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Analysis of Nonlinear Vibrations of a Cylindrical Shell in a Supersonic Gas Flow

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Abstract. In the paper nonlinear vibrations of a drill string's section in a supersonic gas flow are studied. The drill string is modelled in the form of a circular cylindrical shell under the effect of a longitudinal compressing load and torque. In contrast to the previous research, pressure of an unperturbed gas is defined nonlinearly in the third approximation. The eighth order partial differential equation describing the motion of the shell reduces to a nonlinear system of ordinary differential equations with application of the Bubnov-Galerkin technique. An implicit Runge-Kutta method is applied to construct modes of vibrations.

Introduction

Modelling of vibrations and analysis of stability of drill strings for ensuring the successful process of drilling the oil and gas wells are pressing problems of the modern world and have been in the special sight of scientists in the last few decades. Various dangerous modes to which drilling columns are exposed during the drilling works such as torsional stick-slip oscillations, high-amplitude lateral motions and longitudinal displacements may lead to serious technical failures of drilling equipment [1].

In many scientific works and papers devoted to research of well drilling process, a drill string is modelled as a one-dimensional rod or a system of rods in view of big length of the drill string, as, for instance, in [2-4]. However, analyzing the stability and vibrations it is also important to consider the drill string in the form of a cylindrical shell which is affected by diverse loadings and factors of the environment.

It is known when shells buckle, there occur both flexible stress and additional stress in the median surface [5]. This property qualitatively distinguishes the behaviour of shells at the loss of stability from the behaviour of rods and plates.

In [6,7] wide review of literature on stick-slip oscillations of drill strings and geometrically nonlinear vibrations of shells was carried out. V.I. Gulyaev et al. [8] studied critical states of cylindrical shells under the influence of longitudinal and centrifugal inertial forces as a result of rotation. It was shown that while these forces were interacting to each other, the dynamic loss or quasistatic loss of stability could be observed. Nonlinear parametric vibrations of a cylindrical shell undergone to the effect of the uniformly distributed compressing load were investigated in [9] by the use of the method of multiple scales.

Dynamics of nonlinear cylindrical shells in a supersonic gas flow was studied in the works of A.S. Volmir [10], E.A. Kurilov and Yu.V. Mikhlin [11], E.L. Jansen [12]. In these works initial imperfections of the shell were considered, and the aerodynamic pressure of the flow was determined using the linear piston theory. For discretization of the governing equations the Bubnov-Galerkin method was applied [11]. The stability of a circular cylindrical shell under the influence of an axial supersonic flow was studied by M. Amabili and F. Pellicano [13], and also by K.N. Karagiozis et al [14].

This work studies lateral vibrations of a cylindrical shell taking into account nonlinearity of a gas flow and external loadings. The numerical solution of a system of equations obtained by the multimode expansion of the buckling function is built up and the analysis of results is carried out.

Mathematical Model

As the governing equations describing the motion of a drill string in a supersonic flow of gas, let us consider a system of symmetrically constructed fourth order linear equations in terms of the function of buckling w and stress function Φ in the median surface:

$$\frac{D}{h}\nabla^4 w = \nabla_k^{\ 2}\Phi + \frac{q}{h} - \rho\frac{\partial^2 w}{\partial t^2},$$
$$\frac{1}{E}\nabla^4\Phi = -\nabla_k^{\ 2} w, \tag{1}$$

where $D = \dfrac{Eh^3}{12\left(1-\mu^2\right)}$ is the cylindrical rigidity, h the width of the shell's well, E Young's modulus, R the curvature radius of the shell's median surface, μ Poisson's ratio, ∇^4 the biharmonic operator, ρ the material density, q the intensity of the distributed loading taking into account the longitudinal force, torque and aerodynamic pressure of a supersonic flow, $\nabla_k^{\ 2}$ is defined by the following operator

$$\nabla_k^{\ 2} = k_y\frac{\partial^2}{\partial x^2} + k_x\frac{\partial^2}{\partial y^2}. \tag{2}$$

