ABSTRACT BOOK

ISBN: 978-605-2124-29-1

6^{th} INTERNATIONAL CONFERENCE OF MATHEMATICAL SCIENCES ICMS 2022

20-24 JULY 2022

ISTANBUL, TURKIYE

Supported by

Edited by

Özkan Değer and Hüseyin Çakallı

CONTENTS

1. FOREWORD

On behalf of the Organizing Committee, we are very pleased to welcome you to the 6th International Conference of Mathematical Sciences (ICMS 2022) to be held between 20-24 July 2022 via face-to-face and online Conference supported by Maltepe University in Istanbul. We hope that, ICMS 2022 will be one of the most beneficial scientific events, bringing together mathematicians from all over the world, and demonstrating the vital role that mathematics plays in any field of science. Welcome to our conference at Maltepe University.

Hüseyin Çakallı
Chairman of the Organizing Committee

2. COMMITTEES

HONORARY COMMITTEE

Hüseyin Şimşek (Founder of Maltepe University, Chairman of the Board of Trustees, Turkey)

Edibe Sözen (Rector, Maltepe University, Turkey)

ORGANIZING COMMITTEE

Huseyin Cakalli, Conference Chairman, Maltepe University, Istanbul, Turkey

Ozay Gürtug, Maltepe University, Istanbul, Turkey

Dragan Djordjevic, University of Nis, Faculty of Sciences and Mathematics, Serbia

Ljubisa D.R. Kocinac, University of Nis, Faculty of Sciences and Mathematics, Serbia

Robin Harte, Trinity College, Dublin, Ireland

Mehmet Dik, Rockford University, IL, USA

Ravi P. Agarwal, Texas, USA

Richard F. Patterson, North Florida University, Jacksonville, FL, USA

Pratulananda Das, Jadavpur University, India

Mohammad Kazim Khan, Kent State University, Ohio, USA

Ekrem Savas, Usak University, Turkey

Naim Braha, University of Prishtina, Kosova

Habib Mazharimousavi, North Cyprus

LOCAL COMMITTEE

Emin Murat ESİN (Maltepe University)

Hamit TOPUZ (Maltepe University)

Önder ŞAHİNASLAN (Maltepe University)

Tugba AKYEL (Maltepe University)

Bahriye KARACA (Maltepe University)

İffet TAYLAN (Maltepe University)

Özay GÜRTUĞ (Maltepe University)

Hüseyin ÇAKALLI (Maltepe University)

Sibel ERSAN (Maltepe University)

İlhan GUL (Maltepe University)

Filiz CAGATAY UÇGUN (Maltepe University)

Aykut KAYHAN (Maltepe University)

INTERNATIONAL SCIENTIFIC COMMITTEE

A. Duran Türkoğlu (Turkey)
Abdizhahan Sarsenbi (Kazakhstan)
Adil Güler (Turkey)
Alaeddin Malek (Iran)
Alexander Abanin (Russia)
Allaberen Ashyralyev (Turkey)
Amalia Pielorz (Poland)
Ayşe Sönmez (Turkey)
Billy Rhoades (USA)
Bipan Hazarika (India)
Boyan Dimitrov (USA)
Charyyar Ashyralyyev (Turkey)
David Herrera Carrasco (Mexico)
Dejan Ilic (Serbia)
Dragan Djordjevic (Serbia)
E. Fokoue (USA)
E. Alexov (USA)
Ekrem Savaş (Turkey)
Emin Murat Esin (Turkey)
Emin Ozcag (Turkey)
Emrah Evren Kara (Turkey)
Erhan Guzel (Turkey)
Evren Hincal (Turkey)
Fuat Usta (Turkey)
George Anastassiou (USA)
H. A. El-Metwally (Egypt)
Hacer Şengul Kandemir (Turkey)
H. Elsalloukh (USA)
H. Nour Eldin (Denmark)
Hamdullah Şevli (Turkey)
Hamit Topuz (Turkey)
Hari Mohan Srivastava (Canada)
Hemen Dutta (India)
Hongde Hu (USA)
Hüseyin Bor (Turkey)
Hüseyin Çakallı (Turkey)
Hüseyin Kaplan (Turkey)
I. G. Avramidi (USA)
Ivan Jeliazkovn (USA)
İbrahim Çanak (Turkey)
İzzet Sakallı (Turkey)
J. Diblik (Czech Republic)
J. Gerardo Ahuatzi Reyes (Mexico)
J.M. Cushing (USA)
J.Z. Farkas (United Kingdom)
Javier F. Rosenblueth (México)
Jean Horgan (Ireland)
Jiling Cao (New Zealand)
Kadri Ulaş Akay (Turkey)
K. Fahem (Algeria)
Ljubisa D. R. Kocinac (Serbia)
M. Kazim KHAN (USA)
M.F. Shaughnessy (USA)
Makhmud Sadybekov (Kazakhstan)
Marcelo Moreira Cavalcanti (Brazil)
Mark Burgin (USA)
Martin Buntinas (USA)
Matid Matejdes (Slovakia)
Mehmet Dik (USA)
Mikail Et (Turkey)
Murat Esin (Turkey)
Mustafa Bayram (Turkey)
Müjgan Tez (Turkey)
Osman Mucuk (Turkey)
Ozay Gurtug (Turkey)
Öner Çakar (Turkey)

3. SESSIONS

The lectures in the following parallel sessions are to be held after the plenary speakers lectures.

0. **"Plenary"** organized by Hüseyin Çakallı,
1. **"Topology"** organized by Ljubisa D.R. Kocinac,
2. **"Analysis and Functional Analysis"** organized by Hacer Şengül Kandemir and Nazlım Deniz Aral,
3. **"Sequences, Series, Summability"** organized by İbrahim Çanak and Sefa Anıl Sezer,
4. **"Fixed Point Theory"** organized by Duran Türkoğlu and Hakan Şahin,
5. **"Numerical Functional Analysis"** organized by Allaberen Ashyralyev and Charyyar Ashyralyyev,
6. **"Computer Science and Technology"** organized by Şahin Uyaver and Önder Şahinaslan,
7. **"Mathematical Methods in Physics"** organized by Özay Gürtuğ and Filiz Çağatay Uçgun,
8. **"Applied Statistics"** organized by Müjgan Tez and Kadri Ulaş Akay,
9. **"Differential Geometry"** organized by Zerrin Şentürk,
10. **"Algebra"** organized by Leyla Bugay,

4. ACKNOWLEDGMENTS

We thank firstly the founder of Maltepe University, Hüseyin ŞİMŞEK, the rector of Maltepe University, Edibe SÖZEN. We also thank the parallel session organizers, and then all scientific committee members who reviewed abstracts which made the conference better.

There are many people who spent a lot of time and effort to make this conference possible. We would like to thank especially to the following colleagues who had contributed to the success of this conference in various ways:

Özkan Değer, Istanbul University, Istanbul, Turkey,
Goncagül Balki Yıldız, Maltepe University, Istanbul, Turkey,
Ahmet Usta, Maltepe University, Istanbul, Turkey,
Erdal Yaşlıca, Maltepe University, Istanbul, Turkey.
İlkün Orbak , Maltepe University, Istanbul, Turkey.
Fikriye İnce Dağcı, Kültür University, Istanbul, Turkey

Hüseyin Çakallı
Chairman of the Organizing Committee

5. PLENARY SPEAKERS

The abstracts of the plenary lectures are given in the following pages.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Numerical solution of the continuation problem for the one-dimensional acoustics equation

Janar Askerbekova, Syrym Kasenov, Almas Temirbekov, Aigerim Tleulesova
D.Serikbaev East-Kazakhstan Technical University, Ust-Kamenogorsk, Republic of Kazakhstan, ask-janar@mail.ru
Al-Farabi Kazakh National University, Almaty, Republic of Kazakhstan, syrym.kasenov@gmail.com
Al-Farabi Kazakh National University, Almaty, Republic of Kazakhstan, almas_tem@mail.ru
Al-Farabi Kazakh National University, Almaty, Republic of Kazakhstan, aigerim1985_06@mail.ru

Abstract

Currently, one of the most difficult areas of research in applied mathematics is the inverse problem for wave propagation with an important field of application in geophysics and medicine [1-5]. Of course, solving the inverse problem requires a very efficient and very accurate tool for solving the corresponding forward problem, which means, in our case, numerical simulation of the propagation of acoustic and elastic waves in inhomogeneous media [6-9]. Consider the continuation problem in the domain $\Delta(L_x) = \{(x,t) : x \in (0, L_x), t \in (x, 2L_x - x)\}$:

$$v_{tt} = v_{xx} - r(x)v \tag{1}$$
$$v_x(0,t) = \phi(t), \tag{2}$$
$$v(0,t) = f(t). \tag{3}$$

This work presents the continuation problem for one-dimensional equations of acoustics. One such ill-posed problem is the continuation problem. The continuation problem is based on finding the value of the desired function in the rest of the boundary using additional data in a certain part of the boundary. In this paper, we construct a finite-difference scheme for this inverse problem and find an unknown function on the characteristic from this difference equation by inverting the difference scheme. The effectiveness of this method lies in a simple and accurate fast calculation algorithm.

Keywords: Numerical methods, inverse problems, continuation problem.
2020 Mathematics Subject Classification Numbers: 65M32, 65N21, 49N45.

References

[1] Kabanikhin, S., Nurseitova, A., Kasenov, S. Stability estimation of the generalized solution to the direct problem for the acoustic equation, *Journal of Physics: Conference Series*, 2021, 2092(1), 012005.

[2] S.I. Kabanikhin. Inverse and Ill-Posed Problems. Theory and Applications. *De Gruyter*, Germany, 2012.

[3] M.A. Shishlenin, S.E. Kasenov, Zh.A. Askerbekova, Numerical algorithm for solving the inverse problem for the Helmholtz equation, *Communications in Computer and Information Science*, 9th International Conference on CITech 2018. Volume 998, 2019, Pages 197-207.

[4] S.E. Kasenov, A. Nurseitova, D.B. Nurseitov, A conditional stability estimate of continuation problem for the Helmholtz equation, *AIP Conference Proceedings* 1759, 020119 (2016).

