Radiation of a vertical Hertzian dipole above a lossless medium with flat interface : calculation of the transmitted Electromagnetic (EM) field by using a Geometrical Optics (GO) novel image theory approach

S. Sautbekov[1] , P. Frangos[2] , F. Karagiannopoulos[2] , B. Massinas[2] and N. Moraitis[2]

[1]*Department of Physics and Technology, Al-Farabi Kazakh National University*
Almaty, Kazakshtan
e-mail : sautbek@mail.ru

[2]*School of Electrical and Computing Engineering, National Technical University of Athens,*
9, Iroon Polytechniou Str., 157 73 Zografou, Athens, Greece
Tel. : +30 210 772 3694;   e-mail : pfrangos@central.ntua.gr

Abstract

In this paper we examine the problem of the radiation of a vertical Hertzian dipole over a lossless medium with flat interface in high frequency regime. Here the goal is to calculate the electromagnetic (EM) field value below the flat interface. For this purpose, a novel Geometrical Optics (GO) method is adopted here, and in this paper the first step to the problem solution is presented by calculating the position of the virtual image for given coordinates of the observation point below the flat interface. In order to accomplish the above, Snell's law and simple trigonometric relations for the given geometry of the problem are used.

1. Introduction

The problem of electromagnetic (EM) wave propagation over the flat terrain (or over a lossy medium with flat interface) is well – known in the literature as the 'Sommerfeld antenna radiation problem' [1-23]. Even in more cases the interest may be for observation points over the flat interface [1-14], also the problem solution below the flat interface can be formulated [see e.g. 15-22], and this is performed in an exact EM wave formulation [15-22]. On the contrast, in this paper we examine the problem of calculating the EM field value for an observation point below the flat interface ($z<0$), where we assume that the medium at the region $z<0$ (see Fig. 1) is a *lossless* and non-magnetic medium, that is ε_{r2} (relative dielectric constant) o medium 2 is real ($\varepsilon_{r2} > 1$) and $\mu_{r2} = 1$ (μ_{r2} is the relative magnetic constant of medium 2, below the interface). The source of EM radiation in this paper (see also Fig. 1) is assumed to be a vertical Hertzian dipole (i.e. dipole of length much smaller from the wavelength of EM radiation, $2l \ll \lambda$).

Furthermore, here we assume that the frequency of EM radiation from the vertical Hertzian dipole is sufficiently high, so that in this paper we can use a 'Geometrical Optics' (GO) approach, i.e. a 'ray optics – high frequency' approximation method.

Finally, in this paper we will just concentrate on the calculation of the exact position of the 'virtual image' for an observation point B lying in the region $z<0$ (see Fig. 1). Once the location of the image point C has been specified, then the value of EM field at the observation point B below the interface (Fig. 1) can be easily calculated by standard image theory approached (see e.g. [23]).

2. Problem Geometry and Snell's law of refraction

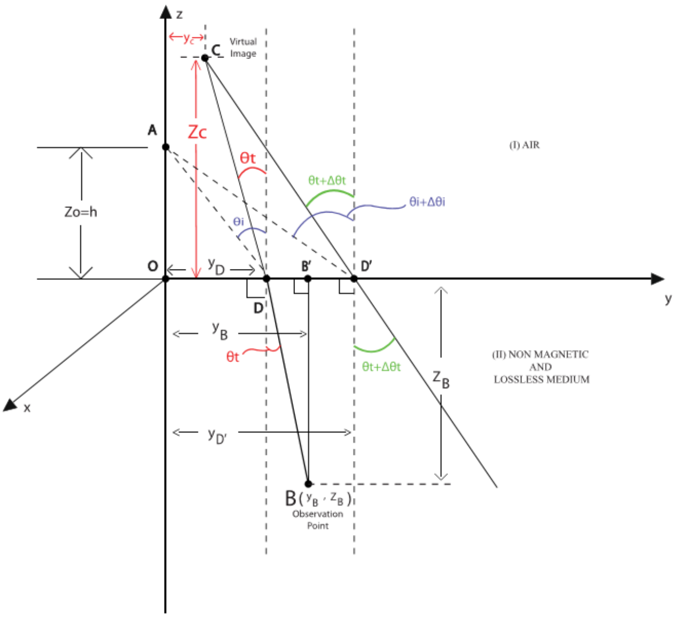

Figure 1. Geometry of the problem.