Apply operator ∇^4 to the first equation in (1), and operator $\nabla_k^{\ 2}$ to the second equation. Then in the case of a circular cylindrical shell ($k_x = 0$, $k_y = 1/R$) the following eighth order governing equation is obtained:

$$\frac{D}{h}\nabla^8 w + \frac{E}{R^2}\frac{\partial^4 w}{\partial x^4} - \nabla^4\frac{q}{h} + \nabla^4\rho\frac{\partial^2 w}{\partial t^2} = 0. \tag{3}$$

Intensity of the distributed loading q in the shell is given as follows

$$q = -h\left(P\frac{\partial^2 w}{\partial x^2} + 2S\frac{\partial^2 w}{\partial x\partial y}\right) - \Delta p, \tag{4}$$

where P is the compressing load, $S = \dfrac{M}{2\pi R^2 h}$ is the tangential load from the twisting moment M, Δp is the pressure of the supersonic gas flow.

Depending on the number of terms retained in the expansion of the gas flow pressure, linear and nonlinear ratios joining pressure and speed of the flow can be obtained. For nonlinear dependences in the third approximation we have the following:

$$\Delta p = -A\frac{\partial w}{\partial x} + \frac{(\kappa+1)h}{\kappa p_\infty}A^2\left(\frac{\partial w}{\partial x}\right)^2 - \frac{(\kappa+1)h^2}{12\kappa^2 p_\infty^{\ 2}}A^3\left(\frac{\partial w}{\partial x}\right)^3, \tag{5}$$

where P_∞ is pressure of the unperturbed gas, $\overline{M}$ is the Mach number, $A = \dfrac{\overline{M}\kappa P_\infty}{h}$ is the reduced Mach number, κ is the polytropic exponent.

Aerodynamic nonlinear dependence between normal pressure and speed of the gas flow brings nonlinearity in Eq. (3). Comparing to [15], in this paper the correction to the expression for pressure (5) is made.

Kirchhoff-Love hypotheses [16] underlying the model considered allow to reduce a three-dimensional strain problem to two-dimensional one. Research of the section of a shell, in turn, reduces to investigation of the behaviour of its median surface.

The boundary conditions for the shell with pinned ends are written in the form

$$w=0,\quad \frac{\partial^2 w}{\partial x^2}=0 \qquad \left(x=0,\, x=l\right), \tag{6}$$

allowing for buckling does not depend on coordinate y along the ends considered.

Discretization of the Nonlinear Differential Equation

As conducting the research of the vibrations basing on the partial differential equations is extremely difficult, we will reduce Eq. (3) to a system of ordinary differential equations. To achieve that, we will apply the Bubnov-Galerkin method. The approximation function for buckling $w(x,y,t)$, which must meet all the boundary conditions (6), is specified as follows:

$$w\left(x,y,t\right)=\sum_{i=1}^{m}\sum_{j=1}^{n} f_{ij}\sin\frac{i\pi x}{l}\sin\frac{jy}{R}, \tag{7}$$

where m is the number of half-waves along the generatrix, n is the number of waves along the circle.

To correctly describe the nature of undulation in the median surface of the shell, it is enough to retain the first three terms of the series in terms of x in the expression (7):

$$w\left(x,y,t\right)=f_{11}\left(t\right)\sin\frac{\pi x}{l}\sin\frac{y}{R}+f_{21}\left(t\right)\sin\frac{2\pi x}{l}\sin\frac{y}{R}+f_{31}\left(t\right)\sin\frac{3\pi x}{l}\sin\frac{y}{R}. \tag{8}$$