[5] Syrym Kasenov, Janar Askerbekova, Aigerim Tleulesova, Algorithm construction and numerical solution based on the gradient method of one inverse problem for the acoustics equation, *Eastern-European Journal of Enterprise Technologies*, Vol. 2, No. 5 (116), 2022, pp. 43-52.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On the ubique solvability of a multipoint boundary value problems of functional differential equations with a conformable derivative

Kairat Usmanov, Kulzina Nazarova
Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkestan, Kazakhstan , kairat.usmanov@ayu.edu.kz
Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkestan, Kazakhstan , kulzina.nazarova@ayu.edu.kz

Abstract

It is known that one of the special cases of integro-differential equations is the so-called differential equations of fractional order. One of the variants of the fractional derivative, the so-called "conformable derivative", was introduced in [1]. In this paper, a multipoint boundary value problem for systems of functional differential equations with a conformable derivative

$$T_\alpha x\left(t\right)+AT_\alpha x\left(T-t\right)=\sum_{k=1}^{N}\int_0^a \phi_k(t)\psi_k(s)x\left(s\right)ds+f(t),\quad t\in\left[0,T\right],\ x\in R^n, \tag{1}$$

$$\sum_{i=1}^{m} B_i x(\theta_i)=d, d\in R^n, \tag{2}$$

$$0=\theta_0<\theta_1<\ldots<\theta_{m-1}<\theta_m=T,$$

is considered in the segment, $[0,T]$, where $0<\alpha<1$, the matrix $K(t,s)$ is continuous in $[0,T]\times[0,T]$, $f(t)$ is an n-dimensional vector function continuous in $[0,T]$, A is a symmetric matrix, $B_i,\ i=\overline{1,m}$ are constant matrices of $n\times n$ dimension.

Using the property of an involutive transformation, the problem is reduced to a multipoint boundary value problem for integro-differential equations. Further, the parametrization method proposed by Professor D. Dzhumabaev [2] is applied to the obtained problem, i.e. the segment under consideration is divided into parts and the values of the desired function at the points of the partition are denoted by a special parameter. Using this parameter, it is possible to transfer to new variables and to obtain initial conditions for the original equation. Determining the unique solution of the Cauchy problem and substituting the resulting solution into the boundary conditions, we obtain a system of linear equations with respect to the introduced parameters. Thus, a connection is established between the reversibility of the matrix of the resulting system and the unique solvability of the original multipoint boundary value problem.

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan Grant No. AP09259137.

Keywords: System of functional differential equations, parametrization method, multipoint boundary condition, unique solvability.
2020 Mathematics Subject Classification Numbers: 45J05, 45B05, 34K28.

References

[1] R. Khalil, M. A. Horani, A. Yousef and M. Sababheh, A new definition of fractional derivative, *J. Comput. Appl. Math.*, 264, 65–70 (2014).

[2] D.S.Dzhumabaev, Criteria for the unique solvability of a linear boundary value problem for systems of differential equations *Journal of calc. math. & math. phys.*, 29, 50–66 (1989)

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Qualitative Properties of One Second Order Differential Equation

Kordan Ospanov
L.N. Gumiliov Eurasian National University, Nur-Sultan, Kazakhstan, ospanov_kn@enu.kz

Abstract

The work is devoted to the study of the following singular differential equation:

$$L_0 y = -s(x)\left(\rho(x)y'\right)' + r(x)y' + q(x)y = f(x), \tag{1}$$

where $x \in R = (-\infty, +\infty)$, and $f \in L_2(R)$. We will assume that s and ρ are twice continuously differentiable, r is continuously differentiable, and q is a continuous functions. The equation (1) is singular in the sense that its coefficients, generally speaking, can be unbounded functions.

We denote by L the closure in $L_2(R)$ of the differential operator $L_0 y = -s(x)\left(\rho(x)y'\right)' + r\left(x\right)y' + q\left(x\right)y$, with $D(L_0) = C_0^{(2)}(R)$.
The function y is called a solution of equation (1) if $y \in D(L)$ and $Ly = f$.

The purpose of this work is to obtain conditions on the coefficients under which
(a) there exists a solution y of equation (1) for any $f \in L_2(R)$,
(b) the solution y of equation (1) is unique,
(c) the following, so-called, coercive estimate holds for y:

$$\left\| s\left(\rho y'\right)' \right\|_2 + \|ry'\|_2 + \|(1+|q|)y\|_2 \le C\left(\|f\|_2 + \|y\|_2\right), \tag{2}$$

where $\|\cdot\|_2$ is the norm in $L_2(R)$.
In addition, we present the application of estimate (2) to find of one property of the resolvent L^{-1}.

If $r = 0$, then (1) is the Sturm-Liouville equation. In this case the estimate (2) was stadied by B. Everitt and M. Giertz (the case of smooth $q(x)$), M. Otelbaev, K.Kh. Boimatov (the case of nondifferentiable $q(x)$) and others. In the case $s(x) = \rho(x) = 1$, and r is a weakly oscillating function that does not obey the coefficient q, the estimate (2) was obtained in [1].

In contrast to these works, we discuss the case where the higher coefficients $s(x)$ and $\rho(x)$ can grow near infinity independently of each other and of $r(x)$ and $q(x)$.

Keywords: Differential equation, generalized solution, maximal regularity.
2020 Mathematics Subject Classification Numbers: 34A30, 34C11, 34L05.

References

[1] K.N. Ospanov, R.D. Akhmetkalieva, Separation and the existence theorem for second order nonlinear differential equation, *Electron J. Qual. Th. Dif. Equat.*, 1, 1-12 (2012).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On the solvability of an initial-boundary value problem for a nonlocal hyperbolic equation

Maira Koshanova, Moldir Muratbekova
Khoja Akhmet Yassawi International Kazakh-Turkish University,Turkistan,Kazakhstan, maira.koshanova@ayu.edu.kz
Khoja Akhmet Yassawi International Kazakh-Turkish University,Turkistan,Kazakhstan, moldir.muratbekova@ayu.edu.kz

Abstract

This work is devoted to the study of the solvability of the initial-boundary value problem for a hyperbolic equation with involution. In the problem under consideration, the order of the boundary operators exceeds the order of the equation. Theorems on the existence and uniqueness of the problem solution under study are proved. The problem is studied by the Fourier method and the explicit form of the solution of the problem is built in the form of a series. According to the classification given in A.M. Nakhushev's book [1], nonlocal equations include equations in which an unknown function and its derivatives appear, generally speaking, for different values of the arguments. Among nonlocal differential equations, a special place is occupied by equations in which the deviation of the arguments has an involutive character. A mapping is usually called an involution if $I^2 = E$, E - is the identity mapping. To date, for differential equations with various types of involution, the well-posedness of boundary and initial-boundary value problems, the qualitative properties of solutions and spectral questions, as well as inverse problems for heat equations and their fractional analogs, have been studied quite well.
Let $\Omega = \{(x,t) : 0 < x < p, 0 < t < T\}$, a_0, a_1 - be some real numbers. In this paper, we have studied the following problem

$$u_{tt}(x,t) - a_0 u_{xx}(x,t) - a_1 u_{xx}(p-x,t) = f(x,t), (x,t) \in \Omega, \tag{1}$$

$$u(0,t) = u(p,t) = 0, 0 \le t \le T, \tag{2}$$

$$u_t^{(k)}(x,0) = \varphi_k(x), u_t^{(k+1)}(x,0) = \psi_k(x), 0 \le x \le p, \tag{3}$$

where $k \ge 1$, $f(x,t), \varphi_k(x)$ and $\psi_k(x)$ predefined functions. Note that problem (1)-(3) in the case $a_0 = 1, a_1 = 0$ was studied in [2].
For problem (1)-(3) theorems on the existence and uniqueness of the solution are proved.
This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP09259074).

Keywords: involution, nonlocal equation,hyperbolic equation.
2020 Mathematics Subject Classification Numbers: 34K06, 35L10,35L20.

References

[1] A.M. Nahushev, *Equations of Mathematical Biology*, Moscow: Nauka, 1995 (in Russian).

[2] D.Amanov, G.Ibragimov, A. Kiliçman, Generalization of the Initial-Boundary Problem for the Vibrating String Equation, *Symmetry* 11, 1-10 (2019).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Direct and inverse problems for a two-dimensional parabolic equation with involution

Moldir Muratbekova, Zhazira Yerkisheva
Khoja Akhmet Yassawi International Kazakh-Turkish University,Turkistan,Kazakhstan, moldir.muratbekova@ayu.edu.kz
Khoja Akhmet Yassawi International Kazakh-Turkish University,Turkistan,Kazakhstan,zhazira.erkisheva@ayu.edu.kz

Abstract

The work is devoted to the study of solvability of direct and inverse problems for a two-dimensional parabolic equation with involution. The studied problems are solved by reducing them to direct and inverse problems for classical two-dimensional differential equations of parabolic type. On the basis of well-known theorems obtained for auxiliary problems, theorems on the existence and uniqueness of the solution of the studied problems are proved. The explicit form of solutions of the studied problems is constructed in the form of a series.

In this paper, using mappings of the involution type, we introduce a nonlocal analogue of the two-dimensional Laplace operator and consider the corresponding two-dimensional differential equation of parabolic type with involution. For this equation, the direct and inverse problems of finding the factors of the right-hand side, depending on the spatial variables, are studied.

It should be noted that considered problems for the classical case were studied in [1].

This research is funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08855810).

Keywords: involution, Direct and inverse problems,parabolic equation.
2020 Mathematics Subject Classification Numbers: 34K06, 35K20,80A23.

References

[1] K. B. Sabitov,A. R. Zainullov, Inverse problems for a two-dimensional heat equation with unknown right-hand side, *Russian Mathematics*, 65:3, 75–88 (2021).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Parallel algorithm for solving the inverse problem of identifying the right-hand part of the time-fractional diffusion equation

Murat A. Sultanov[1], Vladimir E. Misilov[2,3], Yerkebulan Nurlanuly[1]
[1]Khoja Akhmet Yassawi International Kazakh-Turkish University, Turkistan, Kazakhstan, murat.sultanov@ayu.edu.kz, yerkebulan.nurlanuly@ayu.edu.kz
[2]Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg, Russia, v.e.misilov@urfu.ru
[3]Ural Federal University, Ekaterinburg, Russia

Abstract

The paper considers the parallel algorithm for solving the inverse problem of identifying the time-dependent right-hand part of a time-fractional diffusion equation. After discretization and approximation of the auxiliary loaded equation, the problem is reduced to a system of linear algebraic equations with a large tridiagonal matrix. On the basis of the parallel sweep method, a parallel algorithm is implemented for multicore processors.
Keywords: Fractional differential equations, inverse problems, parallel algorithms.

2020 Mathematics Subject Classification Numbers: 35R11, 35R30, 65Y05.

This work considers the parabolic partial differential equation with time-fractional derivative

$$\frac{\partial^\alpha U(x,t)}{\partial t^\alpha} = a(x)\frac{\partial^2 U(x,t)}{\partial x^2} + b(x)\frac{\partial U(x,t)}{\partial x} + c(x)U(x,t) + f(x,t),$$

where $U(x,t)$ is the unknown function, $a(x), b(x), c(x)$ are known coefficient functions, $0 < \alpha < 1$ is the parameter defining the fractional order of the time derivative, $f(x,t)$ is the right-hand part function. The derivative $\partial^\alpha U(x,t)/\partial t^\alpha$ is the Caputo fractional derivative [1].

The problem is on the space interval $0 \le x \le \ell$ and time interval $0 \le t \le T$. The boundary conditions and the initial condition are

$$U(0,t) = g_1(t), \quad U(\ell,t) = g_2(t), \quad U(x,0) = g_0(x),$$

where $g_0(x), g_1(t), g_2(t)$ are the given functions.

