation based on the Crank Nicolson method. The solvability, stability and convergence of the proposed method are analyzed using the technique of Fourier analysis. The convergence order of the method is  $O(\tau^{2-\alpha} + h^2)$  where  $\tau$  is the temporal grid size and  $h$  is the spatial grid size. Some numerical results, figures and comparisons are presented to demonstrate the effectiveness of theoretical analysis and accuracy of the method.

**Keywords.** Crank-Nicholson Difference Schemes, Initial Value Problems for Time-Fractional Heat Equations, Caputo Derivative, Stability, Convergence, method of Fourier analysis.

**MSC 2010.** 65M12, 65M06 .

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## Analysis of Differential Equation with Finite Difference Scheme

Ibrahim Senturk<sup>1</sup> and Muhammet Kurulay<sup>2</sup>

**Abstract.** In this study, we solve differential equations by using finite difference methods. The different finite difference schemes are explicit scheme and Crank-Nicholson implicit scheme. The Analysis of stability of the explicit scheme is done. Crank-Nicholson implicit scheme is more accurate than explicit scheme and the differences of Analytic and Approximate solutions are discussed with using numerical examples. Without any numerical integration the differential equation is reduced to a system of algebraic equations via new accurate explicit approximations of the inner products, the evaluation of which is needed to solve matrix system. The solution is obtained by constructing the linear (or nonlinear) matrix system using Maple and the accuracy is compared with the exact solutions. The technique presented here is that the solution obtained is valid for various boundary conditions both linear and nonlinear equations.

**Keywords.** Dirichlet Boundary Conditions, Burger's Equation, Finite Difference Method.

**AMS 2010.** 65N06, 35M12.

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## **A Nonlinear Integral Equation of Convolution Type with Application in Astronomy**

Juergen Batt <sup>1</sup>

**Abstract.** The present talk contributes to the theory of the VLASOV-POISSON System of partial differential equations which describes mass systems subject to their own gravity. The existence of stationary spherical symmetric solutions have been proven in J.Batt, W.Faltenbacher, E.Horst, *Rational Mech. Anal.* 93 (1986). We prove the existence of "flat" such solutions (with the mass extended in a plane) by solving a nonlinear integral equation of convolution type with a singular kernel in the plane (joint work with E.Joern and Y.Li).

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## Selfadjoint Realization and Green' s Function of One Boundary-Value Problem with Singularities

Kadriye Aydemir<sup>1</sup> and Oktay Muhtaroglu<sup>2</sup>

**Abstract.** The purpose of this paper, is to show Green function and Resolvent operator of new type discontinuous boundary-value problems, which consists of a Sturm-Liouville equation together with eigenparameter-dependent boundary and transmission conditions. Some of the mathematical aspects necessary for developing own technique presented. By applying this technique we construct some special solutions of the homogeneous equation and present a formula of Green' s function. Further by using this results we investigate the resolvent operator the considered problem.

**AMS 2010.** 34B24, 34B27.

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## Spectrum of a New Class Boundary Value Problems

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**Abstract.** In this paper we are concerned with a new class of BVP' s consisting of Sturm-Liouville equation on two disjoint intervals together with eigendependent boundary conditions and supplementary transmission conditions at interface points. By suggesting an own approaches we investigate some spectral properties of the considered problem. In particular we find asymptotic formulas for the eigenvalues and eigenfunctions.

**Keywords.** Sturm-Liouville problems, eigenvalue, eigenfunction, asymptotics of eigenvalues and eigenfunction.

**AMS 2010.** 34B24, 34L15

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## **Consistent Sampling**

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**Abstract.** Sampling is the process of representing a continuous time signal by a discrete set of measurements, which are usually sample values of a signal at some instances. Using discrete sample values is ideal but in a more practical situation, nonideal samples can be given as inner products of a signal with a set of sample functions associated with the acquisition devices. Here we consider the problem of recovering any signal  $f(t)$  in  $L^2(R)$  from nonideal samples. The reconstructed signal  $\tilde{f}(t)$  of  $f(t)$  is sought in a shift invariant space with multi generators in  $L^2(R)$ . Hence in general, we can not expect the exact reconstruction of  $f(t)$  but we seek an approximation  $\tilde{f}(t)$  of  $f(t)$ , which is consistent with the input signal in the sense that it produces exactly the same measurements as the original input signal when it is reinjected into the system. It means that  $f(t)$  and  $\tilde{f}(t)$  are essentially the same to the end users, who can observe signals only through the given measurements. We also discuss the performance analysis of the proposed generalized sampling scheme.

**Key words.** sampling, consistency, shift-invariant space.

**AMS 2010.** 42C15, 94A20.

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## Symbolic Computing High-Accuracy Algorithm for the Separation of the Fractional-Rational Matrices

L.F. Agamalieva<sup>1</sup>

**Abstract.** In the work high-accuracy algorithm is developed for the separation of the fractional-rational matrices with respect to imaginary axis and unit circle on the base of MATLAB symbolic computing procedures.

Consider the separation algorithm in the case of proper fraction. Let the transfer function  $V$  be a proper function,

$$V_+ + V_- = \left[ \frac{b_{ij}(s)}{a_{ij}(s)} \right] = \left[ \frac{b_{1ij}(s)}{a_{1ij}(s)} \right] + \left[ \frac{b_{2ij}(s)}{a_{2ij}(s)} \right],$$

zeros of the polynomials  $a_1(s)$ ,  $a_2(s)$  lie in the opposite sides of the imaginary axis,  $n_l$  is an order of the polynomial  $a_l(s)$ ,  $l = 1, 2$ ,  $n_1 + n_2 = n$ ,  $n_1 \leq n_2$ .

Suppose that the polynomial  $a_1(s)$  is computed (we omit the indexes  $i, j$  опускаются). From  $a(s) = a_1(s)a_2(s)$  we obtain

$$a_2(s) = \frac{a(s)}{a_1(s)}$$

After defining  $a_1(s)$  и  $a_2(s)$  the plynomials  $b_1(s)$  и  $b_2(s)$  are found as a solution of the polynomial equation

$$b(s) = a_2(s)b_1(s) + a_1(s)b_2(s).$$

**Keywords.** separation, fractional-rational matrices, high-accuracy algorithm.

**AMS 2010.** 65Y04.

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## A New Metaheuristic Cricket-Inspired Algorithm

Murat Canayaz<sup>1</sup> and Ali Karci<sup>2</sup>

**Abstract.** Meta-heuristic algorithms enhanced by inspiring from the behaviors of livings in nature take very important roles in solutions of optimization problems. In this study, a new meta-heuristic algorithm having been constituted by inspiring from the attitudes of a kind of insect known as cricket is presented. In the phase of composition of new algorithm, some meta-heuristic algorithms such as Firefly and Bat are studied and the similar ways of these algorithms are applied. Further, a new approach applied for the analyses of optimization problems by using its own authentic characteristics such as making a forecast of the weather by courtesy of the number of fluttering of cricket and the audio frequency in this temperature is developed. The results of applications enhanced in concern with algorithms are compared with the results of algorithms including Firefly, Bat, Particle Swarm, Genetic Algorithm.

**Keywords.** Metaheuristic Algorithms, Firefly, Bat Algorithm

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## A Comparative Analysis of Galerkin and Hybridizable Discontinuous Galerkin Method On Second Order Differential Equation

Mehmet Fatih Karaaslan<sup>1</sup>, Fatih Çeliker<sup>2</sup>, Muhammet Kurulay<sup>3</sup>

**Abstract.** In this study, we are mainly concerned with Hybridizable Discontinuous Galerkin method which is a technological numerical method for differential equations. For that purpose, we investigate this method on boundary value problems for second order differential equations to see its action. Therefore, we compare the results of HDG and continuous Galerkin method. This gains us a different point of view to HDG method because of its efficient implementation.

**Keywords.** Hybridizable Discontinuous Galerkin Method, Local Solvers, Transmission Condition.

**AMS 2010.** 65L10, 65L60.

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## A Modified Piecewise Variational Iteration Method for Solving a Neutral Functional-Differential Equation with Proportional Delays

Mehmet Gıyas Sakar<sup>1</sup>, Fevzi Erdoğan<sup>2</sup>

**Abstract.** In this paper, a modified piecewise variational iteration method is reintroduced with the enhancement of Padé approximants to lengthen the interval of convergence of variational iteration method when used alone in solving a neutral functional-differential equation with proportional delays. Some examples are displayed to demonstrate the computation efficiency of the method. We also compare the performance of the method with that of a particular Runge-Kutta method, variational iteration method and other methods.

**Keywords.** A modified piecewise variational iteration method, Neutral functional-differential equation, proportional delay, Padé approximation.

AMS. 34K28, 34K40

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## **Basins of Attraction of Equilibrium Points and Periodic Solutions of a Certain Rational Difference Equation with Quadratic Terms**

M. Garić-Demirović, M.R.S. Kulenović and M. Nurkanović

**Abstract.** We investigate the basins of attraction of equilibrium points and period-two solutions of the difference equation of the form

$$x_{n+1} = \frac{x_{n-1}^2}{ax_n^2 + bx_n x_{n-1} + cx_{n-1}^2}, \quad n = 0, 1, 2, \dots$$

where the parameters  $a, b, c$  are positive numbers and the initial conditions  $x_{-1}, x_0$  are arbitrary nonnegative numbers. We show that the boundaries of the basins of attractions of different locally asymptotically stable equilibrium points are in fact the global stable manifold of neighboring saddle or non-hyperbolic equilibrium points.

**Keywords.** Keywords: attractivity, basin, difference equation, invariant sets, periodic solutions, stable set

**AMS 2010.** 53A40, 20M15.

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## Numerical-Analytical Method for Determining Points of Branching for Circular Corrugated Membranes

Mikhail Karyakin<sup>1</sup>, Georgy Mostipan<sup>2</sup> and Yuri A. Ustinov<sup>3</sup>

**Abstract.** The paper presents results of investigation of the equilibrium and stability for circular membranes with arbitrary profile along the meridian. Spherical dome with a possible deviation from the ideal surface can be considered as a special case. We consider the general problem for a circular shell of revolution assuming that its' profile is given by the single-valued elevation function in cylindrical coordinates. To describe the large elastic strains of the membrane under hydrostatic pressure we use two-dimensional nonlinear equations [1], based on the Kirchhoff's hypotheses. The axisymmetric behavior of the shell can be described by the 6th order boundary value problem for a ODE system. Special approaches to its' effective numerical solving are discussed.

The scheme for investigating of stability of the constructed axisymmetric solution is based on the bifurcation approach. To eliminate the bifurcation points for nonaxisymmetric modes that introduce an uncertainty in the type and nature of the stability loss and post-critical behavior of the perfect spherical dome special small shape modulations can be used. As an illustration, we present a simple model of such modulation scheme. The important distinction is the absence of bifurcation points on the rising portion of the loading diagram for the modulated dome. In contrast to the comparable spherical domes the modulated membrane has no bifurcation points for modes with  $n > 1$ .

**Keywords.** Membrane, stability, bifurcation.

**AMS 2010.** 74K15, 74G60, 65L10

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### On Uniqueness Classes in Heat Conduction Problems

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**Abstract.** In many applications there is a need of the solution of the model boundary value problems for the heat equation in regions with mobile boundaries.

**Statement of the problem.** For the homogeneous boundary value problem:

$$u_t(x,t) - a^2 u_{xx}(x,t) = 0, \quad u(x,t)|_{x=0} = 0, \quad u(x,t)|_{x=bt} = 0, \quad (1)$$

considered in the region  $G = \{(x,t) : t > 0, \quad 0 < x < bt\}$ , it is required to find a function class of existence of the non-trivial solution where  $0 < b = \text{const} < \infty$ .

**Theorem (main result).** *In a weight class of essentially bounded functions*

$$U = \left\{ u \mid \exp\left(x^2 / (4a^2 t)\right) [\gamma(x,t)]^{-1} u(x,t) \in L_\infty(G) \right\}, \quad (2)$$

where  $\gamma(x,t) = \max_{\{x,t\} \in G} \left\{ \sqrt{bt} / (bt - x); \quad \exp\left((2bt - x)^2 / (4a^2 t)\right) \right\}$ , the boundary value problem (1)

has only one non-trivial solution.

The boundary value problem (1) reduces to studying of special Volterra integral equation of the second kind for which, applying a method of Carleman-Vekua's regularization, we receive Abel's non-homogeneous equation of the second kind. Uniqueness of the solution of the last equation [1] establishes a existence of non-trivial solution of the boundary value problem (1) from weight class of essentially bounded functions (2).

**Remark.** The numerous literature is devoted to uniqueness classes for the parabolic boundary value problems: Holmgren E., Tikhonov A.N., Tacklind S., Ladyzhenskaya O.A., Oleinik O.A., Radkevich E.V., Mikhailov V.P., Kozhevnikova L.M. et.al.

**Keywords.** Uniqueness classes, heat boundary value problem, non-trivial solution.

**AMS 2010.** 35K05; 45D05.

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## Stability "In the Large" of Movement of Models of Phase Systems on a Finite Interval of Time

M.N.Kalimoldayev<sup>1</sup> and L.S.Kopbosyn<sup>2</sup>

**Abstract.** In report sufficient stability conditions "in the large" of models of phase systems are obtained.

Consider the general model of phase systems:

$$\frac{d\delta_i}{dt} = s_i, \quad \frac{ds_i}{dt} = w_i - D_i s_i - f_i(\delta_i) - \psi_i, \quad w_i = c_i^* x_i, \quad (1)$$

$$\frac{dx_i}{dt} = A_i x_i + q_i s_i + b_i u_i, \quad i = \overline{1, l}, \quad t \in [t_0, T], \quad (2)$$

where  $\delta_i$  is angular coordinate;  $S_i$  is angular velocity;  $x_i$  is  $n_i$ - vector of the state regulator;  $u_i$  is feedback control. Let  $f_i(\delta_i)$  – nonlinearity in the control object, is  $2\pi$  – continuously differentiable periodic function. In a specific power system the function  $\psi_i(\delta_{i^*})$  defines the interaction of the  $i$ -th generator with other generators in the system.

The task is to study the stability "in the large" of the system (1), (2).

**Theorem.** Let the following conditions hold: 1) function  $f_i(\delta_i)$  satisfies condition

$$f_i(\delta_{0i}) = \frac{1}{T_i} [P_{ij} \sin(\delta_{i0} + \delta_{ji}) - P_i \sin \delta_{i0}], \quad i = \overline{1, l}; \quad 2) \text{ function } P_{ij}(\lambda) d\lambda \text{ satisfies conditions}$$

$$P_{ij}(\lambda) d\lambda = P_{ij}(\lambda), \quad P_{ij}(\lambda) = -P_{ij}(-\lambda), \quad P_{ij}(\delta_{ij}) \delta_{ij} \geq 0; \quad 3) \text{ constants } \alpha_i, D_i > 0 \text{ such that}$$

$$a) \alpha_i = \frac{K}{D_i}, \quad 0 < K < \min\{D_1, \dots, D_l\}, \quad b) f_i^1(0) \neq \alpha_i D_i^2 (1 - \alpha_i). \text{ Then the zero equilibrium } T_0$$

is asymptotically stable in the Lyapunov sense and internal evaluation of the domain of attraction of a singular point  $T_0$  is determined by the area bounded by the surface  $V(\delta, s) = T$ , where  $T = \min_{1 \leq i \leq N} V(T_i)$ , if  $T_i, i = \overline{1, N}$  are unstable singular points of the system (2).

A concrete example of system "the synchronous generator – the steam turbine" is reviewed.

**Keywords.** Model, phase systems, stability, control.

**AMS 2010.** 53A40, 20M15.

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## Solution to the Problem of Global Asymptotic Stability of Dynamical Systems

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**Abstract.** The problem of global asymptotic stability of dynamical systems, which is one of the key problems in the synchronization theory of various control systems, is solved.

We consider the dynamical system with cylindrical phase space  $Z_i^{n+2}(\delta, S, x)$ :

$$\frac{d\delta_i}{dt} = S_i, \quad \frac{dS_i}{dt} = W_i - K_i S_i - f_i(\delta_i), \quad w_i = c_i^* x_i, \quad i = \overline{1, l}, \quad (1)$$

$$\frac{dx_i}{dt} = A_i x_i + q_i S_i + b_i u_i, \quad i = \overline{1, l}, \quad (2)$$

where  $\delta_i$  is angular coordinate;  $S_i$  is angular velocity;  $x_i$  is  $n_i$ - vector of the state regulator;  $u_i$  is feedback control.

Equations (1) describe the operation of the control object, and (2) - the state regulator.

The task is to study the global asymptotic stability of the system (1), (2).

**Theorem.** Let there exist scalar  $D_i > 0$  such that  $D_i > (D_i)_{cr}$ , the matrix  $\tilde{A}_i$  is Hurwitz, the pair  $(\tilde{A}_i, q_i)$  is completely controllable, the pair  $(\tilde{A}_i, C_i)$  is completely observable,  $\tilde{D}_i > 0$ ,  $Z_i \geq 0$ ,  $d_i - \kappa_i h_i^* \tilde{D}_i^{-1} h_i \neq 0$ ,  $\Gamma_i + \text{Re} W_i(j\omega) \geq 0$ ,  $(\omega \in (-\infty, +\infty))$ ,  $i = \overline{1, l}$ . Then the control

$$u_i = a_i^* x_i + \theta_i S_i + \frac{S_i \psi_i(\delta_{i^*})}{x_i H_i b_i}, \quad Z_i \in \Sigma_i,$$

$$u_i = \bar{a}_i x_i + \bar{\theta}_i S_i, \quad Z_i \notin \Sigma_i$$

provides global asymptotic stability of the motion of system (1), (2).

A concrete example of two-machine power system is reviewed.

**Keywords.** Dynamical system, global asymptotic stability, synchronization, control, power system.

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## Modified Trial Equation Method for Solving Some Nonlinear Evolution Equations

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**Abstract.** In this paper we investigate the exact solutions of the nonlinear partial differential equations. An improved form of the trial equation method [1-3], a modified trial equation method is applied to solve nonlinear evolution equations arising in mathematical physics. Using this method, some exact solutions of the Gilson-Pickering equation [4] and the generalized Fisher equation [5] are obtained. We believe that this method is applicable for the other nonlinear evolution equations with generalized evolution.

**Keywords.** Nonlinear evolution equations, The modified trial equation method, Gilson-Pickering equation, Generalized Fisher equation.

**AMS 2010.** 47J35, 35A25, 35Q99.

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## On The Global Mass of Electro-Scalar Spacetimes

Martin Scholtz

**Abstract.** Perhaps the most frustrating long-standing problem in the general theory of relativity is the concept of the energy, the momentum and the angular momentum. By the diffeomorphism invariance of the theory, there is no way how to introduce these quantities at the local level. Hence, an enormous effort have been spent in order to replace the notion of local energy (momentum, angular momentum) by the notion of *quasi-local* energy which is supposed to be associated with a finite domain of the spacetime (e.g., globally hyperbolic domains with compact closure). A lot of suggestions in this direction have been made, many of them being given as the surface integral of the spinorial Nester-Witten form: the Penrose mass, the Hawking mass, the Brown-York mass.

Nevertheless, there is a well-defined concept of *global energy* of the spacetime provided the spacetime is asymptotically flat. In this case there is an asymptotic group of symmetries (the Bondi-Metzner-Sachs group) which resembles the Poincare group of the flat Minkowski spacetime. The so-called Bondi mass then corresponds to the global energy of the spacetime as measured at the conformal boundary of the spacetime. Expressions for the Bondi mass of vacuum spacetimes and spacetimes with electromagnetic sources are well known. In [1] we have derived corresponding expressions for the spacetimes with scalar and conformal-scalar field sources.

In this talk we briefly review the concept of the Penrose quasi-local mass which is motivated by the twistorial techniques and discuss the connection between the Penrose mass and the Bondi mass. Then we present new calculation of the Bondi mass for asymptotically flat spacetimes with interacting electromagnetic and scalar field sources (gauge theory with  $U(1)$  as the gauge group) and sketch possible physical implications in the theory of the boson stars.

**Keywords.** Quasi-local mass, spinors, asymptotically flat spacetimes.<sup>1</sup>

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## Neumann Boundary Control in a Parabolic System

Murat Subaşı<sup>1</sup>, S. Şule Şener<sup>2</sup> and Hülya Durur<sup>3</sup>

**Abstract.** Neumann boundary control problems with different objectives take great deal in last years[1-5]. In this study, we aim to control the Neumann boundary condition  $g(t)$  in the parabolic system:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + h(x,t), \quad t \in (0,T), \quad x \in (0,l)$$

$$u(x,0) = u_0(x), \quad x \in (0,l)$$

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(l,t) = g(t), \quad t \in (0,T).$$

We use the weak solution approach to get the solution of the parabolic system. The cost functional we consider is

$$J(g) = \int_0^l [u(x,T;g) - y(x)]^2 dx$$

which minimizes the difference between the solution of the system at a final time  $t = T$  and the desired target  $y(x)$  in  $L_2$  norm. Obtaining adjoint problem and Frechet derivative, we get the optimal solution constituting minimizing sequences by gradient method.

**Keywords.** Linear parabolic equation, Optimal boundary control problem, Ill-posed problems.

**AMS 2010.** 49K20, 49M05,47A52

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### Ill-Posed Problem for the Biharmonic Equation

M.T. Jenaliyev<sup>1</sup>, K.B. Imanberdiyev<sup>2</sup> and K.A. Aimenova<sup>3</sup>

**Abstract.** Recently, among the experts on the equations of mathematical physics is considerably increased interest to problems that are ill-posed by Hadamard J. [1]. Problems of this kind are always attracted the attention of researchers. First of all, this is due not only to their importance in theory, but also to the fact that they have to be faced in many applications in various fields of science and technology. Due to the ill-posed problems can be noted classical works of Hadamard J. [1], Tikhonov A.N. [2], Lavrentiev M.M. [3] and many others, who paid the attention of researchers to the ill-posed problems and brought a significant contribution to the development of this important area of mathematics.

We consider the following boundary value problem in domain  $Q = \{x, y \mid 0 < x < 2\pi, 0 < y < 1\}$ :

$$\Delta^2 u = f, \{x \in (0, 2\pi), y \in (0, 1)\} = Q; \quad (1)$$

$$u(0, y) = u_x(0, y) = 0, \quad u(2\pi, y) = u_x(2\pi, y) = 0; \quad (2)$$

$$u(x, 0) = 0, \quad u_y(x, 0) = \varphi_1(x), \quad u_{yy}(x, 0) = 0, \quad u_{yy}(x, 1) = 0, \quad (3)$$

$$u(x, 1) \in U_g - \text{convex closed set of } H_0^{3/2}(0, 2\pi). \quad (4)$$

It is supposed that known functions in the problem (1) – (3) satisfy the following condition:

$$f \in (\tilde{H}^2(Q))', \quad \varphi_1(x) \in H_0^{1/2}(0, 2\pi). \quad (5)$$

In this paper we use optimal control method to solve problem (1) – (5).

**Keywords.** Biharmonic equation, ill-posed problem, optimal control.

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## Hermitian Determinantal Representations of Polyomial Curves

Mao-Ting Chien<sup>1</sup>

**Abstract.** Let  $p(\theta) = \sum_{k=-n}^n c_k \exp(i k \theta)$ ,  $0 \leq \theta \leq 2\pi$ , be a trigonometric polynomial. We propose algorithms by Bezout method and Sylvester elimination which produce a determinantal representation for the polynomial curve in the sense that there exist  $2n$ -by- $2n$  matrices  $S_1, S_2, S_3$  such that  $\det(S_1 + \operatorname{Re}(p(\theta)) S_2 + \operatorname{Im}(p(\theta)) S_3) = 0$ ,  $0 \leq \theta \leq 2\pi$ . For typical trigonometric polynomials, we assert that  $S_1$  is positive definite.

**Keywords.** Determinantal representation, Bezout method, Sylvester elimination.

**AMS 2010.** 15A15, 13P15, 42A05.

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## **Inverse Problem for Optimal Regulator Over the Part of Phase Coordinate**

N. I. Velieva<sup>1</sup>

**Abstract.** In the work inverse problem for stationary system by static output feedback is formulated. Calculating algorithm to solution of this problem basing on the linear matrix inequities (LMI) method is suggested. The results are illustrated by examples.

Direct problem for linear-quadratic (LQ) optimal stabilizing regulators over all phase coordinates was considered enough well with using both time –invariant, frequency methods .In direct problem it needs to define the regulator under condition of the asymptotically stability corresponding closed-loop system, which optimize performance index. The algorithm for solving the direct problem has been reduced to the standard procedure of the construction stabilizing solution of the algebraic Riccati equation (ARE). And for inverse problem (the determine weight matrices square-low functional, corresponding to optimal regulator) the picture other. For the first time the inverse problem have been considered in work [1] and further was investigated in [2]. These works in the main has theoretical character. The algorithm, basing on the calculation of eigenvalues and eigenvectors of the corresponding matrix, which realization meets some difficulties. However in work [2] with using LMI method has been suggested the calculating algorithm for solving inverse problem of the linear –quadratic optimal regulator, which more effective with calculating point of view. In the case, when is measured not all phase coordinates for determine stabilizing regulator in stationary case there are as theoretical, as calculating methods. The inverse problem for optimal regulator over the part of phase coordinate, have been considered in work [3] for problems of the controlling mechanical system with flexible structure. There authors considered the special case, i.e., it is supposed that the optimal value of the functional over of the part of phase coordinate and over all phase coordinate coincide. Therefore there is meaning to consider this problem again. For solving of the inverse problem of synthesis of the optimal stabilizing regulator to output variable the approach based on the LMI methods will be used. The results are illustrated by examples.

**Keywords.** static output feedback, periodic systems, inverse problem, numerical methods, Lyapunov's equation.

**AMS 2010.** 65Y04.

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## One BVP's Consisting of Differential Operator Equation and Boundary-Transmission Conditions

Oktay Muhtaroglu<sup>1</sup>, Merve Coğan<sup>2</sup> and Kadriye Aydemir<sup>3</sup>

**Abstract.** In this study some spectral properties of one BVP consisting of differential-operator equation and boundary-transmission conditions are investigated. At first we construct some eigensolutions of the pure differential part of the considered problem. Then by using an operator formulation and applying comparison theorems we derive an asymptotic formulas for eigenvalues of main problem.

**Keywords.** Sturm-Liouville problem, eigenvalue, eigenfunction, differential-operator equation

**AMS 2010.** 34B24, 34L15

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## The Asymptotic Behaviour of the Linear Transmission Problem in Viscoelasticity

Octavio Paulo Vera Villagran<sup>1</sup>

**Abstract.** We consider a transmission problem with localized Kelvin-Voigt viscoelastic damping. Our main result is to show that the corresponding semigroup  $S_{\{A\}}(t)$  is not exponentially stable, but the solution of the system decays polynomially to zero as  $1/t^{\{2\}}$  when the initial data are taken over the domain  $D(A)$ . Moreover, we prove that this rate of decay is optimal.

**Keywords.** Kelvin-Voigt, Viscoelastic damping, Transmission problem, Semigroup polynomial decay.

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## An Initial Value Problem In Fuzzy Theory

Ömer Akın<sup>1</sup>, Tahir Khaniev<sup>2</sup>, Fikri Gökpinar<sup>3</sup> and I.Burhan Turksen<sup>4</sup>

**Abstract.** Buckley and Feuring [1] have considered the initial value problem for n-th order fuzzy differential equations. In their problem only the initial values are fuzzy numbers. In Akın et. al. [2], a similar fuzzy initial value problem, which has fuzzy coefficients and fuzzy forcing functions has been solved according to the signs of solution and the first and second derivatives of the solution.

In this study, we are investigating the initial value problem for the second order differential equation when both initial values and forcing function are fuzzy. Here, we applied Zadeh's Extension Principle and obtained the analytical form of alpha cuts for the solution of the fuzzy initial value problem with the aid of an indicator operator.

**Keywords.** Second Order Fuzzy Differential Equation, Fuzzy Initial Value Problem, Fuzzy Forcing Function, Zadeh's Extension Principle

**AMS 2010.** 34A07, 03B52.

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## Application of the Exp-function Method to Some Space-Time Fractional Partial Differential Equation

Özkan Güner<sup>1</sup>, Ahmet Bekir<sup>2</sup> and Adem C. Çevikel<sup>3</sup>

**Abstract.** Fractional differential equations (FDEs) are viewed as alternative models to nonlinear differential equations. Varieties of them play important roles and serve as tools not only in mathematics, but also in physics, biology, control theory, systems identification and fractional dynamics to create the mathematical modeling of many nonlinear phenomena.

In recent decades, some effective methods for fractional calculus were appeared in open literature, such as the exp-function method [1], the  $(G'/G)$ -expansion method [2], the first integral method [3] and the fractional sub-equation method [4]. The fractional complex transform [5] has been suggested to convert fractional order differential equations with modified Riemann-Liouville derivatives into integer order differential equations, and the reduced equations can be solved by symbolic computation. The exp-function method [6] can be used to construct the exact solutions for fractional differential equations. The present paper investigates for the applicability and efficiency of the exp-function method on fractional nonlinear differential equations. We applied exp-function method with fractional complex transform for space-time fractional mBBM equation [7], space-time fractional ZK-BBM equation [7] and obtained some new results.

**Keywords.** Fractional differential equation, Exact solutions, Exp-function method, Space-time fractional mBBM equation, Space-time fractional ZK-BBM equation

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## Determining Modes For The $g$ -Navier Stokes Equations

Özge Kazar<sup>1</sup>, Meryem Kaya<sup>2</sup>

**Abstract.** Using the results of the weak solutions of the  $g$ -Navier Stokes equations are given by Roh [4-5], we derive upper bounds for the number of determining modes for the following 2D  $g$ -Navier Stokes equations:

$$\frac{\partial u}{\partial t} - \nu \Delta u + (u \cdot \nabla) u + \nabla p = f$$
$$\nabla \cdot (gu) = 0.$$

**Keywords.**  $g$ -Navier Stokes equations, determining modes

**AMS 2010.** 35Q30, 35Q35, 76D05.

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## Rates of Approximation of Bounded Variation Functions by an Operator Involving Laguerre Polynomial of Degree n

Özlem Öksüzer<sup>1</sup>, Harun Karslı<sup>2</sup> and Fatma Taşdelen Yeşildal<sup>3</sup>

**Abstract.** In this work, we estimate the rate of convergence on function of bounded variation for an operator which is involving Laguerre polynomial of degree n. To prove our main result, we have used some methods and techniques of probability theory.

**Keywords.** Laguerre polynomial, rate of convergence, function of bounded variation, Lebesgue Stieltjes integration

**AMS 2010.** 53A40, 20M15.

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## **On Hamiltonian Realizations of Real Linear Differential Systems**

Răzvan M. Tudoran<sup>1</sup>

**Abstract.** The purpose of this talk is to show that any non-degenerate real  $n$ -dimensional linear differential system, can be explicitly realized as a completely integrable Hamiltonian differential system on a Poisson manifold. As a consequence, one obtain that the phase portrait of any real  $n$ -dimensional linear differential system is determined by the momentum map associated with his completely integrable Hamiltonian realization.

**Keywords.** Hamiltonian dynamics; linear differential equations.

**AMS 2010.** 37J35, 37K10, 70H06.

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## **The Application of Electromagnetism-like Algorithm for the Dynamic Deployment Problem in Wireless Sensor Networks**

Recep Özdağ<sup>1</sup> and Ali Karci<sup>2</sup>

**Abstract.** As the usage of wireless sensor networks is increasing day by day the problems regarding these networks are becoming more noticeable. The sensors in wireless networks are causing dynamic deployment problems inefficiently. The dynamic deployment of the sensors is one of the most important issues that affects the performance of the wireless network sensors. Because of an effective dynamic deployment is actualized, the detection capacity of the sensor increases and as the coverage capacity goes up the efficiency of the wireless sensor networks rises accordingly. It is estimated that realistic outcomes will be obtained while an effective coverage area is calculated with binary detection model.

In this study, with Electromagnetism-like Algorithm (EMA), which contains population based and optimization algorithm, the coverage area of the Networks is tried to increase and dynamic deployment has been applied hoping to have better performance. EMA is a relatively new population-based meta-heuristic algorithm and introduced to solve continuous optimization models using bounded variables. The Electromagnetism-like algorithm is based on the attraction and repulsion of charged particles and it is designed to optimize non-linear, real-valued problems. Movement based on attraction and repulsion is introduced by Coulomb's law, i.e., the force is inversely proportional to the distance between the particles and directly proportional to the product of their charges. In EMA methodology, every particle represents a solution and carries a certain amount of charge which is proportional to the solution quality. Solutions are in turn defined by position vectors which give the real positions of particles in a multi-dimensional space. It has the advantages of multiple search, global optimization, and faster convergence and simultaneously evaluates many points in the search space. Simulation outcomes have shown that electromagnetism-like algorithm could be preferred in dynamic deployment of wireless network sensors.

**Keywords.** Electromagnetism-like algorithm, wireless sensor networks, dynamic deployment.

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## Explicit Solution of Singular Differential Equation by means of Fractional Calculus Operators

Resat Yilmazer<sup>1</sup> and Okkes Ozturk<sup>2</sup>

**Abstract.** In this study, using fractional calculus operator, we obtain explicit solution of the following second order linear ordinary differential equation

$$z^2 \frac{d^2 \phi}{dz^2} + z \frac{d\phi}{dz} + \left[ \left( \frac{\sqrt{4\sigma-1}}{2} z \right)^2 - \left( \frac{\tau-1}{2} \right)^2 \right] \phi(z) = 0,$$

where  $\sigma, \tau$  are parameters.

**Keywords.** Fractional calculus; Ordinary differential equation; Explicit solution.

**AMS 2010.** 26A33, 34A08

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## **Fuzzy Difference Equations in Finance**

Sabri Ali Ümekkan<sup>1</sup>, Aylin Bayrak<sup>2</sup> and Emine Can<sup>1</sup>

**Abstract.** Fuzzy difference equations initially introduced by Kandel and Byatt [1,2]. An important effort to study such of equations has been made by Buckley [3]. In this work we apply the method of fuzzy difference equations to study some problems in finance. As a source of different cases of finance equations we use the work by Chrysafis, Papadopolous, Papaschinopoulos [4]. To illustrate the applicability of the method we give some numerical examples.