The geometry of the problem is shown in Fig. 1, above, where we examine here only the two – dimensional (2D) case (i.e. calculations only on the yz-plane, or $x=0$ plane / one can easily extend our proposed method to the three – dimensional (3D) space). The radiating vertical Hertzian dipole is at height $z_0 = h$ above the lossless dielectric medium with flat interface [which lies in the region $z<0$, i.e. lossless and non – magnetic medium for $z<0$, that is ε_{r2} =

dielectric constant = real ($\varepsilon_{r2} > 1$), $\mu_{r2} = 1$, index of refraction $n_2 = (\varepsilon_{r2})^{1/2} > 1$].

Considering a particular point of refraction D (see Fig. 1) the well – known *Snell's law for refraction* holds (see e.g. [23]) :

$$sin\theta_t = \frac{n_1}{n_2} \cdot sin\theta_i = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \cdot sin\theta_i \qquad (1)$$

Furthermore, let us consider a second incident ray AD' at a point D' close to point D , that is : DD' << OD . Then, we obtain a second refracted ray D'F (besides the first one, which is DBE ray), which intersects the first ray at point C (which is the location of the '*virtual image*'. For this second ray, also Snell's law for refraction holds, that is :

$$sin(\theta_t + \Delta\theta_t) = \frac{n_1}{n_2} \cdot sin(\theta_i + \Delta\theta_i) = \\ = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \; sin(\theta_i + \Delta\theta_i) \qquad (2)$$

Next, the geometric problem that we have to solve is as following :

Step 1 : Given the coordinates (y_B, z_B) of the observation point B, we will calculate the coordinate y_D for given coordinates of the source ($y = 0$, $z = z_0 = h$) and of the observation point B. This task will be presented at Section 3, below (see Fig. 1, and in particular Fig. 2).

Step 2 : Once the coordinate y_D of the refracting point D is calculated (Section 3), the coordinates (y_C, z_C) of the '*virtual image*' C can be calculated by simple geometry (see Section 4 / that is see Fig. 1, and, in particular, Fig. 3).

3. Calculation of position of the unique refracting point, given the coordinates of the observation point.

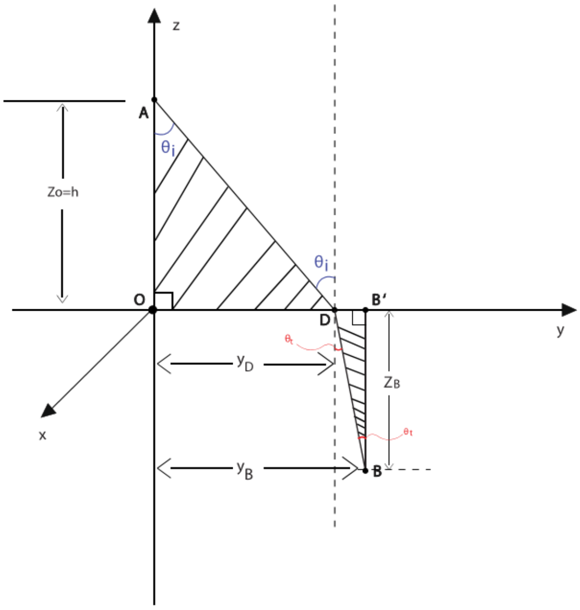

Figure 2. Problem geometry with coordinates of the observation point (B).

From simple *geometrical considerations* of Fig. 2, and working along the horizontal (Oy – axis) we have :

$$DB' = \; y_B - h \cdot \tan\theta_i = \; z_B \tan\theta_t \qquad (3)$$

where $y_D = h \tan\theta_i$. Therefore, from eq. (3):

$$\tan\theta_t \; = \frac{y_B - h \cdot \tan\theta_i}{z_B} \qquad (4)$$

which is a function between the unknown quantities θ_t and θ_i (quantities h, y_B and z_B are considered known, here). Moreover, we repeat here (just for our convenience) *Snell's law for refraction*, eq. (1) :

$$\sin\theta_t \; = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \cdot sin\theta_i \qquad (1)$$

