Spectral Grounds of Diffraction Radiation for the Open Structure Electron Beam - Metamaterial Interface

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***Abstract*—The paper deals with electromagnetic problem of the radiation generated due to the excitation of a periodic interface of an artificial material with dispersive constitutive parameters by an electron beam moving over this interface. It is demonstrated that the possibility of excitation of eigen waves of periodic surface has the principal influence on the radiation characteristics, which show anomalously high levels of the secondary filed. It is also shown that an accurate solution to the spectral problem, allowing numerical modeling of eigen regimes, provides the profound description and physical explanation of the complicated electromagnetic phenomenon.**

Keywords—diffraction radiation, dispersive materials, eigen fields, eigen values, eigen waves, leaky waves, metamaterials, multifold Riemann surface, periodic interface, Smith-Purcell radiation, spectral problems, unusual true eigen waves, Vavilov-Cherenkov radiation

I. Introduction

The paper investigates the nature of the effect of diffraction radiation arising from the uniform motion of a plane, density-modulated electron beam near the periodic boundary of certain artificial material with dispersive constitutive parameters – metamaterial, Fig. 1. The effective permittivity and permeability depend on the modulation frequency of the electron beam and can take negative values.

The process of diffraction radiation is modeled by the boundary value problem of diffraction of the electromagnetic field of the electron flow over the periodic boundary of the metamaterial (approximation of a given current). The solution of this boundary value problem is constructed using the regularization method [1], [2]. This method, in contrast to the previously known ones, allows to perform the analytical continuation of the solution of the diffraction problem into the domain of complex parameter values, in particular, complex modulation frequencies of the electron beam. This approach enables the unambiguous association of the resonant behavior of diffraction radiation, appearing with a change in the modulation frequency, with the excitation of natural oscillations (waves) of the periodic boundary of the metamaterial, as an open resonant structure [2]–[6].

The corresponding sophisticated numerical algorithms and programs served as an efficient and reliable tool for extensive numerical simulation that has illustrated analytical results and made up rather exhaustive and picturesque demonstration of complicated physical phenomena.

II. Formulation of the Problems

The works [1]–[3] contain several fundamental statements related to the formulation, construction of the solution, and physical analysis of the results of the solutions to the boundary value problem

$$\begin{cases} \left[\partial_y^2+\partial_z^2+\varepsilon(g,k)\mu(g,k)k^2\right]U(g,k)=0; \\ \quad g=\{y,z\}\in\Omega_{\text{int}} \\ \mathbf{E}_{\text{tg}}(q,k),\ \mathbf{H}_{\text{tg}}(q,k) \text{ are continuous when} \\ \quad \text{crossing } \Sigma^{\varepsilon,\mu}=\Sigma_x^{\varepsilon,\mu}\times(-\infty<x<\infty) \\ \quad \text{and virtual boundaries } y=0,\ y=-h; \\ \quad q=\{x,y,z\} \\ U\{\partial_z U\}(y,l,k)=\exp(2\pi i\zeta)U\{\partial_z U\}(y,0,k) \\ \quad \text{for } -h\le y\le 0, \end{cases} \tag{1a}$$

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$$U(g,k)=V_0(g,k)+U^+(g,k)$$
$$=V_0(g,k)+\sum_{n=-\infty}^{\infty}U_n^+(g,k)$$
$$=\exp\left(-i\Gamma_0^+y\right)\varphi_0(z)+\sum_{n=-\infty}^{\infty}R_n(k)\exp\left(i\Gamma_n^+y\right)\varphi_n(z); \tag{1b}$$
$$g\in\overline{\mathrm{A}},$$

$$U(g,k)=U^-(g,k)=\sum_{n=-\infty}^{\infty}U_n^-(g,k)$$
$$=\sum_{n=-\infty}^{\infty}T_n(k)\exp\left(-i\Gamma_n^-(y+h)\right)\varphi_n(z); \tag{1c}$$
$$g\in\overline{\mathrm{B}}.$$

The corresponding geometry is presented in Fig. 1.

Relaying on the results of these papers, the efficient numerical-analytical tool for investigation of these problems had been constructed.