**Keywords.** Difference Equations, Interval Arithmetic, Interest Rate

**AMS 2010.** 53A40, 20M15.

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## A Numerical Solution of The Modified Regularized Long Wave (MRLW) Equation Using Quartic B-Splines

Seydi Battal Gazi Karakoc<sup>1</sup>, Turabi Geyikli<sup>2</sup> and Ali Bashan<sup>3</sup>

**Abstract.** In this study, a numerical solution of the modified regularized long wave (MRLW) equation is obtained by subdomain finite element method using quartic B-spline functions. Solitary wave motion, interaction of two and three solitary waves and the development of the Maxwellian initial condition into solitary waves are studied using the proposed method. Accuracy and efficiency of the proposed method are tested by calculating the numerical conserved laws and error norms L2 and L1 . The obtained results show that the method is an effective numerical scheme to solve the MRLW equation. In addition, a linear stability analysis of the scheme is found to be unconditionally stable.

**Keywords.** MRLW equation, Finite element method, Subdomain, Quartic BSplines, Solitary waves.

**AMS 2010.** 97N40, 65N30, 65D07, 76B25, 74S05,74J35

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## **Comparison Between the Homotopy Analysis Method and Homotopy Perturbation Method to Schrödinger Equation**

Sema Gülbahar<sup>1</sup> and Yaşar Aslan<sup>2</sup>

**Abstract.** In this study, first homotopy analysis method and perturbation method homotopy basic concepts expressed and then these methods are used to obtain exact solutions of Schrödinger equation . Homotopy analysis and homotopy perturbation methods were compared. Appropriate values were determined and the results were compared with each other and with the exact solutions.  $\hbar$  values were determined and a suitable results were compared with each other and exact solutions. Advantages and limitations are discussed at the end.

**Keywords.** Exact-Numerical solutions, Homotopy analysis method.

**AMS 2010.** 34K28, 65H20, 83C15.

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## On Approximate Solutions of a Boundary Value Problem with Retarded Argument

Seda İğret<sup>1</sup> and Arzu Aykut<sup>2</sup>

**Abstract.** In this paper, we applied approximate methods for the solution of a boundary value problem with retarded argument:

$$\left. \begin{aligned} x''(t) + a(t)x(t - \tau(t)) &= f(t), \quad (0 < t < T) \\ x(t) &= \varphi(t) \quad (\lambda_0 \leq t \leq 0) \quad x(T) = x(c) \quad (0 < c < T) \end{aligned} \right\} \quad (1)$$

where  $(0 \leq t \leq T)$  and  $a(t) \geq 0, f(t) \geq 0, \tau(t) \geq 0, (0 \leq t \leq T)$  and  $\varphi(t), (\lambda_0 \leq t \leq 0)$  are known continuous functions.

**Keywords.** Fredholm-Volterra integral equations, differential equation with retarded argument, successive approximation method.

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## The Influence of Volterra Type Integral Perturbations to the Boundedness of Solutions of The Weak Nonlinear Third Order Differential Equation with Delays

S.Iskandarov<sup>1</sup> and M.A. Temirov<sup>2</sup>

**Abstract.** All presented functions of  $t, (t, \tau)$  are continuous and relations is true in  $t \geq t_0, t \geq \tau \geq t_0$ ; functions of  $x_j, y_j, z_j$  ( $j = 1..m$ ) are continuous in  $R$ ;  $J = [t_0, \infty)$ ; IDE – integro-differential equation; DE – differential equation.

**PROBLEM.** To establish sufficient conditions for the boundedness  $x^{(k)}(t)$  ( $k = 0, 1, 2$ ) on  $J$  solutions and their first, second order derivatives of the Volterra type IDE:

$$\begin{aligned}
 & x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) + \\
 & + \int_{t_0}^t [Q_0(t, \tau)x(\tau) + Q_1(t, \tau)x'(\tau) + Q_2(t, \tau)x''(\tau)] d\tau = F_0(t) + \\
 & + \sum_{j=1}^m F_j(t, x(\alpha_j(t)), x'(\beta_j(t)), x''(\gamma_j(t))), \quad t \geq t_0, \tag{1}
 \end{aligned}$$

in the case, when the corresponding DE with delays:  $x'''(t) + a_2(t)x''(t) + a_1(t)x'(t) + a_0(t)x(t) =$

$$= F_0(t) + \sum_{j=1}^m F_j(t, x(\alpha_j(t)), x'(\beta_j(t)), x''(\gamma_j(t))), \quad t \geq t_0 \tag{1_0}$$

can have unboundedness solutions on  $J$ .

In (1), (1<sub>0</sub>) functions  $F_j(t, x_j, y_j, z_j)$  satisfies the condition of "weak nonlinearity":

$$|F_j(t, x_j, y_j, z_j, u_j)| \leq g_{0j}(t)|x_j| + g_{1j}(t)|y_j| + g_{2j}(t)|z_j|, \tag{F}$$

with non-negative "Lipschitz coefficients"  $g_{kj}(t)$  ( $k = 0, 1, 2; j = 1..m$ ); functions

$\alpha_j(t), \beta_j(t), \gamma_j(t)$  ( $j = 1..m$ ) satisfies the condition of delays:

$$\alpha_j(t) \leq t, \beta_j(t) \leq t, \gamma_j(t) \leq t, \sigma_j(t) \leq t, \delta_j(t) \leq t, \mu_j(t) \leq t \quad (j = 1..m). \tag{d}$$

The initial set consists of one point  $t_0$ . It's about solutions  $x(t) \in C^3(J, R)$  IDE (1) with any initial data  $x^{(k)}(t_0)$  ( $k = 0, 1, 2$ ). As we know from the works of Yu.A. Ved and L.N. Kitaeva (1965, 1984) such solutions exist.

Thus, the problem is to identify the Volterra type integral perturbation for boundedness of solutions DE (1<sub>0</sub>). To our knowledge, this problem has not been studied

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previously by anyone. To solve this problem we used nonstandard method of reduction to the system (Iskandarov S. Khalilov AT:  $x'(t) = W(t)x(t)$  ), V.Volterra method of conversion equations, S.Iskandarov's method of cutting functions and S.Iskandarov's lemma of generalized integral inequality of the first kind with delays.

**Keywords.** integro-ordinary differential equation with delays, boundedness, influence.

**AMS 2010.** 53A40, 20M15.

## **The Modified Kudryashov Method for Solving Some Fractional Order Nonlinear Equations**

Serife Muge Ege<sup>1</sup> and Emine Misirli<sup>2</sup>

**Abstract.** The travelling wave solutions of nonlinear evolution equations have important role in many fields of applied sciences. In this study, we construct the analytical solutions of some nonlinear evolution equations involving Jumarie's modified Riemann–Liouville derivative in mathematical physics; namely the space–time fractional modified Benjamin-Bona-Mahony(mBBM) equation and the space–time fractional Potential Kadomstev–Petviashvili (PKP) equation by using modified Kudryashov method. This method is useful and applicable for handling nonlinear wave equations.

**Keywords.** modified Kudryashov method, Benjamin-Bona-Mahony(mBBM) equation, the space–time fractional Potential Kadomstev–Petviashvili(PKP) equation, symbolic computation.

**AMS. :** 35G20, 35G50

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## Lie Symmetry Analysis and Conservation Laws of Zoomeron Equation

Sait San<sup>1</sup> and Emrullah Yaşar<sup>2</sup>

**Abstract.** In this paper; we analyze the Zoomeron equation using the theory of Lie point symmetry reductions of partial differential equations. We show that applicability of the functional variable method for finding the exact solutions of the Zoomeron equation. We calculate the travelling wave solutions in terms of hyperbolic and periodic functions. Furthermore we found the new conservation laws by nonlocal conservation theorem method.

**Keywords.** Conservation laws, Symmetry Generators, Lie Symmetry Analysis, Zoomeron Equation, Functional Variable Method, Exact Solution.

**AMS 2010.** 34C14, 35L65, 83C15.

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## Energy Decay of Solutions of Semilinear Plate Equation on Unbounded Domain

Sema Şimşek<sup>1</sup> and Azer Hanmehmetli<sup>2</sup>

**Abstract.** In this work, we investigate the semilinear damped plate equation  $u_{tt} + \Delta^2 u + a(x)u_t + \alpha u + f(u) = 0$  on unbounded domain and we show that for the weak solution of this problem there exist some constants  $C > 1$  and  $\gamma > 0$  such that  $E(t) \leq CE(0)e^{-\gamma t}$  where  $E(t)$  is the energy functional.

**Keywords.** Plate Equation, Energy Decay, Local Dissipativity, Weak Solution.

**AMS 2010.** 35B40, 35L30, 74H40

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## On One Procedure for Searching a Stable Member

Şerife Yılmaz<sup>1</sup>, Taner Büyükköroğlu<sup>2</sup>, Vakıf Dzhafarov<sup>3</sup>

**Abstract.** A polynomial ( or a square matrix ) is called Hurwitz stable if all roots ( all eigenvalues ) lie in the left half of the complex plane. In linear systems theory Hurwitz stability play an important role.

In this report, we consider the problem of finding a stable member for polynomial and matrix families.

A member of examples are considered.

**Keywords.** Hurwitz stability, matrix family, polynomial family, stable member.

**AMS 2010.** 37B25, 93D20.

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## A Solution of KdVB Equation by Using Quintic B-Spline Differential Quadrature Method

Turabi Geyikli<sup>1</sup>, Ali Bashan<sup>2</sup> and Seydi Battal Gazi Karakoc<sup>3</sup>

**Abstract.** In this paper, the numerical solutions of the KdVB equation have been obtained by using Quintic B-spline Differential Quadrature Method (QBDQM). If  $\nu=0$  the equation turns into KdV equation, if  $\mu=0$  the equation turns into Burgers equation and turns into KdVB when both  $\nu$  and  $\mu$  are different from zero. The lowest three invariants  $I_1, I_2, I_3$  and the error norms  $L_2$  and  $L_\infty$  have been computed. To show the accuracy of the method, several test problems have been chosen for each equation. The numerical results are found in good agreement with exact solutions.

**Keywords.** Quintic B-spline, KdVB equation, Differential Quadrature Method, Partial Differential Equation, Fourth-order Runge-Kutta method.

**AMS 2010.** 65D07, 65M99, 65L06.

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## Existence Results For a Class of Nonlinear Elliptic Equation With Variable Exponent

Uğur Sert<sup>1</sup> and Kamal N. Soltanov<sup>2</sup>

**Abstract.** In this talk, we study the Dirichlet problem for the nonlinear filtration equation with variable nonlinearity in a bounded domain  $\Omega$ . We obtain sufficient conditions for existence of weak solution in some nonlinear space which is generated by the considered problem (called pn-spaces) and prove existence theorem by using the embedding and compact embedding results for these nonlinear spaces.[6]

We have to point out that, in the recent years there has been an increasing interest in the study of equations with variable exponent, the interest for differential equations involving variable nonlinearity is based on the multiple possibilities to apply them. A special attention in study of these equations is motivated by their applications in elastic mechanics, the mathematical modeling of non-Newtonian fluids and in image recovery (for example see [1], [2]). To be more specific, we recall that some of the first applications are concerning the electrorheological fluids (sometimes referred to as ‘smart fluids’).

**Keywords.** filtration equation, nonlinear elliptic equation, pn-spaces, nonstandart growth conditions

**AMS 2010.** 35J60, 35J66.

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## Global Existence, Uniqueness of Weak Solutions and Determining Functionals for Nonlinear Wave Equations

Ülkü Dinlemez<sup>1</sup>

**Abstract.** We consider the initial-boundary value problem for a nonlinear wave equation with strong structural damping and nonlinear source terms in  $\mathbb{R}$ . We prove the global existence and uniqueness of weak solutions of the problem and then we will study the determining modes on the phase space  $H^2(0,1) \cap H_0^1(0,1)$  by using energy methods and the concept of the completeness defect.

**Keywords.** Existence, uniqueness, determining modes

**AMS 2010.** 35L70

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## The Computation of the Frequency-Dependent Magnetic and Electric Green's Functions in Layered Rectangular Parallelepiped

Valery G. Yakhno<sup>1</sup> and Sengul Ersoy<sup>2,3</sup>

**Abstract.** Frequency-dependent Maxwell's equations in a layered rectangular parallelepiped with the perfect conducting boundary and arbitrary finite number of layers are considered in the paper. The main problem is to compute the magnetic and electric Green's functions for these Maxwell's equations. A new method for computing these Green's functions is proposed. This method is based on the Fourier series expansion of the vector generalized functions and has the following steps. The first step is the computation of the eigenvalues and corresponding vector-eigenfunctions for the vector operator  $\Delta_x I_3$  ( $\Delta_x$  is the scalar Laplace operator;  $I_3$  is the  $3 \times 3$  identity matrix) subject to the some boundary conditions and matching conditions on the interface of layers. The second step is the presentation of the generalized functions  $\text{curl}_x \left[ e^{\vec{k}} \delta(x_1) \delta(x_2) \delta(x_3) \right]$ ,  $k = 1, 2, 3$ , ( $\delta(\cdot)$  is the Dirac delta function;  $x_1, x_2, x_3$  are space variables;  $\vec{e}^1 = (1, 0, 0)$ ,  $\vec{e}^2 = (0, 1, 0)$ ,  $\vec{e}^3 = (0, 0, 1)$ ) in the form of the vector Fourier series relative to the set of found vector-eigenfunctions. Third step is finding columns of the approximate magnetic Green's function in the form of the vector Fourier series with finite number of terms. The last step is the derivation of columns of the electric Green's function using the elements of the magnetic Green's function. Computational experiments confirm the robustness of the suggested method. The presented paper continues the research of works [1].

**Keywords.** Maxwell's equations, frequency domain, perfect conducting boundary, matching conditions, Green's function, analytical method, simulation.

AMS 2010.35Q61, 35D99, 35L57, 35L20

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## Fractional Supersymmetric $sl(2)$ : $U_3^5(sl(2))$ and $U_3^6(sl(2))$

Y. Uçan<sup>1</sup>, R. Köşker<sup>2</sup>, R. Tekercioğlu<sup>3</sup> and A. Turan Dincel<sup>4</sup>

**Abstract.** Fractional superalgebras based on  $S_n$  invariant forms were first introduced in [1, 2]. There are other approaches to fractional supersymmetry in the literature [3-6]. In this study, our aim is to perform fractional supersymmetric  $U_3^5(sl(2))$  and  $U_3^6(sl(2))$  algebras by using the approach introduced in [6]. Thus, from the commutation relations for the algebra  $sl(2)$  we read

$$\sum_{\sigma=1}^N (a_{\alpha\sigma}^i a_{\sigma\beta}^j - a_{\alpha\sigma}^j a_{\sigma\beta}^i) = \sum_{k=1}^3 c_{ij}^k a_{\alpha\beta}^k \quad (1)$$

According to the Eq. (1), we obtain  $N = 5$  dimensional and  $N = 6$  dimensional matrix representations of the  $sl(2)$  algebra. By using these representations and following equations

$$\sum_{\sigma=1}^N (a_{\alpha\sigma}^k b_{\sigma\beta\gamma}^i + a_{\beta\sigma}^k b_{\sigma\alpha\gamma}^i + a_{\gamma\sigma}^k b_{\sigma\beta\alpha}^i) = \sum_{k=1}^3 c_{jk}^i b_{\alpha\beta\gamma}^j \quad (2)$$

$$\sum_{k=1}^3 (b_{\alpha\beta\gamma}^k a_{\sigma\tau}^k + b_{\sigma\alpha\beta}^k a_{\gamma\tau}^k + b_{\gamma\sigma\alpha}^k a_{\beta\tau}^k + b_{\beta\gamma\sigma}^k a_{\alpha\tau}^k) = 0 \quad (3)$$

$b_{\alpha\beta\gamma}^i$  is fully determined and  $U_3^5(sl(2))$  and  $U_3^6(sl(2))$  algebras are formed.

**Keywords.** Fractional, supersymmetric, Lie algebras.

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## **Thermoelastic Response of a Long Tube Subjected to Periodic Heating**

Yasemin Kaya<sup>1</sup> and Ahmet N. Eraslan<sup>2</sup>

**Abstract.** In this study, the thermoelastic behavior of an infinitely long tube subjected to periodic heating is investigated. The heating of the tube is realized by the linearly increasing temperature from its inner surface. When the inner surface reaches a certain temperature, it is kept at that temperature for some time, and then lowered with the same rate to the initial temperature. The outer surface of the tube is assumed to be isolated. The temperature distribution in the tube is obtained by the analytical solution of the heat conduction equation and by the application of Duhamel's theorem afterwards. The thermoelastic behavior of the tube is modeled by considering radially free inner and radially constrained outer surfaces. Based on the plain strain conditions, the thermoelastic analytical solution of the tube is derived. The results of this solution are compared to those of a numerical solution based on the shooting method. This work outlines analytical and numerical solution procedures and presents some important results of relevance to engineering.

**Keywords.** Thermoelasticity, Periodic Heating, Duhamel's Theorem, Shooting Method.

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### Algorithm to Solution of the Shape Optimization Problem for the Eigenvalues

Yusif S. Gasimov<sup>1</sup>, Natavan Allahverdiyeva<sup>2</sup>

**Abstract.** Consider the following eigenvalue problem

$$-\Delta u(x) + q(x)u(x) = \lambda u(x), \quad x \in D, \quad (1)$$

$$u(x) = 0, \quad x \in S_D, \quad (2)$$

where  $\Delta$  is Laplace operator,  $D \in R^n$ -bounded convex domain with boundary  $S \in C^2$ ,  $q(x)$ -given non negative differentiable function.

By  $M$  we define the set of bounded convex domains. Let  $K = D \in M, S_D \in C^2$ . Our aim is to find  $D^* \in K$  that gives minimum to the eigenvalue  $\lambda_j$  of the problem (1), (2).

Note that such problems have a strong relation with applications in different fields of mechanics and physics, where eigenvalues of the different operators describe corresponding mechanical characteristics of the systems. For example, eigenvalues of the Schrodinger operator indeed are energetic levels of the quantum particle in the electromagnetic field [1]; of the Laplace operator- eigenfrequency of the membrane; of the operator  $\Delta^2$  - eigenfrequency of the plate under across vibrations [2]. We calculate the first variation of the eigenvalue  $\lambda_j(D)$  with respect to  $D$  and got the following formula

$$\delta\lambda_j(D_0, D) = -\max_{u_j^0} \int_{S_{D_0}} |\nabla u_j^0(x)|^2 [P_D(n(x)) - P_{D_0}(n(x))] ds,$$

where  $P_D(x)$  is a support function of the domain  $D$ . On the base of this formula we offer the algorithm below for the numerical solution of the stated problem.

#### Алгоритм

**Step 1.** Choose the initial domain  $D_0 \in K$ . Suppose that  $D_i \in K, i = 1, 2, \dots$  are known.

**Step 2.** In the domain  $D_i$  solve the problem (1), (2) and find eigenfunction  $u_i(x)$ .

**Step 3.** Solve the variational problem  $\int_{S_{D_i}} |\nabla u^{(i)}|^2 P(n(x)) ds \Lambda_i \rightarrow \max$ , and find convex

positively-homogenous function  $\bar{P}_i(x)$ .

**Step 4.** Find the intermediate domain  $\bar{D}_m$  as a subdifferential of the function  $\bar{P}_i(x)$  in the point 0.

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**Step 5.** Define the next domain from the relation

$$D_{i+1} = (1 - \alpha_i)D_i + \alpha_i \overline{D}_i, \text{ where } 0 < \alpha_i < 1.$$

**Step 4.** Check up the error criteria. Take  $D_i = D_{i+1}$  if it is not satisfied and go to Step 1. Otherwise the iteration is over.

**Example.** It needs to find a domain  $D^* \in K$  that gives minimum to the first eigenvalue of the problem

$$-\Delta u = \lambda u, \quad x \in D,$$

$$u(x) = 0, \quad x \in S_D.$$

Condition on the domain  $D$  we give by relation  $D' \subset D \subset D''$ , where  $D'$  is a circle with radius 1, and  $D''$  - circle with radius 3.

Applying the offered above algorithm we find out that the minimal first eigenvalue  $\lambda_1^{\min} = 0.65526$  is obtained in the circle with radius 3 (see Fig.1).

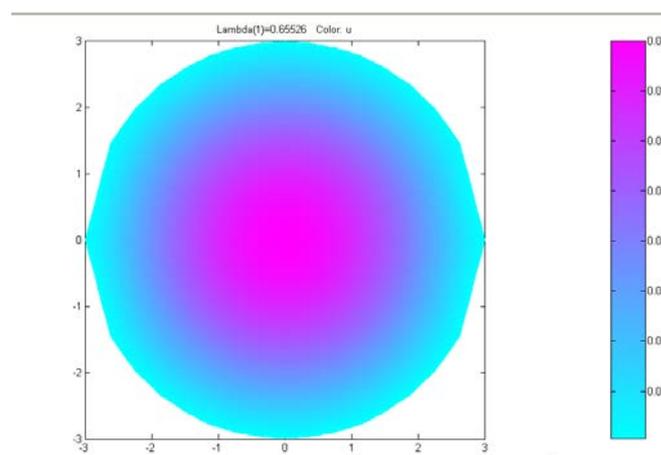


Fig.1

**Keywords.** shape optimization, support function, eigenvalue, convex domains

**AMS 2010.** 49J45, 49Q10

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## Optimal Control via the Initial Velocity in a Hyperbolic Problem

Yeşim Saraç<sup>1</sup> and Hakkı Güngör<sup>2</sup>

**Abstract.** We consider an optimal control problem for a hyperbolic equation with Neumann boundary conditions, the control being the initial velocity of the system. The cost functional consists of the deviation in the  $L_2$  – norm of the velocity of the system at the final time from a given target, plus the  $L_2$  – norm of the control. These kinds of problems have been studied by Lions [1] and more recently by Hasanov [2], Kowalewski [3], and Subaşı and Saraç [4]. Sufficient conditions for the existence of a unique weak solution of the hyperbolic equation considered and a unique optimal solution of the control problem studied are presented. An iterative algorithm is constructed to compute the required optimal control as limit of a suitable subsequence of controls. Finally, this iterative algorithm is implemented in a MAPLE<sup>®</sup> program and symbolic solution is obtained in a numeric example.

**Keywords.** Optimal control, hyperbolic equation.

**AMS 2010.** 49J20, 35L10.

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## **Oscillation of Some Kinds of Functional Differential Equations of First Order**

Yeter Sahiner<sup>1</sup>

**Abstract.** Some oscillation results on first order functional differential equations are represented. Similar results are obtained for differential equations of first order both containing both positive and negative term. Obtained results are questionized for oscillation of first order differential equations containing delay and/or advance term.

**Keywords.** oscillation, first order, delay.

**AMS 2010.** 34C10, 34K11.

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## The Stability of Solutions of Fourth-order Fixed Coefficient Differential Equations

Yalçın Yılmaz<sup>1</sup> and Işıl Arda Kösal<sup>2</sup>

**Abstract.** In this study, more general conditions than Hurwitz conditions on coefficients are obtained for stability of solutions of fourth-order ordinary differential equations. Besides, asymptotic stability of solutions are shown with the same conditions.

**Keywords.** Stability result, Hurwitz conditions.

**AMS 2010.** 34C11, 34A99.

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## On the Continuous Algebraic Riccati Matrix Equation and Bounds for the Solution Matrix

Zübeyde Ulukök<sup>1</sup> and Ramazan Türkmen<sup>2</sup>

**Abstract.** The continuous algebraic Riccati matrix equation plays an important role in design and analysis problems of control and dynamic systems. The continuous algebraic Riccati matrix equation CARE is defined as following:

$$PA + A^T P - PBB^T P = -Q \quad (1)$$

where  $A \in \mathbb{R}^{n \times n}$  is a constant matrix,  $Q \in \mathbb{R}^{n \times n}$  is a given positive semidefinite matrix,  $B \in \mathbb{R}^{n \times m}$ , and the matrix  $P \in \mathbb{R}^{n \times n}$  is the unique symmetric positive semidefinite solution of the CARE (1).

To treat many control problems and in some applications of stability analysis, it is only needed to bounds. for solution matrix. So, in this talk, we first summarize bounds for the solution of the continuous algebraic Riccati matrix equation (CARE) that has been reported in last years. Then, by utilizing block matrix techniques and matrix inequalities, we present new results for the solution bounds of the CARE (1). Depending on these bounds, more sharper estimations can be achieved by choosing appropriate parameters. Finally, we give illustrative examples to make comparisons for the results proposed in this work and previous some results.

**Keywords.** Continuous algebraic Riccati matrix equation, matrix bound, matrix inequalities.

**AMS 2010.** 15A24.

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*DISCRETE*  
*MATHEMATICS*



## Connected Cubic Network Graph and Its Hamiltonian Properties

Burhan Selçuk<sup>1</sup> and Ali Karci<sup>2</sup>

**Abstract.** Hypercube is a popular and more attractive interconnection networks. The attractive properties of hypercube caused the derivation of more variants of hypercube. In this study, we have proposed one variant of hypercube which was called as “Connected Cubic Network Graphs”, and we have investigated the Hamiltonian-like properties of Connected Cubic Network Graphs (CCNG). Firstly, we define Connected Cubic Network Graphs (CCNG). Further, we show the construction and characteristics analyses of  $CCNG(n, m, r)$ . Furthermore,  $CCNG(n, m, r)$  is a Hamilton graph which is obtained by using Gray Code. Finally, we have obtained a recursive algorithm which is used to label the nodes of  $CCNG(n, m, r)$ .

**Keywords.** Hamilton Path, Hypercube, Gray Code, Interconnection Network, Connected Cubic Network Graph

**AMS 2010.** 05C45, 05C65, 90B18

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## Average Lower Domination Numbers of Some Graphs

Derya Doğan<sup>1</sup>

**Abstract.** The stability of communication network, composed of processing nodes and communication links, is the prime importance of network designers. Many graph theoretical vulnerability measures are defined, most studied and best known vulnerability measures in graph theory are connectivity, integrity, tenacity, toughness. If we consider a graph as modeling a network, the average lower domination number of a graph is one of the parameters for graph vulnerability. Let  $G = (V;E)$  be a graph. For a vertex  $v$  of  $G$ , the domination number of  $G$  relative to  $v$ , denoted by  $\gamma_v(G)$  as the minimum cardinality of a dominating set in  $G$  that contains  $v$ . The average lower domination number of  $G$ , denoted by  $\bar{\gamma}(G)$ , can be written as:

$$\frac{1}{|V(G)|} \sum_{v \in V(G)} \gamma_v(G)$$

In this paper, we we show that average lower domination number of some graphs.

**Keywords.** Graph Theory, Dominating set, Domination number.

**AMS 2010.** 68R10, 05C69, 05C99.

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**A Study on the Number of Minimal Access Sets in the Secret Sharing Scheme Based on the Dual Code of the Code of a Symmetric  $(v, k, \lambda)$ -Design**

Erol Balkanay<sup>1</sup> and Selda Çalkavur<sup>2</sup>

**Abstract.** Let  $p$  be a prime and denote the finite field of order  $p$  by  $F_p$ . Suppose that  $C$  is the  $F_p$ -code of a symmetric  $(v, k, \lambda)$ -design. If  $p|(k - \lambda)$  and all of the nonzero codewords of  $C$  are minimal, then the number of the minimal access sets  $m$  in the secret sharing scheme based on  $C^\perp$ , satisfy the inequality

$$p \leq m \leq p^{\frac{v-1}{2}}.$$

In this study, we give also two other theorems about the number of the minimal access sets.

**Keywords.** Linear code, the code of a symmetric design, secret sharing scheme, minimal access set.

**AMS 2010.** 14G50, 94A60, 94C30.

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## The Number of $Z_2Z_{2s}$ Additive Codes and Some Combinatorial Relations

İrfan Şiap<sup>1</sup>, İsmail Aydoğdu<sup>2</sup> and Elif Segah Öztaş<sup>3</sup>

**Abstract.** Recently some studies on  $Z_2Z_4$  additive codes are initiated by Borges et al. in [1]. A generalization of additive codes are presented by Aydogdu and Siap in [2]. In this work, we establish a formula that gives the number of particular matrices that are generator matrices of additive codes on  $Z_2Z_{2s}$ . This is also a natural generalization of the number of such matrices presented for codes over  $Z_2Z_8$  [3]. We also relate this formula to 2-binomial Gaussian numbers. Further, we also present some combinatorial relations and point some further open problems on this subject.

**Acknowledgment:** This research is supported partially by Yıldız Technical University Research Support Unit No: 2012-01-03-YL01.

**Keywords.** Additive codes, 2-binomial Gaussian numbers.

**AMS 2010.** 94B05, 11T71.

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## Graph Coloring and Matching

Zakir Deniz<sup>1</sup>

**Abstract.** In this study, path-matching being generalizations of the matching and some application on coloring in graphs get involved. Our main goal is to examine the notion of path-matching in graphs, and to present some new findings related to some well-known theorems and conjectures in graph theory such as theorems of König and Vizing on edge colorings, Total Coloring Conjecture.

**Keywords.** Matching, coloring of graphs.

**AMS 2010.** 05C70, 05C15.

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# *GEOMETRY*



## The Energy of a Hypersurface

Ayşe Altın<sup>1</sup>

**Abstract.** In this paper, we compute the energy of a unit normal vector field on a hypersurface  $M$  in  $(n + 1)$  –dimensional manifold  $\bar{M}$ . We show that the energy of a unit normal vector field may be expressed in terms of the principal curvatures functions of  $M$ . To this end we define the energy of the hypersurface.

**Keywords.** Energy, Energy of a unit normal vector field, Energy of a Hypersurface, Sasaki metric.

**AMS 2010.** 53A04, 53C04.

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## On The Curves of Constant Breadth In Four Dimensional Galilean Space

Abdullah Mağden<sup>1</sup> and Süha Yılmaz<sup>2</sup>

**Abstract.** In this work, we obtained characterization curves of constant breadth in four dimensional Galilean space in terms of Frenet-Serret vector fields. First we investigate an explicit characterization of curves of constant breadth of four dimensional Galilean space. In addition, it has been observed that differential equation of 4<sup>th</sup> order of a constant breadth curve in four dimensional Galilean space. Finally, some differential and integral characterizations of the mentioned curves are expressed by the classical differential geometry methods.

**Keywords.** 4D Galilean space, classical differential geometry, curves of constant breadth.

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## On Bicomplex-Holomorphic 4-Manifolds

A. Salimov<sup>1</sup> and Rabia Çakan<sup>2</sup>

**Abstract.** The algebra of bicomplex numbers  $B(1, i_1, i_2, e)$  is the first non-trivial complex Clifford algebra. By a bicomplex-holomorphic  $r$ -manifold  $X_r(B(1, i_1, i_2, e))$  we mean a real manifold  $M_{4r}$  with an integrable regular bicomplex structure  $\Pi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$ , where  $\varphi_1 = id_{M_{4r}} = (\delta_v^u C_{1\beta}^\gamma)$ ,  $\varphi_2 = (\delta_v^u C_{2\beta}^\gamma)$ ,  $\varphi_3 = (\delta_v^u C_{3\beta}^\gamma)$ ,  $\varphi_4 = (\delta_v^u C_{4\beta}^\gamma)$ ,  $u, v = 1, \dots, r$ ;  $\gamma, \beta = 1, \dots, 4$ ,  $C_{\alpha\beta}^\gamma$  are structural components of  $B(1, i_1, i_2, e)$ . We note that the transition functions of atlas on manifold  $X_r(B(1, i_1, i_2, e))$  are bicomplex-holomorphic functions. The main purpose of this paper is to study some properties of bicomplex-holomorphic 4-manifolds endowed with a holomorphic double Walker-Norden (-anti-Hermitian) metric.

**Keywords.** Bicomplex algebra, Holomorphic connections, Walker manifolds.

**AMS 2010.** 53C15, 53C50.

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## Curvature Properties of Anti-Kähler-Codazzi Manifolds

Arif Salimov<sup>1</sup> and Sibel Turanli<sup>2</sup>

**Abstract.** In this paper we shall consider a new class of integrable almost anti-Hermitian manifolds, which will be called anti-Kähler-Codazzi manifolds, and we will investigate their curvature properties.

**Keywords.** Nearly-Kähler manifold, Anti-Kähler manifold, Anti-Kähler-Codazzi manifold.

**AMS 2010.**32Q15, 53C55

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## **Ricci Solitons in Generalized Kenmotsu Manifolds**

Aysel Turgut Vanlı<sup>1</sup> and Ayşe Ayhan

**Abstract.** In this paper, we study Ricci solitons in generalized Kenmotsu manifolds. We consider quasi conformal, conharmonic and projective curvature tensors in a generalized Kenmotsu manifold admitting Ricci solitons and prove the conditions for the Ricci solitons to be shrinking, steady and expanding.

**Keywords.** Ricci soliton, generalized Kenmotsu manifolds.

**AMS 2010.** 53A25, 53C15

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## Some Symmetry Conditions on Kenmotsu Manifolds

Ahmet Yıldız<sup>1</sup> and Azime Çetinkaya<sup>2</sup>

**Abstract.** The object of the present paper is to study some symmetry conditions on a Kenmotsu manifold with the semi-symmetric non-metric connection.

**Keywords.** Kenmotsu manifolds, the semi-symmetric non-metric connection, the Weyl conformal curvature tensor.

**AMS 2010.** 53C25, 53C35, 53D10

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## On Curve Couples of a Biharmonic Curve in Cartan-Vranceanu (2n+1)-dimensional Space

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**Abstract.** Biharmonic curves in Cartan-Vranceanu (2n+1)-dimensional space are studied in [1]. We characterize parametric equations of evolute and involute curve couple of the biharmonic curve in the Cartan-Vranceanu (2n+1)-space.

**Keywords.** Cartan-Vranceanu space, biharmonic curve, evolute curve, involute curve.

**AMS 2010.** 53A40, 20M15.

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## A New Approach on Spherical Conic Equations

Bülent Altunkaya<sup>1</sup>, Yusuf Yaylı<sup>2</sup> and H.Hilmi Hacısalihoğlu<sup>3</sup>

**Abstract.** On the search of the E.Study Map of Spherical Conics. Mathematicians tried to find one-parameter equations of the spherical conics [1], [4]. Although many equations related to spherical conics have been known, those equations depend on many parameters. In this study, we have mentioned about the method that we have developed for equations which depends on one parameter of the spherical conics.

**Keywords.** Spherical Parabola, Spherical Ellipse, Spherical Hyperbola

**AMS 2010.** 14Q05, 53A17.

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## Some Characterizations of Paraquaternionic Kahler Manifolds

Beran Pirinççi<sup>1</sup>, Mehmet Erdoğan and Gülşen Yılmaz

**Abstract.** In this work we present some characterizations of paraquaternionic space forms in the class of paraquaternionic Kahler manifolds in terms of the curvature tensor and we introduce the notion of paraquaternionic Kahler Frenet curves. Using the notion of such curves, we characterize some immersions into paraquaternionic space forms.