Furthermore, since $\tan\theta_t$ appears at eq. (4), we can easily transform eq. (1) to a form, where $\tan\theta_t$ (instead of $\sin\theta_t$) appears (this can be very easily performed, by using elementary trignomteric calculus). Then, in this way, *Snell's law for refraction*, instead of eq. (1), takes the form :

$$B(\theta_i) = tan^2(\theta_t) = = \frac{1}{1 - \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \cdot sin^2(\theta_i)} - 1 \quad (5)$$

while, by just squaring eq. (4), we obtain :

$$C(\theta_i) = tan^2\theta_t = (\frac{y_B - h \cdot \tan\theta_i}{z_B})^2 \quad (6)$$

Note here that C(θi), eq. (6), is a *monotonic* function, *decreasing* with θ_i . where DB'= $=y_B - y_D = y_B - h \cdot \tan\theta_i > 0$, then $\tan\theta_i <$ $<(y_B / h)$, and where $0 \leq \theta_i \leq \pi/2$, while $B(\theta_i)$, eq. (5), is a *monotonic* function, *increasing* with θ_i $(0 \leq \theta_i \leq \pi/2)$. Then, by using, e.g. MATLAB, we can easily calculate the angle θ_i (Fig. 2) for which $B(\theta_i) = C(\theta_i)$, yielding the solution $\theta_i = \theta_i^0$. Once θ_i^0 has been calculated (numerically, as described just above), one can easily calculate, successively, the following quantities :

$$y_D = h \cdot \tan\theta_i^0 \quad (7)$$

$$\sin\theta_t^0 = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \cdot sin\theta_i^0 \quad (8)$$

(which is Snell's law, once again), and

$$DB' = y_B - y_D \quad (9)$$

[see Fig. 2 and eq. (3)].

4. Calculation of the coordinates of the image point.

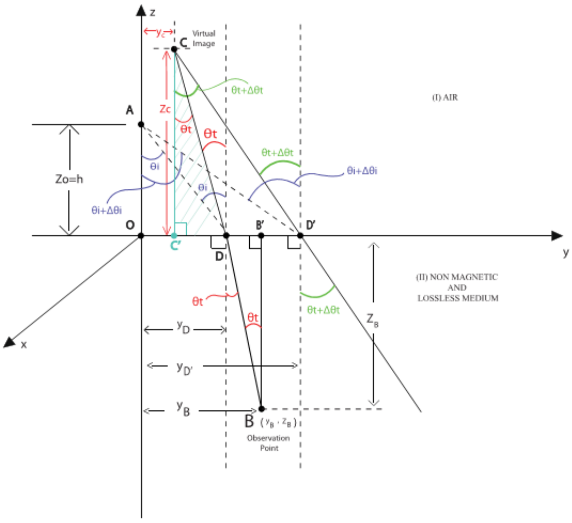

Figure 3. Geometry for the calculation of the image point coordinates (C)

Finally, given the calculation of the quantities mentioned at Section 3, above, *the calculation of the coordinates* (y_C, z_C) *of the 'virtual image' C* can be easily performed by simple trigonometric calculations on triangle CC'D (1st refracted ray of Fig. 3) and triangle CC'D' (2nd refracted ray of Fig. 3), that is by working (simple trigonometric calculations) in half – space z>0. Then, it can be easily proved that the coordinates (y_C, z_C) of the 'virtual image' C are provided by the following equations :

$$y_C = y_D - \frac{\sin(2\theta_t^0)}{2} \cdot \frac{\Delta y_{2rays}}{\Delta\theta_t^0} \quad (10)$$

and

$$z_C = \frac{\Delta y_{2rays}}{\Delta\theta_t^0} \cdot cos^2\theta_t^0 \quad (11)$$

where

$$\frac{\Delta y_{2rays}}{\Delta\theta_t^0} \approx CD \approx CD' \quad (12)$$

where Δy_{2rays} = DD' = chosen small quantity (known, DD' << OD), θ_t^0 has been calculated from eq. (8) and $\Delta\theta_t^0$ is θ of the 2nd ray, rewritten here for our convenience :

$$\sin(\theta_t^0 + \Delta\theta_t^0) = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}} \cdot \sin(\theta_i^0 + \Delta\theta_i^0) \quad (13)$$

and where angle $(\theta_i^0 + \Delta\,\theta_i^0)$ is calculated by (see Fig. 3) :

$$\theta_i^0 + \Delta\theta_i^0 = tan^{-1}\left(\frac{y_D + \Delta y_{2rays}}{z_0}\right) = known \quad (14)$$

Then, from eq. (13), $\Delta\theta_i^0$ is also known.