We consider the inverse problem, which consists in finding the unknown function $U(x,t)$, as well as, the right-hand part $f(x,t)$. Assume that the function $f(x,t)$ has the form $f(x,t) = \eta(t) \cdot \psi(x)$, where $\psi(x)$ is a known function and $\eta(t)$ is the sought time-dependent function. To find this function, we use the idea [2] of introducing the *a priori* information about the solution in some inner spatial point $0 < x^* < \ell$

$$U(x^*,t) = \varphi(t).$$

Then, the inverse problem may be formulated as the initial boundary problem for the auxiliary loaded equation. After discretizing space and time on a uniform grid and approximating the equations using an implicit finite difference scheme, the problem is reduced to a system of linear algebraic equations with a large tridiagonal matrix. To solve it, this work uses the parallel sweep algorithm [3].

The parallel algorithm is implemented for multicore processors. Numerical experiments were carried out to evaluate the efficiency of parallelization.

The first author (M.A.S.) and third author (Y.N.) were financially supported by the Ministry of Education and Science of the Republic of Kazakhstan (project AP09258836).

References

[1] Y. Zhang, A finite difference method for fractional partial differential equation, *Appl. Math. Comput.* 215, 524–529 (2009).

[2] A. A. Samarskii, P. N. Vabishchevich, *Numerical Methods for Solving Inverse Problems of Mathematical Physics*, De Gruyter, Berlin, 2007.

[3] M. A. Sultanov, E. N. Akimova, V. E. Misilov, Y. Nurlanuly. Parallel Direct and Iterative Methods for Solving the Time-Fractional Diffusion Equation on Multicore Processors, *Mathematics* 10(3), art. no. 323 (2022).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Estimates of eigenvalues of a semiperiodic dirichlet problem for a class of degenerate elliptic equations

Mussakan Muratbekov, Sabit Igissinov
M.Kh. Dulaty Taraz Regional University, Taraz, Kazakhstan, musahan_m@mail.ru
M.Kh. Dulaty Taraz Regional University, Taraz, Kazakhstan, igisinovsabit@mail.ru

Abstract

In this paper we consider a class of degenerate elliptic equations with arbitrary power degeneration. The issues about the existence, uniqueness, and smoothness of solutions of the semiperiodic Dirichlet problem for a class of degenerate elliptic equations with arbitrary power degeneration are studied. The two-sided estimates for singular numbers (s-numbers) are obtained. Note that estimates of singular numbers (s-numbers) show the rate of approximation of the found solutions by finite-dimensional subspaces. Here we also obtain estimates for the eigenvalues.

The results of this work are close to those of M.B. Muratbekov [1-2], where differential operators of mixed and hyperbolic types were investigated. In contrast to the above papers, here we investigate previously unconsidered degenerate elliptic equations with an arbitrary power-law degeneracy on the degeneracy line.

Let $\Omega = \{(x, y) : -\pi < x < \pi, 0 < y < 1\}$. Consider the following problem

$$Lu = -k(y)u_{xx} - u_{yy} + a(y)u_x + c(y)u = f(x, y) \in L_2(\Omega), \tag{1}$$

$$u(-\pi, y) = u(\pi, y), u_x(-\pi, y) = u_x(\pi, y), \tag{2}$$

$$u(x, 0) = u(x, 1) = 0, \tag{3}$$

where $a(y), c(y)$ are piecewise continuous functions in $[0, 1]$, $k(y) > 0$ as $y \in (0, 1]$ and $k(0) = 0$. Let $C^{\infty}_{0,\pi}(\bar{\Omega})$ is a class of infinitely differentiable finite functions in $\bar{\Omega}$ and satisfying the conditions (2)-(3).

Closure of the operator L by the norm of $L_2(\Omega)$ we also denote by L.

Theorem Let $a(y), c(y)$ are piecewise continuous functions in $[0, 1]$ and satisfying the conditions

$$i)|a(y)| \geq \delta_0 > 0, c(y) \geq \delta > 0.$$

Then for the eigenvalues of L^{-1} the following estimate

$$|\lambda_k| \leq \frac{c \cdot e^{\frac{1}{2}}}{k^{\frac{1}{2}}}, k = 1, 2, 3, ...,$$

holds, where λ_k are the eigenvalues of the operator L^{-1}.

Keywords: degenarate elliptic operator; boundary value problem; eigenvalues.
2020 Mathematics Subject Classification Numbers: 35J70.

References

[1] M.B. Muratbekov, M.M. Muratbekov, A.M. Abylayeva, On existence of the resolvent and discreteness of the spectrum of a class of differential operators of hyperbolic type, *Electronic Journal of Theory of Differential Equations*, 63, 1-10 (2013).

[2] M.B. Muratbekov, M.M. Muratbekov, Estimates of spectrum for a class of mixed type operators, *Differential equations*, 43, 143-146 (2007).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On eigenvalues of the perturbed differentiation operator on a segment

Nurlan S. Imanbaev
Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan
South Kazakhstan State Pedagogical University, Shymkent, Kazakhstan, imanbaevnur@mail.ru

Abstract

In the functional space $W_2[-1,1]$, we consider the eigenvalue problem of the loaded differential operator

$$L_1 y = y'(t) + \lambda y(-1)\Phi(t) = \lambda y(t), \quad -1 \le t \le 1 \tag{1}$$

with the boundary value condition

$$y(-1) = y(1), \tag{2}$$

where Φ is a function with bounded variation and $\Phi(-1) = \Phi(1) = 1$, $\lambda \in \mathbb{C}$ is a spectral parameter.
It is required to find the complex values λ for which the operator equation (1) has non-zero solutions.
One of features of the considered problem, adjoint to (1)-(2), is the spectral problem with occurrence of the spectral parameter $\overline{\lambda}$ into the boundary value condition with the integral perturbation:

$$L_1^* v = v'(t) = \overline{\lambda} v(t), \quad -1 \le t \le 1 \tag{3}$$

with the boundary value condition

$$v(-1) - v(1) = -\overline{\lambda} \cdot \int_1^1 v(t)\Phi(t)dt, \tag{4}$$

where Φ is a function with bounded variation and $\Phi(-1) = \Phi(1) = 1$, $\overline{\lambda} \in \mathbb{C}$ is a spectral parameter.
Lemma 1. The characteristic determinant $\Delta_1(\lambda)$ of the spectral problems (1)-(2) and (3)-(4) is represented as follows

$$\Delta_1(\lambda) = e^{-\lambda} - e^{\lambda} + \lambda \cdot \int_{-1}^{1} e^{\lambda t}\Phi(t)dt \tag{5}.$$

Due to the formula (5), conclusions about eigenvalues of the first-order differential operators L_1 and L_1^* are established. We get the following result.
Theorem 1. If Φ is a function of bounded variation and $\Phi(-1) = \Phi(1) = 1$, then all zeros of the entire function $\Delta_1(\lambda)$, that is, all eigenvalues of differentiation operators L_1 and L_1^* belong to the strip $|Re\lambda| = |x| < k$, for some k, where $\lambda = x + iy$, and also form a countable set and have asymptotics $\lambda_n^1 = in\pi + O(1)$ as $n \to \infty$.

Keywords: Loaded differential operator, eigenvalue, boundary value condition.
2020 Mathematics Subject Classification Numbers: 34B09, 15A18, 34L20.

References

[1] B.E. Kanguzhin, M.A. Sadybekov, *Differential operators on a segment. Distribution of eigenvalues. Shymkent*, Gylym, 1996.

[2] N.S. Imanbaev, M.A. Sadybekov, Basic properties of root functions of loaded differential operators of the second order, *Reports of the National Academy of Sciences of the Republic of Kazakhstan*, 2, 11-13 (2010).

[3] N.S. Imanbaev, On nonlocal perturbation of the problem on eigenvalues of differentiation operator on a segment, *Vestnik Udmurtskogo Universiteta. Matematika. Mekhanika. Komp'yuternye Nauki*, 31:2, 186-193 (2021).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Numerical method for solving the conformable time-space fractional convective heat equation with a source using shifted Legendre collocation method

Saad Abdelkebir[1] Brahim Nouiri[2]
Department of Mathematics, University of M'sila, Algeria.
e-mail: saad.abdelkebir@univ-msila.dz
Laboratory of Pure and Applied Mathematics, University of M'sila, Box 166, Ichbilia, 28000, M'sila, Algeria.
e-mail: brahim.nouiri@univ-msila.dz

Abstract

In this paper our aim is to find the solutions of time and space fractional convective heat equations with a source by using new definition of fractional derivative called **Conformable fractional derivative.**
In order to find approximate solutions to the convective heat equations with a source, we use Legendre collocation method, along with Euler's method to solve the first order differential equation. We provide illustrative examples of the convective heat equations with a source and give the exact solution. Using Legendre collocation method, we extract the approximate solution and the error made between the exact solution and the approximate solution.

Keywords: Conformable fractional derivative, Legendre polynomials, Legendre collocation method, Euler method.
2020 Mathematics Subject Classification Numbers: 39B82, 44B20, 46C05.

References

[1] T. Abdeljawad. On conformable fractional calculus. Journal of computational and Applied Mathematics, 279:57-66, 2015.

[2] S. Abdelkebir and B. Nouiri. Analytical solution for the conformable fractional telegraph equation by fourier method. Proceedings of International Mathematical Sciences, 2(1):1-6.

[3] B. Nouiri. Numerical approach of the nonlinear reaction-advection-diffusion equa- tion with time-space conformable fractional derivatives. AIP Conference Proceedings, 2334(1):060012, 2021.

[4] A. Saad and N. Brahim. An efficient algorithm for solving the conformable time-space fractional telegraph equations. Moroccan Journal of Pure and Applied Analysis, 7(3):413-429, 2021.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Adomian Decomposition Method for First Order Linear PDE Systems with Unprescribed Data

Tzon-Tzer Lu
Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung, Taiwan, ttlu@math.nsysu.edu.tw

Abstract

In this paper we like to explore the full power of Adomian decomposition method (ADM), specially its symbolic capability [1,2]. We will demonstrate this method, together with splitting technique, to compute the explicit closed-form solutions of first order linear systems of partial differential equations with unprescribed initial conditions, and even with parameters. These features are those normal numerical methods fail to do. Our examples include many prototype hyperbolic and elliptic systems possessing analytical solutions, e.g. the linearised equations of gas dynamics. We conclude that ADM is far more powerful than existing numerical methods.

Keywords: Adomian decomposition method, splitting technique, closed-form solution, power series, unprescribed data, PDE system.
2020 Mathematics Subject Classification Numbers: 35C10, 35E15, 35F40, 35L45, 41A58.

References

[1] T.T. Lu and W.Q. Zheng, *Adomian decomposition method for first order PDEs with unprescribed data, Alexandria Engineering Journal*, 60(2): 2563-2572 (2021).