On applying the Bubnov-Galerkin method, we obtain the following ordinary differential equations in terms of functions $f_{ij}\left(t\right),\, i=\overline{1,3};\; j=1$:

$$\begin{aligned}
&f_{11}''+\omega_1^2 f_{11}+\alpha_1 f_{21}+\alpha_2 f_{11}f_{21}f_{31}+\alpha_3 f_{11}^2 f_{21}+\alpha_4 f_{21}f_{31}^2+\alpha_5 f_{21}^3=0,\\
&f_{21}''+\beta_1 f_{11}+\omega_2^2 f_{21}+\beta_2 f_{31}+\beta_3 f_{11}f_{21}^2+\beta_4 f_{11}f_{31}^2+\beta_5 f_{11}^2 f_{31}+\beta_6 f_{21}^2 f_{31}+\beta_7 f_{11}^3+\beta_8 f_{31}^3=0,\\
&f_{31}''+\gamma_1 f_{21}+\omega_3^2 f_{31}+\gamma_2 f_{11}f_{21}f_{31}+\gamma_3 f_{11}^2 f_{21}+\gamma_4 f_{21}f_{31}^2+\gamma_5 f_{21}^3=0,
\end{aligned} \tag{9}$$

where $\omega_1^2=\dfrac{\alpha}{\delta_1}$, $\omega_2^2=\dfrac{\beta}{\delta_2}$, $\omega_3^2=\dfrac{\gamma}{\delta_3}$, $\alpha_i=\dfrac{\tilde{\alpha}_i}{\delta_1}$, $\gamma_i=\dfrac{\tilde{\gamma}_i}{\delta_2}$, $i=\overline{1,5}$, $\beta_j=\dfrac{\tilde{\beta}_j}{\delta_3}$, $j=\overline{1,8}$;

$$\delta_1=\frac{\pi\left(l^2+\pi^2R^2\right)^2\rho}{2l^3R^3},\quad \delta_2=\frac{\pi\left(l^2+4\pi^2R^2\right)^2\rho}{2l^3R^3},\quad \delta_3=\frac{\pi\left(l^2+9\pi^2R^2\right)^2\rho}{2l^3R^3},$$

$\alpha, \beta, \gamma, \alpha_i, \beta_j, \gamma_i$ are not provided in this paper in view of their bulkiness.

Numerical Results

In order to obtain the numerical solution to the problem on lateral vibrations of the circular cylindrical shell in a supersonic gas flow, we apply an implicit Runge-Kutta method. This method is known to be efficient and imposes no restrictions on time step compared to explicit schemes.

All calculations were carried out at the following values of parameters of the system:

$h = 0.01m$, $R/h = 19$, $M = 10^4 Hm$, $P = 2.2\times10^6 H$, $P_\infty = 1.013\times10^6 \frac{H}{m^2}$, $\kappa = 1.4$, $\bar{M} = 3$,

$E_{st} = 2.1\times10^5 MPa$, $\rho_{st} = 7800\frac{kg}{m^3}$, $E_{dur} = 0.7\times10^5 MPa$, $\rho_{dur} = 2700\frac{kg}{m^3}$, $\mu_{st} = 0.28$, $\mu_{dur} = 0.34$.

For a shell of length $l = 15\ m$ we built the graph demonstrating a contribution of each mode into the nature of undulation. As supposed, the first mode has the greatest influence on the amplitude of shell vibrations (Fig. 1). The second and third forms of vibrations are presented in Fig. 2. To compare the vibrations of drill string's sections made of steel and duralumin, dependences in the first approximation were constructed (Fig. 3).