Finally, note that because of eq. (12) and CD ≈ CD' [for small DD' , as explained below eq. (11), above], the *coordinates (y_C, z_C) are 'almost independent' of the chosen length DD'= =Δy_{2rays} , provided DD' << OD* , as explained above.

5. Numerical example : calculation of the position of the refracting point (D) and of the 'virtual image' (C) for given source and observation point coordinates.

Referring to Fig. 2, in this numerical example we choose : $z_0 = h = 10$ m (position of the radiating Hertzian dipole, i.e. of the source of EM radiation), (y_B = 10 m, z_B = = -5 m, hence $|z_B|$ = 5 m), i.e. the coordinates of the observation point, and ε_{r2} = 1.6. Then, by calculating at MATLAB eqs. (5) and (6), i.e. the quantity $B(\theta_i)$ [Snell's law for refraction] and quantity $C(\theta_i)$ [equation coming from the geometry of Fig. 2], and setting

$$B(\theta_i) = C(\theta_i) \quad (15)$$

we find that eq. (15) holds for $\theta_i = 36.32^0$ [once again, we emphasize here that we look here for angles θ_i in the interval (0^0, 90^0) for which $C(\theta_i)$ is a *decreasing function* of θ_i , see eq. (6) and remarks below that].
Then, from eq. (7) we find : y_D = OD = 7.35 m (see Fig. 2), and from eq. (9) : DB' = OB' – OD = $y_B - y_D$ = 2.65 m.
Furthermore, by choosing (see Fig. 3) : DD'= Δy_{2rays} = 3 m (which is 'much smaller' than OD = 7.35 m , that is DD' <<OD , while we chose here DD' = 3m > DB' = 2.65 m , as it is the case in Fig. 3), we make the following successive calculations :
Moreover, from eq. (14) : $\theta_i^0 + \Delta\theta_i^0 = 45.99^o$, therefore $\Delta\theta_i^0 = 9.66^o$. Further, from eq. (8), i.e. Snell's law : $\theta_t^0 = 27.92^o$, and from eq. (13) : $\theta_t^0 + \Delta\theta_t^0 = 34.65^o$, therefore : $\Delta\theta_t^0 = 6.73^o$. Finally, from eqs. (10) and (11) we calculate the *coordinates of the 'virtual image'* : y_C = -3.22 m , z_C = 24.01 m . .

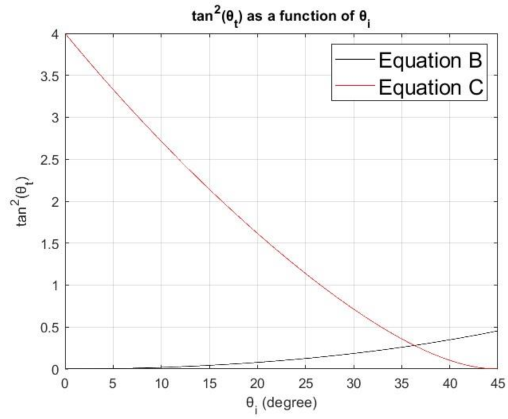

Figure 4. Graphs of monotonic functions B(θi), eq. (5), which is variant form of *Snell's law,* and C(θi), eq. (6), which comes from problem geometry.

6. Conclusion – Short Discussion – Future Research

In this paper we considered the problem of a vertical short (i.e. Hertzian) dipole antenna in air radiating above a flat interface. Below the flat interface a lossless dielectric medium lies (unbounded for z<0), with dielectric constant ε_r (ε_r = real).

In high frequencies the solution to the above EM problem can be solved in an *approximate fashion* by applying the *'Geometrical Optics' (GO)* approach, that is '*ray representation'* of the EM waves. In this paper, as a first step, the refraction of rays on the flat interface is considered (by applying the well – known '*Snell's law of refraction'*) and for given position (coordinates) of the radiating dipole (above the flat interface) and of the observation point (below the flat interface) the coordinates (y_C , z_C) of the '*virtual image'* C of the radiating source are calculated.