[2] D. Zeidan, C.K. Chau and T.T. Lu, *On the development of Adomian decomposition method for solving PDE systems with non-prescribed data*, Computational and Applied Mathematics, 41: 87 (2022).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Approximations with discrete boundary value problems

Vladimir Vasilyev, Alexander Vasilyev, Anastasia Khodyreva
Belgorod State National Research University, Belgorod, Russia, vladimir.b.vasilyev@gmail.com

Abstract

Let $K = \{x \in \mathbb{R}^2 : x = (x_1, x_2), x_1 > 0, x_2 > 0\}$ be first quadrant in a plane. We consider the following boundary value problem

$$\begin{cases} (Au)(x) = 0, \quad x \in K, \\ \int\limits_0^{+\infty} u(x_1, x_2)dx_1 = f(x_2), \quad \int\limits_0^{+\infty} u(x_1, x_2)dx_2 = g(x_1), \\ \int\limits_K u(x)dx = 0. \end{cases} \tag{1}$$

where A is a pseudo-differential operator with the symbol $A(\xi), \xi = (\xi_1, \xi_2)$ satisfying the condition

$$c_1(1 + |\xi)^\alpha \le |A(\xi)| \le c_2(1 + |\xi)^\alpha,$$

and admitting the wave factorization [1] with respect to K with the index æ such that $æ - s = 1 + \delta, |\delta| < 1/2$. A unique solvability for the problem (1) in Sobolev–Slobodetskii space $H^s(K)$ is proved in [2] under the condition $f, g \in H^{s+1/2}(\mathbb{R}_+)$.

Let $\mathbb{Z}^2$ be an integer lattice in a plane. Let us denote $K_d = h\mathbb{Z}^2 \cap K, h > 0$. We introduce functions of a discrete variable $u_d(\tilde{x}), \tilde{x} = (\tilde{x}_1, \tilde{x}_2) \in h\mathbb{Z}^2$. According to [3] we define the discrete Schwartz space $S(h\mathbb{Z}^2)$, the discrete Sobolev–Slobodetskii space $H^s(K_d)$ and the digital pseudo-differential operator A_d.

Further, we study a solvability of the discrete boundary value problem in the space $H^s(K_d)$

$$\begin{cases} (A_d u_d)(\tilde{x}) = 0, \quad \tilde{x} \in K_d, \\ \sum\limits_{\tilde{x}_1 \in h\mathbb{Z}_+} u_d(\tilde{x}_1, \tilde{x}_2)h = f_d(\tilde{x}_2), \quad \sum\limits_{\tilde{x}_2 \in h\mathbb{Z}_+} u_d(\tilde{x}_1, \tilde{x}_2)h = g_d(\tilde{x}_1), \\ \sum\limits_{\tilde{x} \in h\mathbb{Z}_{++}} u_d(\tilde{x}_1, \tilde{x}_2)h^2 = 0. \end{cases} \tag{2}$$

and give a comparison between solutions of problems (1) and (2).

Using a special choice for discrete functions f_d, g_d and elements of periodic wave factorization we can obtain the following result.

Theorem. *Let $f, g \in S(\mathbb{R}), æ > 1$. Then the discrete boundary value problem (2) is uniquely solvable and for solutions u and u_d of continuous problem (1) and its discrete variant (2) the following estimate*

$$|u(\tilde{x}) - u_d(\tilde{x})| \le C(f, g)h^\beta,$$

holds, where $C(f, g)$ depends on functions f, g, $\beta > 0$ is an arbitrary number.

Keywords: Digital pseudo-differential operator, discrete boundary value problem, approximation property.
2020 Mathematics Subject Classification Numbers: 35S05, 35S15, 47G30.

References

[1] V.B. Vasil'ev, *Wave Factorization of EllipticSymbols: Theory andApplications*, Kluwer Academic Publishers, 2000.

[2] V.B. Vasil'ev, On some new boundary value problems in nonsmooth domains, *Journal of Mathematical Sciences*, 173, 225-230 (2011).

[3] A.V. Vasilyev, V.B. Vasilyev, Pseudo-differential operators and equations in a discrete half-space, *Mathematical Modelling and Analysis*, 23, 492-506 (2018).

6.6 Computer Science and Technology

Session Organizers: Şahin Uyaver and Önder Şahinaslan

The session of "Computer Science and Technology" of International Conference of Mathematical Sciences organized by Maltepe University of Istanbul, Turkey was held between 20-24 July 2022.

The session was attracted by many local and international scientists. During the talks the participants had the chance to ask their questions or make their contributions. The talks covered many trending problems from fundamental science and engineering sciences involved in computer science and technology. In this respect the session of the conference is believed to make a good contribution to the related literatures.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

A Solution to the Remote Secure Digital Identification Problem: The Case of Turkey

Abdullah Koksal, Önder Sahinaslan, Ender Sahinaslan
Maltepe University, Department of Informatics, Istanbul, Turkey
abdkoksal@yahoo.com, ondersahinaslan@maltepe.edu.tr, dr.endsa@gmail.com

Abstract. With the rapid developments in information technologies and the current pandemic, rapid changes are experienced in the expectations, habits and needs of the society. One of these changes is the explosion of demand for services offered or received remotely. Solutions and services to be produced in response to increasing demands have to be legal and safe, as well as individual expectations. At the same time, remote transactions must be as secure as face-to-face transactions. It is important to control and detect the security elements on the identity card in order to make the correct identification of the person through the qualified documents provided during these processes. However, remote control of digital ID cards is very difficult and prone to human error. This situation also has the potential to turn into security problems such as forgery and fraud. In the solution of such problems, it is necessary to develop computer-assisted control environments in the determination of qualified identity documents. This study was carried out on digital identity cards, which is one of the qualified identity documents in Turkey. A solution has been developed to detect the authenticity of the identity presented by using the security elements on this digital card. Thanks to this computer-aided solution, it has contributed to the prevention of many incidents such as forgery and fraud before they occur. At the same time, it has become a solution to overcome an obstacle in front of the services to be provided remotely, to prevent human-induced errors, to the efficiency of operational processes and to securely remote control of digital identities.

Keywords: Information Technologies, TR Digital Identity, Remote Acquisition, Secure Identity Detection, Smart Control
2020 Mathematics Subject Classification Numbers: 68M25, 68U05, 68U10, 68W40

References

[1] Bataineh B. A fast and memory-efficient two-pass connected-component labeling algorithm for binary images. Turkish Journal of Electrical Engineering & Computer Sciences 2018, (2019) 27 pp.1243-1259 doi:10.3906/elk-1703-351

[2] Mutlugün M, Adalier O. Turkish national electronic identity card. In Proceedings of the 2nd international conference on Security of information and networks (SIN '09). Association for Computing Machinery, New York, NY, USA, 2009, pp.14–18. doi: 10.1145/1626195.1626201

[3] Rusli F M, Adhiguna K A, Irawan H. Indonesian ID card extractor using optical character recognition and natural language post-processing 2021. 9th International Conference on Information and Communication Technology (ICoICT), 2021, pp. 621-626, doi: 10.1109/ICoICT52021.2021.9527510.

[4] Kareem M. A, Abed D. M, Jaber A. M, Rodhan A. Improving security of ID card and passport using cubic spline curve. Iraqi Journal of Science,2022, 57(4A), pp.2529–2538. ISSN: 0067-2904

[5] Król M, Kowalska D, Kościelniak P. Examination of polish identity documents by laser-induced breakdown spectroscopy, Analytical Letters, 2018. 51:10, pp.1592-1604, doi: 10.1080/00032719.2017.1384833

[6] Mahmoud Mohamed Khalil M. The latest security techniques used in passport design. Journal of Architecture, Art & Humanistic Science 2019. doi: 10.12816/mjaf.2019.25814

[7] Sahinaslan O, Sahinaslan E. Cross-object information security: A study on new generation encryption. AIP Conference Proceedings 2019; vol. 2086, 030034. doi: 10.1063/1.5095119

[8] Sahinaslan O, Sahinaslan E, Gunes E. Review of the contributions of contactless payment technologies in the COVID-19 pandemic process. AIP Conference Proceedings 2021; vol.2334, 070002. doi: 10.1063/5.0042225

[9] Hartl A, Arth C, Schmalstieg D. AR-based hologram detection on security documents using a mobile phone. Advances in Visual Computing. ISVC 2014. Lecture Notes in Computer Science, 2014 vol 8888. Springer, Cham. doi: 10.1007/978-3-319-14364-4_32

[10] M. Yücel, "T.C. Kimlik kartı yönetim ve dağıtım sistemi", UEKAE Dergisi, vol. 2, no. 4, pp. 35-41, 2010. (in Turkish).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Comparative Analysis of First and Second Order Methods for Optimization in Neural Networks

Auras Khanal[1], Mehmet Dik[2],
[1] Beloit College, Beloit, USA, hanalab@beloit.edu
[2] Beloit College, Beloit, USA, dikm@beloit.edu

Abstract

Artificial Neural Networks are fine tuned to yield the best performance through an iterative process where the values of their parameters are altered. Optimization is the preferred method to determine the parameters that yield the minima of the loss function, an evaluation metric for ANN's. However, the process of finding an optimal model which has minimum loss faces several obstacles, the most notable being the efficiency and rate of convergence to the minima of the loss function. Such optimization efficiency is imperative to reduce the use of computational resources and time when training Neural Network models. This paper reviews and compares the intuition and effectiveness of existing optimization algorithms such as Gradient Descent, Gradient Descent with Momentum, RMSProp and Adam that implement first order derivatives, and Newton's Method that utilizes second order derivatives for convergence. It also explores the possibility to combine and leverage first and second order optimization techniques for improved performance when training Artificial Neural Networks.

Keywords: Optimization, Artificial Neural Networks, Loss Function, Gradient Descent, Newton's Method.
2020 Mathematics Subject Classification Numbers: 68T07

References

[1] Bre, F., Gimenez, J., and Fachinotti, V. (2017). Prediction of wind pressure coefficients on building surfaces using artificial neural networks. Energy and Buildings, 158.

[2] Brownlee, J. (2021). Code adam optimization algorithm from scratch. Machine Learning Mastery.

[3] Deepanshi (2021). Artificial neural network: Beginners guide to ann. Analytics Vidhya.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Vulnerability of Banana Trees

Hande Tunçel Gölpek
Dokuz Eylul University, Izmir, Turkey, hande.tuncel@deu.edu.tr

Abstract

One of the most important research topics about complex networks is examination of their vulnerability. Therefore, there are many studies in the literature about analyzing the robustness and reliability of networks using graph theoretical parameters. Among these parameters, the centrality parameters play an important role. The closeness parameters and its derivatives are widely discussed. In this study, the closeness parameter and the more sensitive parameter residual closeness which is based on closeness parameter have been considered. Furthermore, the closeness and residual closeness of banana tree structure have been calculated.