Above and further $U(g,k)=H_x(g,k)$ and

$$E_y(g,k)=-\frac{\eta_0}{ik\varepsilon(g,k)}\partial_z H_x(g,k),$$
$$E_z(g,k)=\frac{\eta_0}{ik\varepsilon(g,k)}\partial_y H_x(g,k) \tag{2}$$

are the nonzero components of the total H -polarized field $\{\mathbf{E}(g,k),\mathbf{H}(g,k)\}$, $g=\{y,z\}$, $\partial_x=0$, formed by the system "boundary $\Sigma^{\varepsilon,\mu}$ - electron beam"; $\Omega_{\text{int}}=\{g\in\mathrm{R}:-h<y<0\}$, $\mathrm{A}=\{g\in\mathrm{R}:y>0\}$, $\mathrm{B}=\{g\in\mathrm{R}:y<-h\}$, $\mathrm{R}=\{g=\{y,z\}\in\mathrm{R}^2:0<z<l\}$, and $\overline{\mathrm{G}}$ is the closure of the domain G . $V_0(g,k)$ is H_x -component of electron beam field; $W_n^+(k)$ and $W_n^-(k)$ are the functions determining efficiency of diffraction radiation at spatial harmonics $U_n^+(g,k)$ and $U_n^-(g,k)$, which are outgoing upward (into vacuum half-space) and downward (into dispersive medium half-space) from the boundary, respectively; $\Gamma_n^+=\sqrt{k^2-\Phi_n^2}$, $\operatorname{Re}\Gamma_n^+\geq0$, $\operatorname{Im}\Gamma_n^+\geq0$ and $\Gamma_n^-=\sqrt{k^2\varepsilon(k)\mu(k)-\Phi_n^2}$, $\varepsilon^{-1}(k)\operatorname{Re}\Gamma_n^-\geq0$, $\operatorname{Im}\Gamma_n^-\geq0$ are vertical propagation constants of these harmonics; functions $\varphi_n(z)=l^{-1/2}\exp(i\Phi_n z)$, $n=0,\pm1,\pm2,...$ form a complete (in space $\mathrm{L}_2(0,l)$) orthonormal system in the cross section of Floquet channel R , $\Phi_n=(n+\zeta)2\pi/l$, $\zeta\,2\pi/l=\Phi_0=k/\beta$ (with this value of Φ_0 , $\operatorname{Re}\Gamma_0^+=0$ and $\operatorname{Im}\Gamma_0^+>0$, $V_0(g,k)$ is an inhomogeneous plane wave component); k and $0<\beta<1$ are modulation frequency and relative beam velocity; $k=2\pi/\lambda$ is a frequency parameter, which is set by modulation frequency; λ is wavelength of radiation field in free space, l and h are period and height of the boundary $\Sigma_x^{\varepsilon,\mu}=\{g:y=f(z),-h\leq f(z)\leq0\}$. The more detailed description of the problem (1) is given in [1], [2]. The choice of the branches of two-valued functions $\Gamma_n^\pm(k,\zeta)$ was made and justified ibid, and relied on the so-called partial radiation conditions [1], [2], [8], [9], requiring that fields $U^\pm(g,k)$ should not contain harmonics coming (carrying energy) from $y=\pm\infty$ to the boundary $\Sigma_x^{\varepsilon,\mu}$.

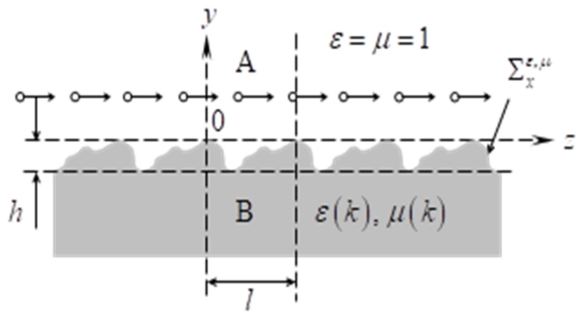

Fig. 1. Periodic interface: upper half-space is vacuum, lower half-space is dispersive material

