A rigorous solution to the problem of an open resonator consisting of a coaxial section of a pipe located inside the infinite waveguide: the Wiener - Hopf method

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The Wiener-Hopf (W-H) method [1–6] is used to consider the resonant case of diffraction of an H_{0n}-wave incident on a segment of an ideally conducting cylinder coaxially located inside an infinite waveguide of radius a. The boundary value problem is first reduced to the paired integral W-H equations and then to a system of linear algebraic equations. The parameters of eigenoscillations of an open cylindrical resonator are determined from the condition that the determinant of the system of algebraic equations is equal to zero.

To simplify the problem, a single-mode resonant mode of a pipe section is considered. In this case, the resonance condition is

$$\frac{L_+(a_1,h)}{L_-^*(a_1,h)}\frac{e^{ihl}}{2h} = \pm 1, \tag{1}$$

where

$$L(a_1,h) = \frac{\mathrm{J}_1(va_1)(a_1',a')}{\mathrm{J}_1(va)},\ (a_1',a') = \mathrm{J}_1(va_1)\mathrm{N}_1(va) - \mathrm{J}_1(va)\mathrm{N}_1(va_1),$$
$$L = L_+L_-,\ L_-^*(a_1,h) = \lim_{w\to h}(w-h)^{-1}L_-(a_1,w), \tag{2}$$

h is a single real zero of the Bessel function $\mathrm{J}_1(va_1)$ on the complex plane $w = \sqrt{k^2 - v^2}$, l is the length of the cylinder of radius a_1. We assume for convenience that the zeros of the function (a_1', a') on the complex plane w are purely imaginary.

A rigorous solution allows us to consider modes in cases where spatial modes in a coaxial line are non-propagating or when resonance occurs in the outer region of a pipe section.

The W-H method can also be applied to TM wave types in solving similar resonance problems. It should be noted that the W-H method can easily be extended to external problems of electrodynamics, as the core of the corresponding integral equations is the product of the Bessel function and the Hankel function, the factorization of which is well known. However, one can be guided by the fact that the solution of the internal problem tends to the solution of the external problem when the radius of the external waveguide can be considered large enough.

The solution to such resonance problems can also have practical application, for example, in the accelerator technology of charged particles, as well as in creation of eddy currents in plasma clots, nanotubes and in other cases.

References

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