Keywords: Vulnerability, Graph Theory, Closeness, Residual Closeness.
2020 Mathematics Subject Classification Numbers: 68R10, 05C40, 05C05, 05C12, 05C76.

References

[1] Aytac A., Odabas Z.N. Residual closeness of wheels and related networks. IJFCS 2011;22(5):1229-1240. URL https://doi.org/10.1142/S0129054111008660.

[2] Aytaç A., Odabaş Berberler Z.N., TWMS Journal of Applied and Engineering Mathematics (TWMS J. of Apl. & Eng. Math.), " Residual Closeness For Helm And Sunflower Graphs ", Vol. 7, issue No 2, pp. 209-220, 2017.

[3] Aytaç A., Odabaş Berberler Z.N., RAIRO-Operations Research, " Network robustness and residual closeness", Vol. 52, issue No 3, pp. 839-847, 2018.

[4] Aytaç A., Odabaş Berberler Z.N., International Journal of Foundations of Computer Science, " Robustness of regular caterpillars", Vol. 28, issue No 7, pp. 835-841, 2017.

[5] Aytac V., Turaci T. Closeness centrality in some splitting networks. Computer Science Journal of Moldova. 2018;26(3):251-269. ID: 57760763.

[6] Berberler ZN, Yigit E., Link Vulnerability in Networks. IJFCS. 2018; 29(03):447-456. URL https://doi.org/10.1142/S0129054118500077.

[7] Cormen T. H., Leiserson C. E., & Rivest, R. L. Introduction to algorithms. MIT Press, Cambridge, MA; McGraw-Hill Book Co., New York (1990)..

[8] Dangalchev Ch., Residual closeness in networks, Physica A Statistical Mechanics and Its Applications, vol. 365,, 556-564, (2006).

[9] Dangalchev Ch., Residual Closeness of Generalized Thorn Graphs. Fundamenta Informaticae. 2018;162(1), 2018, p 1-15. di: 10.3233/FI-2018-1710.

[10] Dangalchev Ch. Closeness of Splitting Graphs. C.R. Acad. Bulg. Sci. 2020; 73(4): 461-466.

[11] Dangalchev Ch. Residual closeness and generalized closeness. IJFCS. 2011;22(8):1939-1947. doi:10.1016/j.physa.2005.12.020.

[12] Freeman L.C., Centrality in social networks: conceptual clarification, Social Networks 1 (1979) 215.

[13] Latora V., Marchiori M., Efficient behavior of small-world networks, Phys. Rev. Lett. 87 (2001).

[14] Odabas Z.N, Aytac A. Residual closeness in cycles and related networks. Fundamenta Informaticae.2013;124 (3): 297-307. doi:10.3233/FI-2013-835.

[15] Turaci T., Okten M. Vulnerability of Mycielski graphs via residual closeness, Ars Combinatoria. 2015; 118: 419-427.

[16] Turacı T. and Aytaç V.; " Residual closeness of splitting networks ", Ars Combin. 130 (2017), 17–27.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Variable Elimination Algorithm in Bayesian Networks: An Updated Version

Linda Smail,
College of Interdisciplinary Studies, Zayed University, Dubai UAE, linda.smail@zu.ac.ae

Abstract

Given a Bayesian network [1, 2] relative to a set of random variables $(X_i)_{i\in I}$, we are interested in computing conditional probabilities of events related to one another. This kind of computations is called inference in Bayesian Networks (BNs) [3]. Using Bayes' theorem [4], we reduce the computation of conditional probabilities to the ratio of two joint probabilities, then we compute each of the marginal probabilities apart. This is essentially an optimization calculation problem, as it becomes increasingly heavy following the complexity of the graph relative to both the number of variables and the number of values taken by these variables.
One approach for eliminating variables from BNs is considered here, the Variable Elimination algorithm (VE) of Dechter [5], which appeared for the first time in Zhang and Poole [6]. So called because it eliminates by marginalization variables one after the other, the VE algorithm aims to compute an arbitrary joint distribution on a subset of variables, given a set of evidence variables. The main idea of this approach is to sum over a set of variables from a list of factors one by one; an ordering of these variables is required as an input and is called an elimination ordering. The computation depends on the order of elimination, i.e., different elimination orders produce different factors.
In this work, we propose an updated version to the Variable Elimination algorithm that will allow writing all intermediate computations as probability distributions and not as simple potentials as the case with the VE algorithm. This property is very important, as at any step of the computation, the resulting probability distribution can still be factored as a product of conditional probability distributions and not in an extensive form.

Keywords: Bayesian Networks, Variable Elimination Algorithm, Inference.

2020 Mathematics Subject Classification Numbers: 62F15

References

[1] F. V. Jensen, *An Introduction to Bayesian Networks* UCL Press, London 1996..

[2] F. V. Jensen, *Bayesian Networks and Decision Graphs* Springer, 2001.

[3] J. Pearl, *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*, Morgan Kaufmann Publishers Inc., San Francisco, CA, 1988.

[4] A. Paz, J. Pearl, Axioms and Algorithms for Inferences Involving Probabilistic Independence, *Information and Computation*,91, 128-141, (1991).

[5] R. Dechter, Bucket Elimination: A Unifying Framework for Probabilistic Inference, *In UAI, eds. E. Horvitz and F. Jensen, San Francisco, CA, Morgan Kaufmann* 211-219, (1996).

[6] N. L. Zhang, D. Poole, Exploiting causal independence in Bayesian network inference, *Journal of Artificial Intelligence Research*, 5, 301-328, (1996).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

The Use and Success of Machine Learning Algorithms in Improving the Customs Declaration Process

Mustafa Gunerkan, Önder Sahinaslan, Ender Sahinaslan
Maltepe University, Department of Informatics, Istanbul, Turkey
mgunerkan@gmail.com, ondersahinaslan@maltepe.edu.tr, dr.endsa@gmail.com

Abstract. It is important that the customs declaration preparation process is carried out completely and without errors in international transportation. This process step is a serious business that requires knowledge, experience and expertise, and legislation and updates are strictly followed. Customs declarations must be submitted accurately and completely to the relevant customs office on time. Otherwise, it may cause many serious risks and problems such as compliance, delays, extra operations and workforce. On the other hand, the preparation of customs declarations has a very complex structure and requires detailed knowledge and experience. In the absence of competent and sufficient human resources, there are many operational errors. Due to these mistakes, serious fines are encountered and in some cases, they may even cause legal problems. There is a need to create intelligent control structures in order to reduce the errors in the customs declaration preparation process, to improve the current process and to work with the least possible errors. Existing data on the way to establish intelligent systems and the use of this data with machine learning applications has now become a necessity. In this study, known common machine learning algorithms were run on the data of an international logistics company and high success rates were obtained. The successful results revealed that machine learning algorithms can be used effectively in this field. Additional control points supported by artificial intelligence have been put in order to improve the process. A structure has been developed that can offer the user a suggestion for each model. Thus, many operational errors and risks that cause various problems were controlled and detected in advance, and this process contributed to the improvement. This study is an example of the successful use of current technologies such as machine learning and artificial intelligence in customs transactions. This will also lead to smarter new studies and encourage such studies.

Keywords: Machine Learning Algorithms, Artificial Intelligence, International Transport, Customs Declaration, Smart Control
2020 Mathematics Subject Classification Numbers: 68P30, 68T07, 90B06, 94A29

References

[1] Sahinaslan, E., Sahinaslan, O. (2019), "A Study on Modern Information Technologies in the Journey of Industrial Transformation", International Conference of Mathematical Sciences, ISBN 9786052124291, Maltepe University, Istanbul

[2] Ilgun, F., Sahinaslan, O., Sahinaslan, E. (2021), "EDI Technology Contributing to Logistics Service Performance and an Application Example", International Conference of Mathematical Sciences, ISBN 9786052124321, Maltepe University, Istanbul

[3] N. M. Abdulkareem, A. M. Abdulazeez, Machine Learning Classification Based on Radom Forest Algorithm: A Review, International Journal of Science and Business, IJSAB International, 5(2), 128-142.B (2021)

[4] Alasadi, S. A., & Bhaya, W. S. (2017). Review of Data Preprocessing Techniques in Data Mining. Journal of Engineering and Applied Sciences, 12(16), 4102-4107.

[5] Barua, L., Zou, B., & Zhou, Y. (2020). Machine learning for international freight transportation management: A comprehensive review. Research in Transportation Business & Management, 100453

[6] Sahinaslan, O. , Dalyan, H. & Sahinaslan, E. (2022), "Multilingual Sentiment Analysis on YouTube Data Using Naive Bayes Classifier", Journal of Information Technologies, 15 (2) , 221-229 . DOI: 10.17671/gazibtd.999960

[7] Roshan, S. E., & Asadi, S. (2020). Improvement of Bagging performance for classification of imbalanced datasets using evolutionary multi-objective optimization. Engineering Applications of Artificial Intelligence, 87, 103319

[8] Calp, M., & Akcayol, M., 2020. Design and Implementation of Web Based Risk Management System Based on Artificial Neural Networks for Software Projects: WEBRISKIT. Pamukkale Univ Muh Bilim Derg., 26(5), 993-1014

[9] Huang, J.-C., Ko, K.-M., Shu, M.-H., & Hsu, B.-M. (2019). Application and comparison of several machine learning algorithms and their integration models in regression problems. Neural Computing and Applications

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On the efficiency of LSTM in classifying musical impressions from EEG recordings

Burak Kaya[1], M. Gökhan Habiboğlu[2], Sanam Moghaddamnia[3]
[1] Turkish-German University, Istanbul, Turkey, kayaburaak1@gmail.com
[2] Turkish-German University, Istanbul, Turkey, habiboglu@tau.edu.tr
[3] Turkish-German University, Istanbul, Turkey, moghaddamnia@tau.edu.tr

Abstract

The objective of this study is the classification of musical impressions with LSTM approach using EEG recordings of 20 subjects, while listing to 10 different music genres [1]. For this purpose, a deep learning model was developed, where relevant features extracted from intrinsic mode functions (IMF) of the clean EEG data are used as the input signals. The classification accuracy of the proposed model is evaluated with various feature sets.

Keywords: EEG, LSTM, EMD, Classification.
2020 Mathematics Subject Classification Numbers: 68T07

References

[1] Krishna Prasad Miyapuram and Pankaj Pandey and Nashra Ahmad and Bharatesh R Shiraguppi and Esha Sharma and Prashant Lawhatre and Dhananjay Sonawane and Derek Lomas (2021). Music Listening- Genre EEG dataset (MUSIN-G). OpenNeuro. [Dataset] doi: 10.18112/openneuro.ds003774.v1.0.0

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Parameter and feature selection in decision trees for the classification of musical impressions from EEG records

Emir Atakan Özaltun[1], Sanam Moghaddamnia[2], M. Gökhan Habiboğlu[3],
[1] Turkish-German University, Istanbul, Turkey, atakanozaltun@gmail.com
[2] Turkish-German University, Istanbul, Turkey, moghaddamnia@tau.edu.tr
[3] Turkish-German University, Istanbul, Turkey, habiboglu@tau.edu.tr

Abstract

Reliable classification of different emotions is an important issue for emotional interaction between humans and computers. Therefore, this study aims at assessing the performance of decision trees in classifying musical impressions from EEG records of 20 subjects listened to 10 songs of different music styles [1]. First, features extracted from the clean EEG data are used to train the classifier, where different feature combinations and parameter settings are considered. Next, the impact of various hyperparameter values on the classification accuarcy is examined and the relevant feature combination is specified.