Applying this tool,

- We have studied the principal energy characteristics

$$W_n^+(k)=|R_n|^2\frac{\operatorname{Re}\Gamma_n^+}{|\Gamma_0^+|},\quad W_n^-(k)=\varepsilon^{-1}(k)|T_n|^2\frac{\operatorname{Re}\Gamma_n^-}{|\Gamma_0^+|} \tag{3}$$

  of diffraction radiation (Vavilov-Cherenkov radiation [7] or Smith-Purcell radiation [8]), generated by a plane density-modulated electron beam flying over a periodically corrugated boundary $\Sigma_x^{\varepsilon,\mu}$ (see Fig. 1) separating vacuum ($\varepsilon\equiv\mu\equiv1.0$) and a dispersive plasma-like medium with material parameters $\varepsilon(k)=1-k_\varepsilon^2/k^2$, $\mu(k)=1-k_\mu^2/k^2$, [1]–[6]; or $\varepsilon(k)=1-k_\varepsilon^2/k^2$, $\mu(k)=1-\theta k^2/(k^2-k_\mu^2)$; $0<\theta<1$. as in [9], [10].
- We have introduced the basic classification of the electromagnetic waves [3]–[6] associated with non-trivial solutions of the problem (1) for $V_0(g,k)\equiv0$.
- We have found out and proved that the characteristics of diffraction radiation are significantly affected by the "synchronism" of the electron beam with one of the *unusual regular surface waves* of the media interface, that is

$$\zeta=kl/2\pi\beta=\overline{\zeta}_n^\pm$$
$$=-n\pm k\frac{l}{2\pi}\sqrt{\frac{\varepsilon(k)\left(\mu(k)-\varepsilon(k)\right)}{1-\varepsilon^2(k)}}.$$

  The fulfillment of this condition is necessary for the existence of the corresponding eigenwaves of the boundary $y=f(z)$, it is written in the form $\operatorname{Im}\Gamma_n^+\left(\overline{\zeta}_n^\pm\right)>0$ and $\varepsilon(k)<0$.

From these relations we obtain:

$$\frac{\varepsilon(k)\left[\varepsilon(k)-\mu(k)\right]}{\varepsilon^2(k)-1}>1 \;\Rightarrow$$
$$\Rightarrow \begin{cases}\varepsilon(k)\mu(k)<1; & |\varepsilon(k)|>1\\ \varepsilon(k)\mu(k)>1; & |\varepsilon(k)|<1\end{cases} \tag{4}$$

We study the complex eigen waves of the interface of various dispersive artificial materials and, as we have proved in numerous electromagnetic problems, their decisive influence on radiation characteristics.

When $V_0(g,k)\equiv 0$ (see 1(b)) and k is fixed, we obtain from (1) a homogeneous (spectral) problem with nontrivial solutions $H_s(g,\varsigma)=\exp(i\varsigma 2\pi z/l)\bar{H}_s(y,\varsigma)$ existing for no more than a countable set of eigenvalues $\{\varsigma=\bar{\varsigma}\}\in \mathrm{F}$ (F is a multi-folded Riemann surface, see [1]) and determining fields of the eigenwaves $\vec{U}(g,\varsigma)=\exp(i\varsigma 2\pi z/l)\{\vec{E}(y,\varsigma),\vec{H}(y,\varsigma)\}$ of the periodic media interface. To solve the problem, it is necessary to find out complex valued roots of the equation:

$$\det\left[I+H(\varsigma)\right]=0 \tag{5}$$

The operator function $H(\varsigma)$ has the form described in details in [1] and [2], and depends analytically on ς on the Riemann surface F. It has to be noted that for the finding complex valued roots of (5) rather complicated numerical routines had been developed. These algorithms are based on the adaptive conjunction of linear and quadratic interpolations of direct and inverse functions, Traub's interpolation, and modified Newton scheme [11], [12]. As initial approximation we have used the values of eigen number for plane boundary:

$$\bar{\zeta}_n^{\pm}=-n\pm k\frac{l}{2\pi}\sqrt{\frac{\varepsilon(k)\left(\mu(k)-\varepsilon(k)\right)}{1-\varepsilon^2(k)}}.$$

For numerical reconstruction of the electromagnetic field, density patterns in the vicinity of the interface of the corresponding eigen vectors have been found numerically using inverse iterations. The situation gets more complicated as we have to consider solutions on multifold Riemann surface and to watch the change of the sheet's number.