The above solution is based, for given coordinates of the observation point below the flat interface, on the calculation of the position of the unique refraction point D on the interface. In this way, by applying our proposed method, a *unique* position (y_C , z_C) for the '*virtual image'* C is calculated (our formulation described above is a 2D formulation of the problem, where one can easily extend that to a 3D problem formulation).

Concerning corresponding *future research* by our group to this paper (in the near future), the well – known EM radiation formula from a vertical Hertzian dipole will be used, so that by applying *standard image theory techniques* an approximate EM formula at the receiver position (below the flat interface) will be derived (i.e. approximate value of the EM field at that point).

Finally, regarding *possible applications* of this research can be, for example, for the calculation of EM field values below the surface of the sea or of lakes, when a vertical antenna radiates above sea or lake surface (the rather small value of sea or lake water conductivity can be neglected, as a first approximation, provided that the depth below sea or lake surface is sufficiently small).

7. Acknowledgment

The authors would like to thank the Kazakshtan Ministry of Education and Research for a research grant, which supported this research.

References

[1] A. N. Sommerfeld, "Propagation of Waves in Wireless Telegraphy", *Ann. Phys.* 1909, 28, pp. 665–737.

[2] K.A. Norton, "The Propagation of Radio Waves over the Surface of the Earth and in the Upper Atmosphere", *Proc. Inst. Radio Eng.* 1936, 24, pp. 1367–1387, doi:10.1109/JRPROC.1936.227360.

[3] A.K. Norton, "The Propagation of Radio Waves over the Surface of the Earth and in the Upper Atmosphere", *Proc. Inst. Radio Eng.,* 1937, 25, pp. 1203–1236, doi:10.1109/JRPROC.1937.228544.

[4] J. Wait, "Launching a surface wave over the Earth", *Electronics Letters* 1967, 3, pp. 396–397, doi:10.1049/el:19670307.

[5] R.J. King, "Electromagnetic Wave Propagation Over a Constant Impedance Plane", *Radio Sci.* 1969, 4, pp. 255–268, doi:10.1029/RS004i003p00255.

[6] T.K. Sarkar; W. Dyab, M.N. Abdallah; M. Salazar-Palma, M.V.S.N. Prasad, S.W. Ting, S.Barbin, "Electromagnetic Macro Modeling of Propagation in Mobile Wireless Communication: Theory and Experiment", *IEEE Antennas Propag. Mag.*, 2012, 54, pp. 17–43, doi:10.1109/MAP.2012.6387779.

[7] J.G.V. Bladel, *The Sommerfeld Dipole Problem. In Electromagnetic Fields*, J. Wiley and Sons, Inc.: Hoboken, NJ, USA, 2007; Section 9.3, pp. 448–452.

[8] G. Tyras, *Field of a Dipole in a Stratified Medium. In Radiation and Propagation of Electromagnetic Waves*; Academic Press, Inc., New York, NY, USA, 1969; Section 6, pp. 133–160.

[9] Y. Rahmat-Samii, R. Mittra, P. Parhami, "Evaluation of Sommerfeld Integrals for Lossy Half-Space Problems". *Electromagnetics,* 1981, 1, pp. 1–28, doi:10.1080/02726348108915122.

[10] R.E.Collin, "Hertzian dipole radiating over a lossy earth or sea: some early and late 20th-century controversies". *IEEE Antennas Propag. Mag.,* 2004, 46, pp. 64–79, doi:10.1109/MAP.2004.1305535.

[11] K.A. Michalski, "On the efficient evaluation of integral arising in the sommerfeld halfspace problem". *IEE Proc.-Microwaves, Antennas Propag.,* 1985, 132, pp. 312–318, doi:10.1049/ip-h-2.1985.0056.

[12] G. Pelosi, J.L. Volakis, "On the Centennial of Sommerfeld's Solution to the Problem of Dipole Radiation Over an Imperfectly Conducting Half Space", *IEEE Antennas Propag. Mag.,* 2010, 52, pp. 198–201, doi:10.1109/MAP.2010.5586629.