**Keywords.** Paraquaternionic Kahler manifolds, paraquaternionic Frenet curves, totally geodesic immersions.

**AMS 2010.** 53C15, 53C40.

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## A Connection Proposal for Generalized Geometry

Bayram Şahin<sup>1</sup>

**Abstract.** We introduce a new operator on generalized tangent bundle and investigate its properties. We show that this operator is torsion free with respect to Courant bracket but it is not compatible with neutral metric on the generalized tangent bundle. We also calculate curvature tensor field of this connection and investigate invariance property of this operator with respect to the generalized complex structure of complex type.

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## On Spherical Slant Helices in Euclidean 3-Space

Çetin Camcı<sup>1</sup>, Mesut Altınok<sup>2</sup> and Levent Kula<sup>3</sup>

**Abstract.** In this paper we investigate spherical slant helices. Moreover, we give some examples about spherical slant helix.

**Keywords.** Slant helix, Spherical curve.

**AMS 2010.** 53A04, 14H50.

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## Homothetic Cayley Formula and its Applications

Doğan Ünal<sup>1</sup>, Mehmet Ali Güngör<sup>2</sup> and Murat Tosun<sup>3</sup>

**Abstract.** In this study, homothetic Cayley mapping for a skew-symmetric matrix is defined. Some relations between skew-symmetric matrices and vectors are given. Then, homothetic Rodrigues Equation and Euler Parameters are obtained. Moreover, homothetic rotation matrix is given and with the help of these some results and applications are obtained.

**Keywords.** Homothetic Cayley mapping, rotation, homothetic motions.

**AMS 2010.** 53A17, 15A30.

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## On the Elastica in Lorentzian Plane $L^2$

Esra Betul Koc Ozturk<sup>1</sup>, Ufuk Ozturk<sup>2</sup> and Matthias Kawski<sup>3</sup>

**Abstract.** In this article, we study the Euler elastica problem in Lorentzian plane  $L^2$ . We use the Pontryagin Maximum Principle and a non-null curve in Lorentzian plane  $L^2$  to solve the Euler elastica problem. Also, we give the differential equations for the curvature of a non-null elastic curve in Lorentzian plane  $L^2$

**Keywords.** Elastica, Lorentzian plane, Optimal control, Pontryagin maximum principle.

**AMS 2010.** 49J15, 49K15, 53B30.

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## Modified Translation Surfaces in $\text{Heis}_3$

Essin Turhan<sup>1</sup> and Gülden Altay<sup>2</sup>

**Abstract.** In this paper, a new type of translation surfaces in 3-dimensional Heisenberg group generated by two curves have been studied. It has made some characterizations for these surfaces.

**Keywords.** Heisenberg Group, Translation Surface.

**AMS Classifications.** 53B, 22E.

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## Half Lightlike Submanifold with Non-Degenerate Planar Normal Sections in $R_2^5$

Feyza Esra Erdoğan<sup>1</sup> and Rıfat Güneş<sup>2</sup>

**Abstract :** We investigate half-lightlike submanifolds with non-degenerate planar normal sections of five dimensional pseudo Euclidean space. We obtain necessary and sufficient conditions for a half-lightlike submanifolds of  $R_2^5$  such that it has non-degenerate planar normal sections.

**Keywords.** Half-lightlike submanifold, planar normal sections.

**AMS 2010.** 53C42, 53C50.

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## Homothetic Motions and Surfaces with Pointwise 1-Type Gauss Map in $E^4$

Ferdağ Kahraman Aksoyak<sup>1</sup> and Yusuf Yaylı<sup>2</sup>

**Abstract.** In this paper, we determine a surface  $M$  by means of homothetic motion in  $E^4$  and we give necessary and sufficient conditions for flat surface  $M$  with flat normal bundle to have pointwise 1-type Gauss map.

**Keywords.** Homothetic Motion, Gauss map, Pointwise 1-type Gauss map, Surfaces in Euclidean space

**AMS 2010.** 53A05, 53B25, 53C40

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## The Fermi-Walker Derivative in Minkowski Space

Fatma Karakuş<sup>1</sup> and Yusuf Yaylı<sup>2</sup>

**Abstract.** In this study, Fermi-Walker derivative and Fermi-Walker parallelism are analyzed for any spacelike curve with a spacelike or timelike principal normal in Minkowski 3-space and the necessary conditions to be Fermi-Walker parallel are explained.

**Keywords.** Fermi-Walker derivative, spacelike curve, timelike curve.

**AMS 2010.** 53B20, 53B21, 53Z05, 53Z99.

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## Projective Coordinate Spaces Over Local Rings

Fatma Özen Erdoğan<sup>1</sup> and Süleyman Çiftçi<sup>2</sup>

**Abstract.** In this paper we deal with projective coordinate space over a local ring  $R$ . We use the construction of projective coordinate space given in [1] and [2]. We show that  $n$ -dimensional coordinate projective Klingenberg space over the ring  $R$  given in [2] is an adaptation of projective coordinate space given in [1]. Then we give an example of projective coordinate space. Also we generalize the concept of projective space over a vector space to a space over a module. We give an isomorphism between the space over a module and the  $n$ -dimensional projective coordinate space given in [2]. Finally, we obtain some combinatoric results for finite projective coordinate space and we verify these results on some examples.

**Keywords.** projective coordinate space, local ring, projective space, module.

**AMS 2010.** 51C05.

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## Horizontal Lift Problems in the Semi-Cotangent Bundle

Furkan Yıldırım<sup>1</sup>

**Abstract.** The main purpose of this paper is to investigate horizontal lifts for semi-cotangent bundle. Let  $M_n$  be an  $n$ -dimensional differentiable manifold of class  $C^\infty$  and  $\pi_1: M_n \rightarrow B_m$  the differentiable bundle determined by a submersion  $\pi_1$ . Suppose that  $(x^i) = (x^a, x^\alpha)$ ,  $a, b, \dots = 1, \dots, n-m$ ;  $\alpha, \beta, \dots = n-m+1, \dots, n$ ;  $i, j, \dots = 1, 2, \dots, n$  is a system of local coordinates adapted to the bundle  $\pi_1: M_n \rightarrow B_m$ , where  $x^a$  are coordinates in  $B_m$ ,  $x^\alpha$  are fibre coordinates of the bundle  $\pi_1: M_n \rightarrow B_m$ . Let  $T_x^*(B_m)$  ( $x = \pi_1(\tilde{x}), \tilde{x} = (x^a, x^\alpha) \in M_n$ ) be the cotangent space at a point  $x$  of  $B_m$ . If  $p_\alpha$  are components of  $p \in T_x^*(B_m)$  with respect to the natural coframe  $\{dx^\alpha\}$ , i.e.  $p = p_i dx^i$ , then by definition the set of all points  $(x^I) = (x^a, x^\alpha, x^{\bar{\alpha}})$ ,  $x^{\bar{\alpha}} = p_\alpha$ ,  $\bar{\alpha} = \alpha + m$ ,  $I = 1, \dots, n+m$  is a semi-cotangent bundle  $t^*(B_m)$  over the manifold  $M_n$ . We note that semi-tangent bundle and its properties were studied in [1], [2], [3]. The aim of this paper is to study semi-cotangent bundle and its horizontal lift problems.

**Keywords.** Vector field, Complete lift, Horizontal lift, Semi-cotangent bundle.

**AMS 2010.** 53A45, 53C55.

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## Bislant Submanifolds of Semi-Riemannian Manifolds

Gülşah Aydın<sup>1</sup> and Abdilkadir Ceylan Çöken<sup>2</sup>

**Abstract.** In this study, we investigate bislant submanifolds of product semi-Riemannian manifold. We analysis bislant definition for Riemannian submanifolds, semi-Riemannian submanifolds, lightlike submanifolds of semi-Riemannian manifold. Besides, we state the theorems which give necessary conditions to do slant submanifold from bislant submanifold. Moreover, we present exercises with the respect to bislant submanifold.

**Keywords.** Riemannian manifold, semi-Riemannian manifold, slant submanifold, lightlike manifold, bislant submanifold, semi-Riemannian product structure.

**AMS 2010.** 53C50, 53C40.

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## **Holditch's Theorem for Two Different Lorentzian Spheres in 3-Dimensional Lorentzian Space**

Gülay Koru Yücekaya<sup>1</sup> and H. Hilmi Hacısalihoğlu<sup>2</sup>

**Abstract.** The present study expresses and proves Holditch's theorem for two different Lorentzian spheres in three-dimensional Lorentzian space through a new method.

**Keywords.** Future pointing time-like vector, Pure triangle, Lorentzian sphere, Holditch's Theorem.

**AMS 2010.** 53B30,53C50,53C99.

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## Holditch's Theorem for a Lorentzian Sphere in 3-Dimensional Lorentzian Space

Gülay Koru Yücekaya<sup>1</sup> and H. Hilmi Hacısalihoğlu<sup>2</sup>

**Abstract.** The present study expresses and proves Holditch's theorem for a Lorentzian sphere surface in three-dimensional Lorentzian space through a new method.

**Keywords.** Future pointing time-like vector, Pure triangle, Lorentzian sphere, Holditch's Theorem.

**AMS 2010.** 53B30,53C50,53C99.

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## Lorentzian Orthogonal Triangles in Lorentzian Plane

Gülay Koru Yücekaya<sup>1</sup>, İsmail Gök<sup>2</sup> and Yusuf Yaylı<sup>3</sup>

**Abstract.** In this study, some characterizations have been given for Lorentzian orthogonal triangles in Lorentzian plane.

**Keywords.** Lorentzian plane, Time-like vector, Orthogonal triangle.

**AMS 2010.** 53B30, 53C50.

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## AW(k)-Type Curves According to the Bishop Frame

Günay Öztürk<sup>1</sup> and İlim Kişi<sup>2</sup>

**Abstract.** In this study, we consider AW(k)-type curves according to the Bishop Frame in Euclidean space  $E^3$ . We give the relations between the Bishop curvatures  $k_1, k_2$  of a curve in  $E^3$ .

**Keywords.** Bishop frame, Frenet frame, AW(k)-type.

**AMS 2010.** 53A04, 53C40.

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## On Helices with Constant Curvature in Sol Space

Hülya Başığmez<sup>1</sup>, Mesut Altınok<sup>2</sup> and Levent Kula<sup>3</sup>

**Abstract.** In this paper, we consider Sol Space, one model space of Thurston's 3-dimensional geometries. We obtain helix with constant curvature and we give some examples.

**Keywords.** Special Curves, Sol Geometry.

**AMS 2010.** 53A35, 14H50.

**Acknowledge.** This work is supported by Ahi Evran University BAP Project with Project number FBA-11-23.

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## Commutative Quaternion Matrices

Hidayet Hüda Kösal<sup>1</sup> and Murat Tosun<sup>2</sup>

**Abstract.** In this study, we introduce the concept of commutative quaternions and commutative quaternion matrices. Firstly, we give some properties of commutative quaternions and their fundamental matrices. After that we investigate commutative quaternion matrices using properties of complex matrices. Then we define the complex adjoint matrix of commutative quaternion matrices and give some of their properties.

**Keywords.** Commutative Quaternions, Commutative Quaternion Matrices, Adjoint Matrix.

**AMS 2010.** 11R52; 15A33.

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## Quaternionic Smarandache Curves According to Parallel Transport Frame

Hatice Parlaticı<sup>1</sup>, Mehmet Ali Güngör<sup>2</sup>

**Abstract.** In this study, we define Smarandache curves of quaternionic and spatial quaternionic curves according to parallel transport frame in  $\mathbb{E}^3$  and  $\mathbb{E}^4$ . In addition, we calculate the parallel transport and Frenet apparatus of the Smarandache curve in terms of the parallel transport apparatus of the curve it is related to.

**Keywords.** Quaternions, quaternionic curve, parallel transport frame, Smarandache curve, Euclidean space.

**AMS 2010.** 53A40, 11R52, 14H45.

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## Anti-Invariant Riemannian Submersions from Cosymplectic Manifolds

İrem Küpeli Erken<sup>1</sup> and Cengizhan Murathan<sup>2</sup>

**Abstract.** We introduce anti-invariant Riemannian submersions from cosymplectic manifolds onto Riemannian manifolds. We survey main results of anti-invariant Riemannian submersions defined on cosymplectic manifolds. We investigate necessary and sufficient condition for an anti-invariant Riemannian submersion to be totally geodesic and harmonic. We give examples of anti-invariant submersions such that characteristic vector field  $\xi$  is vertical or horizontal. Moreover we give decomposition theorems by using the existence of anti-invariant Riemannian submersions.

**Keywords.** Riemannian submersion, Cosymplectic manifold, Anti-invariant submersion.

**AMS 2010.** 53C25, 53C43, 53C55, 53D15.

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## Complete System of Invariants of Subspaces of Lorentzian Space

İdris Ören<sup>1</sup> and Djavvat Khadjiev<sup>2</sup>

**Abstract.** Let  $E_1^{n+1}$  be the  $(n + 1)$ -dimensional Lorentzian space,  $O(n, 1)$  be the group of all pseudo-orthogonal transformations of  $E_1^{n+1}$  and  $S(n + 1, 1)$  be the set of all subspaces of  $E_1^{n+1}$ . The following action of the group  $O(n, 1)$  on  $S(n + 1, 1)$  is considered:  $\alpha(F, V) = F(V)$ , where  $F \in O(n, 1)$  and  $V \in S(n + 1, 1)$ . For  $U \in S(n + 1, 1)$ , denote the number of linearly independent light-like vector in  $U$  by  $k(U)$ . Main results:

- (1) A proper subspaces  $U$  is non-degenerate if and only if  $k(U) \neq 1$ ;
- (2) The system  $\{dimU, k(U), indexU\}$  is a minimal complete system of  $O(n, 1)$  – *invariant* functions on the set  $S(n + 1, 1)$ ;
- (3)  $rankU = dimU - 1$  if and only if  $k(U) = 1$ ;
- (4) Canonical representations of  $O(n, 1)$  – *orbits* of subspaces of  $E_1^{n+1}$  with respect to the action  $\alpha$  are given.

**Keywords.** Invariant, Subspace, Light-like vector, pseudo-Euclidean space.

**AMS 2010.** 15A21, 15A72, 51B20, 83A05.

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## Certain Type of Surfaces in 4-dimensional Euclidean Space $E^4$

Kadri Arslan<sup>1</sup> and Betül Bulca<sup>2</sup>

**Abstract.** In the present study we consider surfaces in 4-dimensional Euclidean space  $E^4$ . We give curvature conditions of some surfaces given with the Monge patch in  $E^4$ . We also give some examples.

**Keywords.** Monge patch, Gauss curvature, Mean curvature.

**AMS 2010.** 53A40

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## **New Results On Circular Helix in Minkowski 3-Space**

Kazım İlarıslan<sup>1</sup> and Çetin Camcı<sup>2</sup>

**Abstract.** In this paper, we obtain new results for a circular helix (or a  $W$ -curve) in Minkowski 3-space. We also give the parametric equations of circular helix.

**Keywords.** Minkowski 3-space, helix, slope axis.

**AMS 2010.** 53C40, 53C50

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## Smarandache Curves on $S^2$

Kemal Taşköprü<sup>1</sup> and Murat Tosun<sup>2</sup>

**Abstract.** In this paper, we introduce special Smarandache curves according to Sabban frame on  $S^2$  and we give some characterization of Smarandache curves. Besides, we illustrate examples of our results.

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## The Napoleon Theorem in Taxicab Geometry

Mahmut Akyiğit<sup>1</sup>

**Abstract.** In this work, we study the median theorem, known very well in the literature, in taxicab geometry. Also we obtain the number of baselines passing through any triangle by using the definition of baseline given by R. Kaya and H. B. Colakoglu in [2]. Moreover, we investigate in the taxicab geometry the equivalent of “Napoleon’s theorem in the sense of Euclidean” and proved that Napoleon's theorem states that if we construct taxicab equilateral triangles on the sides of any triangle the centers of those taxicab equilateral triangles themselves form an taxicab equilateral triangle.

**Keyword.** Napoleon theorem, median theorem

**AMS 2010.** 51K05, 51K99

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## On the Synectic Metric in the Tangent Bundle of a Riemannian Manifold

Melek Aras<sup>1</sup>

**Abstract.** The purpose of this paper is to investigate applications the covariant derivatives of the covector fields and killing vector fields with respect to the synectic lift  ${}^S g = {}^C g + {}^a a$  in a the Riemannian manifold to its tangent bundle  $T(M_n)$ , where  ${}^C g$  -complete lift of the Riemannian metric,  ${}^a a$  -vertical lift of the symmetric tensor field  $a$ .

**Keywords.** Tesor bundle; Metric connection; Covector field; Levi-Civita connections; Killing vector field.

**AMS 2010.** 53C05, 53B05, 53C07

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## On The Two Parameter Homothetic Motions in Hyperbolic Plane

Muhsin Çelik<sup>1</sup> and Mehmet Ali Güngör<sup>2</sup>

**Abstract.** In this article, two parameter planar homothetic motion in the hyperbolic plane are investigated. Also, some definitions, theorems and corollaries about velocities, accelerations and their poles (and hodograph) of a point in hyperbolic planar motion are given. Moreover, in the case of homothetic scale  $h$  identically equal to 1, the results given in [2] are obtained as a special case.

**Keywords.** Two Parameter Motion, Hyperbolic Motion, Homothetic Motion, Planar Kinematics

**AMS 2010.** 53A17, 11E88.

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## Two Parameter Hyperbolic Motions

Muhsin Çelik<sup>1</sup> and Mehmet Ali Güngör<sup>2</sup>

**Abstract.** In this article analogous to planar motion in the complex plane given by [6] two parameter planar motion is defined in the hyperbolic plane. Also, some theorems and corollaries about velocities, accelerations and their poles (and hodograph) of a point in hyperbolic planar motion are obtained.

**Keywords.** Two Parameter Motion, Hyperbolic Motion, Planar Kinematics

**AMS 2010.** 53A17, 11E88.

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## **On Matrix Representations of Split Quaternions and Complex Split Quaternions**

Melek Erdođdu<sup>1</sup> and Mustafa Özdemir<sup>2</sup>

**Abstract.** In this work, we present some important properties of split and complex split quaternions. Firstly, we introduce  $4 \times 4$  real matrix representations of split quaternions. And we also give  $8 \times 8$  real matrix representations of complex split quaternions. Moreover, the relations between these representations are stated. Finally, we solve some certain linear split quaternionic equations by using these representations.

**Keywords.** Split quaternion, Complex split quaternion.

**AMS 2010.** 11R52, 15A66, 14A06.

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## 2-manifolds of Sol Space and Their Geometries

Mahmut Ergüt<sup>1</sup> and Muhittin Evren Aydın<sup>2</sup>

**Abstract.** In this talk, we study one of the eight homogeneous geometries of Thurston, the Sol space. Also, we give the structure equations of the Sol space and classify 2-manifolds with respect to the orthonormal vectors of Sol space to be the normal to these manifolds.

**Keywords.** Homogeneous manifolds, Sol space, structure equations,

**AMS 2010.** 53C17, 58A05.

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## Scalar Curvature of Screen Homothetic Lightlike Hypersurfaces of a Lorentzian Manifold

Mehmet Gülbahar<sup>1</sup>, Sadık Keleş<sup>2</sup> and Erol Kılıç<sup>3</sup>

**Abstract.** We establish some inequalities involving  $k$ -Ricci curvature,  $k$ -scalar curvature, the screen scalar curvature on a screen homothetic lightlike hypersurface of a Lorentzian manifold. Using these inequalities, we obtain some characterization on screen homothetic lightlike hypersurfaces of a Lorentzian manifold.

**Keywords.** Curvature, Lightlike Hypersurface, Lorentzian Manifold

**AMS 2010.** 53C40, 53C42, 53C50.

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## On the Orthogonal Invariants of Dual Planar Bezier Curves

Muhsin İncesu<sup>1</sup> and Osman Gürsoy<sup>2</sup>

**Abstract.** Recently the invariants of control points have had an important role in the systems CAD and CAM. Especially Bezier and B- spline curves and surfaces and NURBS modelling are based on control points belongs to these curves and surfaces. The invariants of these control points mean the invariants of curves and surfaces determined by these control points.

The dual planar Bezier Curves means that the Bezier curves of which control points are dual vectors in  $D^2$ . In this study we examined the orthogonal invariants of control points of dual planar Bezier curves.

**Keywords.** Orthogonal invariants, dual planar Bezier Curves.

**AMS 2010.** 20H15, 12D99, 12Y05, 14H05, 26C15, 14P05, 13J30.

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## The Complete System of Rational Invariants of Bezier Surfaces in Similarity Geometry

Muhsin İncesu<sup>1</sup> and Osman Gürsoy<sup>2</sup>

**Abstract.** Recently the invariants of control points have had an important role in the systems CAD and CAM. Especially Bezier and B- spline curves and surfaces and NURBS modelling are based on control points of these curves and surfaces. The invariants of these control points mean the invariants of curves and surfaces determined by these control points.

In this study we obtained any invariants of control points of Bezier surfaces based on generators of the field of rational invariant functions according to similarity group  $S(3)$  and an important subgroup of its i.e. linear similarity group  $LS(3)$  in  $\mathbb{R}^3$ .

**Keywords.** Rational Functions, Bezier Surfaces.

**AMS 2010.** 20H15, 12D99, 12Y05, 14H05, 26C15, 14P05, 13J30.

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## The LS(n)- and S(n) – Equivalence Conditions of Control Points

Muhsin Incesu<sup>1</sup> and Osman Gürsoy<sup>2</sup>

**Abstract.** In this study we investigated the equivalence conditions of control points for the similarity group S(n) and the important subgroup of its, linear similarity group LS(n), in  $\mathbb{R}^n$  and also stated the LS(n)- and S(n) – Equivalence Conditions of control points in terms of the generator invariants of real algebra of LS(n)- and S(n) – invariant rational functions  $R(x_1, x_2, \dots, x_k)^{LS(n)}$  and  $R(x_1, x_2, \dots, x_k)^{S(n)}$ .

**Keywords.** Rational Functions, Linear similarity group, Equivalence Conditions.

**AMS 2010.** 20H15, 12D99, 12Y05, 14H05, 26C15, 14P05, 13J30.

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## **On the Eigenvalues and Eigenvectors of a Lorentzian Rotation Matrix Which is Generated by a Timelike Quaternion**

Mustafa Özdemir<sup>1</sup> and Melek Erdoğan<sup>2</sup>

**Abstract.** In this paper, we find eigenvalues and eigenvectors of a Lorentzian rotation matrix in Minkowski 3 space by using split quaternions. We express the eigenvalues and the eigenvectors of a rotation matrix in terms of the components of the corresponding unit timelike split quaternion. Moreover, we show that the casual characters of rotation axis depend only on first component of the corresponding timelike quaternion..

**Keywords.** Quaternions, Split Quaternions, Rotation Matrix.

**AMS 2010 15A18, 53A35, 51N20.**

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## Silver Differential Geometry

Mustafa Özkan<sup>1</sup> and Betül Peltek<sup>2</sup>

**Abstract.** Golden structure defined by (1,1) tensor field  $\phi$  satisfying  $\phi^2 - \phi - I = 0$  have been studied by Crasmareanu and Hretcanu [2]. The purpose of this paper is to study Silver structure defined by (1,1) tensor field  $\phi$  satisfying  $\phi^2 - 2\phi - I = 0$  and geometry of the Silver structure on a manifold is investigated by using a corresponding almost product structure.

**Keywords.** Riemannian manifold, silver structure, silver proportion.

**AMS 2010.** 53C15.

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## Smarandache Curves on $S_1^2$ and $H_0^2$ in Minkowski 3-Space

Murat Savas<sup>1</sup>, Atakan Tuğkan Yakut<sup>2</sup> and Tuğba Tamirci<sup>3</sup>

**Abstract.** In this study we introduce special Smarandache curves according to Sabban frame on  $S_1^2$  and we give some characterization of Smarandache curves on the de Sitter surface. In addition, the existence of duality between Smarandache curves on the de Sitter space and Smarandache curves on hyperbolic space is shown. Besides, we illustrate examples of our main results.

**Keywords.** de sitter space, Smarandache curves, Sabban frame Minkowski space

**AMS(2000).** 53A04

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## On the Osculating Spheres of a Dual Quaternionic and Dual Split Quaternionic Curve

Nurten Bayrak Gürses<sup>1</sup>, Özcan Bektaş<sup>2</sup>, Mücahit Akbıyık<sup>3</sup> and Salim Yüce<sup>4</sup>

**Abstract.** In this paper, we investigate the osculating spheres of a dual quaternionic and dual split quaternionic curve. The study is based mainly the concept of osculating spheres studied by [10] for quaternionic curve in  $E^4$  and studied by [11] for semi real quaternionic curve in  $E_2^4$ .

**Keywords.** Osculating sphere, Frenet equations, quaternion, dual split quaternionic curve

**AMS 2010.** 14Q05, 53A35.

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## On the Quasi-Minimal Surfaces in the 4-Dimensional de Sitter Space-Time with 1-type Gauss map

Nurettin Cenk Turgay<sup>1</sup>

**Abstract.** In this paper, we study the Gauss map of the quasi-minimal surfaces in the 4-dimensional de Sitter space-time  $S^4_1(1)$ . We obtain the complete classification of the quasi-minimal surfaces with 1-type Gauss map. We also give a characterization of quasi-minimal surfaces with proper pointwise 1-type Gauss map.

**Keywords.** quasi-minimal surface, finite type Gauss map, de Sitter space-time

**AMS 2010.** 53B25, 53C50.

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## **$G_2$ Structures on 7-dimensional 3-Sasakian Manifolds**

Nülifer Özdemir<sup>1</sup> and Şirin Aktay<sup>2</sup>

**Abstract.** In this study, we give eight new  $G_2$  structures on a 7-dimensional 3-Sasakian manifold which are all in the class  $W=W_1+W_2+W_3+W_4$ , with the notation of Fernandez and Gray [1]. Then we show that these fundamental 3-forms of general type can be deformed into the  $G_2$  structures of type  $W_1+W_3+W_4$ , which have a unique metric connection with skew-symmetric torsion.

**Keywords.**  $G_2$  structure, 3-Sasakian manifold.

**AMS 2010.** 53C10, 53C25.

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## **Lightlike Submanifolds of Indefinite Kähler Manifolds with Quarter-symmetric Non-Metric Connection**

Oğuzhan Bahadır<sup>1</sup>

**Abstract.** In this paper, we study lightlike submanifold of indefinite Kähler manifolds. We introduce a class of lightlike submanifold called semi-invariant r-lightlike submanifold. We consider lightlike submanifold with respect to a quarter-symmetric non metric connection which is determined by the complex structure. We give some equivalent conditions for integrability of distributions with respect to the Levi-Civita connection of semi-Riemannian manifold and the quarter symmetric non metric connection and some results.

**Keywords.** Lightlike Submanifolds, Indefinite Kähler manifolds, Quarter-Symmetric Non-Metric Connections.

**AMS 2010.** 53C15, 53C25, 53C40.

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## Quaternionic Osculating Curves in Semi Euclidean Space $E_2^4$

Özcan Bektaş<sup>1</sup> Nurten Bayrak Gürses<sup>2</sup> and Salim Yüce<sup>3</sup>

**Abstract.** In this study, the osculating curves in semi Euclidean space  $E_1^3$  and  $E_2^4$  are studied through the agency of quaternions. We discourse basic definitions of quaternionic osculating curves in  $E_1^3$ , then we figure out the quaternionic osculating curve in  $E_2^4$  as a quaternionic curve whose position vector always lies in the orthogonal complement  $N_2^\perp$  (or  $N_3^\perp$ ) of its first binormal vector field  $N_2$  (or  $N_3$ ) in semi Euclidean space  $E_2^4$  with the Frenet frame  $\{T, N_1, N_2, N_3\}$ . We describe quaternionic osculating curves with refence to their curvature functions  $K$ ,  $k$  and  $(r-K)$  and serve the necessary and the sufficient conditions for arbitrary quaternionic curve in  $E_2^4$  to be a quaternionic osculating. Moreover, we conduct an explicit equation of a quaternionic osculating curve in  $E_2^4$ .

**Keywords.** Osculating curve, Frenet equations, curvature, real quaternion.

**AMS 2010.** 11R52, 14Q05

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## On the Quaternionic Normal Curves in the Euclidean Space

Önder Gökmen Yıldız<sup>1</sup> and Siddika Özkaldı Karakuş<sup>2</sup>

**Abstract.** In this paper, we define the quaternionic normal curves in Euclidean 3-space  $E^3$  and four dimensional Euclidean space  $E^4$ . We obtain some characterizations of quaternionic normal curves in terms of their curvature functions. Moreover, we give necessary and sufficient condition for a quaternionic curve to be a quaternionic normal curves in  $E^3$  and  $E^4$  respectively.

**Keywords.** Normal curves, real quaternion, quaternionic curve, position vector.

**AMS 2010.** 53A04, 11R52.

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## Dual Spacelike Curves Lying on Dual Lightlike Cone

Pınar Balkı Okullu<sup>1</sup> and H. Hüseyin Uğurlu<sup>2</sup>

**Abstract.** In this paper we give the representation formulas for dual spacelike curves lying on dual lightlike cone. Using these formulas we present the properties and structures of dual cone curves in dual lightlike cone. Some examples are also given.

**Keywords.** Dual cone curve, Dual structure function, Dual representation formula, Riccati equation.

**AMS 2010.** 51B20.

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## On a Cotangent Bundle with Deformed Riemannian Extension

Seher Aslanci<sup>1</sup> and Rabia Cakan<sup>2</sup>

**Abstract.** Let  $(M_n, g)$  be a Riemannian manifold with metric  $g$  and  ${}^cT(M_n)$  its cotangent bundle with local coordinates  $(\pi^{-1}(U), x^i, x^{\bar{i}} = p_i), i = 1, \dots, n; \bar{i} = n + i = n + 1, \dots, 2n; U \subset M_n$ . A new (pseudo) Riemannian metric  ${}^\nabla g \in \mathfrak{S}_2^0({}^cT(M_n))$  on  ${}^cT(M_n)$  is defined by the equation  ${}^\nabla g({}^cX, {}^cY) = -\gamma(\nabla_X Y + \nabla_Y X)$  for any  $X, Y \in \mathfrak{S}_0^1(M_n)$ , where  $\gamma(\nabla_X Y + \nabla_Y X)$  is a function in  $\pi^{-1}(U) \subset {}^cT(M_n)$  with local expression  $\gamma(\nabla_X Y + \nabla_Y X) = p_h(X^i \nabla_i Y^h + Y^i \nabla_i X^h)$ . We call  ${}^\nabla g$  the Riemannian extension of the Levi-Civita connection  $\nabla_g$  to  ${}^cT(M_n)$ . The main purpose of this paper is to study deformed Riemannian extension  ${}^\nabla g + {}^V g$  in the cotangent bundle. The curvature properties of metric connections for deformed Riemannian extensions is also investigated.

**Keywords.** Riemannian extension, cotangent bundle, vertical and complete lift, horizontal lift, deformed metric.

**AMS 2010.** 55R10, 53C07.

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## On $\phi$ -Concircularly Symmetric and $\phi$ -Conharmonically Symmetric Co-Symplectic Manifolds

Saadet Doğan<sup>1</sup> and Müge Karadağ<sup>2</sup>

**Abstract.** The importance of concircular transformation and concircular curvature tensor is very well known in the differential geometry of certain F-structure such as complex, almost complex, Kaehler, almost Kaehler, contact and almost contact structure etc.

Concircular and conharmonic curvature tensors are studied by some authors with certain symmetry conditions on some manifolds.

If a conformal transformation transforms a harmonic function into a harmonic function, then it is called a conharmonic transformation, and the conharmonic curvature tensor is an invariant under conharmonic transformations.

We study on  $\phi$ -concircularly symmetric and  $\phi$ -conharmonically symmetric co-symplectic manifolds.

It is shown that  $\phi$ -concircularly symmetric co-symplectic manifolds are  $\phi$ -symmetric co-symplectic too.

In addition to  $\phi$ -conharmonically symmetric co-symplectic manifolds -under the certain condition- are conharmonically conservative.

**Keywords.** Cosymplectic manifold, concircular curvature tensor,  $\phi$ -concircularly symmetric manifolds,  $\phi$ -symmetric manifolds, conharmonic curvature tensor,  $\phi$ -conharmonically symmetric manifolds, conservativeness

**Mathematics Subject Classification(2000).** 53C05, 53C15, 53D15

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## Curves on Parallel Surfaces in Euclidean 3-Space

Sezai Kızıltuğ<sup>1</sup> and Ali Çakmak<sup>2</sup>

**Abstract.** In this paper, we study curves on parallel surface. Using the definition of parallel surface we find image of curve which lying on surface. Then we obtain relationships between the geodesic curvature, the normal curvature, the geodesic torsion of curve and its image curve. Besides, we give some characterization for its image curve.

**Keywords.** Parallel Surface, Darboux Frame, Geodesic Curvature, Normal Curvature, Geodesic Torsion.

**AMS 2010.** 53A40, 53A05.

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## Generalized Bicomplex Numbers and Lie Groups

Siddika Özkaldı Karakuş<sup>1</sup> and Ferdağ Kahraman Aksoyak<sup>2</sup>

**Abstract.** In this paper, we define the generalized bicomplex numbers and give some algebraic properties of them. Also, we show that some hyperquadrics in  $\mathbb{R}^4$  and  $\mathbb{R}_2^4$  are Lie groups by using generalized bicomplex number product and obtain left invariant vector fields of these Lie groups.

**Keywords.** Bicomplex numbers, Lie groups, Surfaces in Euclidean space and Surfaces in pseudo- Euclidean space.

**AMS 2010.** 30G35, 43A80, 53C40, 53C50.

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## The Natural Lift Curves and Geodesic Curvatures of the Spherical Indicatrices of the Spacelike-Timelike Bertrand Curve Couple

Süleyman Şenyurt<sup>1</sup> and Ömer Faruk Çalışkan<sup>2</sup>

**Abstract.** In this paper, when  $(\alpha, \alpha^*)$  spacelike-timelike Bertrand curve couple is given, the geodesic curves and the arc-lengths of the curvatures  $(T^*), (N^*), (B^*)$  and the fixed pole curve  $(C^*)$  which are generated over the  $S_1^2$  Lorentz sphere or the  $H_0^2$  hyperbolic sphere by the Frenet vectors  $\{T^*, N^*, B^*\}$  and the unit Darboux vector  $C^*$  have been obtained. The condition being the naturel lifts of the spherical indicatrix of the  $\alpha^*$  is an integral curve of the geodesic spray has expressed.