For our purposes, that is to establish correspondence between excitation of eigen waves and resonances in radiation field and to investigate the regularities of such electromagnetic scenario, it is necessary to consider the eigen wave problem in the first (physical) sheet of Riemann surface and to focus the study on the investigation of

Unusual true eigen waves: for the media without losses ($\mathrm{Im}(\varepsilon)=0,\ \mathrm{Im}(\mu)=0$); $\mathrm{Im}(\varsigma)=0$;

and

Something leaky waves, $\mathrm{Im}(\varsigma)\neq 0$.

In order to meet boundary and radiation conditions in the presence of artificial material having negative or even double negative constitutive parameters $\mathrm{Re}(\varepsilon)<0$ and/or $\mathrm{Re}(\mu)<0$, it is necessary to choose the branches of the function $\Gamma_n^-(k,\varsigma)$, $\mathrm{Im}(\varsigma)=0$; $k>0$ according the rules:

1. $\mathrm{Re}(\varepsilon)>0$, $\mathrm{Re}(\mu)>0$ $\rightarrow$ $\mathrm{Re}\left(\Gamma_n^-(k,\varsigma)\right)\geq 0$, $\mathrm{Im}\left(\Gamma_n^-(k,\varsigma)\right)\geq 0$;
2. $\mathrm{Re}(\varepsilon)<0$, $\mathrm{Re}(\mu)>0$ (or $\mathrm{Re}(\varepsilon)>0$, $\mathrm{Re}(\mu)<0$) $\rightarrow$ $\mathrm{Re}\left(\Gamma_n^-(k,\varsigma)\right)\geq 0$, $\mathrm{Im}\left(\Gamma_n^-(k,\varsigma)\right)\geq 0$; $\varsigma>0$
3. $\mathrm{Re}(\varepsilon)<0$, $\mathrm{Re}(\mu)<0$ $\rightarrow$ $\mathrm{Re}\left(\Gamma_n^-(k,\varsigma)\right)\leq 0$, $\mathrm{Im}\left(\Gamma_n^-(k,\varsigma)\right)\geq 0$; $\varsigma>0$

The scheme illustrating the conditions formulated above and supplied with detailed legend is presented in Fig. 2.

The cuts are defined by the equality $k^2\varepsilon\mu-\left(\mathrm{Re}(\varsigma)+n\right)^2+\left(\mathrm{Im}(\varsigma)\right)^2=0$; functions $\Gamma_n^-(k,\varsigma)$ are even ones: $\Gamma_n^-(k,-\varsigma)=\Gamma_n^-(k,\varsigma)$.

The treatment of the electromagnetic problem for the system "periodic interface of artificial material - electron beam" and the establishment of the one-to-one connection of the excitation of eigen surface waves with anomalously high levels of electromagnetic radiation is also restricted by the properties of the electron beam, in particular its velocity (parameter β) and the limits for the existence of propagating (transferring energy) waves in the upper free space. In [2]–[6], the special attention had been payed to the possible combinations of forbidden zones, zones of synchronism, and their correlation with the regimes of propagation of various modes.

The investigation of the influence of the frequency parameter k variation on radiation characteristics may begin from the study of regularities of a plane boundary. The over mapping of the chart for values $\varsigma_n^{\pm}$ of a plane boundary and chart of the limits of various regimes defined by the values of $\mathrm{G}_n^{\pm}=\{k,\beta\}:\Gamma_n^{\pm}(k,\beta)=0$ in the plane (k , β) of propagating waves in upper and low half space allows to find out the points k , where $kl/2\pi\beta=\bar{\zeta}_n^{\pm}$.

Moving along the frequency parameter k , the line $\zeta=k/\beta$ crosses various curves of eigen waves with different numbers and propagation regimes, that is accompanied by the change in the eigen field configuration. The frequency points of radiation bursts are rather close to the crossing points' projection onto the axis k on the abovementioned charts.