Keywords: Decision trees, Classification, EEG.
2020 Mathematics Subject Classification Numbers: 68T05

References

[1] Krishna Prasad Miyapuram and Pankaj Pandey and Nashra Ahmad and Bharatesh R Shiraguppi and Esha Sharma and Prashant Lawhatre and Dhananjay Sonawane and Derek Lomas (2021). Music Listening- Genre EEG dataset (MUSIN-G). OpenNeuro. [Dataset] doi: 10.18112/openneuro.ds003774.v1.0.0

6.7 Mathematical Methods in Physics

Session Organizers: Özay Gürtuğ and Filiz Çağatay Uçgun

The session “Mathematical Methods in Physics” is organized in ICMS 2022, Maltepe University, Istanbul, Turkey, on 20th - 24th July, 2022. The programme of this session is mainly oriented towards some recent developments in nonlinear systems, In-plane waves, special functions, boundary problems and some relevant mathematical methods.

We hope that all attending this meeting will recall it as a useful and pleasant event. We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. And special thanks to our sponsors for their financial supports, which were very significant for realization of this scientific activity.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Approximate analytical solutions of the Schrödinger equation in central potential field

Aysel Özfidan
Tarsus University, Mersin, Turkey, ayselozfidan@tarsus.edu.tr

Abstract

Approximate l-state solutions of the Schrödinger equation with spherical symmetric potentials play a crucial role in quantum mechanical models. From a systematic point of view, asymptotic iteration approach[1-3] for any l-state solutions of non-relativistic wave equation is a powerful computational method. For this reason, we present approximate analytical solutions of the Schrödinger equation with Hulthén plus a class of Yukawa potential[4] in the framework of Greene-Aldrich approximation[5] and asymptotic iteration method. We obtain the energy spectrum and the radial wavefunction for considered potential in the framework of non-relativistic theory. Bound-state wavefunction solution is expressed in terms of Gauss hypergeometric function.

Keywords: Asymptotic iteration method, Yukawa potential, Hulthén potential.
2020 Mathematics Subject Classification Numbers: 81Q05, 33C20, 35Q40.

References

[1] H. Ciftci, R. L. Hall, N. Saad, Asymptotic iteration method for eigenvalue problems,*Journal of Physics A: Mathematical and General*, 36, 11807-11816 (2003).

[2] H. Ciftci, R. L. Hall, N. Saad, Construction of exact solutions to eigenvalue problems by the asymptotic iteration method, *Journal of Physics A: Mathematical and General*, 38, 1147-1155 (2005).

[3] H. Ciftci, R. L. Hall, N. Saad, Iterative solutions to Dirac equation, *Physical Review A*, 72, 022101-7 (2005).

[4] A. I. Ahmadov et al., Analytical bound state solutions of the Dirac equation with the Hulthén plus a class of Yukawa potential including a Coulomb-like tensor interaction, *The European Physics Journal Plus*, 136, 208-29 (2021).

[5] R. L. Greene, C. Aldrich, Variational wave functions for a screened Coulomb potential, *Physical Review A*, 14, 2363-2366 (1976).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Propagation of Linear In-plane Waves in a Layer with Rough Surfaces

Ekin Deliktaş-Özdemir, Ayşe Peker Dobie
Piri Reis University, Istanbul, Turkey, edeliktas@pirireis.edu.tr
Istanbul Technical University, Istanbul, Turkey, pdobie@itu.edu.tr

Abstract

Problem of the propagation of linear in-plane waves in an elastic medium consisting of a single layer with corrugated surfaces and nonuniform thickness is investigated. The following assumptions are made for the problem in consideration:

- The constituent material of the layer is homogeneous, isotropic and incompressible.
- The linear shear and longitudinal velocities, c_L and c_T, satisfy $c_T < c_L$, and the phase velocity c satisfies $c_T < c < c_L$.
- During the motion, the strain functions on free surfaces are zero.

Under these assumptions, the dispersion relation of linear waves are derived in terms of wave number and angular frequency as well as the change in free surfaces. The effect of variations of free surfaces on the wave propagation is observed.

Keywords: In-plane waves, corrugated surfaces, perturbation methods
2020 Mathematics Subject Classification Numbers: 74B05, 74J05, 74G10

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Fractional viscoelastic contact problem with Tresca's friction

Leila Ait Kaki
Ecole Normale Supérieure de Constantine Laboratory MAD, 25000 Constantine, Algéria, leilaitkaki@yahoo.fr

Abstract

We consider a mathematical model that describes the quasi-static process of contact between viscoelastic body and a foundation. The constitutive law is assumed to be linear fractional viscoelastic and the process is quasistatic. The contact is modelled with Tresca's law. We establish the existence and uniqueness result of the weak solution of the model. The proofs are based on arguments of time-dependent variational inequalities, differential inclusion, Rothe's method and fixed point theorem.

References

[1] M. Rochdi,M. Shillor, M. Sofonea, *Quasistatic viscoelastic contact compliance and friction, Journal of elasticity* **51** (1998) 105-126

[2] J. Simon, *Compact sets in the space* $L^p(0,T;B)$, Ann. Mat. Pura. Appl., **146** (1987), 65-96.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

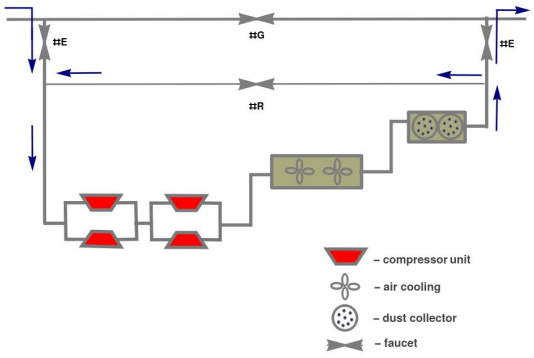
Minimizing compressor fuel cost on large natural gas pipeline transmission networks

Saule Burgumbayeva, Dinara Zhussupova
L. N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan, saulenai@yandex.ru
L. N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan, zhus.dinara@gmail.com

Abstract

Natural gas is the most important source of energy in the world. Currently, natural gas consumption is increasing the most compared to other non-renewable energy sources. In gas transmission networks designed to collect, transport and distribute natural gas, compressor stations are used to supply the energy needed to overcome friction between the gas and the pipes' inner wall pressure losses while maintaining gas flows transmission through the network.

Figure 1: *Compressor station scheme*



However, a significant part of the transported gas (estimates in range 3%- 5%) is consumed by series-parallel compressor units installed in the network before the gas reaches the receiving units. Minimizing this consumption is a task that not only has great financial value for the industry, but also has important environmental implications. This leads to the fuel cost minimization problem (FCMP). The approach by dynamic programming are presented in [1]. The problem of protecting the compressor from stalling into the surging mode is one of the most important problems for the reliable and safe operation of the comressor units (see [2]). The Fig. 1 shows a simplified flow diagram of a compressor station. It consists of 4 comressor units, air cooling and dust collector. Two couples of the compressors are connected sequentially, and each is connected parallel. The full scheme is much more complicated than the one presented and does not contain all the technological equipment but such a representation is sufficient for mathematical modeling of the operation of the compressor station.

We present optimization idea for fuel cost minimization using regaulation of the faucet #R which is shown in Fig. 1. The #R is recirculatian valve which is used to protect the compressor units from falling into the surge zone when it is opened. When valve is opened fully or partially the part of the output gas flow joins to the input flow increasing the volume of the input flow.

The calculations were performed using real data from a compressor station that operates in one of the gas transmission networks of Kazakhstan for the cases of a closed and an open #R valve. Real data is taken within one month, and for each fixed value, the parameters of the operation envelope of one compressor unit are calculated. According to the obtained results opening the faucet #R was necessary only in some cases.

Keywords: Natural gas transportation, mathematical modeling, optimization, fuel cost minimization problem.
2020 Mathematics Subject Classification Numbers: 65M06, 76N06, 76N25.

References

[1] Xiaorui Zhang, Changchun Wu, Lili Zuo. Minimizing fuel consumption of a gas pipeline in transient states by dynamic programming *Journal of Natural Gas Science and Engineering*, Volume 28, 193-203 (2016).

[2] S. A. Sardanashvili, Computational Techniques and Algorithms (Pipeline Gas Transmission) [in Russian]. *FSUE Oil and Gaz, I.M. Gubkin, Russian State University of Oil and Gas* (2005).

6.8 Applied Statistics

Session Organizers: Müjgan Tez and Kadri Ulaş Akay

Today, it is of great importance to evaluate and summarize the data obtained from many different disciplines and to use them as preliminary information in our future studies. At this stage, statistical techniques are needed to evaluate the data. In order to disseminate these techniques, symposiums are organized in which ideas are exchanged. One purpose of this session is to provide an environment where the latest developments in applied statistics are discussed. In particular, it is aimed to contribute to the development of science by targeting scientific interaction among the participants. In this session, besides applied statistics, studies on limit theorems and the applications of maximum likelihood estimators were presented.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

FIFA/Coca-Cola World Rankings on the Predictability of the Men's and Women's FIFA World Cup: A Comparative Analysis

Brandon Joly, Thomas Stojsavljevic, Mehmet Dik
Beloit College, Beloit, U.S.A, jolybs@beloit.edu
Beloit College, Beloit, U.S.A, stojsavljevictg@beloit.edu
Beloit College, Beloit, U.S.A, dikm@beloit.edu

Abstract

Since 1992, the International Federation of Association Football (FIFA) has been ranking senior men's national soccer teams based on a variety of criteria. In 2003, FIFA extended the FIFA/Coca-Cola World Rankings into ranking senior women's national soccer teams. The FIFA/Coca-Cola World Rankings published just before the 1994 FIFA World Cup USA, 1998 FIFA World Cup France, 2002 FIFA World Cup Korea/Japan, 2006 FIFA World Cup Germany, 2010 FIFA World Cup South Africa, 2014 FIFA World Cup Brazil, 2018 FIFA World Cup Russia, 2003 FIFA World Cup USA, 2007 FIFA World Cup China, 2011 FIFA World Cup Germany, 2015 FIFA World Cup Canada, and the 2019 FIFA World Cup France were considered. These rankings were compared to the final results of those FIFA World Cups based on two different methods of displaying the teams finish and were analyzed. Of the top 16 teams in each of the Men's FIFA World Cups, 74.1% of those teams advanced to the Round of 16. Meanwhile, 83.9% of the top 12 teams in each of the Women's FIFA World Cups advanced to the Round of 16 or Quarterfinals. The Pearson correlation coefficient between the Pre-Tournament rankings and final results was calculated using both ranking methods. The Women's World Cups had higher Pearson correlation coefficients for both methods than the Men's World Cups. In addition, the Women's World Cups had higher t-values and z-scores than the Men's World Cup when tested for independence and association between the Pre-Tournament rankings and final results using both ranking methods. These findings indicate that the Women's World Cups were more predictable than Men's World Cups based on the FIFA/Coca-Cola World Rankings.