**Keywords.** Lorentz space, spacelike-timelike Bertrand Curve Couple, Natural Lift, Geodesic Spray

**AMS 2010.** 53A04

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## Ricci Solitons in 3-Dimensional Normal Almost Paracontact Metric Manifolds

Selcen Yüksel Perktaş<sup>1</sup> and Sadık Keleş<sup>2</sup>

**Abstract.** In the present paper we study 3-dimensional normal almost paracontact metric manifolds admitting Ricci solitons and gradient Ricci solitons. An example of a 3-dimensional normal almost paracontact metric manifold is given. It is shown that if in a 3-dimensional normal almost paracontact metric manifold the metric is Ricci soliton where potential vector field  $V$  collinear with the characteristic vector field  $\xi$ , then the manifold is  $\eta$ -Einstein provided  $\alpha, \beta = \text{constant}$ . We also prove that a 3-dimensional normal almost paracontact metric manifold with  $\alpha, \beta = \text{constant}$  and  $V = \xi$  admits a Ricci soliton. Furthermore we show that if a 3-dimensional normal almost paracontact metric manifold admits a Ricci soliton  $(g, \xi, \lambda)$  then the Ricci soliton is shrinking.

**Keywords.** Normal almost paracontact metric manifold, Ricci soliton, gradient Ricci soliton,  $\eta$ -Einstein manifold.

**AMS 2010.** 53C15, 53C50.

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## On Some Regular Polygons in the Taxicab Space

Süleyman Yüksel<sup>1</sup> and Münevver Özcan<sup>2</sup>

**Abstract.** The Taxicab distance function between the points  $A(a_1, a_2, a_3)$ ,  $B(b_1, b_2, b_3)$  in the analytical 3-space is defined by  $d_T(A, B) = |a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3|$ . In this study, the Euclidean regular polygons which are also Taxicab regular polygons in the analytical 3-space were determined and the existence of Taxicab regular polygons which are not Euclidean regular polygons in the analytical 3-space were researched and identified.

**Keywords.** Taxi geometry, Taxi regular polygons, Taxi regular polyhedrons, Line segment with equal Taxi lengths

**AMS 2010.** 51B20, 51F99, 51K05, 51K99, 51N25

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## **On the Fundamental Forms of the Involute Rectifying Developable Surface in the Euclidean 3-space**

Şeyda Kılıçoğlu<sup>1</sup> and H. Hilmi Hacısalihoğlu<sup>2</sup>

**Abstract.** Deriving curves based on the other curves is a subject in geometry. Involute-evolute curves, Bertrand curves are this kind of curves. By using the similar method, we produce a new ruled surface based on the other ruled surface. In an earlier paper [16], by using that method we also work on B-scroll and involutive B-scroll.

In this paper, we consider the special ruled surfaces D-scroll which are also known rectifying developable surface of any curve in Euclidean 3-space. Further, we define the involute D-scroll, associated to a space curve  $\alpha$  with curvature  $k_1 \neq 0$  and involute curve  $\beta$ . We also have investigated the differential geometric elements (such as Normal vector field  $N$ , Weingarten map  $S$ , curvatures  $K$  and  $H$ ) of the D-scroll and involute D-scroll relative to each other. In this paper we examined the first three fundamental forms too.

**Keywords.** Darboux vector, fundamental forms, rectifying developable surface, evolute-involute curves..

**AMS 2010.** 53A04, 53A05.

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## A New Distance Function Reduced from Dodecahedron

Temel Ermiş<sup>1</sup>, Özcan Gelişgen<sup>2</sup> and Rüstem Kaya<sup>3</sup>

**Abstract.** There are only five Platonic solids, the cube, the octahedron, the icosahedron, the tetrahedron and the dodecahedron. In the previous studies, the metrics of which unit spheres are cube and octahedron were given by [1], [2], [3]. So, the relationship of octahedron and cube with metric geometries has been observed in many studies of the metric geometry. In this work, we give new metric of which unit sphere is the dodecahedron .

**Keywords.** Platonic Solids, Metric Geometry, Metric, Distance Function.

**AMS 2010.** 51M20, 51K99, 51K05

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## Spinor Representation of Sabban Frame on $S^2$

Tülay Soyfidan<sup>1</sup>, Mehmet Ali Güngör<sup>2</sup> and Murat Tosun<sup>3</sup>

**Abstract.** In this study, firstly we introduce spinors and spherical curves in  $S^2$ . Then, we give spinor representation of Sabban frame of curves on unit sphere  $S^2$  and obtain some results about spinor representation of this frame.

**Key words.** Spinor, Sabban frame, spherical curve, unit sphere.

**AMS 2010.** 53A04, 15A66.

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## Smarandache Curves According to Curves on a Spacelike Surface in Minkowski 3-Space

Ufuk Ozturk<sup>1</sup> and Esra Betul Koc Ozturk<sup>2</sup>

**Abstract.** In this study we define Smarandache curves according to curves on a spacelike surface in Minkowski 3-space. We obtain the Frenet frame, the curvature and the torsion of the Smarandache curves. Finally, we give an example.

**Keywords.** Smarandache curves, Pseudohyperbolic space, Sabban invariants, Geodesic curvature, Minkowski space.

**AMS 2010.** 53B30, 53A35, 53C22.

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# *MATHEMATICS* *EDUCATION*



## **The Factors Affecting Middle School Mathematics Teacher Candidates' Concerns Towards GeoGebra**

Bekir Kürşat Doruk<sup>1</sup>, Muharrem Aktümen<sup>2</sup>, Yasemin Kıymaz<sup>3</sup>, Zekiye Morkoyunlu<sup>4</sup>  
Avni Yıldız<sup>5</sup> and Serdal Baltacı<sup>6</sup>

**Abstract.** In today's world in which following the progression of technology is getting harder the integration of technology to mathematics classes has still limited [1], [2]. The key role of the teacher is seen when the research related to the use of ICT in classes are examined. Teachers' beliefs and attitudes about the use of ICT have also impact on the level of getting benefit from this technology [3]. Teacher candidates should be educated through the use of technology for mathematics education as well as introducing the candidates to new technology during teacher candidates' professional development process [2],[4]. One of the problems that can be encountered is that teachers are not concerned enough with the use of this new technology when it is decided to have this type of education. In this study, revealing the factors identifying the level of concerns of teacher candidates for the GeoGebra which is developed in 2002 [5] combining the properties of computer algebra systems and dynamic geometry software is aimed. This study is planned as a case study which is one of the qualitative research design. Interviews and surveys consisting of open ended questions are used as data collection tools. The study is conducted with 70 teacher candidates who are their second and third years in their undergraduate education in the department of middle school mathematics teacher training. After applying the survey to all the candidates, semi-structured interviews are designed with 16 of the candidates. This study is an ongoing research in which we conduct content analysis using the transcripts of interviews and written questionnaires. If this paper is accepted for presentation we are willing to present overlapping themes emerged and discuss our results with respective literature.

**Keywords.** Dynamic mathematics software, GeoGebra, Concern, Teacher candidates  
**AMS 2010.** 53A40, 20M15

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## **Estimating the Effect of Class Nonattendance on Academic Success by Piecewise Linear Regression**

Cüneyt Toyganözü<sup>1</sup>

**Abstract.** In higher education, class attendance is important for students to understand the main idea of particular class. Generally, the relationship between class nonattendance and academic success is negatively linear. In this study, we show that the effect of class nonattendance on academic success can be different after a threshold by using piecewise linear regression.

**Keywords.** Linear regression, piecewise regression, class nonattendance.

**AMS 2010.** 62J05, 62-07, 97K80.

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## **A Study of Middle School Students Proof Schemes: Gleaning from Clinical Interviews**

Dilek Tanışlı<sup>1</sup> and Deniz Eroğlu<sup>2</sup>

**Abstract.** The purpose of this study is to investigate middle school students' proof scheme and whether their proof schemes differ in terms of their and grade level and academic achievement. In this study the proof scheme framework is adapted based on the proof scheme categories of other researchers (Balacheff, 1988; Harel&Sowder, 1998; Knuth, 1999; Waring, 2000). In this framework, three primary schemes acquired from Harel and Sowder's proof schemes that are external, empirical and analytical proof schemes. In addition to these categories, other proof scheme categories (Balacheff, 1988; Knuth, 1999; Waring, 2000) were built upon these three primary categories. This framework provides a lens for interpreting and analyzing our students' approaches to proving. The 13 participants for the study were middle school students enrolled in 6<sup>th</sup>, 7<sup>th</sup>, and 8<sup>th</sup> grade. At the beginning of the study, the proof questionnaire (Knuth, Choppin, Slaughter ve Sutherland, 2002) consists of 9 open-ended items had administered to 89 students from 6<sup>th</sup>-8<sup>th</sup> students. Each student written response was coded with respect to Waring's (2000) levels of proof concept development. In the previous analysis, majority of the students were classified under level 0 and level 1. After this classification, 2 middle-achieving and 2 high-achieving students from level 0 and level 1 were selected in the 6<sup>th</sup> and 7<sup>th</sup> grades and 1 high achieving from level 0, 1 middle-achieving and 1 high-achieving from level 1, 1 middle-achieving from level 2, and 1 high-achieving from level 4 were selected from 8<sup>th</sup> grade students. Each student was asked about their written responses during about 45 minutes clinical interviews. Interviews were audio taped and protocols were transcribed. The verbal protocols were analyzed within the proof scheme framework described above. The results of this study revealed that majority of the students were classified under the authoritarian and naïve empiricism proof scheme. They mostly believed that using given rules and empirical examples were sufficient to justify their solutions and statements. Furthermore, results also suggested that there is no difference among students in terms of their grade level and academic achievement with regard to their proof schemes. The detailed findings from the study and discussion of findings will be presented during the presentation session.

**Keywords.** Proof scheme, middle school students, grade level, academic achievement.

**AMS 2010.** 53A40, 20M15.

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## **Diagrams Produced by High School Students in Multiplicative Comparison**

González Fany Markela<sup>1</sup> and Castro Enrique<sup>2</sup>

**Abstract.** The research community in the field of teaching and learning mathematics recognizes the usefulness of using different kinds of representations for mathematical thinking and problem solving. Several researchers propose that simultaneous work with different types of representations allows better conceptual construction and improved problem solving (Cuoco & Curcio, 2001). This leaves many open questions concerning the role and relationship of the different representations in the mind of the problem solver.

Our goal is to determine the role of graphical and diagrammatic representations (Diezmann & English, 2001; Pantziara, Gagatsis, & Elia, 2009), specifically in solving arithmetic word problems. Therefore, we have chosen a set of problems and applied multiplicative comparison (Lewis, 1989; Stern, 1993).

In this paper, we analyzed the responses of high school students to the translation of multiplicative comparison word problems to representation graphs. We have used the responses of 12 -14 year old students (freshman year of secondary school) to represent multiplicative comparison word problems to identify and categorize the students responses, which allowed us identify categories for each type of representation and hypothesize priority order and subordination between the categories.

We describe in detail the integrated quantitative diagram types produced by these students. The students are not familiar with building diagrams that integrate existing relations in word problems. Most of the students do not use all the quantitative information contained in the word problem, therefore draw diagrams referring to the subject or context of the problem without relating to the data in it. We have identified four kinds of integrated mapping utilized by the students in which they have used to represent multiplicative comparison problems with inconsistent procedures, which correspond to four strategies addressing the diagram construction.

**Keywords.** External representation, diagrammatic representations, diagrams.

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## Effective Use of Dynamic Geometry Software in Teaching Various Calculus Concepts

Muharrem Aktümen<sup>1</sup>, Bekir Kursat Doruk<sup>2</sup> and Zekiye Morkoyunlu<sup>3</sup>

**Abstract.** Teachers have an important role for the use of technology in mathematics classes. The factors which are related to teachers can be ordered as accessing the suitable teaching materials, technical support, the knowledge explaining how technology can be used in mathematics classes, sufficient time for computer aided education, accepting that computer aided education is a necessity, the experiences that the teachers have; and teachers' attitudes and beliefs [1]. In this study, the issue of how technology can be used in mathematics classes was the focus; and GeoGebra Software which integrates the Dynamic Geometry Software and Computer Algebra Systems was used [2]. In the study, the effective use of GeoGebra in class atmosphere will be discussed through the support of the literature. There will be several examples providing to examine several calculus concepts conceptually by using the properties of GeoGebra which are based on Dynamic Geometry Software and Computer Algebra Systems; and there will be design process of these examples.

**Keywords.** GeoGebra, calculus, manipulative materials.

**AMS 2010.** 97U60, 97U70.

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## Calculating Area of Circle with Different Integrations

Murat Altun<sup>1</sup> and Hatice Kübra Güler<sup>2</sup>

**Abstract.** Although the aims of mathematics education vary according to the different levels of education, the basic aim of mathematics education is to improve the mathematics literacy of the students. Mathematics literacy does not imply only the knowledge and skills of the traditional education. Additional ability to functionally use the mathematical knowledge within various contexts, and to reflect the inner perspective, and creativity are also implied [1]. The mathematics programmes in Turkey attach importance to provide students with conditions that students will develop creativity and reasoning [2]. Creativity to an important degree is fed by flexible thinking. Therefore, in order to improve creativity, there is need to allow place for spontaneous/self-induced thoughts, and opportunities for students to think flexibly, and to discuss their ideas [3]. The present study aimed at investigating flexible thinking and inclusion of creativity through focusing on ‘measuring the field of a circle by the help of integral’ as a topic. What underlies “integration” is the idea that particular magnitude greatness can be estimated by dividing it into similar little pieces; measuring the properties (length, field, volume) of the pieces, and adding these together [4]. The present study focused on the field estimation that is under the curve “the summation of similar infinite small parts” that could be calculated in various ways. “Estimating through integral” method was developed while explaining (helping students to comprehend), at elementary level, why the field of a circle is  $\pi r^2$  through an exemplar activity. It was shown that the area of a circle could be estimated in three ways: by dividing into rectangles, circles, and circle slices.

Through the current study it is aimed; to show that many integration methods can be produced by the learners once the ‘covering separate infinite small parts’ be designed, and to reinforce the idea (thought) of “integration”.

**Keywords.** Integration, area under the curve, creativity.

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## **Pre-Service Mathematics Teachers' Problem Solving Strategies with Gsp: Mirror Problem**

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**Abstract.** Problems and problem posing have been trend topics in mathematics education for many years. Main reasons lie behind on the significance of actually explained in their definitions. As known, problems are specific tasks that necessitate steps to get the solution [2] and problem solving is core of any ideas of functionality with mathematics and as well has a special importance in the study of mathematics. In parallel with developing technology, problem solving tools has become diversified. One of these tools is geometers sketchpad software giving chance to visualize modeling variations. It is dynamic mathematics visualization software used for exploring algebra, geometry, calculus, and other areas of mathematics and sciences [1]. This study specifically draws attention to the analysis of pre-service elementary mathematics teachers' problem solving processes in a given problem with the help of geometer's sketchpad. They used Geometer's Sketch Pad (GSP) to solve the mirror problem. It was carried out in spring term during 2011-2012 academic year with 56 freshman pre-service elementary mathematics teachers in Ağrı İbrahim Çeçen University. According to their works on GSP and reflections about their works, there appeared two different solution methods. In addition, they could not visualize the problem in their mind and apply it to GSP. More generally, they found the problem hard to solve.

**Keywords.** Dynamic geometry software, problem solving, pre-service mathematics teachers.

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## **A Comparison of the Instruction of ‘Circle Analytics’ in a Geogebra Setting In terms of Vocational and Anatolian High School Students**

Mihriban Hacısalihoğlu Karadeniz<sup>1</sup> and Ümit Akar<sup>2</sup>

**Abstract.** The scope of Geometry and its division into various branches in itself enabled Geometry to develop much faster than expected. Today, with the emergence of computers and the development of computer technology, the variety of educational and instructional tools and materials used for Geometry instruction increased in the past and still continues to increase [1]. Traditional Geometry instruction, which has survived for hundreds of years, has recently undergone a kind of revolution with the introduction of technology into education. Computers are used as an efficient tool of learning for the instruction of many courses and subjects from the first level of elementary school to the university level [2].

The concept of ‘Computer-Assisted Instruction (CAI) began to be used with the introduction of computers, a product of advanced technology, into the process of learning and instruction [3]. Computer-assisted Mathematics instruction is based on the use of computers at an appropriate and fixed time during the instruction of Mathematics and the exploitation of the materials prepared with the use of purpose-oriented software [4]. One of the most important software developed for CAI is GeoGebra. Because Geogebra works on such Mathematics terms as points, the right pieces, lines, conic sections and the like, it can be viewed as a dynamic Geometry software in a sense [5].

The aim of this study is to make, in a GeoGebra setting, a comparison between the success of Vocational High School and Anatolian High School students in circle analytics in terms of the gains in parallel with the suggestion of ‘Students achieve the general, vectoral and standart equation of the circle and do practices’ in ‘Circle equation’ a sub-learning section of eleventh-grade course ‘Circle’. In this study, a pretest was first given to the students in both groups. Second, based on the results obtained, the subject matter ‘Circle Analytics’ was studied with the help of GeoGebra software.

Based on the results of the post-test given to the students in both groups, students’ achievements were measured. In the sample group of the study were involved totally 34 eleventh-grade students, 17 of whom studied in the Field of Computer at Bulancak Vocational Technical and Industry High School and the rest of whom studied at Bulancak

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Bahçelievler Anatolian High School. An Achievement Test, which consists of 10 open-ended questions and which was developed in parallel with the views of experts and teachers, was given to students in the form of pre-and post-test. Independent t-test was given during the binary comparisons between different groups, whereas dependent t-test was applied during binary comparisons in the same group. No significant difference was found between pre-test achievement scores applied to both groups before the study [ $t(32) = .017, p > .05$ ]. The result obtained indicated that students' prior knowledge before instruction was equal. The data obtained from the study showed that instruction carried out in GeoGebra-based setting increased student-success in both groups. However, Anatolian High School students were able to learn the subject matter more creatively and effectively than Vocational High School students [ $t(32) = -3.557, p < .05$ ].

**Key Words:** Dynamic Geometry Software, GeoGebra, Circle Analytics, Eleventh-grade Students.

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## **Prospective Primary School Teachers' Technology Use During Mathematics Teaching Activities**

Mihriban Hacısalihoğlu Karadeniz<sup>1</sup> and Sayım Aktay<sup>2</sup>

**Abstract.** Today, since information is rapidly developing and changing, technology has become something all individuals need in their daily life. order for individuals to be able to make use of technology, they need to acquire necessary knowledge, attitudes, and behaviors. Equipping individuals with technological skills is among the primary goals of educational systems in the world. Accordingly, it is proper to conjecture that using technology during the instruction of mathematics activities will ensure effective and permanent learning. According to National Council of Teachers of Mathematics, the existence of technology and its multi way use made it possible to review that how students should learn mathematics. [1]. NCTM suggests that teachers should quit their duties as a sole authority and they should guide students to construct their mathematical knowledge and enrich their learning environments [1], [2]. Furthermore, teacher training programs emphasize the use of technology in mathematics in schools [3], [4]. In this context, technology use of prospective primary school teachers while teaching mathematics activities is very important.

In this study, how the prospective primary school teachers make use of technology while teaching mathematics, and whether they see themselves adequate along with the problems they encountered was investigated. This study was conducted in “Mathematics Teaching-I” course with participation of 32 prospective primary school teachers attending 3<sup>rd</sup> grade of faculty of education in Giresun University in Turkey in 2013 spring term. For the study, students were organized into 4-6 people groups and each group presented their assignment every week. Data was gathered through observation of the prospective teachers for 7 week. Furthermore, at the end of the application, semi-structured interviews were conducted with the prospective teachers. Content analysis technique along with NVivo qualitative data analysis software has been used in the analysis of interview data. As a result of the study, it was found that teacher candidates are experiencing some difficulties integrating technology into teaching mathematics. In addition, teacher candidates mentioned some technical problems undermining technology use such as defective cables, electrical problems.

**Keywords:** Technology, Mathematic teaching, Primary school teacher candidates.

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## **The Difference Between Trainee Teachers in the Departments of “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Science and Technology” in Terms Of Critical Thinking**

Mustafa Terzi<sup>1</sup> and İbrahim Yuksel<sup>2</sup>

**Abstract.** Critical thinking is an objective and productive form of reasoning. This form of reasoning seems to be essential, in everyday life, at work, at school, in short, in every environment that there are people for the productive purposes, it is considered to be a process that an individual reveals his/her complex mental activity. It's stated in society that individuals having advanced critical thinking skills are freed from dogmas, give the right direction to his/her beliefs and behaviors, they can trigger social change for the future in accordance with the requirements of the era, they are open-minded, objective and creative [1]. This study is important for comparison of trainee mathematics and science and technology teachers' critical thinking skills.

In the statistical analysis part of this study, it is tested whether there is a meaningful statistical difference between the average degrees of 84 trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” according to the CEDTDX test results. The test is composed of four parts. Part 1: Induction (23 Questions), Part 2: Reliability of Sources and Observations (24 Questions), Part 3: Deduction (14 Questions) and Part 4: Identification of hypotheses (10 Questions). In hypotheses, null hypothesis means that there is no difference between the average points of trainees from “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences”; and the alternative hypothesis means there is a difference. The answer to each question is assigned as binary variable: if the answer is right, 1 is assigned. If the answer is wrong 0 is assigned. Total point of a trainee is decided according to the sum of the right answers. According to this data it is investigated whether there is a meaningful statistical difference between the two group or not. In order to investigate that t-test which is frequently used in parametric statistic is used. Because the number of participants in each group is above 30, using this method is decided.

It is compared whether there is a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of induction. As a result of the T test applied to the groups, the significant value is found 0,071. Accordingly, there is not a

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meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of induction.

It is compared whether there is a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of Reliability of Sources and Observations. As a result of the T test applied to the groups, the significant value is found 0,941. Accordingly, there is not a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of Reliability of Sources and Observations.

It is compared whether there is a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of deduction. As a result of the T test applied to the groups, the significant value is found 0,232. Accordingly, there is not a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of deduction.

It is compared whether there is a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of identification of hypothesis. As a result of the T test applied to the groups, the significant value is found 0,083. Accordingly, there is not a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of identification of hypothesis.

According to the study results, it is supported that there is not a meaningful statistical difference between the average degrees of trainee teachers from the parts “Teacher Training in Mathematics at Primary School Level” and “Teacher Training in Sciences” in terms of critical thinking.

**Keywords.** Thinking, Thinking skills, Critical thinking, Reasoning

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## **Meanings that Secondary School Students Attributed to the “Mathematic” Concepts: Metaphorical Analysis**

Mustafa Terzi<sup>1</sup>

**Abstract.** The negative attitudes constituted against Mathematics are regarded as a significant factor influencing the emergence of the mathematic skills especially in the students. The students who are under the influence of such an attitude cannot acquire the mathematic knowledge in the demanded level and they attempt to memorize the mathematic knowledge they acquired without understanding, internalizing and comprehending (Isık, Ciltas Bekdemir, 2008). Due to this reason, putting forward the perceptions of the students regarding mathematics is thought to be as a need. Studies carried out regarding metaphors or through the use of metaphors in the different disciplines of the social science have increased quite a lot in the recent years. According to Lakoff and Johnson (2005) “the essence of the metaphor is to understand and experience a kind of thing according to another kind of thing.” In the recent years, many researches revealing the fact that metaphors are a strong research tool in determining the perceptions are held out (Inbar, 1996; Guerrero Villamiş, 2002; Saban, Kocbekar Saban, 2006). Researching the metaphors as one of the most significant perception tools is considered to be significant in revealing the students’ point of view on the mathematics. In this aspect, the research can be evaluated as a struggle to fill in the gaps in the literature. The purpose of this research is to reveal the perceptions of the secondary school students towards the concept of mathematic through metaphors.

The study group of the research is comprised of 220 students in total who are getting education in 6<sup>th</sup>, 7<sup>th</sup> and 8<sup>th</sup> classes. Seven open-ended questions were asked to each student participating in the research to reveal the mental images regarding the concept of mathematic. Data acquired in the research was analyzed through “content analysis”, which is one of the qualitative research methods. In the research it was acquired that the metaphors developed by the students towards the question, “What is mathematics? How do you define it?” “change basing upon the fact whether they like mathematics or not. In addition, 12% of the students responded to the question “In which subject did you have the most difficulty in learning in mathematics?” as “exponential numbers” 26% of the students responded to the question, “Why are we learning mathematics? What is the relation of the mathematics with the real life?” as “we are learning it as it is asked in the exam.” 39% of the students responded to

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question “Does your love towards your teacher influence your love towards the course? Why?” as “I don’t love mathematics as I don’t love my teacher.”; 25% of the students responded the question “How would you want the mathematics to be taught?” as “Through games, pictures and competition” and 20% of the students responded to the question “what would be lack in your life without mathematics “as “nothing would be a lack in my life”. Positive or negative attitudes and behaviors that the students develop against mathematics influence their success in the course. These results acquired from the research indicate that generally students have a negative attitude towards mathematics

**Keywords:** Mathematic, Metaphor

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## **The Evaluation of Pre-service Teachers' Studies on Geometric Thinking**

Nilüfer Y. Köse<sup>1</sup>

**Abstract.** In this study, it is aimed that the pre-service teachers' studies for improving geometric thinking are analyzed. For this aim, it is determined that the focus of this study is on how they prepare worksheets, how they apply these worksheets on a student with interactive interviews and how they assess these worksheets for students, themselves and geometric thinking. Participants of this study are of 29 pre-service teachers of a state university. The participants are those who enrolled "geometric thinking and its development" course for the spring term of the academic year. The data of the research consist of worksheets about geometry problems, the video records of the students on whom these worksheets are applied and detailed reports containing the evaluation of these video records. These pre-service teachers were working with groups of 3-4 and the "geometric habits of mind" framework learned during spring term was taken a base for their analysis. The "geometric habits of mind" framework consists of 4 components. These are "reasoning with relationships", "generalizing geometric ideas", "investigating invariants" and "balancing exploration and investigation". "Reasoning with relationships" means thinking about geometric figures, exploring geometric relationships and using special reasoning skills. "Generalizing geometric ideas" is understanding and defining geometric facts. In this process, steps, results and properties of the figures are generalized. "Investigating invariants" means analyzing properties and cases, either transforming or not in geometric structure. "Balancing exploration and investigation" is trying different solution methods and periodically turning preliminary steps in a problem to review the situation [1]. The qualitative data collected were analyzed based on these components of the framework and an evaluation of the worksheets prepared by pre-service teachers is carried out by the visualization of the results obtained.

**Keywords.** Geometric thinking, teacher education, geometric habits of mind.

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## **Secondary School Students' Levels of Visual Mathematics Literacy Self-Efficacy Perception**

Ömer F Tutkun<sup>1</sup>, Betül Öztürk<sup>2</sup> and Duygu Gür Erdoğan<sup>3</sup>

**Abstract.** The aim of this study is to determine the secondary school students' levels of visual mathematics literacy self-efficacy perception. In addition, the students' levels of visual mathematics literacy self-efficacy perception were examined through gender, levels of perceived mathematics achievement, grade level, levels of perceived income and educational level of parents. Descriptive survey model was used as the method of the research. "A visual mathematics literacy self-efficacy perception scale (VMLSPS)" developed by Bekdemir and Duran (2012) was used as the data collection tool. The sample of the study consists of a total of 342 secondary school students including 160 female and 182 male students. The findings obtained from the study are the followings: 1- The top group of the students' levels of visual mathematics literacy self-efficacy perception was found to 46.2% while the medium group to 41.2% and the lowest group to 12.6%. 2- It was observed that the levels of visual mathematics literacy self-efficacy perception differentiated in terms of gender, levels of perceived mathematics achievement, levels of perceived income and educational level of father. 3- It was observed that the levels of visual mathematics literacy self-efficacy perception did not differentiate in terms of grade level and educational level of mother.

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## **Orthic Triangle of a Triangle and the Examination of Relation between the Elements of These Two Triangles through Cabri II Plus Geometry Program**

Recep Aslaner<sup>1</sup> and Kübra Açıkgül

**Abstract.** Triangles have peculiar special points. Some of these points are: the intersection points of medians (center of gravity), intersections of perpendicular bisectors (center of circumferential circle) and intersection of bisecting angles (center of internal tangent circle) and these topics are taught in the geometry classes in the curriculum of secondary schools. Today, however, hundreds of points peculiar to triangles have been identified and interesting relationships among those points have been revealed. A triangle formed by a triangle's corners' perpendicular projection points on the opposite edge is called the orthic triangle of the given triangle. In this study we have examined the relation between fundamental elements of triangle and its orthic triangle's fundamental elements. Special conditions such as the impact of a triangle's being right angled triangle or obtuse angled triangle and its being equilateral triangle or isosceles on the findings of a particular triangle have been analyzed through Cabri II Plus geometry program.

**Keywords.** Altitude, Orthic Triangle, Fundamental Elements of Triangle.

**AMS 2010.** 53A40.

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## A Comparison of Software used for Mathematics Education

Selman Hizal<sup>1</sup>, Muhsin Çelik<sup>2</sup> and Ahmet Zengin<sup>3</sup>

**Abstract.** Teaching and learning in mathematics curriculums in universities by using mathematical software is a difficult and demanding task, especially for novice learners. Using computer-aided software offers practical solutions to instructors and students. Thus the technology can strengthen students' learning process by presenting content numerically, graphically, as well as symbolically without extra burden of spending time to calculate the complex computational problems by hand. There are several popular mathematical software that are widely used in mathematics education for enhancing students' skills and reinforcing teachers' pedagogical role. In this study, mathematical software which are used in teaching undergraduates are examined. These software are divided into two groups by their licenses based on closed source (Magma, Maple, Mathcad, Mathematica, Microsoft Mathematics, Matlab, Wolfram Alpha) and open source (Axiom, CoCoA, Mathomatic, Maxima, Octave, OpenAxiom, Sage, Xcas). After introducing each software in general, their functionalities are compared. Also, the operating system platform support (Windows, Mac OS, Linux, BSD, Solaris) is given. Thus, the choice of mathematics software for education is simplified.

**Keywords.** Mathematical software, computer programs.

**AMS 2010.** 97N80.

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## Developing Concept of Parabola: A Case Study

Tuba Ada<sup>1</sup>, Aytaç Kurtuluş<sup>2</sup>, H. Bahadır Yanık<sup>3</sup>

**Abstract.** The purpose of this case study is to examine how a 10th grade student developed understanding of parabola which is part of conic sections. The study was conducted in Eskisehir, Turkey as part of a school project. The researchers designed a learning environment in which both Euclidean and Taxicab geometries were used. The researchers first designed tasks that required the student to use her past experience and knowledge she gained in Euclidean geometry and identified some preconceptions. Once the researchers and the student worked altogether on these preconceptions, Taxicab geometry was introduced to the student. The student worked on tasks that helped her developed concept of parabola in Taxicab geometry. The findings of the study showed that the student transferred some knowledge from Euclidean geometry to solve tasks designed in Taxicab geometry related to parabola. As part of the study, the student also designed a model of a bridge in Euclidean and Taxicab geometries using knowledge of parabola.

**Keywords.** Parabola, Euclidean Geometry, Taxicab Geometry.

**AMS 2010.** 51N, 91C.

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**Problem-Solving Skills of Students at Primary Education Department of  
Mathematics and Science**

Uğur Karaman<sup>1</sup>, Mustafa Terzi<sup>2</sup>, İbrahim Yüksel<sup>3</sup>

**Abstract.** This study aims to examine the perceptions of the problem solving skills of the students who are at Mathematics and Science Education, (Primary Education Department ) Faculty of Education-Gazi University. A total of 235 students, consisting of 168 females and 67 males, have participated in the study. The Problem Solving Inventory, developed by Heppner and Peterson (1982) and adapted into Turkish by Şahin, Sahin and Heppner (1993) has been used in the research. T test was utilized for independent samples on the comparison of the problem solving skills points of the university students by demographic variables in the research.

The problem solving skills points of the students have indicated that there is no significant difference by the gender and department of the students. However, the problem solving skills points of the students have shown a significant difference by the classlevel. According to the seresults, it may be said that the 4th grade students' problem solving skills are higher than 2nd gradestudents'.

**Keywords.:** Problem, Skills, Problem Solving Skills.

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## **The Interpretation Process of Pre-service Elementary Mathematics Teachers' in Concepts of Linear Dependence-Independence and Spaning in $\mathbb{R}^2$ and $\mathbb{R}^3$**

Yasemin Kıymaz<sup>1</sup>, Cahit Aytakin<sup>2</sup>, Tuğba Horzum<sup>3</sup>

**Abstract.** In this study, the interpretation process of pre-service elementary mathematics teachers' in concepts of linear dependence-independence and spaning in  $\mathbb{R}^2$  and  $\mathbb{R}^3$  are examined. For this purpose, a pre-test which included open-ended questions was given to 38 preservice elementary mathematics teachers who attended a course on Linear Algebra. Then, two groups of pre-service teachers consisting of 4 volunteers were interviewed. During the interview the pre-service teachers found an opportunity to see the provisions of these concepts in  $\mathbb{R}^2$  and  $\mathbb{R}^3$  by using GeoGebra. In other words, the association processes of the pre-service elementary mathematics teachers between the formal definitions and provisions of the concepts were investigated.

**Keywords.** Linear dependence, linear independence, spaning.

**AMS 2010.** 97H60, 97D99.

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## Catching the stars: Principles and Extant Examples of Astrolabe

Yasemin Nemlioğlu Koca<sup>1</sup>

**Abstract.** Nowadays, with the development of technology, any location in the world can be found by electronic devices and satellite systems. In the past, this work was done with the astrolabe. Astrolabe which was a device from consisting of working disks with trigonometric principles in astronomy, was used in areas such as the graphical representation of various of problems, measuring the angles of the height of the stars, determining of latitudes, measuring of the time, obtaining information about the zodiac and etc. Astrolabe which was a portable device like a quadrant and used upright on observation. The height and positions of the stars and the sun were measured by an astrolabe on the horizon.