The study of the electromagnetic scenario with the corrugation depth variation h at a fixed frequency or varying parameter k presently lacks required regularities and is in progress. Generally, the increase of corrugation value h essentially influences the Q-factor and radiation characteristics. The investigation of the electromagnetic system "periodic interface - electron beam" with the depth of a periodic interface grooves changing and the corresponding influence on radiation characteristics involves much more challenging problems. In this situation, the study and consideration of the cuts for the function $\Gamma_n^-(k,\varsigma)$, which are

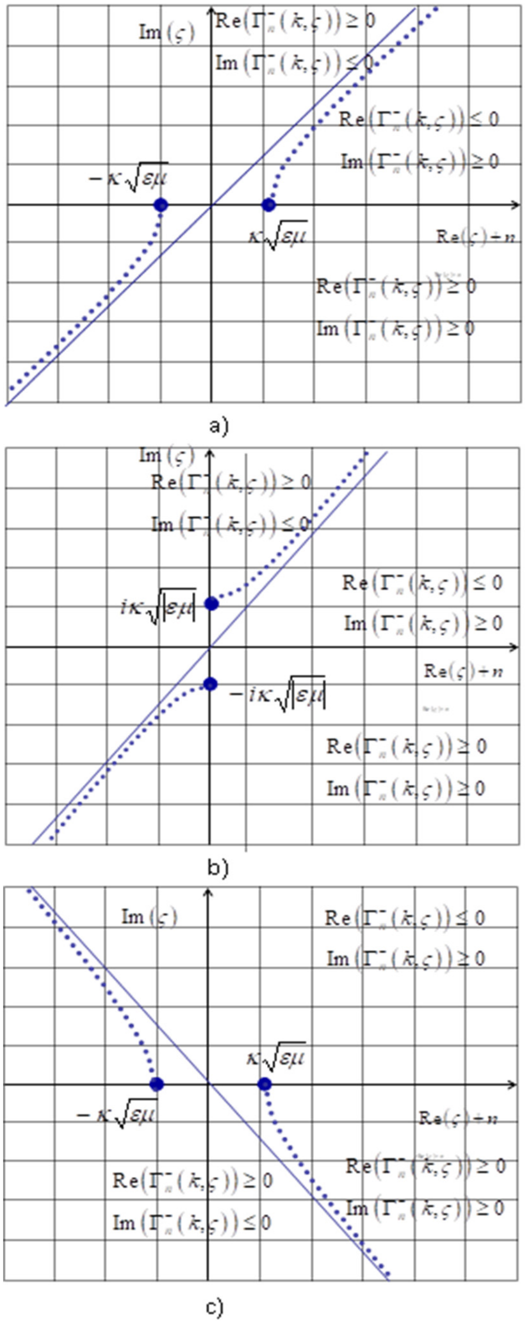

Fig. 2. The choice of cuts and branches for the function $\Gamma_n^-(k,\varsigma)$, $\mathrm{Im}(\varepsilon)=0$: (a) $\mathrm{Re}(\varepsilon)>0$, $\mathrm{Re}(\mu)>0$; (b) $\mathrm{Re}(\varepsilon)<0$, $\mathrm{Re}(\mu)>0$ or $\mathrm{Re}(\varepsilon)>0$, $\mathrm{Re}(\mu)<0$; (c) $\mathrm{Re}(\varepsilon)<0$, $\mathrm{Re}(\mu)<0$

depicted schematically in Fig. 2, acquires the decisive importance. Even the apparatus of catastrophe theory and consideration of Morse critical points [11], [13] may contribute to the understanding of this electromagnetic scenario.

