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# PROCEEDINGS

# Supersonic Flow with Perpendicular Injection of a Hydrogen

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*Abstract*–**A computational fluid dynamics code for multispecies, Favre-Averaged Naveir-Stokes equations is developed to simulate the turbulent supersonic two-dimensional reacting flow. The explicit ENO scheme (third order of accuracy) has been used to solve the system of equations, and algebraic Baldwin-Lomax's turbulence model to calculate the eddy viscosity coefficient. For the description of processes of reaction of hydrogen the seven chemical reactions Spurk's mechanism is adopted. Computer code was validated by the solution of the two-dimensional chemically reacting supersonic hydrogen-air flow, where the main air flow was entering in the planar channel and hydrogen is injected perpendicularly from the slots of the walls. Influence of boundary conditions for the temperature on the walls on turbulent mixture is investigated.**

***Keywords - supersonic flow, hydrogen combustion, ENO-scheme, seven chemical reactions mechanism***

## I. INTRODUCTION

The layer of mixing and combustion length is an important factor in the design of supersonic ramjet (scramjet) engines. The flow field in such devices is very complex. It is known that they are determined by processes of the turbulent fuel-air mixing, chemical reaction, shock waves, separation region ahead of the jet and behind of it. Consequently, the studying of the turbulent reactive flows is of great importance in supersonic combustion. Some of questions of the supersonic flows problem with transverse injection of jets with reactions and without it have been considered in [1-3]. The angle of inclination of the shock waves and separation-region length were found as function of the jet's pressure ratio. The enhancement of mixing have been investigated in [4] by using TVD scheme. Thus, developed computational fluid dynamic code for solution Navier-Stokes equations for supersonic multi-species flow is important task.

The two-dimensional supersonic turbulent flow of multi-component gas, in the presence of cross injection hydrogen from the bottom and top walls of the channel is considered. For convenience of calculation we consider injection stream only from the bottom wall, as show in fig.1.

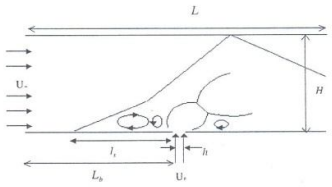


Fig. 1. The flow scheme

## II. MATHEMATICAL MODEL

The two-dimensional Favre-averaged Navier-Stokes equations for multi-species flow with chemical reactions is:

$$\frac{\partial \vec{U}}{\partial t}+\frac{\partial\left(\vec{E}-\vec{E}_v\right)}{\partial x}+\frac{\partial\left(\vec{F}-\vec{F}_v\right)}{\partial z}=\vec{W} \qquad (1)$$

here the vector of the dependent variables and the vector fluxes is given as

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$$\bar{U}=\begin{pmatrix}\rho\\ \rho u\\ \rho w\\ E_t\\ \rho Y_k\end{pmatrix},\ \bar{E}=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uw\\ (E_t+p)u\\ \rho uY_k\end{pmatrix},\ \bar{F}=\begin{pmatrix}\rho w\\ \rho uw\\ \rho w^2+p\\ (E_t+p)w\\ \rho wY_k\end{pmatrix},$$

$$\bar{E}_v=\begin{pmatrix}0\\ \tau_{xx}\\ \tau_{xz}\\ u\tau_{xx}+u\tau_{xz}-q_x\\ J_{kx}\end{pmatrix},\ \bar{F}_v=\begin{pmatrix}0\\ \tau_{xz}\\ \tau_{zz}\\ w\tau_{xz}+w\tau_{zz}-q_z\\ J_{kz}\end{pmatrix},$$

$$\bar{W}=\begin{pmatrix}0\\ 0\\ 0\\ 0\\ W_k\cdot\dot{\omega}_k\end{pmatrix}$$

the viscous stresses and thermal conduction and diffusion of species are:

$$\tau_{xx}=\frac{\mu}{Re}\left(2u_x-\frac{2}{3}(u_x+w_z)\right),$$

$$\tau_{zz}=\frac{\mu}{\mathrm{Re}}\left(2w_z-\frac{2}{3}(u_x+w_z)\right),$$

$$\tau_{xz}=\tau_{zx}=\frac{\mu}{Re}(u_z+w_x),$$

$$q_x=\left(\frac{\mu}{\mathrm{Pr}\,\mathrm{Re}}\right)\frac{\partial T}{\partial x}+\frac{1}{\gamma_\infty M_\infty^{\,2}}\sum_{k=1}^{N}h_kJ_{xk},$$

$$q_z=\left(\frac{\mu}{PrRe}\right)\frac{\partial T}{\partial z}+\frac{1}{\gamma_\infty M_\infty^{\,2}}\sum_{k=1}^{N}h_kJ_{zk},$$

$$J_{kx}=-\frac{\mu}{Sc\,\mathrm{Re}}\frac{\partial Y_k}{\partial x},\ J_{kz}=-\frac{\mu}{Sc\,\mathrm{Re}}\frac{\partial Y_k}{\partial z}.$$

here $Y_k$ is the mass fraction of $k$-th species, $\dot{\omega}_k$ - rate of mass production of species, $k=1...N$, where $N$ - number a component of a mix of gases.