Astrolabe was the most common and best-known device of astronomy and known by the Greeks in 2<sup>nd</sup> - 4<sup>th</sup> centuries BC. [1] It was developed in ancient times, however it was known that its variations and using was increased in the Arab-Islamic culture 8<sup>th</sup> - 9<sup>th</sup> centuries. Astrolabe which was learned from Muslims during the period of the Crusades and the Andalusia was used on open sea especially by the Spanish and Portuguese sailors thanks to its practical usage.

Mathematicians were continued using the Astrolabe until the 18<sup>th</sup> century on the other hand in Ottomans *muvakkithânes*<sup>2</sup> and observatories. There are protected astrolabes in Istanbul by various museums.[2]

The present paper aims to study the examples and principles of the astrolabe in Museums of Istanbul.

**Keywords.** Astrolabe, Astronomy, Istanbul Museums, Geographical Location, Observatory

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<sup>2</sup> Muvakkithanes were a time keeping room and where the time of the day was determined according to the position of the sun, clocks were adjusted and correct prayer times were established and announced to the public. Muvakkithanes acted as small observatory and were usually built alongside Mosques and Kulliyes.

# *STATISTICS*



## A clustering method for risk classification in insurance

Ayşen Apaydin<sup>1</sup>, Furkan Baser<sup>2</sup>, Gultac Eroglu Inan<sup>3</sup>

**Abstract.** Insurance companies operate in an environment which becomes increasingly competitive. In order to succeed in this environment they strive for a combination of market growth and profitability, and these two goals are at times conflicting. This paper considers the problem of classifying policy holders and predicting claim costs in the automobile insurance industry. We differentiate policy holders according to their perceived risk by using fuzzy c-means clustering algorithm.

**Keywords.** Insurance, risk classification, clustering.

**AMS 2010.** 62-07, 62H30, 91C20.

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## **Earthquake Occurrence Probabilities For East Anatolian Fault Zone With Discrete Parameter Markov Chains**

Adem Doğaner<sup>1</sup> and Sinan Çalık<sup>2</sup>

**Abstract.** Markov Chains are one of the probabilistic methods that allow to estimate the next state when present the current state is known. Because of this is that next state only depend on current state. Many natural events occurs as sequence. Occurrence of earthquake is an example for this events. An magnitude of earthquake depend on that energy and stress occurs during a previous earthquake in same region.

In this study, magnitude of earthquake, epicenters, hypocenters were estimated by discrete parameter Markov Chains method for may occurring at future in East Anatolian Fault Zone. Dataset related to earthquakes occurred in East Anatolian Fault Zone between 1900 and 2010 were used for estimates. Markov Chains is convenient method to estimate and modelling for natural events such as earthquake.

**Keywords:** Applications in probability theory and statistics, Applications of Markov chains and discrete-time, Markov chains,

**AMS2010:** 46N30;60J20;60J10

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## **Development of Information Generation Model Based on Hidden Markov Model in Information Systems**

Adem Dođaner<sup>1</sup> and Sinan alık<sup>2</sup>

**Abstract.** Word generation, word sequences in sentence and meaningful information to produce in words are important parameters for development of information systems. Information generation is possible by word classification in database, meaningful word sequence and word obtaining information.

Computer sciences and stochastic models are closely related to information generation. Bayesian network, Markov models and some statistical methods is frequency used for computer sciences.

In this study, information generation model were focused by hidden Markov models. Words in text document were classified for automatic information generation. Word types and elements of sentence were determined as parameters. Algorithms based on hidden Markov model were studied.

As a result, algorithm based on hidden Markov models in information system for automatic information generation were observed to can give successful results.

**Keywords:** Information Systems, Applications of Markov chains and discrete-time, Database theory

**AMS2010:** 68U35; 60J20; 68P15

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## Distributing Data Into Intervals Using Weighed Moving Averages

Alma Krivdić

**Abstract.** Task was to distribute data from telecom files into defined number of intervals in such way that every interval contains the most probable quantity of data according to some „usual“ behavior of subjects. In order to do that, it was decided to apply weighed moving averages principles and Kullback-Leiber divergence. Since profiling was aim of this experiment, measuring chance that certain data (ex: call that is 3 minutes long) would appear was very important. Profiling was designed on principle of comparison between “good” – usual behavior and “bad” – unusual behavior. Data did not follow normal distribution, so it was important to find intervals containing outliers (having small probability).

**Keywords.** fraud profiling, AWKL, weighed average

**AMS 2010.** 53A40, 20M15.

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## A New Approach in Nonlinear Regression for Breakdown Point

Ahmet Pekgör<sup>1</sup> and Aşır Genç<sup>2</sup>

**Abstract.** Breakdown point is one of the robustness measures of an estimator. In robust estimators, the value of this measure is on the other hand in nonrobust estimators, the value of this measure is. Unfortunately in today's statistical packet programmes, the robustness measure of handed estimator can not be provided to the user. This impossibility causes the user to compute the robustness of an estimator of his interested model by hand and theoretically. But on the other hand by this way, computing of the robustness of an estimator is always impossible because of the difficulty of the model and the big size of computing. Therefore in the user's interested model and in the results of an estimator that the user is interested, computing breakdown point is very important. Starting from this point, computing breakdown point by a computer is the main theme of this study. In this study, by defining an alternative new breakdown point[4], computing of breakdown point is provided to the user in computer occasion and also by courtesy of this new definition, sorting of estimators in terms of breakdown points is allowed.

**Keywords.** Breakdown point, Nonlinear regression, Robust estimators, Trespit method.

**AMS 2010.** 62G35, 62J02.

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## **Robust Quadratic Hedging Problem in Incomplete Market Under Parameter Uncertainty : For One Period and Exponential Asset Price Model**

Gultac Eroglu Inan<sup>1</sup> And Aysen Apaydin<sup>2</sup>

**Abstract.** In this study, first the quadratic hedging problem was handled for one period in incomplete market. The optimal solution of the problem, obtained by Föllmer-Schweizer (1989) by the linear regression approach was given. In the stochastic models defined for the asset price,  $\mu$  drift and  $\sigma$  volatility model parameters values allways may not be known exactly. Optimal solution may be affected the parameters uncertainty. Robust optimization methods, used to overcome the uncertainty can be used in the hedging problems. In this study Pınar (2006) robust quadratic hedging problem was handled. For one period and exponential stochastic asset price model (Brownian motion) ; one application of this approach was given in the situation that volatility and drift are unknown. Numerical results show that; classical solution obtained in the situation volatility and drift are known and the solution obtained in the uncertainty situation give the similar values at the objective function. As result; we can say that, the solution obtained in the uncertainty situation, is robust to volatility and drift variation.

**Keywords.** Robust quadratic hedging problem, uncertainty, robust optimization, incomplete market.

**AMS 2010.** 90C20, 90C47, 49K35.

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## **An Application on Estimated Value of Stocks Using Artificial Neural Networks**

İbrahim Demir<sup>1</sup>, Hasan Aykut Karaboğa<sup>2</sup>

**Abstract.** In recent years, artificial neural networks are at the beginning of the most commonly used methods in areas such as management, economics, civil engineering, computer engineering, mapping engineering and data mining. The most important reason on the basis of the principle of artificial neural networks to mimic the human brain works. So, neural networks are run by a system as in the learning model of human brain. With this learning algorithm as well as using the given information inferences are made, estimates are made or comments are made as in the human brain.

Financial sector is one of the most used areas of artificial neural networks. Artificial neural networks is a popular method used in the most crucial issues such as financial failure prediction, the index forecast, price forecast, the financial sector, and bankruptcy prediction. The most important reason of the use of artificial neural networks in these issues is to have the characteristics such as understanding the complex structures, using the past information, and also working with missing information. Financial markets are becoming more complex and gaining a different dimension day by day. Due to the impact of globalization, the emergence of very large financial institutions, and enabling the opportunity to trade in all markets at the same time with the positive effect (moving forward) of technology in this sector made the world's financial markets become even more dependent on each other. Also, new financial instruments such as derivatives give a different dimension to the markets as well as decreased the number of small investors that can make right investment. However, in World markets the interests of small as well as large investors in gold, foreign exchange, commodity, and the stocks continue to increase. Without doubt the most sensitive one from all of these financial products and securities is the stocks.

Stocks which are in the highest risk group and parallel to this provides the greatest return are financial investment instruments providing immediate response to all economic and political issues. A wide variety of studies have been conducted on to what extend the markets and the basic economic factors affect stocks and on what will happen the future prices of stocks. The ability to estimate the stock prices at the end of a certain time attracts the attention of many investors and academics. For this purpose, in this study with the help of data covering years between 1990 and 2012 stocks included in certain basic indexes are used and primarily factors

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affecting the price changes are tried to be identified and later the price of stocks at the end of a certain time are tried to be understood whether it has an upward trend or downward trend by using daily and weekly data trends. In addition to all this, the ratio of price change in the next step is also predicted by artificial neural networks.

**Keywords.** Artificial Neural Networks, Financial Forecast, Stock Price Forecast, Stock Market, Neural Networks.

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## Estimation of Time Varying Parameters in an Optimal Control Problem

Levent Özbek<sup>1</sup> and Esin Köksal Babacan<sup>2</sup>

**Abstract.** In this paper, we employ a non-linear state space model and the extended Kalman filter to estimate the time-varying parameters in an optimal control problem, where the objective (loss) function is quadratic. Our methodology also allows us to derive the difference between the optimal control and the observed control variable. A simulation exercise based on a simple intertemporal model shows that the estimated parameter values are very close to their population values, which provide further support for the estimation methodology introduced in this paper.

**Keywords.** Non-linear state space models, Extended Kalman filter, Optimal linear regulator.

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## Approximate Bayesian Computation Using Modified Loglikelihood Ratios

Laura Ventura<sup>1</sup> and Walter Racugno<sup>2</sup>

**Abstract.** Asymptotic arguments are widely used in Bayesian inference, and in recent years there have been considerable developments of the so-called higher-order asymptotics [1]. This theory provides very accurate approximations to posterior distributions, and to related quantities, in a variety of parametric statistical problems, even for small sample sizes. The aim of this contribution is to discuss recent advances in approximate Bayesian computations based on the asymptotic theory of modified loglikelihood ratios, both from a theoretical and a practical point of view. Results on third-order approximations for univariate posterior distributions, also in the presence of nuisance parameters, are reviewed [2] and a new formula for a vector parameter of interest is presented. All these approximations may routinely be applied in practice for Bayesian inference, since they require little more than standard likelihood quantities for their implementation. Moreover, these approximations give rise to a simple simulation scheme, alternative to MCMC, for Bayesian computation of marginal posterior distributions for a scalar parameter of interest. In addition, they can be used for testing precise null hypothesis and to define accurate Bayesian credible sets [3]. Some illustrative examples are discussed, with particular attention to the use of matching priors [4].

**Keywords.** Asymptotic expansion, Bayesian simulation, Credible set, Laplace approximation, Marginal posterior distribution, Matching priors, Modified likelihood root, Nuisance parameter, Pereira-Stern measure of evidence, Precise null hypothesis, Tail area probability.

**AMS 2010.** 62F15.

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## Discrete Choice Models And Its Applications in Marketing

Melfi Alrasheedi<sup>1</sup> and Ateq Alghamedi<sup>2</sup>

**Abstract.** Scientific research as well as ordinary life situations are often related with choices and decisions. There are many directions of application of choice models, such as transport demand [1,2], marketing research [3,4], adoption decision [5] etc. In particular, the analysis of demand on different goods is one of the most perspective directions. In this paper, the equations for logit model probabilities are derived and analyzed. The logit regression procedure was applied in order to estimate the demand for different goods in the market. The results have shown that the logit with fixed effect is more suitable than simple logit and probit models. The obtained odds ratios for fixed-effect logit show the correspondence with experimentally obtained probabilities. Considered methods can be used for price management in markets in order to increase the demand on the given product.

**Keywords.** Decisions, discrete choice models, marketing research, logit and probit models.

**AMS 2010.** 62-07, 62P20, 90B50.

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## **The Relation Between Energy Consumption and Economic Growth: Summary of Panel Data Analysis**

Mehmet Yusuf Erdoğan<sup>1</sup>, Fatih Kaplan<sup>2</sup>

**Abstract.** In this study, the relation between GDP and energy consumption is examined for 11 countries between 2001 and 2010. Accordingly, the panel data of countries and unit root tests, and cointegration tests and the long-term coefficients have been investigated. In the research, Levin-Lin ve Chu (LLC), Breitung, Im-Pesaran ve Shin (IPS), Fisher ADF, Fisher PP and Hadri unit root tests have been used. In order to test cointegration Pedroni, Kao and Johansen- Fisher cointegration tests have been used; to estimate the long-term coefficients PMGE (Pooled Mean Group Estimation) has been used. As a result of the econometric applications, it has been found out that there is a cointegration relation between energy consumption and GDP and this relation is -0.5 in the long term.

**Keywords:** Energy consumption, Economic Growth, Panel Data, Panel Unit Root Tests, Panel Cointegration.

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## A Study on the Equivalence of BLUEs under General Partitioned Linear Model and its Related Models by the IPM Method

Nesrin Güler<sup>1</sup>

**Abstract.** The general partitioned linear model, denoted by  $\mathcal{A} = \{y, X_1\beta_1 + X_2\beta_2, V\}$  and known as full model, with its submodels, reduced models, and alternative models is considered. Using the inverse partitioned matrix (IPM) method introduced by [14], the formulas for the differences between the best linear unbiased estimators (BLUEs) of  $M_2X_1\beta_1$  with  $M_2$  being an appropriate projector under the considered models are obtained. Furthermore, the necessary and sufficient conditions for the equality between the BLUEs of  $M_2X_1\beta_1$  under the full model and its related models are given.

**Keywords.** BLUE, general partitioned linear model, reduced model, submodel, alternative model, orthogonal projector.

**AMS 2010.** 62J05, 62H12, 62F30.

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## Parameter Estimation in Geometric Process with the Inverse Gaussian Distribution

Özlem Türkşen<sup>1</sup>, Mahmut Kara<sup>2</sup> and Halil Aydoğdu<sup>3</sup>

**Abstract.** A counting process  $\{N(t), t \geq 0\}$  with the interoccurrence times  $X_1, X_2, \dots$  is a geometric process if there exists a positive real number  $a$  such that  $a^{i-1}X_i, i = 1, 2, \dots$  forms a renewal process. In this study, a geometric process is considered with the distribution of the first occurrence time of an event,  $X_1$ , assumed to be Inverse Gaussian. The process parameter  $a$  and distribution parameters  $\mu$  and  $\sigma^2$  are estimated by using the maximum likelihood (ML) method. The asymptotic distributions and consistency properties of the ML estimators are derived. Monte Carlo simulations are performed to compare the efficiencies of the ML estimators with the corresponding modified moment (MM) estimators.

**Keywords:** Geometric process, maximum likelihood, asymptotic normality.

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## **Sharp Bounds on Lifetime Dispersion of Reliability Systems with Exchangeable Components**

Ptryk Miziula<sup>1</sup> and Tomasz Rychlik<sup>2</sup>

**Abstract.** We consider arbitrary mixed reliability systems composed of exchangeable components. Then the lifetime distribution has so called Samaniego representation which is a convex combination of distributions of order statistics of component lifetimes with the combination coefficients dependent merely on the system structure. The vector of the coefficients is called the Samaniego signature. We present universal sharp lower and upper bounds on various measures of dispersion of the system lifetime distribution gauged in the same units of dispersion for the component lifetime distribution. The dispersion measures include variance, median absolute deviation and many others. The bounds are expressed in terms of the Samaniego signature.

**Keywords.** Reliability system, exchangeable components, Samaniego signature, dispersion, sharp bound.

**AMS 2010.** 60E15, 62G30, 62N05.

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## Poisson Regression Analysis in the Presence of Measurement Error

Rukiye Dağalp<sup>1</sup>, Özlem Türkşen<sup>2</sup> and Esin Köksal Babacan<sup>3</sup>

**Abstract.** Regression analysis is a statistical methodology for studying the relationship between dependent ( $Y$ ) and independent variable ( $X$ ) so that  $Y$  can be explained from  $X$  variable. Generally in real world, the independent variable cannot be observed, either because it is too expensive, unavailable, or mismeasured. In this situation, a substitute variable  $W$  is observed instead of  $X$ , that is  $W=X+U$  where  $U$  is a measurement error. This substitution of  $W$  for  $X$  creates problems in the analysis of the data, generally referred to as measurement error problems. The statistical models use to analyze such data are called *measurement error models*. These measurement error problems occurs in many areas such as environmental, agricultural, medical investigations, or electrical engineering.

In this study, the independent variable  $X$  is considered with measurement error and the dependent variable  $Y$  is considered as a Poisson variable. The aim of the work is defining the relationship between  $X$  and  $Y$  by using Poisson regression analysis with measurement error.

**Keywords:** Poisson Regression Analysis, measurement error, regression calibration, SIMEX method.

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## **An Upper Bound for the Ergodic Distribution of an Inventory Control Model When Demands and Inter-Arrival Times are Dependent**

Tahir Khaniyev<sup>1</sup> and Cihan Aksop<sup>2</sup>

**Abstract.** A stochastic process which represents an inventory control model with (s,S)-type policy is constructed when the demands and inter-arrival times are dependent. Under the assumption that the demands can be expressed as a monotone convex function of the inter-arrival times, it is proved that this process is ergodic and an upper bound for the ergodic distribution is given.

**Keywords.** Dependence, ergodic distribution, inventory model.

**AMS 2010.** 60K15.

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## Benford's Law and Invariance Properties

Zoran Jasak<sup>1</sup>

**Abstract.** Benford's law is logarithmic law for distribution of leading digits formulated by  $P[D = d] = \log(1+1/d)$ , where  $d$  is leading digit or group of digits. Benford's law describes probability for leading digits of number. It's named by Frank Albert Benford (1938) who formulated mathematical model of this probability. Before him, the same observation was made by Simon Newcomb. This law has changed usual presumption of equal probability of each digit on each position in number.

Main characteristic properties of this law are base, scale, sum, inverse and product invariance. Base invariance means that logarithmic law is the same for any base. Inverse invariance means that logarithmic law for leading digits holds for inverse values in sample. Multiplication invariance means that if random variable  $X$  follows Benford's law and  $Y$  is arbitrary random variable with continuous density then  $XY$  follows Benford's law. Sum invariance means that sums of significands are the same for any leading digit or group of digits. Term 'significand' is used instead of term 'mantissa' to avoid terminological confusion with logarithmic mantissa.

Special attention, from practical and theoretical point of view, deserves sum invariance property. If we have sample of size  $N$  we need appropriate test to examine how much sample deviates from predicted sums of significands. Main problem is to establish critical values which we need for any statistic test. In my work I propose a method to test this property by use of limit values based on sample size and some of standard statistical tests.

**Keywords.** Benford's law, invariance, statistical tests.

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**On Moments of the Semi-Markovian Random Walk with Delay**  
Zafer Küçük<sup>1</sup>, Rovshan Aliyev<sup>2</sup>, Tahir Khaniyev<sup>3</sup>, Burcu Hasancebi<sup>4</sup>

**Abstract.** Markov processes which one of the important fields of probability theory is used quite often in practice. The semi-Markovian random walk with special barrier considered which belong this class is a stochastic model. In this study, under some weak assumptions the ergodicity of this process is discussed. Furthermore, when  $\{\zeta_n\}$ ,  $n \geq 1$  which describes the discrete interference of chance is random variable with triangular stationary distribution in the interval  $[s, S]$  with center  $a \equiv (S - s)/2$ , the exact and asymptotic formulas for the first four moments of ergodic distribution of the process are obtained. To solve some practical problems for the variance, skewness and kurtosis of the ergodic distribution of the process are established by means of above expansions asymptotic formulas. Finally, by using Monte Carlo experiments it is shown that the given approximating formulas provide high accuracy to exact formulas.

**Keywords.** Semi-Markovian random walk, a discrete interference of chance, asymptotic expansion.

**AMS 2010.** 60G50, 60F05.

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# *TOPOLOGY*



## Almost Menger Property in Bitopological Spaces

A. Emre Eysen<sup>1</sup> and Selma Özçağ<sup>2</sup>

**Abstract.** The notion of almost Menger property was introduced in [1]. In bitopological case: A bitopological space  $(X, \tau_1, \tau_2)$  has almost Menger property if for each sequence  $(U_n : n \in \mathbb{N})$  of  $\tau_1$ -open covers of  $X$  there exist a sequence  $(V_n : n \in \mathbb{N})$  such that for every  $n \in \mathbb{N}$ ,  $V_n$  is finite subset of  $U_n$  and  $\bigcup_{n \in \mathbb{N}} \bigcup_{V \in V_n} cl_{\tau_2}(V) = X$ . [2]

Here we investigate some properties of almost Menger bitopological space.

**Keywords.** Menger, almost Menger, open cover,  $\omega$ -cover.

**AMS 2010.** 54A25, 54C35, 54D20.

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## Near Soft Sets and Near Soft Topology

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**Abstract.** Near set theory provides a formal basis for observation, comparison and classification of perceptual granules. Soft set theory was proposed by Molodtsov as a general framework for modelling vagueness. Feng and Li investigated the problem of combining soft sets with rough sets. The purpose of these paper is obtaining a different model called near soft sets based on a nearness approximation space.

**Keywords.** Soft sets, Near sets, Soft topological space, Near soft sets, Near soft topological space, Near soft interior, Near soft closure

**AMS 2010.** 03E75, 03E99, 54A10

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## Topologies and Approximation Operators Induced by Binary Relations

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**Abstract** Rough set theory is an important mathematical tool for dealing with uncertain or vague information. This paper studies some new topologies which induced by a binary relation on universe with respect to neighborhood operators. Moreover, the relations among them are studied. In addition, lower and upper approximation of rough sets using the binary relation with respect to neighborhood operators are studied and examples are given.

**Keywords:** Rough sets, rough topology

**AMS 2010:** 54A05,54A10

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## Weak Structures and p-Stacks

Aslı Güldürdek<sup>1</sup>

**Abstract.** Recently Csaszar [1] introduced the weak structures. We study the p-stacks in weak structures. We first introduce the convergence of p-stacks on weak structures, then define some separation axioms, and compactness of weak structures. We also characterize these separation axioms and compactness by using the convergence of p-stacks.

**Keywords.** Generalized topological space, weak structure, p-stack.

**AMS 2010.** 54A05, 54C10.

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## More about of Generalized Shift Dynamical Systems

Fatemah Ayatollah Zadeh Shirazi<sup>1</sup>

**Abstract.** For topological space  $X$ , nonempty set  $\Gamma$ , and map  $\varphi: \Gamma \rightarrow \Gamma$ , define  $\sigma_\varphi: X^\Gamma \rightarrow X^\Gamma$  with  $\sigma_\varphi((x_\alpha)_{\alpha \in \Gamma}) = (x_{\varphi(\alpha)})_{\alpha \in \Gamma}$  (for  $(x_\alpha)_{\alpha \in \Gamma} \in X^\Gamma$ ) [2]. We call  $(X^\Gamma, \sigma_\varphi)$  a generalized shift dynamical system. In this talk we have a short look to dynamical properties of generalized shift dynamical system  $(X^\Gamma, \sigma_\varphi)$  for discrete  $X$ . For example if  $X$  is finite discrete space with at least two elements, then the following statements are equivalent [1]:

- the generalized shift dynamical system  $(X^\Gamma, \sigma_\varphi)$  is distal;
- the generalized shift dynamical system  $(X^\Gamma, \sigma_\varphi)$  is equicontinuous;
- all points of  $\Gamma$  are periodic points of  $\varphi$ .

We will continue our study other dynamical properties of generalized shift dynamical systems, e.g. we study scrambled pairs in  $(X^\Gamma, \sigma_\varphi)$  for discrete  $X$  with at least two elements and countable  $\Gamma$ .

**Keywords.** Distal, Equicontinuous, Generalized shift.

**AMS 2010.** 37B99, 54H20.

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## **Textures and The Limit of Inverse Systems**

Filiz Yıldız<sup>1</sup>

**Abstract.** The notion of *Texture* was introduced as a point-based setting for the study of fuzzy sets, and have since proved to be an appropriate setting for the development of complement-free mathematical concepts. In this sense, it will be clear that textures may be regarded as a closed set version of the C-spaces and a *ditopology* on a texture is essentially a "topology" for which there is no a priori relation between the open and closed sets, and thus ditopological texture spaces were conceived as a point-based setting for the study of fuzzy topology, and provide a unified setting for the study of topology, bitopology and fuzzy topology.

In this work, a foundation is laid for a suitable theory of inverse system in plain textures as an analogue of the notion of inverse system in classical set theory.

According to that, first of all, the concepts of "inverse system" and "inverse limit" are defined and some properties of them in the category **ifPTex** of plain textures and w-preserving functions are studied. Following that, the theory of inverse system are considered for the category **ifPDitop** in the context of ditopological plain texture spaces.

**Keywords.** Texture, Ditopology, Category, Inverse system, Inverse limit

**2010 AMS Classification.** 54A05, 54C30, 03E20, 54B30.

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## Simplicial Cohomology Rings of Digital Images

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**Abstract.** In this study, digital versions of some concepts from algebraic topology has been studied. We define the relative cohomology groups of digital images and state its basic properties[1]. When we compute cohomology groups of a space, additional structures in cohomology have been obtained. One of these structures is the multiplication in cohomology, called cup product, which makes the cohomology groups of a space into a ring[2]. We state simplicial cup product for digital images and use it to establish ring structure of digital cohomology. Furthermore we suggest a method for computing the cohomology ring of digital images and give some examples concerning cohomology ring.

**Keywords.** Digital simplicial relative cohomology group, cup product, cohomology ring.

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## Two General Fixed Point Results On Weak Partial Metric Space

Gonca Durmaz<sup>1</sup> and Özlem Acar<sup>2</sup>

**Abstract.** In this work, we obtain two fixed point results on weak partial metric space. Our results are extend and generalize some previous results.

**Keywords :** Fixed point, partial metric, weak partial metric.

**AMS 2010.** 54H25, 47H10

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## A Generalized Polynomial Invariant of Regular Isotopy for Chiral Knots and Links

İsmet Altıntaş<sup>1</sup>

**Abstract.** In this paper, we define a two-variable polynomial invariant of regular isotopy,  $X_L$ , for chiral knot and link diagrams  $L$ . From  $X_L$  by multiplying it by a normalizing factor, we obtain a polynomial invariant of ambient isotopy,  $Y_L$ , for chiral knots and links. We show that the polynomial  $Y_L$  generalizes the Jones polynomial and the normalized bracket polynomial and the normalized polynomial  $N_L$  for chiral knots and links. We also define a three-variable polynomial invariant of regular isotopy,  $E_L$ , for chiral knots and link diagrams  $L$  that generalizes  $X_L$  the polynomial and the Kauffman polynomial  $L$ . From the polynomial  $E_L$  by multiplying it by a normalizing factor, we produce a polynomial invariant of ambient isotopy,  $M_L$ , for chiral knot and link diagrams  $L$ . The polynomial  $M_L$  generalizes the polynomial  $Y_L$  and the Kauffman polynomial  $F$  and the Homfly polynomial for chiral knots and links.

**Keywords:** Bracket polynomial, Jones Polynomial, Kauffman polynomials, Homfly polynomial, regular isotopy, ambient isotopy, chiral knot and link.

**AMS 2010.** 57M25.

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## On Braids and Jones Polynomial

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**Abstract.** In [1], a polynomial invariant of regular isotopy,  $G_L$ , is constructed for oriented knot and link diagrams  $L$ . From  $G_L$  by multiplying it by normalizing factor, an ambient isotopy invariant,  $N_L$ , is obtained, and it is showed that the polynomial  $N_L$  yields The Jones polynomial. In this paper, we demonstrate the normalized polynomial  $N_L$  is a version of the original jones polynomial by way of the theory of braids. For this purpose, by discussing generalities about braids we look directly at the polynomial  $G_L$  on closed braids. In the process, the structure of the Jones polynomial and its associated representations of the braid groups will naturally emerge.

**Keywords.** G-polynomial, N-polynomial, Jones polynomial, braid group, diagram monoid.

**AMS 2010.** 57M25

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## On Intuitionistic Fuzzy Filters

İrem Eroğlu<sup>1</sup> and Erdal Güner<sup>2</sup>

**Abstract.** Let  $X$  be a nonempty set and  $F$  be an intuitionistic fuzzy filter on  $X$ . In this paper, defining  $Q$ -convergence in  $F$  some results on  $Q$ -convergence are obtained. Also, let  $(X_1, F_1), (X_2, F_2)$  be intuitionistic fuzzy filter spaces and  $p_1, p_2$  be intuitionistic points in  $X_1, X_2$  respectively. We will show that if  $p_1$  is a  $N$ -cluster point of  $F_1$  and if  $p_2$  is a  $N$ -cluster point of  $F_2$ , then  $(p_1, p_2)$  is a  $N$ -cluster point of  $F_1 \times F_2$ .

**Keywords.** Fuzzy filter, intuitionistic fuzzy filter,  $Q$ -convergence,  $N$ -cluster point.

**AMS 2010.** 53A40, 20M15.

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## Points of Uniform Convergence

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**Abstract.** Let  $(f_n)_n$  be a sequence of functions defined on a topological space  $X$  with values in a metric space  $(Y, d)$ . We say that the sequence  $(f_n)_n$  uniformly converges at a point  $x \in X$  if for every positive  $\varepsilon$  there exists a neighbourhood  $U$  of  $x$  and a positive integer  $n_0$  such that  $d(f_n(y), f_m(y)) < \varepsilon$  for each point  $y \in U$  and each  $m, n \geq n_0$ . Denote the set of all points of uniform convergence of  $(f_n)_n$  by  $U(f_n)$ .

In [1] it is shown that  $U(f_n)$  is a  $G_\delta$ -set and contains all isolated points of  $X$  (in the case if  $(f_n)_n$  converges). We will give the characterization of the set  $U(f_n)$ . For a metric space  $X$  and a  $G_\delta$ -set  $G$  there are functions  $f_n : X \rightarrow \mathbb{R}$  such that  $(f_n)_n$  converges and  $G = U(f_n)$ . Likewise, there are quasicontinuous functions  $f_n : X \rightarrow \mathbb{R}$  such that  $G = U(f_n)$ . However, we cannot require simultaneously that  $f_n$  are quasicontinuous and  $(f_n)_n$  converges or that  $f_n$  are continuous.

Baire metric spaces can be characterized using the set  $U(f_n)$  too. A metric space  $X$  is Baire if and only if for each convergent sequence  $(f_n)_n$  of quasicontinuous functions, the set  $U(f_n)$  is dense.

We say that a sequence  $(f_n)_n$  is equi-quasicontinuous at a point  $x$  if for every positive  $\varepsilon$  and for every neighbourhood  $U$  of  $x$  there exists a positive integer  $n_0$  and a nonempty open set  $G \subset U$  such that  $d(f_n(y), f_m(y)) < \varepsilon$  for each  $y \in G$  and each  $n \geq n_0$ , [2]. Denote the set of all equi-quasicontinuity points of  $(f_n)_n$  be  $E(f_n)$ . Connections between the sets  $U(f_n)$  and  $E(f_n)$  will be considered.

**Keywords.** Uniform convergence at a point, quasicontinuous function, equi-quasicontinuity.

**AMS 2010.** 54C08, 54C30.

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## Normal Base of Ditopological Texture Space and Its Wallman Compactification

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**Abstract.** In this study, it has been intended to create an approach providing to get Wallman-type compactification of ditopological texture space. For this purpose, the concept of normal base well-known in general topology is relocated to ditopological texture space. We get the Wallman compactification of ditopological texture space by defining normal base ,co-normal base.

**Keywords.** Ditopological texture space, compactification,normal base, Wallman compactification

**AMS 2010.** 54D30, 54E55,54D20.

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## Cardinal Invariants of the Vietoris Topology on $C(X)$

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**Abstract.** Let  $X$  be a topological space,  $C(X)$  be the space of all real valued continuous functions on  $X$  and  $\tau_{\Gamma}$  be the Vietoris topology on  $X \times R$  restricted to  $C(X)$ , where functions are identified with their graphs. The basis of the topology  $\tau_{\Gamma}$  consists of the sets of the form

$$B(f, \varepsilon) = \{g \in C(X); |f(x) - g(x)| < \varepsilon(x) \text{ for } x \in X\},$$

where  $f \in C(X)$  and  $\varepsilon$  is lower semi continuous real valued function on  $X$ . This fact relates the topology  $\tau_{\Gamma}$  to well known  $m$ -topology (or fine topology)  $\tau_w$ . We investigate cardinal invariants of  $(C(X), \tau_{\Gamma})$  and  $(C(X), \tau_w)$  depending on the properties of  $X$ .

**Keywords.** Vietoris topology,  $m$ -topology, fine topology, cardinal invariant.

**AMS 2010.** 54A25, 54C30, 54C35.

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## Relations Between Spaces of Minimal Usco and Minimal Cusco Maps

Ľubica Holá<sup>1</sup> and Dušan Holý<sup>1</sup>

**Abstract.** We give a characterization of set-valued maps which are minimal usco and minimal cusco simultaneously. Let  $X$  be a topological space and  $Y$  be a Banach space. We show that there is a bijection between the space  $MU(X; Y)$  of minimal usco maps from  $X$  to  $Y$  and the space  $MC(X; Y)$  of minimal cusco maps from  $X$  to  $Y$  and we study this bijection with respect to various topologies on underlying spaces.

**Keywords.** minimal cusco map, minimal usco map, quasicontinuous function, subcontinuous function, set-valued mapping, selection, extreme function.

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## Topological Properties of the Graph Topology on Function Spaces

L. Holá<sup>1</sup> and L. Zsilinszky<sup>2</sup>

**Abstract.** The graph topology  $\tau_G$  ([1], [2], [3]) is the topology on the space  $C(X)$  of all continuous functions defined on a Tychonoff space  $X$  inherited from the Vietoris topology on  $X \times R$  after identifying continuous functions with their graphs. It is shown that all completeness properties between complete metrizability and hereditary Baireness coincide for the graph topology if and only if  $X$  is countably compact; however the graph topology is a Baire space, regardless of  $X$ . Pseudocompleteness, along with properties related to 1st and 2nd countability of  $(C(X), \tau_G)$  are also investigated.