[13] J.R. Wait, "The Ancient and Modern History of EM Ground-Wave Propagation.", *IEEE Antennas Propag. Mag.,* 1998, 40, pp. 7–24, doi:10.1109/74.735961.

[14] A. Baños, *Dipole Radiation in the Presence of a Conducting Half-Space*, Pergamon Press, Oxford, UK, 1966; pp. 151–158.

[15] S.S. Sautbekov, R.N. Kasimkhanova,; P.V. Frangos, *"Modified Solution of Sommerfelds's Problem:*, In Proceedings of the CEMA'10 Conference, Athens, Greece, 7–9 October 2010; pp. 5–8. Available online: http://rcvt.tu-sofia.bg/CEMA/proceedings/CEMA_2010_proc.pdf (accessed on May 2021).

[16] S. Sautbekov, "The Generalized Solutions of a System of Maxwell's Equations for the Uniaxial Anisotropic Media. In Electromagnetic Waves Propagation in Complex Matter"; *IntechOpen Limited*: London, UK, 2011; Chapter 1, pp. 1–24, doi:10.5772/16886.

[17] K. Ioannidi,; C. Christakis, S. Sautbekov, P. Frangos, S.K. Atanov, "The Radiation Problem from a Vertical Hertzian Dipole Antenna above Flat and Lossy Ground: Novel Formulation in the Spectral Domain with Closed–Form Analytical Solution in the High Frequency Regime". *Int. J. Antennas Propag. (IJAP)*, 2014, Special Issue on 'Propagation of Electromagnetic Waves in Terrestrial Environment for Applications in Wireless Telecommunications', doi:10.1155/2014/989348.

[18] S. Bourgiotis, K. Ioannidi, C. Christakis, S. Sautbekov, P. Frangos, "The Radiation Problem from a Vertical Short Dipole Antenna Above Flat and Lossy Ground: Novel Formulation in the Spectral Domain with Numerical Solution and Closed-Form Analytical Solution in the High Frequency Regime", *Proceedings of CEMA'14 Conference*, Sofia, Bulgaria, 16–18 October 2014; pp. 12–18, Available online: http://rcvt.tu-sofia.bg/CEMA/proceedings/CEMA_2014_proc.pdf (accessed on May 2021).

[19] S. Bourgiotis, A. Chrysostomou, K. Ioannidi; S. Sautbekov and P. Frangos, "Radiation of a Vertical Dipole over Flat and Lossy Ground using the Spectral Domain Approach: Comparison of Stationary Phase Method Analytical Solution with Numerical Integration Results", *Electronics and Electrical Engineering Journal,* 2015, 21, pp. 38–41, doi:10.5755/j01.eee.21.3.10268.

[20] A. Chrysostomou, S. Bourgiotis, S. Sautbekov, K. Ioannidi and P. Frangos, "Radiation of a Vertical Dipole Antenna over Flat and Lossy Ground: Accurate Electromagnetic Field Calculation using the Spectral Domain Approach along with Redefined Integral Representations and corresponding Novel Analytical Solution", *Electronics and Electrical Engineering Journal,* 2016, 22, pp. 54–61, doi:10.5755/j01.eie.22.2.14592.

[21] S. Sautbekov, S. Bourgiotis, A. Chrysostomou and P. Frangos, "A Novel Asymptotic Solution to the Sommerfeld Radiation Problem: Analytic Field Expressions and the Emergence of the Surface Waves", *PIER M*, 2018, 64, pp. 9–22, doi:10.2528/PIERM17082806.

[22] S. Bourgiotis, P. Frangos, S. Sautbekov and M. Pshikov, "The Evaluation of an Asymptotic Solution to the Sommerfeld Radiation Problem using an Efficient Method for the Calculation of Sommerfeld Integrals in the Spectral Domain", *'Electronics' Journal*, MDPI Publisher, 1, . https://doi.org/10.3390/electronics1010000, https://www.mdpi.com/journal/electronics, Special Issue on 'Propagation of Electromagnetic Waves in Terrestrial Environment for Applications in Wireless Telecommunications and Radar Systems', June 2021.

[23] J. Fikioris, *Introduction to Antenna Theory and Propagation of Electromagnetic Waves;* National Technical University of Athens: Athens, Greece, 1982. (In Greek).