The thermal equation for multi-species gas is:

$$p=\frac{\rho T}{\gamma_\infty M_\infty^{\,2}W},\ W=\left(\sum_{k=1}^{N}\frac{Y_k}{W_k}\right)^{-1},\ \sum_{k=1}^{N}Y_k=1 \qquad (2)$$

where $W_k$ - is the molecular weight of the species.

The equation for a total energy

$$E_t=\frac{\rho h}{\gamma_\infty M_\infty^{\,2}}-p+\frac{1}{2}\rho\left(u^2+w^2\right) \qquad (3)$$

The enthalpy of the gas mixture is calculated according to $h=\sum_{k=1}^{N}Y_kh_k$, $h_k=h_k^0+\int_{T_0}^{T}c_{pk}dT$ - specific enthalpy of $k$-th species.

Specific heat at constant pressure for each component $c_{pk}$ is:

$$c_{pk}=C_{pk}/W,\quad C_{pk}=\sum_{i=1}^{5}\bar{a}_{ki}T^{(i-1)},$$

$$\bar{a}_{pk}=a_{pk}T_\infty^{\,i-1}$$

molar specific heat $C_{pk}$ is given in terms of the fourth degree polynomial with respect to temperature in the JANAF Thermochemical Tables [5].

The system of the equations (1) is written in the conservative, dimensionless form. The flow parameters are $\rho_\infty, u_\infty, w_\infty, T_\infty, h_\infty, W_\infty, R_\infty$, jet parameters are $\rho_0, u_0, w_0, T_0, h_0, W_0, R_0$. The governing parameters are the entrance parameters, the pressure and total energy are normalized to $\rho_\infty u_\infty^2$, the enthalpy - to $R_0T_\infty/W_\infty$, the molar specific heat - to $R_0$ and the considered lengths – to the slot width $l$.

The coefficient of viscosity is represented in the form of the sum $\mu_l$ - molecular viscosity and $\mu_t$ - turbulent viscosity: $\mu=\mu_l+\mu_t$, where $\mu_t$ is defined according to Baldwin-Lomax's algebraic model. Molecular viscosity of a mix is from Wilke's formula.

The chemical reactions of hydrogen ($H_2$) with air are described on the basis of the Spurk's chemical model [5]:

$$H_2+O_2\rightarrow OH+OH$$
$$H+O_2\rightarrow OH+O$$
$$OH+H_2\rightarrow H_2O+H$$
$$O+H_2\rightarrow OH+H$$
$$OH+OH\rightarrow H_2O+O$$
$$H+OH+M\rightarrow H_2O+M$$

$$H + H + M \rightarrow H_2 + M$$

Thus, the model of a chemically reacting gas, which is used in the present work, involves seven components: $(H_2, O_2, H_2O, OH, H, O, N_2)$.

The boundary conditions have the following form:
at the entrance

$$W_k = W_{k\infty},\ p = p_\infty,\ T = T_\infty,\ u = M_\infty\sqrt{\frac{\gamma_\infty R_0 T_\infty}{W_\infty}},$$

$$w = 0,\ Y_k = Y_{k\infty},\qquad x = 0,\ 0 \le z \le H;$$

the boundary layer on the wall is given in the input section, and the profile of velocity approximated by power law;
on the slot:

$$W_k = W_{k0},\quad p = np_\infty,\quad T = T_0,\ w = M_0\sqrt{\frac{\gamma_0 R_0 T_0}{W_0}},$$

$$u = 0,\ Y_k = Y_{k0},\qquad z = 0,\quad L_b \le x \le L_b + h;$$

( $n = p_0 / p_\infty$ is the jet pressure ratio, $p_0$ is the jet pressure, and $p_\infty$ is the flow pressure); on the lower wall the no-slip condition, for the temperature the adiabatic and constant wall condition are imposed; on the upper boundary the condition of symmetry is assumed; on the outflow the nonreflecting boundary condition [1] is used.