**Keywords.** graph topology, hereditarily Baire space, countably compact space.

**AMS 2010.** 54C35, 54E52.

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## Coverings and Actions of Structured Lie Groupoids

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**Abstract.** In this work, we deal with coverings and actions of Lie group-groupoids and Lie ring-groupoids being two sorts of the structured Lie groupoids. After we give definition of a structured Lie groupoid, we prove some characterizations of structured Lie groupoids. Then we present the concept of covering of a structured Lie groupoid. As first main result of this work, we show that the category  $LSCov(M)$  of the smooth coverings of Lie group  $M$  is equivalent to the category  $SLGCov(\pi_1 M)$  of the coverings of structured Lie groupoid  $\pi_1 M$ , where it is assumed that  $\pi_1 M$  is a Lie group-groupoid, specially. Furthermore, we define an action of a structured Lie groupoid on a connected Lie structure. Finally, we show that the category  $SLGCov(G)$  of the coverings of a structured Lie groupoid  $G$  and the category  $SLGOp(G)$  of the actions of  $G$  on Lie structures (group or ring) are equivalent.

**Keywords.** Lie groupoid, covering, action, structured Lie groupoid.

**AMS 2010.** 22A22, 57M10.

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## Affine Singular Control Systems on Lie Groups

Memet Kule<sup>1</sup>

**Abstract.** The purpose of this paper is to prove that an affine singular control system  $S$  on a connected finite-dimensional Lie group  $G$  leads to two subsystems: an affine control system on a homogeneous space  $G/H$  and a differential-algebraic control system on a closed subgroup  $H$  of  $G$ .

**Keywords.** Singular Control System, Homogeneous Space, Algebraic Differential Equations

**AMS 2010.** 93B05, 93B27, 93C10.

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## Generalized Volterra Spaces

Milan Matejdes<sup>1</sup>

**Abstract.** In this paper we deal with a general concept of classification of subsets of a topological space  $X$  with respect to a given nonempty system  $\mathcal{E}$ . Using system  $\mathcal{E}$ , the notions of  $\mathcal{E}$ -Volterra and weakly  $\mathcal{E}$ -Volterra spaces are introduced which cover classical Volterra and weakly Volterra spaces [1], [3] - [6] as well as irresolvable spaces [2].

**Keywords.** Baire space, Volterra space, weakly Volterra space, resolvable space.

**AMS 2010.** 54A05, 54E52.

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## Soft Semi-Topological Groups

Nazan Çakmak<sup>1</sup> and Bekir Tanay<sup>2</sup>

**Abstract.** Nazmul and Samanta [6] gave the definition of soft topological group after initiating the soft set theory by Molodtsov [5] in 1999. In this paper, the concept of soft semi-topological groups are introduced. Moreover, several theorems and properties related to soft semi-topological groups are given. Also, the relation between soft topological groups and soft semi-topological groups, analogously the relation between soft groups and soft topological group (soft semi-topological groups) are studied.

**Keywords.** Soft semitopological group, soft topological group, soft group.

**AMS 2010.** 20N99, 22A99, 03E72.

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## Soft Filter

Naime Tozlu<sup>1</sup>, Şaziye Yüksel<sup>2</sup> and Zehra Güzel Ergül<sup>3</sup>

**Abstract.** We study some soft neighbourhood properties in a soft topological space and introduce soft filters which are defined over an initial universe with a fixed set of parameters. We set up a soft topology with the help of a soft filter. We also introduce the concepts of the greatest lower bound and the least upper bound of the family of soft filters over an initial universe, soft filter subbase and soft filter base. Also, we define limit point and closure point of a soft filter and we compare these concepts. Finally, we study a characterization of soft Hausdorff space related with soft filter.

**Keywords.** Soft topological space, soft neighbourhood system, soft filter.

**AMS 2010.** 06D72, 54A40.

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## A Fixed Point Theorem for New Type Contractions on Weak Partial Metric Spaces

Özlem Acar<sup>1</sup> and İshak Altun<sup>2</sup>

**Abstract.** Recently, Karapinar and Romaguera introduced a new type contraction on partial metric space and give a nonunique fixed point result. Then Romaguera used this contraction to obtain some multivalued fixed point results on partial metric space. In the present work, using this new idea we give a fixed point result on weak partial metric space.

**Keywords.** Fixed point, partial metric space, weak partial metric space

**AMS 2010.** 54H25, 47H10.

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## Neighborhood Structures of Graded Ditopological Texture Spaces

Rıza Ertürk<sup>1</sup> and Ramazan Ekmekçi<sup>2</sup>

**Abstract.** M. Demirci presented two sorts of neighborhood structures of smooth topological spaces [3]. Graded ditopologies on textures as an extension of the notion of ditopology to the case where openness and closedness are given in terms of a priori unrelated grading functions have been introduced and studied by L. M. Brown and A. Šostak. So, the notion of ditopology has been fuzzified in some sense [6].

In this work, the authors study the generalization of two sorts of neighborhood structure defined in [3] to the graded ditopological texture spaces.

**Keywords.** Fuzzy topology, Smooth Topology, Neighborhood, Graded ditopology, Texture.

**AMS 2010.** 54A40, 54A05, 54B99.

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## Applications of Delta-Open Sets

Raja Mohammad Latif<sup>1</sup>

**Abstract.** In 1968 Velicko [2] introduced the concepts of  $\delta$ -closure and  $\delta$ -interior operations. We introduce and study properties of  $\delta$ -derived,  $\delta$ -border,  $\delta$ -frontier and  $\delta$ -exterior of a set using the concept of  $\delta$ -open sets. We also introduce some new classes of topological spaces in terms of the concept of  $\delta$ -D-sets and investigate some of their fundamental properties. Moreover, we investigate and study some further properties of the well-known notions of  $\delta$ -closure and  $\delta$ -interior of a set in a topological space. We also introduce  $\delta$ - $R_0$  space and study its characteristics. We introduce  $\delta$ -irresolute,  $\delta$ -closed, pre- $\delta$ -open and pre- $\delta$ -closed mappings and investigate properties and characterizations of these new types of mappings and also explore further properties of the well-known notions of  $\delta$ -continuous and  $\delta$ -open mappings.

**Keywords.**  $\delta$ -interior,  $\delta$ -closure,  $\delta$ -opens et,  $\delta$ -closed set,  $\delta$ -derived,  $\delta$ -border,  $\delta$ -frontier,  $\delta$ -exterior,  $\delta$ -Hausdorff,  $\delta$ -saturated,  $\delta$ -compact,  $\delta$ -kernel,  $\delta$ - $R_0$ ,  $\delta$ -convergence,  $\delta$ -irresolute mapping,  $\delta$ -continuous mapping,  $\delta$ -open mapping,  $\delta$ -closed mapping,  $\delta$ -open mapping, pre- $\delta$ -open mapping, pre- $\delta$ -closed mapping.

**AMS 2010.** 54A05, 54A10, 54A20, 54F65.

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## **Selection properties of texture structures**

Selma Özçağ<sup>1</sup> and Ljubiša D.R. Kočinac<sup>2</sup>

**Abstract.** We begin the investigation of selection principles in texture spaces. Among other things, we introduce Menger and co-Menger properties in ditopological context and study some notions related to these selection principles. We also present some game-theoretic and Ramsey-theoretic results in this connection.

**Keywords.** Texture space, Selection principles, Topological games,

**AMS 2010.** 54E55, 54A25, 54C35, 54D20.

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## Quotient Structure of Interior-Closure Texture Spaces

Şenol Dost<sup>1</sup>

**Abstract.** By a texturing [2] of a set  $S$  we mean a subset  $\delta$  of the power set  $P(S)$  which is a point separating complete, completely distributive lattice with respect to inclusion which contains  $S$  and  $\emptyset$ , and for which arbitrary meets coincide with intersections and finite joins coincide with unions. The pair  $(S, \delta)$  is then called a texture space, or simply texture.

Direlations [3] arise often in the study of textures. A direlation is a pair  $(r, R)$  where  $r$  (relation) and  $R$  (corelation) are the elements of a textural product satisfying certain conditions. Difunctions are also defined as a special direlation. In [1], the equivalence direlations and the corresponding quotient textures are defined. It also given some properties of quotient difunctions.

A pair  $(\mathbf{int}, \mathbf{cl})$  of the set valued mappings is called a interior-closure space on  $(S, \delta)$  [4] in sense Dikranjan-Giuli, if  $\mathbf{int}, \mathbf{cl}: \delta \rightarrow \delta$  is grounded, isotone, idempotent and  $\mathbf{int}$  is contractive and  $\mathbf{cl}$  is expansive. The category of interior-closure spaces and bicontinuous difunctions is denoted by **dfICL**.

In this study, we introduce quotient structures and quotient difunctions in the interior--closure spaces. The generalizations of several results concerning separation and quotient are presented. It is shown that the category **dfICL** has a  $T_0$  reflection.

**Keywords.** Texture, Equivalence direlation, Quotient texture, Quotient difunction, Interior-closure operator, Reflection.

**AMS 2010.** 03G10, 54B15, 54B30, 54A05.

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## On Soft Compactness and Soft Separation Axioms

Şaziye Yüksel<sup>1</sup>, Zehra Güzel Ergül<sup>2</sup> and Naime Tozlu<sup>3</sup>

**Abstract.** Some authors studied several basic concepts and properties on soft topology such as soft separation axioms, soft compactness, soft Lindelöf, etc. In this paper, investigating the properties of soft separation axioms and soft compactness have been continued. Other types of soft compactness have been introduced. Moreover, the relations between these concepts have been investigated.

**Keywords.** Soft set, soft topological space, soft subspace, soft separation axiom, soft compact, soft countably compact, soft locally compact, soft Lindelöf.

**AMS 2010.** 06D72, 54A40.

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## On Fuzzy Soft Topological Spaces

Tugba Han Dizman (Simsekler)<sup>1</sup> and Saziye Yuksel<sup>2</sup>

**Abstract.** In the present work we contribute the concept of fuzzy soft topology defined in [8]. We develop the fuzzy soft quasi separation axioms. Also we consider fuzzy soft compactness and fuzzy soft continuity and obtain new results.

**Keywords.** Fuzzy soft set, topology, fuzzy soft compactness, quasi separation axioms

**AMS 2010.** 54A40, 06D72.

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## Soft Connected Spaces

Zehra Güzel Ergül<sup>1</sup>, Şaziye Yüksel<sup>2</sup> and Zehra Güven<sup>3</sup>

**Abstract.** Many researchers defined some basic notions on soft topology and studied many properties. In this work, we give some new concepts in soft topological spaces such as soft connected sets, soft connected spaces, soft connected subspaces, soft components, soft totally disconnected spaces and soft locally connected spaces. Also, we investigate many basic properties of these concepts.

**Keywords.** Soft connected space, soft disconnected space, soft connected subspace, soft component, soft totally and locally disconnected space.

**AMS 2010.** 06D72, 54A40.

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# *THE OTHER AREAS*



## Bisection Method and Algorithm for Solving the Electrical Circuits

Alpaslan Ersöz<sup>1</sup> and Mehmet Kurban<sup>2</sup>

**Abstract.** Iteration is the process to solve a problem or defining a set of processes to called repeated with different values [1],[2]. The method mentioned in this paper, the roots of equations which is described, will be found. This method is called bisection [3]. The use of this method will be implement on a electrical circuit. The solution of the problem is only finding the real roots of the equation. In different types of applications, sometimes the real roots can not be found. In this situation, the complex roots of the equation is determined. On the other hand, the finding of the complex roots is needed to make numeric analysis. The numeric analysis is except for this paper. Defined by the flow chart of the method can be present different approach for this method by using Fortran,C, Matlab programming language.

**Keywords.** Iteration, Bisection Method, Fortran, C, MatLab.

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## Artificial Atom Algorithm for Reinforcement Learning

Ahmet Karadoğan<sup>1</sup> and Ali Karci<sup>2</sup>

**Abstract.** The machine learning is a popular subject in artificial intelligence. The machine learning includes supervised learning, unsupervised learning, reinforcement learning, etc. The basic idea of reinforcement learning is that of awarding the learner (agent) for correct actions, and punishing wrong actions. Reinforcement learning (RL), suitable for hunter prey problem, has a difficulty where parameter values can only be determined by trial and error. We proposed to use A<sup>3</sup> – Artificial Atom Algorithm (A new Meta-heuristic Computational Intelligence Algorithm Inspired by Chemical Processes) to obtain optimal or near-optimal parameter values in reinforcement learning.

**Keywords.** Reinforcement learning, Multi-agent learning, Q-Learning, Artificial Atom Algorithm, Hunter Prey Problem.

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## On Double New Integral Transform and Double Laplace Transform

Artion Kashuri<sup>1</sup>, Akli Fundo<sup>2</sup> and Rozana Liko<sup>3</sup>

**Abstract.** In this paper, a relationship between double new integral transform and double Laplace transform was establish and many other results are presented. We consider general linear telegraph equation with constant coefficients and wave equation [1,2]. The applicability of this relatively double new integral transform is demonstrated using some special functions, which arise in the solution of PDEs. First, we transform the partial differential equations to algebraic equations by using double new integral transform method and second, using the inverse double new integral transform we get the solution of PDEs.

**Keywords.** Double Laplace Transform, Double New Integral Transform, Single Laplace Transform, Single New Integral Transform, Inverse Double New Integral Transform, Double Convolution Theorem.

**AMS 2010.** 53A40, 20M15.

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## A Cosemanticness of Positive Jonsson Theories and Their Models

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**Abstract.** We have considered the notion of a cosemanticness in the frame of positive Jonsson theories. We say that Jonsson theories  $T_1$  and  $T_2$  are cosemantic each other ( $T_1 \triangleright \triangleleft T_2$ ), if they have a common semantic model, i.e.  $C_{T_1} = C_{T_2}$ . Let  $A$  – a some infinite model of a signature  $\sigma$ .  $A$  is a  $\Delta - PJ$ -model, if a set of sentences  $Th_{\forall\exists^+}(A)$  will be a  $\Delta - PJ$ -theory. Denoted by  $\forall\exists^+(A)$  the theory  $Th_{\forall\exists^+}(A)$ . Models  $A$  and  $B$  are  $\Delta - PJ$ -equivalent, if for any  $\Delta - PJ$ -theory  $T$   $A \models T$  and  $B \models T$ . We denote this fact as  $A \equiv_{PJ}^{\Delta} B$ . We say that models  $A$  and  $B$  are cosemantic each other ( $A \triangleright \triangleleft_{PJ}^{\Delta} B$ ), if for any Jonsson theory  $T_1$  such that  $A \models T_1$ , there exist Jonsson theory  $T_2$  such that ( $T_1 \triangleright \triangleleft T_2$ ) follows that  $B \models T_2$  and the converse is true too. It is easy to note that next statement is true for any two models:  $A \equiv B \Rightarrow A \equiv_{PJ}^{\Delta} B \Rightarrow A \triangleright \triangleleft_{PJ}^{\Delta} B$ .

And in the frame of this definitions we have following result

Lemma 1. Let  $T_1$  and  $T_2$  –  $\Delta - PJ$ -theories,  $C_1$  is a semantic model of  $T_1$ ,  $C_2$  is a semantic model of  $T_2$ . Then if  $(T_1)_{\forall^+} = (T_2)_{\forall^+}$ , we have that  $T_1 \triangleright \triangleleft_{PJ}^{\Delta} T_2$ .

Lemma 2. Let  $A$  и  $B$  –  $\Delta - PJ$ -models. Then the following conditions equivalent:

- 1)  $A \triangleright \triangleleft_{PJ}^{\Delta} B$ ,
- 2)  $\forall\exists^+(A) \triangleright \triangleleft_{PJ}^{\Delta} \forall\exists^+(B)$ .

Theorem 1.

Let  $T_1$  and  $T_2$  – existential complete, perfect Jonsson  $\Delta - PJ$ -theories, If  $T_1^*$  model consist with  $T_2^*$ , then  $A \triangleright \triangleleft_{PJ}^{\Delta} B$ , where  $A$  and  $B$  are semantic models of  $T_1$  and  $T_2$  correspondingly.

**Keywords.** Positive Jonsson theory, cosemanticness, perfectness.

**AMS 2010.** 03C05,03C10.

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## On the Number of Representation of Positive Integers by Some Octonary Quadratic Forms

Bülent Köklüce<sup>1</sup>

**Abstract.** We find formulae for the number of representation of a positive integer  $n$  by each of the quadratic forms  $x_1^2 + x_2^2 + 2x_3^2 + 2x_4^2 + 3x_5^2 + 3x_6^2 + 6x_7^2 + 6x_8^2$ ,  $x_1^2 + x_2^2 + x_3^2 + x_4^2 + 3x_5^2 + 3x_6^2 + 6x_7^2 + 6x_8^2$ ,  $x_1^2 + x_2^2 + 2x_3^2 + 2x_4^2 + 3x_5^2 + 3x_6^2 + 3x_7^2 + 3x_8^2$ ,  $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_4^2 + 3x_5^2 + 3x_6^2 + 6x_7^2 + 6x_8^2$ ,  $x_1^2 + x_2^2 + 2x_3^2 + 2x_4^2 + 6x_5^2 + 6x_6^2 + 6x_7^2 + 6x_8^2$  by using some known convolution sums of divisor functions and known representation formulae for quaternary quadratic forms. Formulae for some other octonary quadratic forms of these type are given before in [1-5].

**Keywords.** Quadratic Forms, Representation Numbers.

**AMS 2010.** 11A25,11E25.

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## Population Growth and Sustainable Harvest Modeling by means of Leslie Matrices

Candice Louw<sup>1</sup> and Sebastiaan von Solms<sup>2</sup>

**Abstract.** The importance of successful environmental resource management has the potential to make a global impact and has rightfully come under scrutiny in recent years. The natural resources of any environment are limited and the importance of replenishing these resources for future provision is of cardinal importance. Achieving an optimal harvest [1] of resources is thus what is aimed for, but predicting the outcome of a chosen strategy is only visible once it has already succeeded (retrospective) or if it is currently in the process of failing. In the case of the latter, recovery of all resources is highly unlikely and the only accurate prediction that can be made is that of a loss – an undesirable result.

Instead of depending on human predictions, a visual, mathematical software model that can be applied to any natural resource with fecundity and mortality rates, that not only identifies resource growth and decline but also makes suggestions about ideal harvest sizes, would thus be indispensable. Leslie matrices and Eigenvalues for population growth modelling [2] are an ideal combination for exactly this problem.

To firstly expand our knowledge about the problem we are trying to solve, we look at different population modelling approaches (with regard to natural resource management) for a polar bear population [3] and fisheries [4]. Then, by gathering information from these and other approaches, we analyse the pros and cons of each and implement our own population growth model accordingly. Lastly, to contribute both a visual representation of our underlying mathematical techniques and to serve up an effective educational tool to users, we choose to use NetLogo [5] as the platform for developing a prototype system and then compare its results, pros and cons to that of other, similar models.

**Keywords.** Population Growth, Leslie Matrices, Sustainable Harvest Ratio.

**AMS 2010.** 53A40, 20M15.

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## **The Mathematical Analysis of Momentum Production of Five Phases Bipolar 10/8 Switched Reluctance Motor**

Erdal Büyükbıçakcı<sup>1</sup>, Ali Fuat Boz<sup>2</sup> and Zeynep Büyükbıçakcı<sup>1</sup>

**Abstract.** In this study, a new five phases segmental type switched reluctance motor (SARM) having a different rotor and stimulation structure was compared with the classically used Switched Reluctance Motor (ARM) with respect to high momentum productivity ability. The visible inductance profiles of the system were investigated to simplify the complex structure of SARM model. The inductance values were calculated by assuming the current applied to phases created from an ideal current source and by determining the situations of different phases. It was revealed that the SARM produced much more momentum than the classical ARM having the same phase number by using a mathematical model.

**Keywords.** Switched reluctance motor, Momentum production, Mathematical model

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## **The Mathematical Model of a New Five Phases Bipolar 10/8 Configuration Segmental Switched Reluctance Motor**

Erdal Büyükbıçakcı<sup>1</sup>, Ali Fuat Boz<sup>2</sup> and Zeynep Büyükbıçakcı<sup>1</sup>

**Abstract.** In this study, a mathematical model of five phases segmental switched reluctance motor (SARM) was established by using basic electrical motor equations. This five phases SARM has a different rotor and stimulation structure than classical ARM, and its phase currents and magnetic field variations with respect to inductance were calculated by means of magnetic equivalence circuit. The value of inductances was calculated by stating the visible inductance profiles of different phases instantaneously in order to emphasise the difference between to motors. Finally, the momentum production of motor was obtained from inductance equivalent.

**Keywords.** Switched reluctance motor, Mathematical model, Inductance profile

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## Predictive Microbiology

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**Abstract.** Predictive microbiology is modeling of responses (such as growth/inactivation) of microorganisms to factors of growth conditions (such as time, temperature, pH). Although predictive microbiology is a new area of food microbiology, effort of modeling of living things' behaviors, such as population growth, has its basis on some old theories. One of the most popular of them is the Malthus' "Theory of Population". According to the theory, although population of humanbeings increase geometrically, food sources increases arithmetically and as a result of this phenomenon, hunger is inevitable. This theory is unrealistic and was modified by an another demographer, Gompertz. According to Gompertz model, population does not increase infinitely but reaches an asymptote level which can be expressed as "Log(y)=a". To date, Gompertz model has been modified in many ways to explain population dynamics of human, animal, fish and bird. In recent years the model has been used for explaining microbial growth.

Amount of microorganisms can be determined easily when it is at high levels, such as  $10^2$ - $10^6$  g<sup>-1</sup>. But at low levels, such as  $10^0$ - $10^{-6}$  g<sup>-1</sup>, a reliable determination of the level of microorganisms by analitical means is almost impossible. For example, in canned food industry, *Clostridium botulinum* spore number must be lower than  $10^{-12}$  g<sup>-1</sup>. But determination of the spore content at this level is impossible. Control of inactivation of spores at this level has been achieved by simple graphical models since beginning of 1900's. The linear models asumes that "each cell in the population have the same heat resistance". This unrealistic approach was repaired by nonlinear models. One of the non-linear inactivation models is the Weibull model, which was originally developed for failure rates for machinery, in 1950's. The model can be reduced to two parameter forms, inactivation factor can be incorporated to the model easily and the shape of the curve can be changed representing differences in resistance to the factor through different type of microorganisms. In this presentation, modeling of microbial growth and inactivation will be represented by examples of the two models mentioned above.

Today, many models are used to express the response of living things to their environment. Some of them are empiric models which only consist of curve fitting. But contemporary modeling approach is developing into mechanistic models, which are based on physico-

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chemical laws and that try to explain all biological changes in the cell. These lead the way to developing more comprehensive models. However one must keep in mind, that, just like the unpredictability of sub-atomic particles, it is impossible to determine each and every individual enzyme or substrate molecule behaviour under certain circumstances. Models explain us the general state or probability densities of the changes. Nevertheless, even merely the probability values give us more valuable information than experimental studies most of the time.

For this reason, modeling is a developing area in biology, microbiology, food protection, etc. The most important benefit of the modeling or predictive microbiology is that it gives us opportunities using engineering design or modeling in state of trial and error experiments. Therefore, we can use earlier experiences more effectively and solve new problems quicker than we did before. However we can say that the modeling concept is still in its infancy period. Choosing the response for modeling versus environmental factors requires high level of microbiological knowledge, and expression of knowledge into the models requires both mathematical and microbiological knowledge. Insufficient mathematical knowledge limits the possibilities to produce alternatives in solving problems and leads the researcher into repeating the same studies. Another difficulty in modeling is that the algorithms of multi-parameter models are not easy to use or that package programs are not developed to be user-friendly. This difficulty leads the researcher to use simpler models with 2-3 parameters. Since behaviours of living things are shaped not by individual factors but by combinations of numerous factors; limiting the research with less parameter models drives us away from nature or nature of the problem.

In conclusion, mathematicians can contribute very much to this emerging field. Multi-disciplined studies conducted in Turkey can contribute significantly to worldwide literature in this field. This presentation will include comparisons of aforementioned model equations, explanations of mathematical problems and which fields are open for a collaborative approach.

**Keywords.** Predictive microbiology, growth, inactivation.

## Cost – Value – Profit Analysis via Fuzzy Logic Theory

Gökhan Baral<sup>1</sup> and Sinan Esen<sup>2</sup>

**Abstract.** Cost – value – profit (C-V-P) analysis tries to determine the effects on the profit that will be provided of the changes on the factors required for the purpose of profit planning. C-V-P analysis is a type of analysis that the businesses used for the decisions to get profit. Making a correct profit planning and determining the business decisions for the purpose of the control of the business according to these issues is directly related to realistic costing.

It has been thought that the cost will be more realistic when it has applied to the activity based cost with the view of products consume activities and the activities consume the sources. It is not only used to determine the cost but also used to make profit planning because of the positive effect on profit planning of activities. The customers caring the quantity and the quality of the various activities while preferring a product has provided to contribute the activity based cost method to this study.

Cost – value – profit analysis is used to determine the forward decisions. But it is not possible what is going to be or what is going to be changed in the future. Because of that reason the decision of the directors is to be supported via more reliable data. Including fuzzy logic theory and environment of uncertainty to C-V-P analysis can provide increasing the expected benefit from these theories.

**Keywords.** Cost – Value – Profit Analysis, Fuzzy Logic

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## Some New Results on Orderings on Soft Sets

Gözde Yaylalı<sup>1</sup> and Bekir Tanay<sup>2</sup>

**Abstract.** Molodtsov [6] introduced the soft set theory. Moreover Babitha and Sunil [1], [2] introduced partially ordered soft set. In this study we improve orderings on soft sets by giving new definitions such as filtered soft set, soft lattice, complete soft lattice, and some results related these definitions are studied.

**Keywords.** Soft set, Partially ordered soft set, Soft lattice

**AMS 2010.** 06A06, 06B99, 06Fxx

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## On the Structure of Hexagonal Cellular Automata With Reflective Boundary Over the Primitive Field $Z_p$

Hasan Akın<sup>1</sup> Irfan Siap<sup>2</sup> and Selman Uguz<sup>3</sup>

**Abstract.** The hexagonal finite cellular automata (shortly HCA) are two dimensional (2D) cellular automata whose cells are of the form of a hexagonal. Morita et al. [1] have introduced this type of cellular automaton (CA) and they called it hexagonal partitioned CA (HPCA). Hence, they have studied logical universality of a reversible HPCA. In [2], the authors have investigated two-dimensional (shortly 2D) finite cellular automata defined by local rules based on hexagonal cell structure under null boundary. Rule matrix of the hexagonal finite cellular automaton has been obtained. In [3], the authors have presented a study of two-dimensional hexagonal cellular automata (CA) with periodic boundary. Moreover, general algorithms to determine the reversibility of 2D 3-state cellular automata with periodic boundary have been presented.

In this paper, we study the structure of 2D finite cellular automata defined by local rules based on hexagonal cell structure with reflective boundary over the primitive field  $Z_p$ . We obtain the rule matrix corresponding to the hexagonal finite cellular automaton. We compute the rank of rule matrices representing the hexagonal finite cellular automata via an algorithm. It is a well known fact that determining the reversibility of 2D cellular automata is a very difficult problem in general. Here, the reversibility problem of this family of the hexagonal cellular automata is also resolved completely over the primitive field  $Z_p$ . For some given coefficients and the number of columns of hexagonal information matrix, we will prove that the hexagonal cellular automata under reflective boundary are reversible

**Keywords.** Hexagonal Cellular automata, Reversible Cellular Automata, Matrix algebra, reflective boundary.

**AMS 2010.** 37B15, 68Q80.

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## Target Costing via Fuzzy Logic Theory

Hilmi Kırılıođlu<sup>1</sup>, Gökhan Baral<sup>2</sup> and Aydın Şenol<sup>3</sup>

**Abstract.** The aim of Target Costing is determining the costs that will provide the desired profit. Target Costing is related to costing and profit planning. The way providing the desired profits of the businesses is possible to know and manage the costing in the world that the sale cost are stabilized and easy to learn.

It has to be studied on the methods that will calculate cost estimates and profit targets reliably and decrease the difference between planned and real results minimum.

The aim of this study is declaring the existence of new method instead of a traditional method; fuzzy logic theory and showing that it is a practical and simple method that requires minimum resource for cost estimates and profit planning on uncertain situations.

**Keywords:** Target Costing, Fuzzy Logic Theory

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## **Mathematics of Single Crystal Structure Determination by Using X-Rays**

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**Abstract.** Much of our present knowledge of the architecture of molecules has been obtained from studies of the diffraction of X rays or neutrons by crystals. X rays are scattered by the electrons of atoms and ions, and the interference between the X rays scattered by the different atoms or ions in a crystal can result in a diffraction pattern.

This study shows mathematics of single crystal structure determination by using X-rays scattered from each atom in the crystal structure. When X rays are diffracted by a crystal, the intensity of X-ray scattering at any angle is the result of the combination of the waves scattered from different atoms and the manner in which they modify this intensity by various degrees of constructive and destructive interference. A structure determination involves a matching of the observed intensity pattern to that calculated from a postulated model, and it is thus imperative to understand how this intensity pattern can be calculated for any desired model. In order to obtain an image of the material that has scattered X rays and a given a diffraction pattern, which is the aim of these studies, one must perform a three dimensional Fourier summation. Such a set of terms described as a Fourier series, can be used in diffraction analysis because the electron density in a crystal is a periodic distribution of scattering matter formed by regular packing of approximately identical unit cells. The Fourier series that is used provides an equation that describes the electron density in the crystal under study.

Once the approximate positions and identities of all the atoms in the asymmetric unit are known (that is, when the true crystal structure is known), the amplitudes and phases of the structure factor can readily be calculated. Structure factors for the suggested “trial structure” were calculated and compared with those that had been observed. When more productive methods for obtaining trial structures the “Patterson function” and “direct methods” were introduced, the manner of solving a crystal structure changed dramatically for the better.

Refinement of a good trial structure until the calculated and measured intensities agree with each other within the limits of any errors in the observations. This is usually done by a least squares refinement, although difference electron-density maps may also prove useful. The result of the refinement is information on the three-dimensional atomic coordinates in this particular crystal, together with atomic displacement parameters [1].

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For clarity, the true crystal structure is indicated by a line diagram. As can be seen, only the first map correctly gives peaks at atomic positions. An electron-density map with correct phases much more nearly approximates the correct structure than does an electron-density map with incorrect phases, even if each has the correct magnitudes for the  $|F(hkl)|$  values. The analysis of electron-density and Patterson maps has benefited greatly from the improvements in computer graphics so that now it is possible to view the three-dimensional map on a computer screen and rotate and move it at will in order to obtain structural information [2].

**Keywords.** Single crystal, X ray, structure factor, Fourier analysis.

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## Investigation of the Magnetic Properties in the Odd-Mass Deformed Nuclei

Hakan Yakut<sup>1</sup>, Emre Tabar<sup>1</sup> and Ekber Guliyev<sup>2</sup>

**Abstract.** In this study some of the ground and excited state properties of the deformed odd-A nuclei were investigated by using a microscopic method called as Quasiparticle-phonon nuclear model (QPNM). The characteristics of the ground-state and low-lying magnetic dipole (M1) excitations are calculated for several deformed odd-mass nuclei using a separable Hamiltonian within the QPNM. The model operates with the strength function method, calculating the fragmentation of single quasiparticle, one phonon states and the quasiparticle-phonon states over a large number of nuclear levels [1,2]. The calculated magnetic properties such as magnetic moment ( $\mu$ ), intrinsic magnetic moment ( $g_K$ ), effective spin factor ( $g_s^{\text{eff}}$ ), M1 states and reduced transition probabilities  $B(M1)$  are the fundamental characteristic of the odd-mass nucleus and provide key information to understand nuclear structure. The results are compared with the available experimental data and other theoretical approaches. Our calculations indicated that because of the core polarization, the  $g_s$  factors of the nucleons in the nucleus reduce noticeable from its free nucleon value and the spin-spin interactions play an important role in the renormalization of the  $g_s$  factors. Furthermore we found a strong fragmentation of the M1 strength in  $^{163}\text{Dy}$  which was in qualitative agreement with the experimental data.

**Keywords.** QPNM, M1, Quasiparticle-Phonon.

**AMS 2010.** 81S05, 00A79.

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## The Use of Financial Mathematics in The History of Ottoman Economics Research

Hüsnü Yücekaya<sup>1</sup> and Sıddık Arslan<sup>2</sup>

**Abstract.** Numeric data has been widely used in Ottoman historiography day by day. As far as Ottoman Empire is concerned, making analyses only on results of politic events can lead to diverge from historical facts. Today, it is available to get homogeneous and coherent numeric data from Ottoman archival records. For example these archives make it possible to reach facts and figures about determining costs of goods and cost rise on the same goods in different periods within time span. At this step complexity of the problem and getting information from different periods and different goods, in addition to personal attitudes of recorders, influence the reliability of results. In this kind situation it is necessary to apply three different process. At first it is necessary to determine goods appropriate for objective sampling. After that, for each of the product range, yearly percentage increase depending on compound interest on time value of money should be calculated. Then average of the goods which can influence each year should be taken and as a result of these two processes yearly increase in rates should be calculated. The third step is that in order to calculate rate increase periods in decades, exponential calculation should be made. By examining results through coherent mathematical process, it can be possible to reach new perspectives and foresights about economic basis of political and sociological events seen during Ottoman Empire. Using correct calculation method make it easy to build mental construction of historical events. Mispleading in calculations can lead to false foresights.

**Keywords.** Financial Mathematics, Ottoman Economics Research, Price Increase

**AMS 2010.**91G30,91G99,62P25.

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## Wavelet Transform in Power System Analysis

İdil Işıklı Esener<sup>1</sup>, Tolga Yüksel<sup>2</sup> and Mehmet Kurban<sup>3</sup>

**Abstract.** Load forecasting, the first step of power system planning, is of great importance in economic electric power generation and distribution, improvement of system operating conditions, effective system control and energy pricing. In signal processing, the most known method applied to this problem is the Fourier Transform which gives amplitude-frequency information about the signal, but it does not contain any information about the time [1]. Short-Time Fourier Transform which determines the frequency components of the signal at the desired time range in different sized areas is developed. In this method, frequency resolution decreases as time resolution increases, and vice versa. In order not to have any information loss neither on time nor frequency, Wavelet Transform is an appropriate mathematical alternative. Continuous Wavelet Transform is obtained by the convolution of the signal to be analyzed and the scaled and shifted versions of the mother wavelet function. On the other hand, Discrete Wavelet Transform is a filter bank that decomposes the signal into coefficients named as approximation for low frequency components, and details for high frequency components. Transform continues decomposing the approximations until the desired level by reducing the sample number to half at each level. In this study, wavelet transform is applied to load forecasting problem, and Turkey's 24-hour-ahead load forecasting of year 2009 is aimed. Frequency components of the load demand data are obtained by using Daubechies 2 wavelet function at the fifth-level, and these components are used in neural network training [2]. In simulation stage, the frequency components of the load demand of the forecasting date are obtained and reconstructed again using Daubechies 2 wavelet function at the fifth-level. Error results in the study shows a remarkable success.