To demonstrate one of the electromagnetic situations arising in the study of these aspects of the problem, we present several results for the plasma-like interface $\varepsilon(k)=1-k_\varepsilon^2/k^2$, $\mu(k)=1-k_\mu^2/k^2$. To simplify a little the scenario, we have chosen $k_\varepsilon=0.8$, $k_\mu=0.09$, and $\beta=0.7$. Here, $k_\varepsilon>k_\mu$, $\varepsilon(k)<0$, and partial components $U_n^+(g,\bar{\zeta}_n^\pm)$, $U_n^-(g,\bar{\zeta}_n^\pm)$

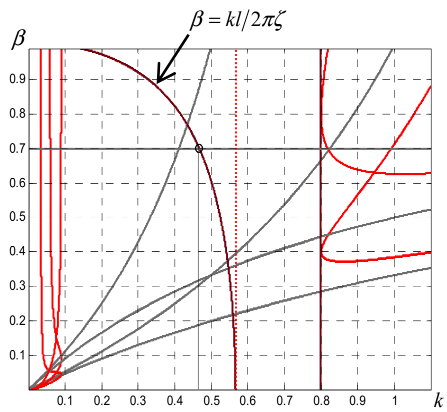

Fig. 3. Boundaries $G_n^\pm$ in the domains of k and $\beta=kl/2\pi\zeta$ variation, separating regions where harmonics $U_n^\pm(g,k)$ propagate without attenuation. Intersection of the curve of β_s with $\beta=0.7$ defines the frequency point $k=0.4703$, which corresponds to the *unusual true eigen wave* of plane boundary, that serves as the first approximation for studying eigen waves when the corrugation depth increase

of the *true eigen waves* $U(g,\bar{\zeta}_n^\pm)$ transfer the energy in opposite along the axis z directions. We call these waves "*unusual true eigen waves*" in contrast to the usual *true eigen waves* $U(g,\bar{\zeta}_n^\pm)$, whose region of existence is limited by the frequencies $k>k_\varepsilon$ for which $\varepsilon(k)>0$, see [4]. So, for the chosen parameters, the *unusual true eigen wave* may exist within the frequency interval $0.0894=k_\varepsilon k_\mu\left(\sqrt{k_\mu^2+k_\varepsilon^2}\right)^{-1}<k<\sqrt{0.5}k_\varepsilon=0.5657$, but our interests are limited by the region of propagation of G_{-1}^+, and there are no propagating waves inside the plasma-like material. Therefore, in this example, we study only the Smith-Purcell radiation, see Fig. 3.

The introduction of the periodic corrugation adds the periodicity of the wavelength scale to the problem and, naturally, brings another challenge to the study of artificial and specially designed smart materials. The grooves depth influence for conventional materials had been studied in numerous works, e.g. see [1], [11] and references there. In the present problem, the periodic dependence of diffraction radiation characteristics of a grating on the depth of grooves had not been observed.

The challenging contour plot of $W_{-1}^+(k,h)=const$ is presented in Fig. 4(a). In this figure, for the sake of clarity, the values of W_0^+ are limited by the level $W_0^+(k,h)=10.0$. The first the most notable thing in this figure is the pronounce resonance around the frequency $k=0.4703$ (it is marked by the white dashed line, which is found out for the plane surface in Fig. 3, $\beta=0.7$), that is around the frequency of possible excitation of the principal *unusual true eigen wave* of the interface. The Q factor of this resonance is essentially changing as the depth h increase. The bunch of resonance lines spring out from the vicinity of the accumulation point $k^{\mathrm{sing}}=k_\varepsilon\sqrt{0.5}\approx 0.5657$ (the more detailed illustration with smaller sampling of parameters k,h is presented in Fig. 4(b). For the plasma-like medium, we have only one accumulation