Keywords: FIFA/Coca-Cola World Ranking, FIFA World Cup, Chi-Square, Fisher Exact Test, Pearson Correlation Coefficient
2020 Mathematics Subject Classification Numbers: 62P99

References

[1] FIFA. (2005, March 8). *FIFA/Coca-Cola World Ranking OVERVIEW OF BASIC PRINCIPLES AND METHOD OF CALCULATION. FIFA.com the official web site of the Fédération Internationale de Football Association.* Retrieved February 15, 2022, from https://web.archive.org/web/20050308034148/http://www.fifa.com/en/mens/statistics/rank/procedures/0,2540,3,00.html

[2] FIFA. (2007, June 4). *FIFA/Coca-Cola World Ranking Schedule.* FIFA. Retrieved February 15, 2022, from https://web.archive.org/web/20070604211354/http://www.fifa.com/worldfootball/ranking/procedure/men.html

[3] FIFA. (2017, January 10). *Unanimous decision expands FIFA World Cup^{TM} to 48 teams from 2026.* FIFA.com. Retrieved March 26, 2022, from https://web.archive.org/web/20170110231324/http://www.fifa.com/about-fifa/news/y=2017/m=1/news=fifa-council-unanimously-decides-on-expansion-of-the-fifa-world-cuptm–2863100.html

[4] FIFA. (2018). *Revision of the FIFA / Coca-Cola World Ranking.* FIFA. Retrieved February 15, 2022, from https://digitalhub.fifa.com/m/f99da4f73212220/original/edbm045h0udbwkqew35a-pdf.pdf

[5] FIFA. (n.d.). *Women's ranking.* Wiegman: England's squad even better than I thought. Retrieved February 16, 2022, from https://www.fifa.com/fifa-world-ranking/women?dateId=ranking_20211210

[6] Hoffman, J. I. E. (2014). *Fisher exact test.* Fisher Exact Test - an overview — ScienceDirect Topics. Retrieved February 17, 2022, from https://www.sciencedirect.com/topics/medicine-and-dentistry/fisher-exact-test

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Limit theorems for dependent random variables with infinite means

Ismahen Bernou
Laboratoire de Statistique et Modélisations aléatoires, Tlemcen, Algeria, ismahen825@gmail.com

Abstract

We provide necessary and sufficient conditions for the convergence in probability of weighted averages of random variables with infinite means. Our results extend and improve the corresponding theorems obtained in the independent setup by Adler (2012) and Nakata (2016).

Consider a sequence $\mathcal{X} = \{X_n, n \geq 1\}$ of real-valued random variables (rv's) defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and satisfying

$$\mathbb{P}(|X_j| > x) \asymp x^{-\alpha} \quad \text{for } j \geq 1 \text{ and some } \ 0 < \alpha \leq 1 \tag{1}$$

and

$$\limsup_{x\to\infty} \sup_{j\geq 1} x^{\alpha}\mathbb{P}(|X_j| > x) < \infty, \quad \text{for some } \ 0 < \alpha \leq 1 \tag{2}$$

The strong law of large numbers fails for these rv's since they have infinite means. Herein, we establish necessary and sufficient conditions for the convergence in probability of

$$W_n := \frac{1}{b_n} \max_{1\leq k\leq n} \left| \sum_{j=1}^{k} a_j (X_j - c_{nj}) \right|,$$

for a suitable sequence $\{c_{nj},\ 1 \leq j \leq n\}$.

Theorem. Let $0 < \alpha \leq 1$ and consider two sequences of positive constants $\tilde{a} = \{a_n,\ n \geq 1\}$ and $\tilde{b} = \{b_n,\ n \geq 1\}$ such that $\sum_{j=1}^{n} a_j^{\alpha} = o(b_n^{\alpha})$. If $\mathcal{X} = \{X_n, n \geq 1\}$ is a sequence of rv's satisfying (1), (2) and a Rosenthal-type maximal inequality, then $\mathcal{W} = \{W_n, n \geq 1\}$ converges in probability.

A necessary condition for the the convergence in probability of $\mathcal{W}$ is also derived.

Keywords: Weak laws, weighted exact laws, Rosenthal–type maximal inequalities.
2020 Mathematics Subject Classification Numbers: 60F15.

References

[1] A. Adler, An exact weak law of large numbers, *Bull. Inst. Math. Acad. Sin.*, 7(3), 417–422 (2012)

[2] I. Bernou, F. Boukhari, Limit theorems for dependent random variables with infinite means, to appear in *Stat. Probab. Lett.* (2022).

[3] T. Nakata, Weak laws of large numbers for weighted independent random variables with infinite mean, *Stat. Probab. Lett.*, 109, 124-129 (2016).

[4] H. Xu, X. Li, W. Yang, F. Xu, Laws of large numbers with infinite mean, *J. Math. Inequal*, 13, 335-349 (2019).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

Exact maximum likelihood estimation of the Box-Cox transformation parameter

Rui Gonçalves
University of Porto, Porto, Portugal, rjasg@fe.up.pt

Abstract

The Box-Cox transformation, [1] is a well known family of transformations used to obtain data suitable for the normality assumption of the residuals. However, the positiveness condition for the Box-Cox (BC) transformation results, after transforming, in data having a truncated distribution. In practice, most BC users consider the transformed data as approximated normal not caring about truncation as suggest in [1].
The inverse BC transformation of the truncated normal (TN) distribution is known as Power Normal (PN) distribution and was first noticed by Goto et al. [2] where some of its properties are presented. In this work, and following [3], we calculate the log likelihood function for the PN distribution and we give the maximum likelihood estimators of the mean and standard deviation. We also present a detailed calculation of the derivatives of the log likelihood function and an algorithm that uses the Newton-Raphson (NR) numerical method. The NR method is very sensitive to initial values so, in practice the estimation process has be carried out on a grid of search values covering the range of an interval.
The proposed algorithm searches the best BC parameter on a grid of values and it is multi-step since that in each step the selected value is used to build a narrower interval until a certain interval range is obtained.
In practice, prior to find the best estimate, the user has to look to the overall results before deciding the values for convergence criteria and also to implement a procedure to eliminate extreme points and outliers from the set of candidate estimates.

Keywords: Statistics, Box-Cox transformation, Maximum likelihood estimation.
2020 Mathematics Subject Classification Numbers: 62E10, 62-07, 62H05.

References

[1] G. E. Box, D. R. Cox, An analysis of transformations, *J. of the Royal Stat. Soc., Series B.*, 26, 211-43 (1964).

[2] M. Goto and T. Inoue,Some properties of power normal distribution, *Journal of Biometrics*, 1, 28-54 (1980).

[3] M. Goto, T. Inoue and Y. Tsushihide, On estimation of parameters in power normal distribution, *Bull. of Informatics and Cybernetics.*, 21, 41-53 (1984).

6.9 Differential Geometry

Session Organizer: Zerrin Şentürk

Differential Geometry plays an important role in the other disciplines such as Physics, Engineering. It uses the techniques of Differential Calculus, Linear algebra, Differential equations to develop new results on the theory of curves, surfaces and manifolds.

The Session of "Differential Geometry" of the "6th International Conference of Mathematical Sciences (ICMS 2022), 20- 24 July 2022" is organized by Maltepe University, Istanbul, Turkey.

The main objective of the Session of "Differential Geometry" to create a platform for presentations of the scientific works in the Differential Geometry and to bring together many Differential Geometers who works different subjects in this area.

The subjects of the presentations include Biconservative Hypersurfaces, Zermelo's navigation problem, Randers and Kropina metrics.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On the Biconservative Hypersurfaces

Aykut Kayhan, Nurettin Cenk Turgay
Maltepe University, Istanbul, Turkey, aykutkayhan@maltepe.edu.tr
Istanbul Technical University, Istanbul, Turkey, turgayn@itu.edu.tr

Abstract

An isometric immersion $x : (\Omega, g) \to \mathbb{E}_s^m$ is said to be biharmonic if the equation

$$\Delta^2 x = 0$$

is satisfied, where (Ω, g) is an m-dimensional semi-Riemannian manifold of the pseudo-Euclidean space $\mathbb{E}_s^m$, [1]. In this case, the immersed submanifold $M = x(\Omega)$ is said to be biharmonic. Because of the well-known Laplace-Beltrami formula $\Delta x = nH$, M is a biharmonic submanifold if and only if

$$\Delta H = 0, \tag{1}$$

where H is the mean curvature vector of M. By splitting ΔH into its normal and tangential components, one can obtain that the condition (1) is equivalent to

$$m\text{grad}\,\langle H, H\rangle + 4\text{tr}\, A_{\nabla^\perp_{(\cdot)}H}(\cdot) = 0, \tag{2}$$

$$\text{tr}\,(h(A_H(\cdot), \cdot)) - \Delta^\perp H = 0, \tag{3}$$

where A, h and $\nabla^\perp$ denote the shape operator, the second fundamental form and normal connection of M, respectively.
On the other hand, M is said to be a biconservative submanifold if (2) is satisfied. Note that if M is a hypersurface, then (2) becomes

$$A(\text{grad}\,\|H\|) = \varepsilon\frac{n\text{grad}\,\|H\|}{2},$$

where $\|H\|$ denotes the mean curvature function of M and ε is the signature of the unit normal vector field of M. In this work, we study on biconservative hypersurfaces with certain shape operator. We obtain a non-existence result.