**Keywords.** Wavelet Transform, Artificial Neural Networks, Load Forecasting.

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## **Implementation of Burke-Shaw Chaotic System on FPGA**

İsmail Koyuncu<sup>1</sup>, Ahmet Turan Özcerit<sup>2</sup> and İhsan Pehlivan<sup>3</sup>

**Abstract.** Chaotic systems are very sensitive to system parameters and initial conditions that can change the dynamic behaviour of the system to a great extent and they exhibit non-periodic behaviours in time series of the system [1]. Due to these features of chaotic systems, the interest towards these systems increases and as a result; significant researches are being conducted in scientific and industrial areas on the implementation of chaotic systems [2], [3]. In this study, Burke-Shaw chaotic system is modelled numerically and the chaos analysis is conducted with phase portraits and time series. The chaotic system is encoded in VHDL for its hardware implementation. After the design process, the chaotic system is tested with Xilinx ISE Design Tools program. The results of the designed FPGA-based chaotic oscillator unit are compared with numeric-based modelling and the results generated by the unit are confirmed. In addition, pursuant to Place and Route process, statistics of FPGA source use and clock speed are analysed. According to simulation results, maximum operating frequency of FPGA-based Burke-Shaw chaotic oscillator unit is 427.615 MHz.

**Keywords.** Chaos, chaotic systems, FPGA, chaotic oscillator.

**AMS 2010.** 53A40, 20M15.

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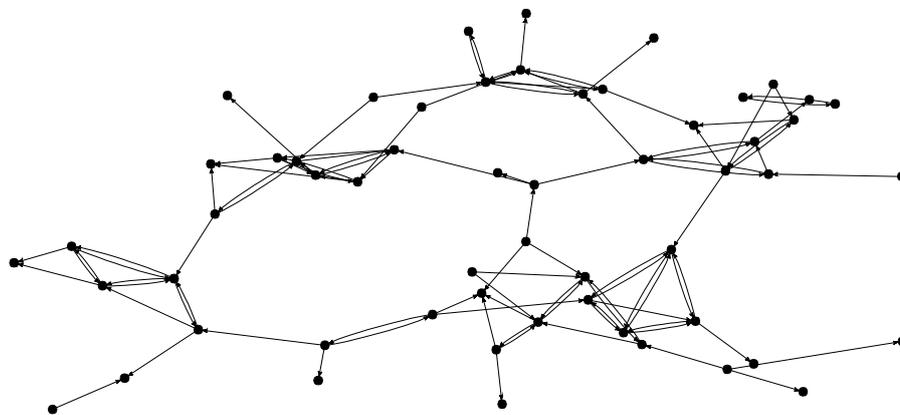
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## The Mathematical Analysis of the One Social Network

Josipa Matotek<sup>1</sup> and Ivanka Stipančić-Klaić<sup>2</sup>

**Abstract.** In the article we will describe social network of group of students on Faculty of Civil Engineering in Osijek. We present some mathematical properties of the network's graph and attached adjacency matrix. Using different centrality measures, degree centrality, eigenvector centrality, closeness and betweenness centrality we weigh importance of vertices and their connections. We will answer to the question: "Thus the belonging to some group influence on transience of the exam of mathematic or not?"



**Keywords.** network's graph, adjacency matrix, centrality.

**AMS 2010.** 91D30, 62P25 .

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## Circuit Models with Mixed Lumped and Distributed Elements for Passive One-Port Devices

Metin Şengül<sup>1</sup>

**Abstract.** In this work, to model measured data obtained from an actual passive one-port device, a circuit modeling method with mixed lumped and distributed elements is proposed [1]. Namely, measured data is modelled by means of its Darlington equivalent, in other words, as a lossless two-port terminated with a resistance [2]. In the proposed modeling method, analytic expression of the input reflection coefficient of the two-port model is obtained by using gradient method [3], and then, after synthesizing this two-variable function, the model is reached [4,5].

**Keywords.** Modeling, gradient method, two-variable functions.

**AMS 2010.** 53A40, 20M15.

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## Ultra-Peripheral Collisions at RHIC and LHC

Melek Yılmaz Şengül<sup>1</sup>

**Abstract.** In relativistic heavy ion collisions, the strong Lorentz-contracted electromagnetic fields can produce various numbers of lepton pairs via the two-photon mechanism. When we make a semiclassical approximation for the motion of the two ions, we use Monte Carlo techniques to reach the exact calculation of production by this mechanism [1]. The bound-free electron-positron pair production is considered for relativistic heavy ion collisions. In particular, cross sections are calculated for the pair production with the simultaneous capture of the electron into the  $1s$  ground state of one of the ions and for energies that are relevant for the relativistic heavy ion collider (RHIC) and the large hadron collider (LHC). In the framework of perturbation theory, we applied Monte Carlo integration techniques to compute the lowest-order Feynman diagrams amplitudes. Calculations were performed especially for the collision of Au+Au at 100 GeV/nucleon and Pb+Pb at 3400 GeV/nucleon [2].

**Keywords.** Relativistic heavy ion collisions, bound-free electron-positron pair production, Monte Carlo method.

**AMS 2010.** 53A40, 20M15.

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## **Calculation of Outdoor Lighting Using Triangular Field Simulation and Point-Point Method**

Nazım İmal<sup>1</sup>, Yılmaz Uyaroğlu<sup>2</sup> and Mehmet Kurban<sup>3</sup>

**Abstract.** Lighting calculations, in general, are performed as two types of interior and exterior lighting calculations. While interior lighting calculations performing lighting type, reflection coefficients and statements which are based on the efficiency of lighting, outdoor lighting calculations, which are based on geometric calculations performed three-dimensional space. Here, the study carried out for outdoor lighting using the triangular simulation and point-point method of calculation has been applied. In this study, the projector, focal point and the lighting level to the desired point on the basis of the virtual triangles, vertical and horizontal lighting calculation equations were obtained and used in the sample application. In this study, among the points of projector, focus point and calculating point for lighting level virtual triangles based on the calculation, vertical and horizontal lighting calculation equations are obtained and used for the sample application. The novelty of the calculation method used in this study according to its counterparts is to be able to calculate lighting levels for each point. Calculation of lighting levels for different planes, it is possible with the method used, where most desirable with regard to lighting and architectural, vertical and horizontal lighting applications are realized.

**Key Words.** Outdoor lighting, virtual triangles, point-point method, vertical and horizontal lighting.

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## An Intuitive Approach of a Sufi to the Natural Numbers

Şaban Can Şenay<sup>1</sup> and Hasan Şenay<sup>2</sup>

**Abstract.** Many mathematicians and philosophers agree that mathematics is not a purely deductive science and that it would be a mistake to deny the importance of intuition in the discovery and the invention of mathematics [4]. According to intuitionistic philosophy, mathematics should be defined as a mental activity that consists in carrying out constructs one after the other which are inductive and effective [3]. In this study, we present a sufi who is known as Shabistari, and his intuitive approach to the natural numbers and we also show the inductiveness and effectiveness of this approach. Sa'd ud din Mahmūd Shabistari was born at Shabistar about A.D. 1250 and he died at Tabriz. *Gulshan i Rāz* is the most famous book of him [2], [5]. According to Shabistari, the phenomenal world has no real "objective" existence like numbers. It is only the repetition of the "One" who is infinite and this repetition is the same as counting ones and if we start to count from one we never come to the end of the numbers. Although the period from the fifth century until the eleventh century is known as the Dark Ages for western Europe [1], it was a Golden Age in every field for the Islamic world. As one of the last representatives of that age and despite the fact that he was not a mathematician or a scientist, Shabistari showed us the strong relation between these sciences and sufism in a very clear wording.

**Keywords.** Intuition, natural numbers.

**AMS 2010.** 03F55, 00A30, 01A30.

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## Consistency of Peano's Arithmetic System and Structural Incompleteness of Peano's Arithmetic System

Teodor J. Stepień<sup>1</sup> and Łukasz T. Stepień<sup>2</sup>

**Abstract.** §1. **Terminology.**  $X \subseteq Y$  denotes that  $X$  is a subset of the set  $Y$ . Next,  $\rightarrow, \sim, \vee, \wedge, \equiv$  denote the connectives of the implication, negation, disjunction, conjunction and equivalence, respectively. We use  $\Rightarrow, \neg, \vee, \&, \Leftrightarrow, \forall, \exists$  as metalogical symbols.

$S_A$  denotes the set of all well-formed formulas of Peano's Arithmetic System (cf. [3]).  $R_{S_A}$  denotes the set of all rules over  $S_A$ . For any  $X \subseteq S_A$  and  $R \subseteq R_{S_A}$ ,  $Cn(R, X)$  is the smallest subset of  $S_A$ , containing  $X$  and closed under the rules of  $R$ .

Next, the couple  $\langle R, X \rangle$  is called a system, whenever  $X \subseteq S_A$  and  $R \subseteq R_{S_A}$ .  $r_0^p$  denotes Modus Ponens for Peano's Arithmetic System and  $r_+^p$  denotes the generalization rule for Peano's Arithmetic System. Hence  $R_{0+}^p = \{r_n^p, r_+^p\}$ .

$L_2^p$  denotes the set of all logical axioms in Peano's Arithmetic System.  $A_r^p$  denotes the set of all specific axioms of Peano's Arithmetic System. Thus,  $\langle R_{0+}^p, L_2^p \cup A_r^p \rangle$  denotes Peano's Arithmetic System.

Let  $X \subseteq S_A$  and  $R \subseteq R_{S_A}$ . Hence,  $Perm(R, X)$  denotes the set of all permissible rules in the system  $\langle R, X \rangle$ . Analogically,  $Der(R, X)$  denotes the set of all derivable rules in the system  $\langle R, X \rangle$ . Thus

DEFINITION 1.1  $r \in Perm(R, X)$  iff

$$(\forall \Pi \subseteq S_A)(\forall \varphi \in S_A)[\langle \Pi, \varphi \rangle \in r \ \& \ \Pi \subseteq Cn(R, X) \Rightarrow \varphi \in Cn(R, X)].$$

DEFINITION 1.2  $r \in Der(R, X)$  iff

$$(\forall \Pi \subseteq S_A)(\forall \varphi \in S_A)[\langle \Pi, \varphi \rangle \in r \Rightarrow \varphi \in Cn(R, X \cup \Pi)].$$

Now, at first, we must notice that the notion of structural rule on the ground of the propositional calculus has been introduced in [1]. The notion of structural rule on the ground of the predicate calculus has been used in [2].

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Let now  $Structs_A$  denotes here the set of all structural rules, where  $Structs_A \subseteq Rs_A$ <sup>3</sup>. Next, let  $X \subseteq S_A$  and  $R \subseteq Rs_A$ . Hence,  $\langle R, X \rangle \in SCpls_A$  denotes that the system  $\langle R, X \rangle$  is structurally complete and  $\langle R, X \rangle \in Cns_A^T$  denotes that the system  $\langle R, X \rangle$  is traditionally consistent. Thus,

DEFINITION 1.3.  $\langle R, X \rangle \in SCpls_A \Leftrightarrow Structs_A \cap Perm(R, X) \subseteq Der(R, X)$ .

DEFINITION 1.4.  $\langle R, X \rangle \in Cns_A^T \Leftrightarrow (\neg \exists \alpha \in S_A) [\alpha \in Cn(R, X) \& \sim \alpha \in Cn(R, X)]$ .

## § 2. The Main Result

THEOREM 1.  $\langle R_{0+}^P, L_2^P \cup A_r^P \rangle \in Cns_{S_A}^T$ .

**Proof:** By using only the axioms of the first-order Logic and the axioms of Peano's Arithmetic System (cf. [5]).<sup>4</sup>

THEOREM 2.  $\langle R_{0+}^P, L_2^P \cup A_r^P \rangle \notin SCpls_A$ .

**Proof:** By using THEOREM 1, the First Gödel's Incompleteness THEOREM and the main result from [4].

**Keywords.** Peano's Arithmetic System, consistency, Structural completeness.

**AMS 2010.** 03F03.

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<sup>3</sup> The definition of the structural rule on the ground of arithmetic will be presented during the talk.

<sup>4</sup> From the construction of this Proof of THEOREM 1, it follows right away that the Second Gödel's Incompleteness THEOREM is invalid.

## Mathematical Instruments and Measuring Technics in History

Zeki Tez<sup>1</sup>

**Abstract** .During the Renaissance period in Italy, the new practices of geometry have led to the rise of different fields of science, including cartography, navigation and astronomy, and various mathematical instruments were developed. Those tools were used by a number of technicians like astronomers, account experts, technical drawers, builders, cartographers, architects, field measurers and artillerymen. In efforts to enhance measurement precision, cross sectioning, ‘*Nonius*’ or vernier and its very best application, micrometer screw, were developed. [2]

In the 16<sup>th</sup> century, measure-quadrant was used for measuring distances in field measurement and astronomy; quadrant for vertical angle measurement; ‘Jacob’s stick’ for measuring heights; astrolabe, compass and tools alike, plumbline, measure-stick, goniometer, protractor, sextant, octant and theodolite was used for measuring distances and angles. Quadrant is equal to one quarter of the circumference of a circle, sextant to one sixth and octant to the eighth of it. Jacob’s stick is recognized as being similar to that of St.Jacob’s pilgrim staff of Middle Ages. To measure time accurately, sundials, nocturnals and mundials were made use of. Celestial sphere, star-wheel, torquetum (‘*Turkish device*’), armillary sphere, triangulum, periscope, binoculars and compass had a significant place in astronomy. Tools that measure angle were of greater importance to navigation, cartography, construction of fortifications and gunnery.

In the early 17<sup>th</sup> century, the concept of logarithm and mechanical calculation instruments were put into effect. John Napier, who is credited with the invention of logarithms, designed the “*Napier’s rods*” in 1617 as mechanical calculation device in his famous book, entitled *Rhabdologie*. They were strung side to side in an appropriate line and used for carrying out multiplication and division. [1] Edmund Gunter developed ‘Gunter’s scale’ (1624), using Napier’s rods, while William Oughtred was the first to use two scales sliding by one another (‘*sliding rule*’) (1632). Calculations were performed more easily with this analogue tool. With the emergence of pocket size digital calculators in 1973, sliding rule were abandoned.

In the early 16<sup>th</sup> century, in order to make a geometrical drawing smaller at a certain ratio, a reduction-compass in the form of letter “X” were used. German artist Albrecht Dürer is known to have introduced a compass used for reduction (‘*Reduktionszirkel*’). There is one

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other tool developed by Dürer himself, ‘*Vergleicher*’. Of such instruments, pantograph is regarded as the one that performs proportional drawing in the fastest way. Pantograph, used to make a figure look smaller or bigger than it is by means of similar triangles, was invented by Christoph Scheiner. French scholar Nicolas Bion elaborated on a wide range of instruments in his book of 1709. [3]

Wenzel Jamnitzer managed to perform polyhedra drawings in large numbers, using his perspective machine. Luca Pacioli gave place to some simple polyhedra drawings produced by Leonardo da Vinci in his book *De divina proportione* (1497), where he strived to explain the secret of beauty with golden ratio ( $\varphi=1,618\dots$ ).

Abacus is known to have appeared thousands of years ago. One of the first calculation machines was developed by William Schickard in 1623. With this machine he named ‘*Calculation Clock*’, he was able to perform four arithmetical operations. French mathematician Blaise pascal, only to facilitate the never-ending tax calculations of his father, a tax-farmer, developed in his 18’s of age a machine named ‘*La Pascaline*’, capable of adding and subtracting eight digit numbers.

**Key Words:** Mathematical instrument, Napier’s rods, compass, calculation machine

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## Efficiency of Turkish Insurance Companies: a Stochastic Frontier Analysis

İlyas Akhisar<sup>1</sup>

**Abstract.** There is a growing interest and concern about the international competitiveness and efficiency of Turkish financial institutions in general and insurance companies in particular. Over the past decade the Turkish financial institutions have gradually deregulated the financial services sector through a series of banking and insurance directives with a view to creating a single Europe an market in financial services [1], [2]..

The objective of this paper is to model and measure cost and profit efficiency in the Turkish insurance sector using stochastic frontier analysis (SFA), and to explore variations in efficiency [3].

In this research, financial efficiency of Turkish Insurance companies doing business in non-life insurance branches, for each company and each year, we are able to construct consistent measures of revenue, profits and operating costs from the accounting data are obtained for the period 2000-2010 and applying the Stochastic Frontier Analysis model which was developed with the FRONTIER 4.1 software.

**Keywords.** Stochastic Frontier Analysis, efficiency , insurance sector, financial ratios.

**AMS 2010.** 62L20, 65C20.

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# *POSTER*



## **Resource capacity of the transport of industrial conglomerates**

Anatoliy Bondarenko<sup>1</sup>

**Abstract.** There are approaches to solving social, transport, technical problems on the basis of harmonious analysis algorithm of ant colonies. The ways of forming new objectives, goals using a simplified interface. Offered a productive function in the analysis of movement of rolling stock and harmonious vibrations path.

Routes of administration are simulations of the distribution, game theory in the centralized traffic control, detection of overheated axle boxes, surface defects skating wheels. Algorithmic structural diagrams show how the individual elements of their relationship, subordination, which provides dynamic coordination process. Functional concepts provide automatic control for variations in the performance of a certain function. Moving described by differential equations of motion of the rolling stock and control the road conditions harmonious vibration equations.

In setting up new tasks proposed conditions of its expression is incorrect, do not uniquely define its solution. Therefore, detailed description of the tasks to redundancy. We express the task of improving the smoothness of the way the linear equations that depend on the load on the network variables: the amount of cargo, passengers, wagons, flux and network density, speed of movement can be regarded as a free radical. Poor conditionals of problem of approximation is calculated by regularization, we reduce the resolution or pseudo solution.

**Keywords.** Ant colony algorithm, the fluctuations of the rolling stock, the smooth path.

**AMS 2010.** 68W25, 94D05.

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## An Extension of Srivastava's Triple Hypergeometric Function $H_A$

A. Çetinkaya<sup>1</sup>, M. B. Yağbasan<sup>2</sup>, İ. O. Kıymaz<sup>3</sup>

**Abstract.** An extension of beta function, containing an extra parameter was used to extend the hypergeometric, confluent hypergeometric and Appell's hypergeometric functions. In this study, we introduce an extension of Srivastava's triple hypergeometric function  $H_A$  by using extended beta function. We also give some integral representations of this new function.

**Keywords.** Beta function, Srivastava's triple hypergeometric function, Appell's hypergeometric function.

**AMS 2010.** 33C60, 33C65, 33C70.

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## Ball and Burmester Points in Lorentzian Sphere Kinematics

Abdullah Inalcık<sup>1</sup> and Soley Ersoy<sup>2</sup>

**Abstract.** In this work, we study Lorentzian spherical motion of rigid bodies by using instantaneous invariants and define Lorentzian inflection curve, the Lorentzian circling points curve and the Lorentzian cubic of twice stationary curve which are the loci of points having same properties during Lorentzian spherical motion of rigid bodies.

Also the intersection points of these curves are called Ball points and Burmester points and we define Lorentzian Ball and Burmester on Lorentzian sphere.

**Keywords.** Lorentz Spherical Kinematics, Instantaneous Invariants

**AMS 2010.** 53B30, 53A17, 53A35.

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**About Solvability of the Cauchy Problem For One-Dimensional Nonlinear  
Nonstationary Six-Moment System Equations of Boltzmann**

A.Sakabekov<sup>1</sup> and E.Auzhani<sup>2</sup>

**Abstract.** The main scaling parameter in kinetic theory is the Knudsen number  $Kn = \lambda/L$ , where  $\lambda$  is the mean free path between collisions,  $L$  is the macroscopic length. Complete equilibrium is given for  $Kn = 0$ , described by the dissipationless Euler equations. In many processes gas dynamics with Navier-Stokes and Fourier fail at  $Kn \approx 0,01$  and sometimes even for smaller values. For large Knudsen numbers the collisions between the particles can be neglected and they move ballistically in a free flight. The range between these limiting cases can be split into kinetic regime and transition regime. Kinetic regime describes by the nonlinear Boltzmann equation. In transition regime a state of gas describes with assistance Boltzmann's moment system equations. There exist an infinite number of Boltzmann's moment equations corresponding to expansion of distribution function in a series.

An approximation strategy in kinetic theory is given by Grad's [1] moment method based on Hilbert expansion of the distribution function in Hermite polynomials. The method is described in Grad (1949) [1].

In work [2] had been received the system of moment equations for the spatially-non-homogeneous Boltzmann equation, which is distinct from Grad's system, by expansion of distribution function of particles by eigenfunctions of the linearized collision operator. Thus the differential part of the moment system has appeared to be linear, and problem statement of boundary condition has been solved. Studying of an initial or initial and boundary value problem for each approximation of Boltzmann's moment system equations represents the big interest.

In this work we prove the existence and uniqueness of the initial value problem for one-dimensional nonlinear nonstationary six-moment system equations of Boltzmann in space of functions, continuous in time and square summable by spatial variable.

**Keywords.** Boltzmann's moment equations; kinetic regime; Cauchy problem.

**AMS 2010.** 53A40, 20M15.

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## **Reconstruction of Unknown Boundary Data for a 3D Elliptic Equation**

Balgaisha Mukanova<sup>1</sup>, Saule Mausumbekova<sup>2</sup> and Magira Kulbai<sup>3</sup>

**Abstract.** In this work, we discuss about the method of solution to the inverse problems for a stationary diffusion process in the cylindrical system of coordinates. The considered problem is classical, there exists multiple references. We mentioned here only the closest to the considered topic [1]–[3]. Unlike work [3] we consider a problem when sources are on inaccessible border of area of the decision, and diffusion process is supposed established. Such problems are formulated in the form of the inverse problems of boundary values determination on inaccessible border for Laplace's equation and reduced to an initial and regional problem. This problem is ill-posed, however, has many practical applications, the methods and programs for its numerical solutions are actual.

**Keywords.** Inverse problem, Laplace's equation, ill-posed problems.

**AMS(MOS) subject classifications.** 35J05, 35P99, 35R25, 35R30, 49K20.

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## Evolutes-Involutes of Spatial Curves and Meusnier Spheres in Minkowski Space

Beyhan Uzunoğlu<sup>1</sup>, Zehra Bozkurt<sup>2</sup>, and Yusuf Yaylı<sup>3</sup>

**Abstract.** From elementary differential geometry it is well known that the locus of the osculating circles of a curve  $\alpha$  is the involute of the curve  $\alpha$ . On the other hand, focal curves of the curve  $\alpha$  are locus of the center of its osculating spheres.

In view of these basic facts, it is natural to ask the following geometric question:

**What is the locus of centers of Meusnier spheres in 3-dimensional Minkowski space?**

In this study, we investigate the answer of the above question.

**Keywords.** Involutes, evolutes, osculating spheres, focal curves, curves in Minkowski space.

**MSC 2010.** 53A04, 51B20.

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## Some Special Solutions of The Differential Equations $y'' + f(x)y = 0$

C. Dane<sup>1</sup>, K. Kasapoglu<sup>2</sup> and H. Akbas<sup>3</sup>

**Abstract.**  $y'' + C(x)y = 0$  the differential equation's solutions which have founded for some special values of  $C(x)$  are compared with special solutions which have founded by means of symmetries. [1], [2], [3], [4], [5], [6].

**Keywords.** Symmetry, analytical solution, differential equation.

**AMS 2010.** 53A40, 20M15.

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## The Pseudo-Holographic Coding Images & $p$ -adic Spheres

Daria Uryvskaya<sup>1</sup> and Vladimir Chernov<sup>2</sup>

**Abstract.** The mathematical background for the holographic encoding technique for images [1], [2] and its relationship with a covering the domain of a digital image by  $p$ -adic spheres with decreasing radius [4] are represented in this work. Resistance of the holographic encoding method and its robustness for loss of chunk of data are investigated. This work is supported by the RFBR grant № 12-01-31316.

**Keywords.** Holographic coding, non – Archimedean metrics.

**AMS 2010.** 11S31, 11T71.

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## Some New Paranormed Fibonacci Difference Sequence Spaces

Emrah Evren Kara<sup>1</sup> and Serkan Demiriz<sup>2</sup>

**Abstract.** In this study, we define new paranormed sequence spaces by the sequence of Fibonacci numbers. Furthermore, we compute the  $\alpha$ -,  $\beta$ - and  $\gamma$ -duals and obtain bases for these sequence spaces. Besides this, we characterize the matrix transformations from the new paranormed sequence spaces to the Maddox's spaces  $c_0(q)$ ,  $c(q)$ ,  $\ell(q)$  and  $\ell_\infty(q)$ .

**Keywords.** Matrix transformations, Sequence spaces, Fibonacci numbers.

**AMS 2010.** 11B39, 46A45, 46B45.

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## One Dimensional Models of Some Dynamic Problems with Viscous Resistance Force

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**Abstract.** In this, study, a new two models has been developed: the classical Tomlinson model and a generalized model that can be used for any long-range potential. In this process, numerical solutions has been obtained and an algorithm has been set to determine the friction forces between the atoms in the chain. Taking the Morse [1] potential into account and considering the mutual interactions of atoms we have reached the following generalized ordinary differential nonlinear equations for the displacements:

$$\frac{d^2 u}{dt^2} = \Phi(t, u, \frac{du}{dt}) = F^R - F^L - F^{ext},$$

$$F_i^R = 2D\alpha \sum_{p=i+1}^N \left\{ \exp[-2\alpha(x_p - x_i - r_e + u_p - u_i)] - \exp[-2\alpha(x_p - x_i - r_e) + u_p - u_i] \right\},$$

$$F_i^L = 2D\alpha \sum_{p=0}^{i-1} \left\{ \exp[-\alpha(x_p - x_i - r_e + u_i - u_p)] - \exp[-2\alpha(x_i - x_p - r_e) + u_i - u_p] \right\},$$

$$F_i^{ext} = 2D\alpha \sum_{p=0}^{i-1} \left\{ \exp[-\alpha(r_{il} - r_e)] - \exp[-2\alpha(r_{il} - r_e)] \right\} \cos \beta_{il} - \gamma \frac{du_i}{dt},$$

where  $i = 1, 2, \dots, N$  is the number of the equation describing the motion of the  $i$ th atom and  $\beta_{il}$  is the angle between the axis Ox and the Radius vector  $r_{il}$  connecting the  $i$ th atom in the deformable chain with the  $l$ th atom in the rigid chain;  $r_{il} = |r_{il}|$ . By definition,

$$r_{il} = [(x_{Tl} + U_c - x_i)^2 + d^2]^{\frac{1}{2}},$$

where  $x_{Tl}$  are the initial coordinates of atoms in the rigid chain [2].

**Key words:** dynamical problems, rigid chain, Morse potential.

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## Cubic B-Spline Finite Element Solution of the Burgers' Equation

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**Abstract.** Generally, it is not easy to solve nonlinear partial differential equation directly. Rarely, it is possible to convert nonlinear partial differential equation to equivalent linear partial differential equation by using a suitable transformation. For instance, Burgers' equation replaces with heat equation by means of the Hopf-Cole transformation. In this paper, by keeping nonlinear structure of Burgers' equation, Burgers' equation was converted to a set of nonlinear ordinary differential equations using the method of discretization in time. Then, each of them was solved by applying the cubic B-spline finite element method. In order to evaluate the efficiency of the presented method, two problems were considered. The numerical solutions obtained for various values of viscosity were compared with the exact solutions. It is seen that the results obtained by this way are quite satisfactory.

**Keywords.** Burgers' Equation, The Method of Discretization in Time, Galerkin Method, Cubic B-spline Finite Element Method.

**AMS 2010.** 65M60, 34K28.

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## A Uniqueness Theorem for Special Eigenvalue Problem

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**Abstract.** In this study, a uniqueness theorem is given for Sturm-Liouville Problem with special singular potential. We prove that singular potential function can be uniquely determined by the spectral set  $\{\lambda_n(q_0, h_m)\}_{m=1}^{+\infty}$ .

**Keywords.** Singular, Sturm Liouville, Uniqueness theorem, Eigenvalue.

**AMS 2010.** 34B05, 47E05.

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## A New Method for Inextensible Flows of Curves In $E^4$

Essin Turhan<sup>1</sup> and Talat Körpınar<sup>2</sup>

**Abstract.** In this paper, we construct a new method for inextensible flows of curves in  $E^4$ . Using the Frenet frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in  $E^4$ .

**Keywords.** Fluid flow,  $E^4$ , partial differential equation.

**AMS Classifications.** 31B30, 58E20.

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## Null W-Slant Helices In $E_1^3$

Fatma Gökçelik<sup>1</sup>, F. Nejat Ekmekci<sup>2</sup> and İsmail Gök<sup>3</sup>

**Abstract.** In this paper, we give the necessary and sufficient conditions for null curves in  $E_1^3$  to be W-slant helix in terms of their curvature functions. In particular, we obtain some relations between null helices and null W-slant helices. Also, a relationship is given between the null W-slant helices and their pseudo-spherical images. Finally, we give some examples of the null W-slant helices in  $E_1^3$ .

**Keywords.** Null curve, Slant helices, Harmonic curvature functions, Lorentzian 3-space.

**AMS 2010.** 14H45, 14H50, 53A04.

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## **Comparison of the Effects of Using Flock Mean and Median as Dependent Variables on the Growth Model Parameters of Slow-Growing Chicken Raised in the Organic System**

Hasan Eleroğlu<sup>1</sup> and Fikret Nafi Çoksöyler<sup>2</sup>

**Abstract.** In recent years mathematical modeling of growth of birds has become a popular research subject. In these studies, sigmoid functions such as Gompertz [ $W=\beta_0\exp(-\beta_1\exp(-\beta_2t))$ ] and Logistic [ $W=\beta_0(1+\beta_1\exp(-\beta_2t))^{-1}$ ] growth models are widely used. Principally, growth of birds is traced individually, although their feed consumption is not individual. This dilemma can be solved by tracing flock medium or flock median as growth variable. Because of inhomogeneity of birds in the same flock, using medium or median causes different effects on the growth parameter values.

Aim of this study is to investigate such effects on growth parameters of  $\beta_0$  (asymptotic mature weight),  $\beta_1$  (scaling constant) and  $\beta_2$  (instantaneous growth rate). In this study, 120 birds are used belonging 6 flocks. Weekly weights of males and females of each flock are averaged as arithmetic mean and medium. Then these averaged values are modeled vs growth period. Arithmetic means and mediums caused some differences on the values of the parameter. Also their effects were different on male and female birds. Most of these differences found to be statistically different ( $p<0.05$ ) when tested by paired t-test. But generally, arithmetic mean resulted with bigger determination coefficient ( $r^2$ ). Differences and their possible reasons are discussed in detail in the submission.

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## The Incomplete Lauricella Functions

İ. O. Kıymaz<sup>1</sup> and A. Çetinkaya<sup>2</sup>

**Abstract.** In 2012, Srivastava et al. introduced the incomplete Pochhammer symbols and used them to define the incomplete hypergeometric functions. In this study, we introduce the incomplete Lauricella functions with the help of incomplete Pochhammer symbols, and investigate some of their properties.

**Keywords.** Incomplete Pochhammer symbols, incomplete Lauricella functions.

**AMS 2010.** 33B20, 33C65, 33C70.

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## Saturation Number in Benzenoid Graphs

Ivana Zubac<sup>1</sup> and Tomislav Došlić<sup>2</sup>

**Abstract.** A *matching*  $M$  in a graph  $G$  is a set of edges of  $G$  such that no two edges from  $M$  have a vertex in common [1]. A matching  $M$  is *maximal* if it cannot be extended to a larger matching. *Saturation number* of a graph  $G$  is the minimum possible size of a maximal matching in  $G$ . In this work we are concerned with determining the saturation number for several classes of benzenoid graphs that serve as mathematical models for benzenoid hydrocarbons. (For a comprehensive introduction to matchings in benzenoid graphs we refer the reader to [2]).

**Keywords.** matching, saturation number, benzenoid

**AMS 2010.** 53A40, 20M15.

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## The Natural Lift of The Fixed Centrode of A Non-null Curve in Minkowski 3-Space

Mustafa Çalışkan<sup>1</sup> and Evren Ergün<sup>2</sup>

**Abstract.** In this study, we dealt with the natural lift curves of the fixed centrode of a non-null curve. Furthermore, some interesting result about the original curve were obtained, depending on the assumption that the natural lift curves should be the integral curve of the geodesic spray on the tangent bundle  $T(S_1^2)$  and  $T(H_0^2)$ .

**Keywords.** Natural Lift, Geodesic Spray, Darboux vector.

**AMS 2010.** 51B20, 53B30, 53C50.

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## On Asymptotically $\lambda$ -Statistical Equivalent of Order $\alpha$ of Sequences of Functions

Mikail Et<sup>1</sup>, Muhammed Çınar<sup>2</sup> and Murat Karakaş<sup>3</sup>

**Abstract.** In this paper we introduce and study the asymptotically  $\lambda$ -statistical equivalent of order  $\alpha$  of sequences of real-valued functions and strong asymptotically  $\lambda$ -equivalent of order  $\alpha$  of sequences of real-valued functions. Asymptotically  $\lambda$ -statistical equivalent of order  $\alpha$  of sequences of real-valued functions and strong asymptotically  $\lambda$ -equivalent of order  $\alpha$  of real-valued functions are given.

**Keywords.** Statistical Convergence, Asymptotically  $\lambda$ -Statistical Equivalent, Sequences of Functions.

**AMS 2010.** 40A05, 40C05, 46A45.

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## **Discontinuous Solutions of Nonlinear Boundary Value Problems of the Nonlinear Theory of Elastic Dislocations**

Karyakin Mikhail<sup>1</sup> and Pustovalova Olga<sup>2</sup>

**Abstract.** The concept of discontinuous solutions originated in the nonlinear theory of elasticity in the experimental work of A. Gent and P. Lindley [1] showed that cavities can arise in the rubber samples subjected to triaxial stress state. In the framework of nonlinear elasticity theory J. Ball [2] showed that the problem of the expansion of the isotropic sphere can have a solution that describes the formation of a cavity in the center of the ball. The existence of discontinuous solutions of the problems of the formation of dislocation and disclination in nonlinear elastic bodies was shown in the works of L. Zubov et al [3].