## III. METHOD OF THE SOLUTIONS

The numerical solution of the system (1) is calculated by the two steps. The first is defined dynamic parameters and second mass species. The approximation of convection terms is performed on the ENO-scheme of third order accuracy [1,6-7]. The ENO scheme is constructed on the basis of Godunov method, where piecewise polynomial function is defined by the Newton's formula of the third degree. In approximation of derivatives in diffusion terms were used second-order central-difference operators. The system of the finite difference equations were solved by the Beam-Warming method with using matrix sweep. Then it is necessary to define Jacobian matrix which in a case of the thermally perfect gas represents difficult task. This problem is connected by explicit representation of pressure through the unknown parameters. Here pressure is determined by introducing an effective adiabatic parameter of the gas mixture.

$$\bar{\gamma} = \frac{h_{sm}}{e_{sm}}, \qquad (4)$$

where $h_{sm} = \sum_{i=1}^{N} Y_i \int_{T_0}^{T} c_{p_i} dT$ , $e_{sm} = \sum_{i=1}^{N} Y_i \int_{T_0}^{T} c_{v_i} dT$ - enthalpy and internal energy of the mixture minus the heat and energy of formation; $T_0 = 293K$ - standard temperature of formation, which allows to write an expression for the pressure

$$p = (\bar{\gamma} - 1)\left[E_t - \frac{1}{2}\rho(u^2 + w^2) - \rho\frac{h_0}{\gamma_\infty M_\infty^2}\right] + \frac{\rho T_0}{M_\infty^2 W}$$

The equations for components are solved by the scalar sweep, where on the first step were included convection and diffusion terms and it was calculated using ENO scheme. In the second step the matrix equation with term ($\dot{W}_k = W_k\dot{\omega}_k$) was solved implicitly. For that the source terms $\dot{W}_k$ was linearized by expansion in a Taylor series.

$$\dot{W}_k^{n+1} = \dot{W}_k^n + \gamma\left(\frac{\partial \dot{W}_k}{\partial Y_m}\Delta Y_m + \frac{\partial \dot{W}_k}{\partial T}\Delta T + \frac{\partial \dot{W}_k}{\partial \rho}\Delta\rho\right)$$

The temperature is found by using the Newton-Raphson iteration from the equations [1].

## IV. NUMERICAL RESULTS

The computation was performed on and 241x181 space grid with respect to spatial coordinates. The first was considered the perpendicular injection of a gas into two-dimensional channel without chemical reactions. The parameters of jet and air flow was adopted as [1]: $M_0 = 1, T_0 = 217K$ , $P_0 = 1.24MPa$ width of slot 0.0559 cm; $M_\infty = 2.9, T_\infty = 108K$ , $P_\infty = 0.0663MPa$ . The channel height and length were 7.62 and 15 cm, respectively. The slot was located at a distance of 10 cm from the input section. The results of the numerical experiment were in satisfactory agreement with experimental data and numerical calculations of authors [1,8-9].

In the next, for testing code the perpendicular injection of jet of hydrogen into planar air flow was numerically calculated. Here, the width of slot is 0,0559 cm, jet parameters are $M_0 = 1{,}1$, $T_0 = 400$ K, $P_0 = 1$ MPa and air $M_\infty = 3$, $T_\infty = 1300$ K, $P_\infty = 0{,}25$ MPa, the boundary condition on the wall was adopted constant. The height of the channel is 7,62 cm, and length - 15 cm. The slot was location on distance of 10 cm from entrance. The formation of the product of the reaction is shown in the

fig. 2. In the fig.2 are resulted isolines of species of hydrogen ( $H_2$ ), oxygen ( $O_2$ ), water vapor ( $H_2O$ ), hydroxyl radical ( $OH$ ). It is visible combustion in the zone of mixture of the hydrogen and flow of air. Numerical experiment showed the zone of combustion is concentrated near wall for adiabatic boundary. Thus the constructed computer code for turbulent supersonic reacting flow allows to study influence parameters to the mixture and combustion.

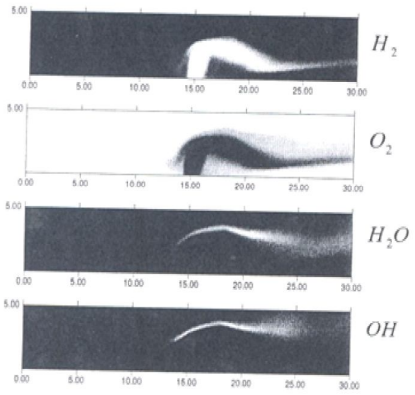


Fig. 2. – Concentration isolines of hydrogen, oxygen, water vapor, hydroxyl radical

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