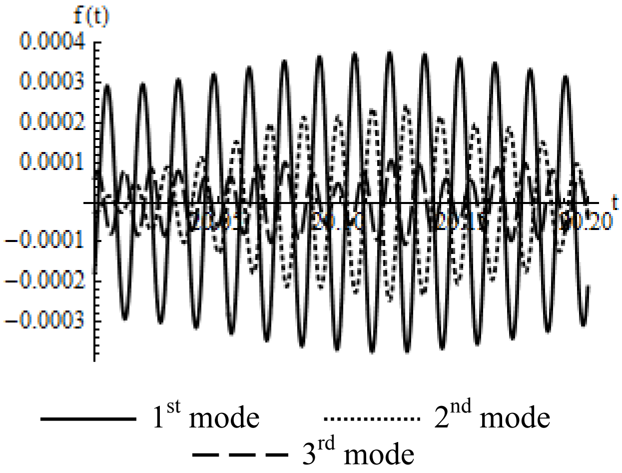

Fig. 1 – Vibration modes of a shell of length $l = 15\ m$

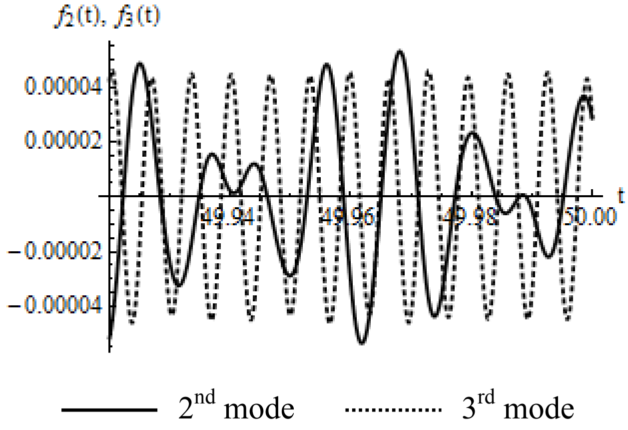

Fig. 2 – Second and third vibration modes of a shell of length $l = 10\ m$

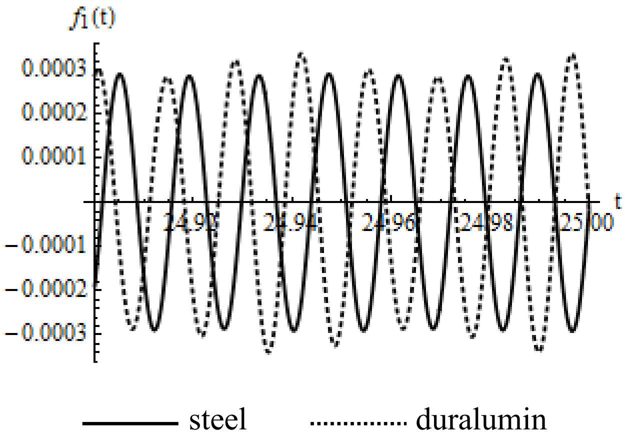

Fig. 3 – Vibrations of shells made of different materials, $l = 10\ m$

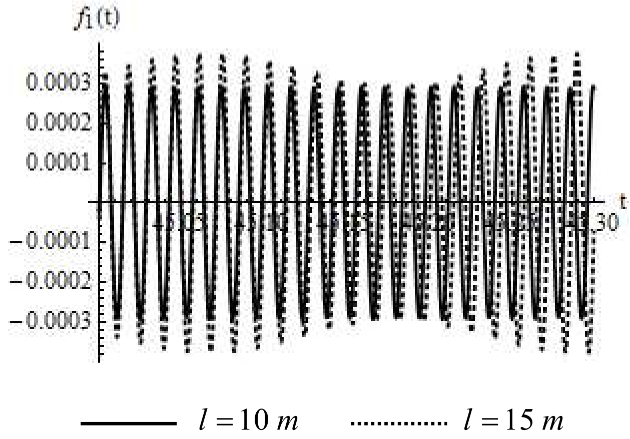

Fig. 4 – Vibrations of shells of various lengths

As regards the shells of various lengths, if the shell becomes longer we observe a small increase in the amplitude of vibrations, whereas the beat frequency decreases (Fig.4).

If the length of a shell is much bigger than its diameter, then the boundary conditions have no considerable impact on the magnitude of critical pressure and the form of the loss of stability [17].

Summary

In this work we studied nonlinear lateral vibrations of a cylindrical shell in a supersonic gas flow. On the basis of multimode expansion of the buckling function the system of nonlinear ordinary differential equations was obtained. The numerical solution was carried out by an implicit Runge-Kutta method. We analyzed the influence of shell's length, pressure of a gas flow, material properties and compressing load on the magnitudes and modes of shell vibrations. As a result, it was found out that the second and third modes substantially affect the nature of undulation in the median surface of a shell.

In future works, investigating vibrations of a drill string, a circular cylindrical shell with initial imperfections of the form of the median surface in the nonlinear gas flow will be considered.

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