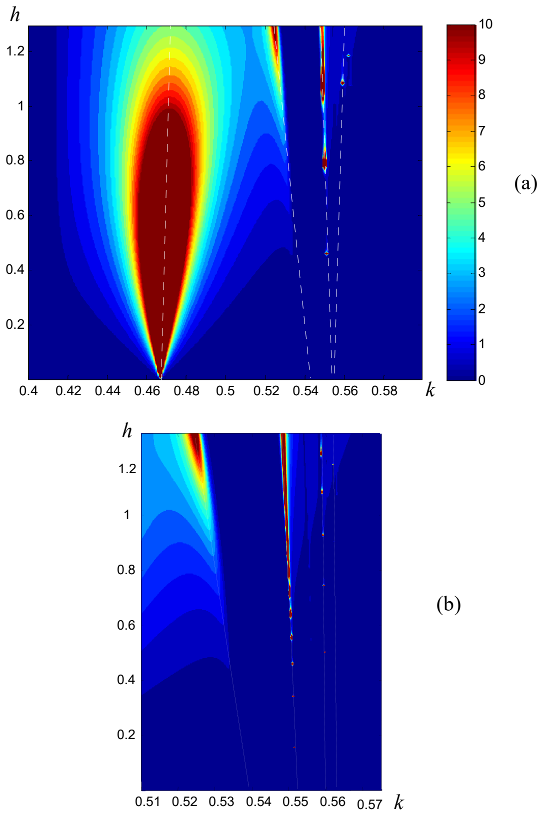

Fig. 4. Contour plots of $W_{-1}^{+}(k,h)=const$ within the parameters' range $k\in[0.4;0.6]$, $h\in[0;1.3]$ corresponding to the allowed zone of *true eigen waves* of the structure: $\beta=0.7$

point. For the media with constitutive parameters like the mentioned above $\varepsilon(k)=1-k_{\varepsilon}^{2}/k^{2}$, $\mu(k)=1-\theta k^{2}/\left(k^{2}-k_{\mu}^{2}\right)$, we have two accumulation points, which are influencing the radiation characteristics. Fig. 4 allows to note that the resonance's peaks of radiation characteristics demonstrate rather different behavior in the domain of the frequency close to the first *unusual true eigen wave* ($0.45\le k\le 0.48$) and in the vicinity of k^{sing} when approaching from the left. This bunch of resonances has considerably higher Q factor and each of the resonance lines in the plane k,h "moves on" in a different way.

Naturally, to find out the explanation or, at least, more profound picture of the behavior of the system "electron beam - periodic dispersive interface", we move to the consideration of the eigen wave problem. In Fig. 5, where the curves of $\overline{\zeta}_{0}^{\pm}(h)$ are presented, we see that while moving along one curve with h increasing, we are staying in the same eigen wave, having the same configuration of eigen fields.

More complicated situation appears in the study of $\zeta_{0}^{\pm}(h)$ while k is approaching $k^{\text{sing}}\approx 0.5657$. The picture of eigen propagation constants $\overline{\zeta}_{m}^{\pm}$ with grooves variation has completely different topology for different frequency parameter and for different type of eigen waves if compared with the scheme detailed in [4].

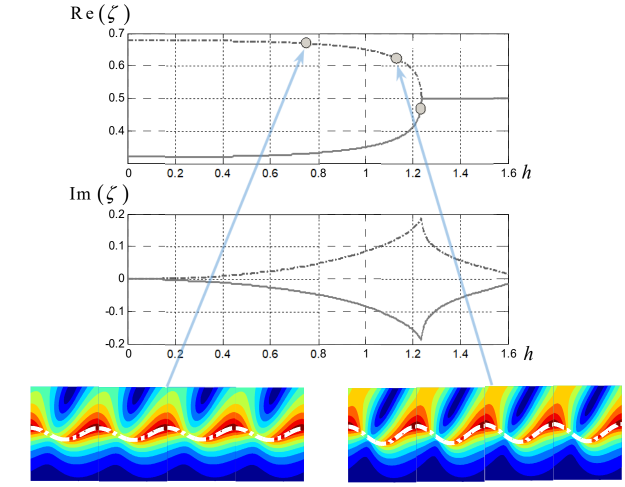

Fig. 5. To the scenario of corrugation h change ($k=0.4703$, $\beta=0.7$): $\overline{\zeta}_{1}^{+}(h)$ (solid line), $\overline{\zeta}_{1}^{-}(h)$ (dashed line), and field patterns of $H_{x}(g,k)$ component calculated for $h=0.706$ and $h=1.1152$ (marked with arrows)

III. Conclusion

The theory of electromagnetic wave scattering (homogeneous and inhomogeneous) with the appearance of new technologies brings new challenges to the researchers. One of such challenges is the investigation of new artificial materials with complex dispersive parameters. This paper confirms that for an investigation of such electromagnetic objects, the spectral theory is rather efficient and promising apparatus.

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