This paper studies different aspects of the cavity formation along the axis of screw dislocation and wedge disclination within the framework of nonlinear elasticity of incompressible media. Some integral relations were obtained that can be used to determine the size of the cavity and the possibility of its formation. For some classes of elastic energy functions the necessary conditions for the existence of discontinuous solutions were determined. It was shown that this existence is not related with well-known Hadamard inequality of nonlinear elasticity.

Several boundary value problems on the formation of a cavity in a nonlinear elastic body with isolated defects were resolved taking into account the microstructure of the material. For this purposes the theory of Cosserat continuum has been used. It was shown that account of microstructure typically reduces the radius of the cavity, until its complete elimination.

**Keywords.** Nonlinear elasticity, dislocation, disclination, Cosserat continuum.

**AMS 2010.** 74B20, 74G40.

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## **Discrete Cosine Transform on Pre-Fractal Domains**

Mikhail Kasparyan<sup>1</sup> and Vladimir Chernov<sup>2</sup>

**Abstract.** A class of discrete cosine transforms (DCT) defined on two-dimensional pre-fractal domains associated with the fundamental domains of canonical number systems in imaginary quadratic fields [1], [2]. JPEG-like compression algorithm using these transformations was investigated [3]. It is shown that the boundary artifacts less visually distinguishable. This work is supported by the RFBR grant № 12-01-31316.

**Keywords.** Discrete cosine transform, JPEG, compression of images.

**AMS 2010.** 65Y15, 65T50, 28A80, 11S31.

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## Bäcklund Transformation for Spacelike Curves in $E_1^4$

Mustafa Özdemir<sup>1</sup>, Melek Erdoğan<sup>2</sup> and A. Aziz Ergin<sup>3</sup>

**Abstract.** The main purpose of this paper is to construct a Bäcklund transformation between two spacelike curves with the same constant curvature in Minkowski 4 space by considering some assumptions. Moreover, We give the relations between curvatures of these two spacelike curves..

**Keywords.** Bäcklund Transformation, Spacelike Curve, Constant Torsion Curve..

**AMS 2010** 53B30, 53A05, 37K35

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## Comparison of Non-Linear Weibull Model with Linear Models in Inactivation of Microorganisms

Müzeyyen Şahinöz<sup>1</sup> and F. Nafi Çoksöyler<sup>2</sup>

**Abstract.** The level of unwanted microorganisms in foods has to be decreased down to 1 kob/g (living, reproducible cells/g), 10 kob/g and even to almost non-existing levels (such as 10-12-10-13 kob/g). Since the success in decreasing microorganism presence to these levels cannot be analytically tested, mathematical models are used to calculate the progress. The first models in this area were used in early 1900's, for the inactivation of bacterial spores which constituted a problem in canned food industry. These models, just like in half-life calculation, were designed assuming that every microorganism spore has equal thermal resistance and that instant inactivation rate is in proportion with living spore concentration (as in first-order reaction kinetics in chemistry). They are easy to apply. Original model is as follows:

$$N = N_0 e^{t \cdot k_d}$$

Though it is exponential, the used form is linear:

$$\ln(N) = \ln(N_0) - t \cdot k_d$$

The importance of linear models, especially in that period, was that they can be performed through very simple calculations (as in linear regression) or by drawing the most accurate curve directly by hand on paper.

In those years, even though it was acknowledged that some spores were more resistant and could stay alive during thermal process and directly distance (tail in graphic), or that a horizontal line could occur since there was no death at the beginning (shoulder in graphic), these were ignored due to calculation difficulties. Therefore the linear part of the inactivation curve was used for the calculations.

Since contemporary computer means made calculations easier, more accurate models to represent the real conditions were developed and used. However, customarily, linear models or the forms of models after lineation are still being used in some studies. Nevertheless, it is broadly observed that non-linear models explain inactivation more accurately. One of the most successful non-linear models is the Weibull model.

In this study, data of the earlier studies, in which the evaluations were made by linear models, were used, and Weibull model was utilized to evaluate if it represented the inactivation pattern in that study more accurately. This evaluation was made based upon visual compatibility, RSD, and determination factor ( $r^2$ ).

**Keywords.** Linear models, non-linear models, microorganisms, inactivation, Weibull model .

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## **The Effect of Conceptual Change Texts on Students of Primary 5th Class Understanding of “Height” Concept**

Nejla Gurefe<sup>1,2</sup>, Saliha Hilal Yazar<sup>3</sup>, Büsra Nur Karaarslan<sup>3</sup> and Hasan Es<sup>3</sup>

**Abstract.** It is important to be provided meaningful and permanent learning in the Primary Mathematics Curriculum based constructivist approach. In order to achieve it, firstly, existing information of students about a subject should be determined and lastly, if they have misconceptions related his subject, these misconceptions are determined and removed. One of the effective methods using to determine these is Conceptual Change Texts (CCT) [1-3]. The purpose of this study was to provide being learnt “Height” concept by the primary fifth class students and to remove misconceptions of these students with CCT. The study was made of the spring semester of the 2012-2013 academic years. Participants were 80 students from two different schools of Ankara. One of these schools was public schools and the other school was private school. A quasi-experimental design was used as the research method. There were two groups being experimental and control in each of schools. CCT was used in the experimental group and traditional instruction was used in the control group. “Height Achievement Test” developed by researchers was applied as pre-test and post-test for the experimental and control group. Diversity of experimental between control groups was analyzed with independent samples t-test. The findings in this study indicated that there was a significant difference between the control and experimental groups in favour of experimental group ( $p < .005$ ) for the both schools. Findings indicated that lessons being studied with conceptual change texts are more effective than traditional instruction.

**Keywords.** Conceptual change text, “height” concept, geometry.

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## **Simulation transformation of harmful substances in the atmosphere**

N.M. Temirbekov<sup>1</sup>, E.A. Malgazhdarov<sup>2</sup>, A.N. Temirbekov<sup>1</sup>

**Abstract.** The given paper is devoted to examining the problem of impurities dispersion from point sources taking into consideration transformations. To account the effect of anthropogenic heat source and nonuniformity of underlying surface we examine the model of atmospheric boundary layer. Modeling impurities dispersion we took into consideration local relief and water surfaces for Ust-Kamenogorsk city. We present numerical modeling of impurities dispersion and transformation against the background of meso-meteorological processes on the example of Ust-Kamenogorsk. Papers [1]-[2] are devoted to studying the dynamics of impurities dispersion. A new method of solving Burgers equation without taking photochemistry into consideration was proposed in work [1]. The given method is very interesting for modeling problems of the environment. Inverse problem was examined for assessing risk of lake Baikal. The model of impurities dispersion from private sector was examined in paper [2]. Issues of impurities effecting visual range and safe aircraft operation were studied there. Research done in this paper show that in the result of incomplete coal combustion and local peculiarities almost all emissions remain in soil atmospheric layer at human height. Paper [3] examined the processes of transformation and described in detail 156 chemical reactions among 82 components of impurities. Mesoscale hydra-thermodynamic processes and transfer of antropogenic impurities in the atmosphere and hydrosphere of lake Baikal region were modeled.

**Keywords.** Transformation, atmosphere, numerical modeling,

**AMS 2010.** 93A30, 97N60.

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## About one Method to Solve of Finite-Difference Equations of Incompressible Fluid in Variables «Stream Function, Vorticity»

Danaev N.T.<sup>1</sup>, Amenova F.S.<sup>2</sup>

**Abstract.** In this paper, we consider the question of convergence of solving two-dimensional operator-difference problem for incompressible fluid in variables “stream function-vorticity” to solving differential problem. For error of solution estimation of convergence rate was got.

**Keywords.** Navier-Stokes equation in variables “stream function-vorticity”, difference problem, iteration algorithms, estimation of convergence rate.

In square domain  $D = \{0 \leq x, y \leq 1\}$  let us study the following system of Navier-Stokes steady-state equations in variables stream function, velocity curl for incompressible fluid [1]

$$\left( \Omega \frac{\partial \Psi}{\partial y} \right)_x - \left( \Omega \frac{\partial \Psi}{\partial x} \right)_y = \nu \Delta \Omega + f(x, y), \quad (1)$$

$$\Delta \Psi = \Omega, \quad (x, y) \in D \quad (2)$$

with following boundary conditions  $\Psi = \frac{\partial \Psi}{\partial n} \Big|_{\partial D} = 0,$  (3)

where  $\nu > 0$  is viscosity coefficient,  $\bar{n}$  is outer normal to domain boundary,  $\Delta$  is two-dimensional Laplace operator,  $f(x, y)$  is some given function,  $\Psi$  is stream function,  $\Omega$  is velocity curl.

For approximation of equations (1)-(2) in finite-difference domain  $D_h = \{(kh_1, mh_2), k, m \in \overline{1, N-1}\}$ , where  $h_1$  and  $h_2$  are grid steps in  $x$  and  $y$  directions, respectively, we examine the following scheme on symmetrical pattern

$$L_h(\Omega)\Psi = \nu \Delta_h \Omega + f, \quad (4)$$

$$\Delta_h \Psi = \Omega, \quad (5)$$

where  $L_h$  is difference operator, which complies with respective approximation of convectional summands of equation (1), for example,  $L_h(\Omega)\Psi = \left( \Omega \Psi_0 \right)_y^x - \left( \Omega \Psi_0 \right)_x^y$ , where

$\Psi_0^x, \Psi_0^y$  are symmetrical difference derivative in  $x$  and  $y$  directions, respectively.

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In our investigation the boundary conditions for velocity curl are taken in the form of Tom's

formulas [2] 
$$\Omega_{0m} = \frac{2}{h_1} \Psi_{x,0m}, \quad \Omega_{Nm} = -\frac{2}{h_1} \Psi_{x,Nm}, \dots \quad (6)$$

It is shown that the solution of the difference problem (4) - (6) converges to the solution of the differential problem (1) - (3).

For the error of the estimate of convergence of solutions  $\|\Delta_h \Phi\| \leq c_0 \cdot h^{3/2}$ , where  $c_0 > 0$  is a uniformly bounded constant.

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## Spinor Frenet Equations in three Dimensional Lie Groups

O. Zeki Okuyucu<sup>1</sup>, Ö. Gökmen Yıldız<sup>2</sup> and Murat Tosun<sup>3</sup>

**Abstract.** In this paper, we study spinor Frenet equations in three dimensional Lie groups with a bi-invariant metric. Also, we obtain spinor Frenet equations for special cases of three dimensional Lie groups.

**Keywords.** Spinor, Frenet equations, Lie groups.

**AMS 2010.** 11A66; 22E15.

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## Inextensible Flows of Curves in Lie Groups

Önder Gökmen Yıldız<sup>1</sup> and Osman Zeki Okuyucu<sup>2</sup>

**Abstract.** In this paper, we study inextensible flows of curves in three dimensional Lie groups with a bi-invariant metric. The necessary and sufficient conditions for inextensible curve flow are expressed as a partial differential equation involving the curvatures. Also, we give some results for special cases of Lie groups.

**Keywords.** Inextensible flows; Lie groups

**AMS 2010.** 53A04; 22E15.

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## On Classification of Binary and Ternary Number Systems for Imaginary Quadratic Rings and Their Applications

Bogdanov P.S.<sup>1</sup> and Chernov V.M.<sup>2</sup>

**Abstract.** In the paper the classification theorem for binary and ternary number systems in imaginary quadratic fields is proved [1], [2]. Generalizing the method of [3] synthesized fast parallel algorithms for "error-free" computing of a discrete cyclic convolution and multiplication of large integers.

**Keywords.** Number system, quadratic fields, discrete cyclic convolution.

**AMS 2010.** 11A05, 11A07, 11A51, 65T50.

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## Explicit Solution of Hydrogen Atom Equation via Fractional Calculus Operators

Resat Yilmazer<sup>1</sup>, Necdet Catalbas<sup>2</sup>, Erdal Bas<sup>3</sup>

**Abstract.** In this study, it is obtained an explicit solution of the following second order linear ordinary differential equation by fractional calculus operator

$$\frac{d^2y}{dz^2} + \frac{\alpha dy}{z dz} + \left(\lambda + \frac{\beta}{z} + \frac{\tau}{z^2}\right)y(z) = 0,$$

where  $\alpha, \lambda, \beta, \tau$  are parameters.

**Keywords.** Fractional calculus; Hydrogen atom equation; Explicit Solutions.

**AMS 2010.** 26A33, 34A08.

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## **A Simple Approach for Estimation of Dynamic State Variable By Using Taylor Series**

### **Approximation**

Saadettin Aksoy<sup>1</sup> and Hakan Kızmaz<sup>2</sup>

**Abstract.** State variables that determine a system's dynamics should be known for analysis and control of dynamical systems [1-2]. The estimation of the state variables is required for dynamic feedback. Specially, it is an important problem in adaptive control applications [3]. Unfortunately, all of the state variables cannot be measured in practice. As a result, the usage of a suitable state observer or estimator is unavoidable in order to obtain immeasurable state variables. There exist various state observers in the literature [4]. The gain matrix  $\mathbf{G}$ , which is necessary for state observer, can be calculated by using one of the methods such as Ackerman, Bass Gura etc. [4].

In this study, a general algorithm that uses only input and output measurements; is proposed for estimation of the state variables of linear, time-invariant multi-input multi-output systems. The proposed algorithm is based on the Taylor series approximation and has an analog solution [5,6]. Hence, it is not affected by rounding errors. As a consequence, accuracy is not a function of the step size. The solution that results from the proposed algorithm gets closer to the true solution when more and more terms are kept in the Taylor series. Finally, the proposed method gives the approximate solution of the estimation vector  $\hat{x}(t)$  as a function of time in the interval  $t \in [0, T]$ ,  $T \leq 1$ . Consequently, the computation of the state and error integral equations for each  $t$  is eliminated.

The algorithm consists of three steps. In the first step, the feedback gain matrix  $\mathbf{G}$ , which will force the estimation error to go to zero in a short time, is determined by using a suitable method [4]. In the second step, the observer state equation is converted into integral equation by integrating the terms on either side of the equation. Then, the unknown state vector together with the Taylor series approximation of known input and output vectors is substituted in the integral equation. After some algebraic manipulations, the time dependent terms on either side of the integral equation are removed. Hence, the problem is reduced to a set of nonlinear equations with constant coefficients. Finally, nonlinear equations for unknown state vector are converted into a recursive form whose solution can be obtained easily by a computer program. Thus, unknown series expansion coefficients for estimation of state variables vector are easily calculated.

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The proposed estimation algorithm was implemented in MATLAB<sup>TM</sup> and it was applied to different cases. The results obtained by the proposed algorithm are in harmony with the actual results.

**Keywords.** State estimation, Taylor series, state observers.

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## **State Variables Computational Technique for Linear Systems by Leguerre Series Approximation**

Saadettin Aksoy<sup>1</sup> and Hakan Kızmaz<sup>2</sup>

**Abstract.** State variables that determine a system's dynamics should be known for analysis and control of dynamical systems [1,2]. Specifically, dynamic feedback for pole placement is required. Furthermore, estimation of state variables in real time is a very important problem in adaptive control applications [3]. Unfortunately, all of the state variables cannot be measured in practice. As a result, usage of a suitable state observer or estimator is unavoidable in order to obtain immeasurable state variables. There exist several of state observers in the literature [4]. The gain matrix  $\mathbf{G}$ , which is necessary for state observer, can be calculated by using one of the methods such as Ackerman, Bass Gura etc. [4].

The Laguerre series are defined on the interval  $t \in [0, \infty)$  and have the orthogonal property like the Walsh, Chebyshev and Legendre series [5,6]. The proposed algorithm uses some important properties such as the operational matrix of integration for Laguerre vector [7-9].

In this study, a general algorithm that uses only input and output measurements is proposed for state variables estimation of linear, time-invariant multi-input multi-output systems. The proposed algorithm is based on the Laguerre series approximation and has an analog solution. Hence, it is not affected by rounding errors. As a consequence, accuracy is not a function of the step size. The proposed method gives the approximate solution of the estimation vector  $\hat{\mathbf{x}}(t)$  as a function of time in the interval  $t \in [0, \infty)$ .

The algorithm consists of three steps. In the first step, the feedback gain matrix  $\mathbf{G}$ , which will force the estimation error to go to zero in a short time, is determined by using a suitable method [4]. In the second step, the state and error equations are converted into integral equations by integrating the terms on either side of the equations. Then unknown state and error vectors together with the Laguerre series approximation of known input vectors are substituted in the integral equations. After some algebraic manipulations, the time dependent terms on either side of the integral equations are removed. Finally, the nonlinear equations are converted into a recursive form whose solution can be obtained easily by a

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computer program. Unknown series expansion coefficients for estimation of state variables vector are easily calculated.

The proposed estimation algorithm was implemented in MATLAB<sup>TM</sup> and it was applied to different cases. Results obtained by the proposed algorithm are in harmony with the real results.

**Keywords.** State estimation, Laguerre series, state observers.

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## On Some Combinatorial Identities Involving the Terms of Generalized Fibonacci and Lucas Sequences

Serpil Halıcı<sup>1</sup> and Zeynep Akyüz<sup>2</sup>

**Abstract.** In this paper, we investigate Horadam sequence and obtain important some identities involving the terms of this sequence. Using by a different method. we give some combinatorial identities for this sequence.

**Keywords.** Horadam Sequence, Recurrence, Matrix Method.

**AMS 2010.** 11B37, 11B39, 11C20.

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## Weighted Quadrature Rules via Grüss Type Inequalities for Weighted LP Spaces

Sanja Tipuric-Spuzevic<sup>1</sup>, Andrea Aglic Aljinovic<sup>2</sup> and Josip Pecaric<sup>3</sup>

**Abstract.** New estimates of Grüss type for weighted Čebyšev functional in weighted Lp spaces are presented by using the Sonin's identity. These results are applied to obtain new Ostrowski type inequalities and further to weighted numerical rules for functions whose derivative belong to weighted Lp spaces.

**Keywords.** Sonin's identity, Čebyšev functional, Grüss inequality, numerical quadrature rule.

**AMS 2010** 26D15, 65D30

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## **Comparison Efficiency of Medium, Median and Geometric Medium at Evaluation of Microbiological Analysis Results of Foods**

Şehriban Uğuz<sup>1</sup>, F. Nafi Çoksöyler<sup>2</sup>

**Abstract.** One of the important problems of microbiological food control is how to evaluate microbial loads of sample units and /or how to evaluate analytical results of repeated measurements in the same sample unit. In cultural or direct count methods, initially sample is homogenized and all the microbial cells are randomly distributed in the sample mass. In this case, probability of microbial densities in the analytical portion of sample unit, obey the poisson distribution. However, distribution of microorganisms densities, in the food lot somewhat larger than the as in the sample unite and than, is accepted as log-normal distribution. In log-normal distribution, geometric mean of microbial loads of sample units is more representative for population mean rather than arithmetic mean. Therefore, in calculation of sample unit load from analytical repeated measurements arithmetic mean is used and in calculation of microorganism concentration of the lot, geometric mean of the load of sample units is used. When distribution of population loses symmetry, median become better for representing of real mean of the population.

In this study, it is intended to investigate the efficiency of medium, median and geometric mean in

1. Calculation of sample unit mean concentration from repeated measurements,
2. Calculation of lot mean concentration from sample units.

The data, used in this study, are produced by simulation regarding to real data held in the microbiological control studies. In this study, it is supposed that distribution of densities of repeated microbial count in Petri dishes (cultural count) and in microscopic fields (direct count) obey poisson distribution, and distribution of densities of microbial loads of samples units taken from same lot obey log-normal distribution. Microsoft Excel-2007 is used for random number generation for simulation. Simulated repeated counts and sample unit microbial loads are evaluated by calculation arithmetic mean, median and geometric mean. Then, values of these three parameters are investigated for representing core values. It has been a general observation in this study that, in proportion with population average, when standard deviation rises and/or average value approaches to 1, the representability of the median gets better than arithmetic mean, however, in these cases generally geometric mean is more succesful.

**Keywords.** Geometric mean, arithmetic mean, median.

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## Constant Angle Spacelike Surface in de Sitter Space

Tuğba Mert<sup>1</sup> and Baki Karlığa<sup>2</sup>

**Abstract.** In this paper, two special class of surface which are called constant timelike angle with spacelike axis and constant timelike angle with timelike axis spacelike surfaces are investigated in de Sitter space  $S_1^3$ . We study constant angle with spacelike axis spacelike surface whose unit normal vector field make constant timelike angle with a fixed spacelike axis in  $R_1^4$ . Similarly we study constant timelike angle with timelike axis surface whose unit normal vector field make constant timelike angle with a fixed timelike axis in  $R_1^4$ .

**Keywords.** Constant Angle, de Sitter, Spacelike

**AMS 2010.** 53040

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## New Modified Trial Equation Method and its Application

Yusuf Ali Tandogan<sup>1</sup> and Yusuf Pandir<sup>2</sup>

**Abstract.** Liu give the trial equation method based on the complete discrimination system for polynomials [1]. Also, an application of this method is included by Jun [2]. Then, Gurefe et al. have proposed some novel approaches to Liu's trial equation method [3,4]. In this study, we define a new modified trial equation method in order to obtain new function classes of traveling wave solutions. From here, we find some interesting results for two dimensional time-fractional Fisher equation [5] by using the proposed method.

**Keywords.** New modified trial equation method, two dimensional time-fractional Fisher equation, soliton solutions, elliptic integral function solutions.

**AMS 2010.** 35R11, 35C08, 33E05.

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## Some New Multiplicative Numerical Methods

Yusuf Gurefe<sup>1</sup> and Emine Misirli<sup>2</sup>

**Abstract.** Bashirov et al. consider a new calculus, called as multiplicative calculus, in the paper [1]. They give some important definitions and properties with respect to this calculus. Then, multiplicative Runge-Kutta methods has been developed by Aniszewska [2]. In 2009, the authors in the paper [3] define multiplicative finite difference methods for solving the multiplicative version of second order boundary value problems. Also, Misirli and Gurefe construct new multi-step methods, that are multiplicative Adams Bashforth-Moulton methods, in order to obtain the numerical solutions of the first order multiplicative initial value problems [4]. In [5], the authors study on modelling of the multiplicative differential equations. Apart from these, we give new numerical algorithms for the multiplicative initial value problems, and also examine the stability analysis of these methods.

**Keywords.** Multiplicative calculus, multiplicative numerical analysis, multiplicative stability.

**AMS 2010.** 65L05, 65L06.

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## A Multiple Extended Trial Equation Method for Nonlinear Physical Problems

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**Abstract.** Real world problems that are modeled by nonlinear differential equations play an extremely important role in mathematical physics. Many researchers are working on obtaining exact solutions of these equations [1, 4]. In this study, we define a new method named multiple extended trial equation method in order to obtain new function classes of traveling wave solutions. As an application, some new solutions to Burger-Fisher equation [5] are given.

**Keywords.** Multiple extended trial equation method, two dimensional time-fractional Fisher equation, soliton solutions, elliptic integral function solutions

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## Magnetic Curves on Sasakian Manifolds

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**Abstract.** In this paper, we investigate a variational approach for the magnetic flows associated with the Killing magnetic vector field and give some characterizations for the magnetic curves (trajectories of the magnetic flow) in 3-dimensional Sasakian manifolds. Moreover, we obtain some results for the magnetic curves in Pseudo-Hermitian manifolds using the Tanaka-Webster connection.

**Keywords.** Special Riemannian manifolds, magnetic vector fields, magnetic curve.

**AMS 2010.** 53C25, 53A04.

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## **Parameter Estimation Based Type-II Fuzzy Logic where the Independent Variables from Pareto Distribution**

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**Abstract.** In the linear regression analysis, while parameter estimation by the classical methods there are a number of assumptions need to be satisfied. One of them is errors are normally distributed. In the case that independent variable has a distribution different from normal distribution there are solution methods which are not affected by the deterioration of assumption. Some of these methods, benefit of membership functions in the process of analysis and thus are based on fuzzy logic. In this work, the case that independent variables have Pareto distribution to be discussed and an algorithm using adaptive networks suggested to parameter estimation where the  $k$  which is one of the parameters of the fuzzy membership functions is fuzzy. Also the parameter of fuzzy membership function is fuzzy the estimation process is based on type-II fuzzy logic.

**Keywords.** Pareto Distribution, Type-II fuzzy logic, membership function.

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285	Lecturer	Anatoliy Bondarenko	Priazowskyi State Technical University
286	Lecturer	Artion Kashuri	University "Ismail Qemali", Vlora
287	Lecturer	Doğan Ünal	Sakarya University
288	Lecturer	Furkan Yıldırım	Republic of Turkey Ministry of National Education
289	Lecturer	Gülşen Ulucak	Gebze Institute of Technology
290	Lecturer	Hakkı Güngör	Ataturk University
291	Lecturer	Hülya Durur	Ardahan University
292	Lecturer	Josipa Matotek	Faculty of Civil Engineering Osijek
293	Lecturer	Kastriot Zoto	University of Gjirokastra
294	Lecturer	Mehmet Fatih Öcal	Agri İbrahim Çeçen University
295	Lecturer	Merve Çoğan	Hitit University
296	Lecturer	Meryem Odabaşı	Ege University
297	Lecturer	Muhsin Çelik	Sakarya University
298	Lecturer	Murat Canayaz	Yuzuncu Yil University
299	Lecturer	Özge Kazar	Hitit University
300	Lecturer	Özkan Güner	Dumlupınar University
301	Lecturer	Recep Özdağ	Yuzuncu Yil University
302	Lecturer	Selman Hızal	Sakarya University
303	Lecturer	Şaban C. Şenay	KTO Karatay University
304	Lecturer	Uğur Karaman	Ministry of National Education
305	Rsc. Assist.	Abdulkadir Karakaş	Siirt University
306	Rsc. Assist.	Adem Doğaner	Firat University
307	Rsc. Assist.	Ahmet Kara Doğan	Inonu University
308	Rsc. Assist.	Ahmet Pekgör	Selcuk University
309	Rsc. Assist.	Ali Çakmak	Ataturk University
310	Rsc. Assist.	Ali Emre Eysen	Hacettepe University
311	Rsc. Assist.	Ayşe Erdoğan Yıldırım	Firat University
312	Rsc. Assist.	Ayşe Tiryakioğlu	Gebze Institute Of Technology
313	Rsc. Assist.	Ayşe Yılmaz	Akdeniz University
314	Rsc. Assist.	Bengi Yıldız	Bilecik Seyh Edebali University
315	Rsc. Assist.	Beyhan Uzunoğlu	Ankara University
316	Rsc. Assist.	Bilge İnan	Inonu University
317	Rsc. Assist.	Burhan Selçuk	Karabuk University
318	Rsc. Assist.	Burhan Tiryakioğlu	Marmara University
319	Rsc. Assist.	Büşra Çiftçi	Middle East Technical University
320	Rsc. Assist.	Çağrı Ataseven	Dumlupınar University

321	Rsc. Assist.	Ekin Varlı	Middle East Technical University
322	Rsc. Assist.	Elçin Gökmen	Mugla Sitki Kocman University
323	Rsc. Assist.	Emel Aslankarayığit	Marmara University
324	Rsc. Assist.	Emre Taş	Ankara University
325	Rsc. Assist.	Faik Gürsoy	Yıldız Technical University
326	Rsc. Assist.	Fatma Gökçelik	Ankara University
327	Rsc. Assist.	Fatma Özen Erdoğan	Uludağ University
328	Rsc. Assist.	Fatma Peker	Erciyes University
329	Rsc. Assist.	Fatmanur Gürsoy	Yıldız Technical University
330	Rsc. Assist.	Ferdağ Kahraman Aksoyak	Erciyes University
331	Rsc. Assist.	Gökhan Çelebi	Anadolu University
332	Rsc. Assist.	Gözde Yaylalı	Mugla Sitki Kocman University
333	Rsc. Assist.	Gülden Altay	Firat University
334	Rsc. Assist.	Gülseli Burak	Pamukkale University
335	Rsc. Assist.	Gülşah Aydın	Suleyman Demirel University
336	Rsc. Assist.	Gültaç Eroğlu İnan	Ankara University
337	Rsc. Assist.	Hakan Kızmaz	Sakarya University
338	Rsc. Assist.	Hasan Aykut Karaboğa	Yıldız Technical University
339	Rsc. Assist.	Hatice Aslan	Firat University
340	Rsc. Assist.	Hatice Parlaticı	Sakarya University
341	Rsc. Assist.	Hidayet Hüda Kösal	Sakarya University
342	Rsc. Assist.	Hüseyin Albayrak	Suleyman Demirel University
343	Rsc. Assist.	İbrahim Karahan	Erzurum Technical University
344	Rsc. Assist.	İbrahim Özbek	Yıldız Technical University
345	Rsc. Assist.	İrem Eroğlu	Ankara University
346	Rsc. Assist.	İrem Küpeli Erken	Uludağ University
347	Rsc. Assist.	Kadir Emir	Osmangazi University
348	Rsc. Assist.	Kadirhan Polat	Ataturk University
349	Rsc. Assist.	Kadriye Aydemir	Gaziosmanpasa University
350	Rsc. Assist.	Kemal Taşkörü	Bilecik Seyh Edebali University
351	Rsc. Assist.	Koray Yılmaz	Dumlupinar University
352	Rsc. Assist.	Mehmet Fatih Karaaslan	Yıldız Technical University
353	Rsc. Assist.	Mehmet Gülbahar	Inonu University
354	Rsc. Assist.	Mehmet Ünver	Ankara University
355	Rsc. Assist.	Mehmet Yusuf Erdoğan	Ankara University
356	Rsc. Assist.	Melek Erdoğan	Necmettin Erbakan University
357	Rsc. Assist.	Murat Çağlar	Ataturk University
358	Rsc. Assist.	Mustafa Sarı	Yıldız Technical University
359	Rsc. Assist.	Müzeyyen Ertürk	Yıldız Technical University
360	Rsc. Assist.	Naime Tozlu	Nigde University
361	Rsc. Assist.	Nazan Çakmak	Mugla Sitki Kocman University
362	Rsc. Assist.	Nejla Gürefe	Gazi University
363	Rsc. Assist.	Neşe İşler Acar	Mehmet Akif Ersoy University
364	Rsc. Assist.	Nihat Akgüneş	Selcuk University
365	Rsc. Assist.	Nihat Yağmur	Erzincan University
366	Rsc. Assist.	Nilüfer Köse	Anadolu University
367	Rsc. Assist.	Nurten Bayrak Gürses	Yıldız Technical University
368	Rsc. Assist.	Onur Saldır	Yuzuncu Yil University
369	Rsc. Assist.	Önder Gökmen Yıldız	Bilecik Seyh Edebali University
370	Rsc. Assist.	Özlem Öksüzer	Ankara University
371	Rsc. Assist.	Pınar Balkı Okullu	Celal Bayar University
372	Rsc. Assist.	Rabia Çakan	Ataturk University
373	Rsc. Assist.	Ramazan Ekmekçi	Canakkale Onsekiz Mart University
374	Rsc. Assist.	Rukiye Öztürk	Ataturk University

375	Rsc. Assist.	Rushadije Ramani Halili	State University of Tetovo
376	Rsc. Assist.	S. Şule Şener	Ataturk University
377	Rsc. Assist.	Sait San	Osmangazi University
378	Rsc. Assist.	Sema Şimşek	Hacettepe University
379	Rsc. Assist.	Serkan Onar	Yildiz Technical University
380	Rsc. Assist.	Sibel Turanlı	Erzurum Technical University
381	Rsc. Assist.	Şehriban Uğuz	Yuzuncu Yil University
382	Rsc. Assist.	Şengül Ersoy	Izmir Institute of Technology
383	Rsc. Assist.	Şerife Müge Ege	Ege University
384	Rsc. Assist.	Şerifenur Cebesoy	Ankara University
385	Rsc. Assist.	Şirin Aktay	Anadolu University
386	Rsc. Assist.	Şükran Konca	Sakarya University
387	Rsc. Assist.	Temel Ermiş	Osmangazi University
388	Rsc. Assist.	Tuğba Han Dizman	Selcuk University
389	Rsc. Assist.	Tuğba Mert	Cumhuriyet University
390	Rsc. Assist.	Tuğba Yurdakadim	Ankara University
391	Rsc. Assist.	Tülay Soyfidan	Sakarya University
392	Rsc. Assist.	Uğur Sert	Hacettepe University
393	Rsc. Assist.	Yasemin Kaya	Middle East Technical University
394	Rsc. Assist.	Yunus Atalan	Yildiz Technical University
395	Rsc. Assist.	Zakir Deniz	Suleyman Demirel University
396	Rsc. Assist.	Zehra Bozkurt	Ankara University
397	Rsc. Assist.	Zehra Güzel Ergül	Ahi Evran University
398	Rsc. Assist.	Zekiye Çiloğlu	Suleyman Demirel University
399	Rsc. Assist.	Zeynep Hande Toyganözü	Suleyman Demirel University
400	Rsc. Assist.	Zübeyde Ulukök	Selçuk University
401	Teacher	Saadet Doğan	Huseyin Koluk Ticaret Meslek Lisesi
402	Student	Almas Temirbekov	Serikbayev EKSTU
403	Student	Attia Mostafa	University of Belgrade
404	Student	Aycan Sinanoğlu	Ege University
405	Student	Burcu Gürbüz	Celal Bayar University
406	Student	Burcu Hasançebi	Karadeniz Technical University
407	Student	Candice Louw	University Of Johannesburg
408	Student	Emre Deniz	Kirikkale University
409	Student	Farida Amenova	D.Serikbayev East Kazakhstan State Technical Uni.
410	Student	Fatma Tokmak	Ege University
411	Student	Gonca Durmaz	Kirikkale University
412	Student	Gülsüm Ulusoy	Kirikkale University
413	Student	Hasan Pişkin	Sakarya University
414	Student	Hülya Başeğmez	Ahi Evran University
415	Student	Işıl Arda Kösal	Sakarya University
416	Student	Kübra Özkan	Ataturk University
417	Student	Magira Kulbai	Al-Farabi Kazakh National University
418	Student	Mesut Altınok	Ahi Evran University
419	Student	Mikhail Kasparyan	Image Processing System Institute of RAS
420	Student	Özlem Acar	Kirikkale University
421	Student	Pavel Bogdanov	Image Processing System Institute of RAS
422	Student	Sabri Ali Ümekkan	Kocaeli University
423	Student	Seda İğret	Ataturk University
424	Student	Sema Gülbahar	Inonu University
425	Student	Şerife Yılmaz	Anadolu University
426	Student	Tuğba Sarışahin	Suleyman Demirel University
427	Student	Tuğba Şenlik	Ege University