Keywords: biconservative submanifolds, hypersurfaces, pseudo-Euclidean space.
2020 Mathematics Subject Classification Numbers: 53C42

References

[1] Chen, B.Y., *S*ome open problems and conjectures on submanifold of finite type, *Soochow J. Math*, 17, 169-188 (1991).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

The Generalization of Zermelo's Navigation Problem using Randers and Kropina metrics

Illatra Khamounejad, Bahman Rezai, Mehran Gabrani.
Urmia University, Urmia, Iran, i.khamonezhad@urmia.ac.ir
Urmia University, Urmia, Iran, b.rezai@urmia.ac.ir
Urmia University, Urmia, Iran, m.gabrani@urmia.ac.ir

Abstract

This paper aims to solve the navagation problem of Zermelo within the perspective of the Kropina and Randers metrics. If F is a Randers metric on an n-dimensional manifold M, and V is a vector field on (M, F); we know that $\tilde{F}$ is a solution of the navigation problem with navigation representation (F, V). By letting $F(x, -V_x) = 1$, we first generlize $\tilde{F}$ to $\tilde{F}^{(m)}$ and find relationship between F and $\tilde{F}^{(m)}$.Furthermore we show the relationship between F and $\tilde{F}^{(m)}$ with certain curvature properties. As well as we show by letting $F(x, -V_x) = 1$, prove that $\tilde{F}$ must be a Kropina metric. Furthermore, we prove the relationship between F and $\tilde{F}$ with certain curvature properties. Finally, we show that if c is a factor of S-curvature for Randers metric F, factor of the conformal vector field V with respect to h is the same as c and show that curvature properties in this case are invariant.

Keywords: Randers metrics, Kropina metrics, Conformal vector field, Navigation problem, S-curvature..
2020 Mathematics Subject Classification Numbers: 53B40.

References

[1] Antonelli P L. Handbook of Finsler Geometry. Kluwer: Academic Publishers,2003

[2] X. Cheng, Z. Shen, Randers metrics of scalar flag curvature, J. Aust. Math. Soc., 87(3)(2009), 359-370.

[3] X. Cheng, Q. Qu, S. Xu, The navigation problems and the curvature properties on conic Kropina manifolds, Differential Geometry and its Applications, 74(2)(2021), 101-709.

[4] X. Cheng, T. Li and L. Yin, The conformal vector fields on conic Kropina manifolds via navigation data, Journal of Geometry and Physics, 131(2018), 138-146.

[5] L. Huang, X. Mo, On geosidesics of Finsler metrics via navigation problem. Proc Amer Math Soc, in press

[6] L.Kang, On conformal fields of (α, β)-space. Preprint, 2011.

[7] X. Mo, L. Huang, On curvature decreasing property of a class of navigation problem. Publ Math Debreceen, 2007, 71: 141–163

[8] V. K. Kropina, Projective two-dimensional Finsler spaces with a special metric (Russian), Trudy Sem. Vektor. Tenzor. Anal., 11(1961), 277-292. 9

[9] Z. Shen, Q. Xia, On conformal vector fields on Randers manifolds, Sci. China Math. 55(9) (2012) 1869–1882.

[10] Z. Shen, Finsler metrics with K=0 and S=0, Canad. J. Math., 55(1)(2003), 112-132.

[11] R. Yoshikawa, K. Okubo, Kropina spaces of constant curvature, Tensor, N.S. 68 (2007) 190–203.

[12] E. Zermelo, Ü ber das Navigations problem bei ruhender oder ver¨anderlicher Windverteilung, Z. Argrew. Math. Mech., 11(1931), 114- 124.

[13] X. Zhang, Y. Shen, On Einstein Kropina metrics, Differential Geometry and its Applications, 31(1)2013, 80-92.

6.10 Algebra

Session Organizer: Leyla Bugay

In this session, a total of four presentations were made on finite groups, cryptography and solvable groups.

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On generating sets and digraphs for certain transformation semigroups

Leyla Bugay
Çukurova University, Adana, Turkey, ltanguler@cu.edu.tr

Abstract

For $n \in \mathbb{Z}^+$ let P_n, T_n and S_n be the partial transformations semigroup, (full) transformations semigroup and symmetric group on a finite chain $X_n = \{1, \ldots, n\}$, respectively. It is well known that every finite semigroup is embeddable in a transformation semigroup T_n for any appropriate n, which is correspond to Cayley's theorem for finite symmetric group S_n. Hence, the studies on transformation semigroups (similarly, on partial transformation semigroups) and their subsemigroups have certain important roles for finite semigroup theory like as the studies on symmetric groups for finite group theory. The subsemigroup generated by A is defined by $\langle A \rangle = \{a_1 \cdots a_n : a_1, \ldots, a_n \in A,\ n \in \mathbb{Z}^+\}$ and it is an important problem to find a method which decides whether an arbitrary non-empty subset X of any semigroup S is a (minimal) generating set of S, or not. With these motivations, we obtain a useful method to respond this lack by using digraphs for certain transformations semigroups. In this talk we present the method that we obtained and also an importance and usefulness of digraphs for finding elements which generate a fixed element.

Keywords: Generating set, transformation semigroup, digraph.
2020 Mathematics Subject Classification Numbers: 20M20.

References

[1] F. Al-Kharousi, R. Kehinde, A. Umar, Combinatorial results for certain semigroups of partial isometries of a finite chain, *Australas. J. Combin.*, 58(3), 365–375 (2014).

[2] F. Al-Kharousi, R. Kehinde, A. Umar, On the semigroup of partial isometries of a finite chain, *Comm. Algebra*, 44, 639–647 (2016).

[3] G. Ayık, H. Ayık, Y. Ünlü, J.M. Howie, Rank properties of the semigroup of singular transformations on a finite set, *Commun. Algebra*, 36, 2581–2587 (2008).

[4] G. Ayık, H. Ayık, L. Bugay, O. Kelekci, Generating sets of finite singular transformation semigroups. *Semigroup Forum*, 86, 59–66 (2013).

[5] G.U. Garba, Idempotents in partial transformation semigroups, *Proc. Royal Soc. Edinburgh*, 116A, 359–366, (1990).

[6] H. Ayık, L. Bugay, Generating sets of finite transformation semigroups $PK(n, r)$ and $K(n, r)$, *Comm. Algebra*, 43, 412-422 (2015).

[7] J.M. Howie, R.B. McFadden, Idempotent rank in finite full transformation semigroups, *Proc. Royal Soc. Edinburgh*, 114A, 161–167 (1990).

[8] J.M. Howie, *Fundamentals of Semigroup Theory*, New York, Oxford University Press, 1995.

[9] L. Bugay, M. Yağcı , H. Ayık, The ranks of certain semigroups of partial isometries, *Semigroup Forum*, 97, 214–222 (2018).

[10] L. Bugay, M. Yağcı, On minimal generating sets of certain subsemigroups of isometries, *Mathematical Sciences and Applications E-notes*, 8 (2), 71-78 (2020).

[11] O. Ganyushkin, V. Mazorchuk, *Classical Finite Transformation Semigroups*, Springer-Verlag, London, 2009.

[12] P. Zhao, V.H. Fernandes, The ranks of ideals in various transformation monoids, *Comm.Algebra*, 43, 674-692 (2015).

[13] V.H. Fernandes, T.M. Quinteiro, Presentations for monoids of finite partial isometries, *Semigroup Forum*, 93, 97–110 (2016).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

The structure of $(1, r)$-potents

Leyla Bugay
Çukurova University, Adana, Turkey, ltanguler@cu.edu.tr

Abstract

The *index* and the *period* of an element a of a finite semigroup are defined as the smallest values of $m \geq 1$ and $r \geq 1$ such that the elements $a, a^2, \ldots, a^{m+r-1}$ are different and $a^{m+r} = a^m$, respectively. Then, an element a with idex m and period r is called (m, r)-potent. The aim of this talk is to present some properties of $(1, r)$-potents (which is also called transformation of index 1) in T_n, where T_n is the transformation semigroup on $X_n = \{1, \ldots, n\}$. First we give the orbit structure of $\alpha \in T_n$ where $\mathrm{im}\,(\alpha^k) = \mathrm{im}\,(\alpha)$ for all $k \in \mathbb{Z}^+$ and prove that $\alpha \in T_n$ is an $(1, r)$-potent if and only if $\mathrm{im}\,(\alpha^k) = \mathrm{im}\,(\alpha)$ for all $k \in \mathbb{Z}^+$. Then we present some important properties of $(1, r)$-potents.

Keywords: Transformations, orbit, index, period
2020 Mathematics Subject Classification Numbers: 20M20.

References

[1] G. Ayık, H. Ayık, Y. Ünlü, J.M. Howie, The structure of elements in finite full transformation semigroups, *Bull. Austral. Math. Soc.*, 71, 69-74, (2005).

[2] J.M. Howie, *Fundamentals of Semigroup Theory*, New York, Oxford University Press, 1995.

[3] L. Bugay, O. Kelekci, On transformations of index 1, *Turk. J. Math.*, 38, 419-425 (2014).

[4] O. Ganyushkin, V. Mazorchuk, *Classical Finite Transformation Semigroups*, Springer-Verlag, London, 2009.

[5] P.M. Higgins, Combinatorial results for semigroups of order-preserving mappings, *Math. Proc. Camb. Phil. Soc.*, 113, 281–296 (1993).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

New cryptographic study of a functional message using a chaotic model

Nour Elhouda Berguellah
ESCF of Constantine, Constantine, Algeria, nberguellah@escf-constantine.dz

Abstract

In this paper we present a chaotic model used for secure transmission of a functional message which is the function $sin\omega t$, without forgetting the role of synchronization mechanisms of chaotic system to the success of these transmissions.

Keywords: Chaos, synchronization, cryptography.
2020 Mathematics Subject Classification Numbers: 37A99.

References

[1] E. N. Lorenz, Deterministic nonperiodic flow. *J. Atmos. Sci.*, 20, 130-141 (1963).

[2] L. M. Pecora, T. L. Carroll, Synchronization in chaotic systems, *Physical Review Letters*, vol. 64, 821-825 (1990).

[3] M. A. Khan, Synchronization of different 3D chaotic systems by generalized active control, *Journal of Information and Computing Science*, 7(4), 272-283 (2012).

[4] N. E. Berguellah, N. E. Hamri, Cryptographic study of functional message using two chaotic models, *Italian Journal of Pure and Applied Mathematic*, 42, 67-79 (2019).

6^{th} International Conference of Mathematical Sciences (ICMS 2022)
20-24 July 2022, Maltepe University, Istanbul, Turkey

On the orbital regular graph of finite solvable groups

Vijay Kumar Bhat, Karnika Sharma, Pradeep Singh
SMVD University, Katra, India, vijaykumarbhat2000@yahoo.com
SMVD University, Katra, India, karnikasharma069@gmail.com
SMVD University, Katra, India, pradeep33singh@gmail.com

Abstract

Let G be a group that acts on a finite set Υ. Then the orbit of $\upsilon \in \Upsilon$ is the subset $O(\upsilon) = \{g\upsilon \mid g \in G, \upsilon \in \Upsilon\}$ [1]. Omer et al. [3] defined orbit as the set of all conjugates of the elements, where G acts on itself by conjugation. Furthermore, by defining orbit graph as a graph whose vertices are non-central orbits under group action on Υ, Omer et al. [3] extended the work on conjugate graphs. Using various group actions, they constructed orbit graphs for various groups, such as finite non-abelian groups, finite p-groups, and groups of order pq. They also used a regular action to introduce the orbit graph for some finite solvable groups. In this article, we use regular action on a finite set Δ. We define orbitals of G as the orbits of the regular action of G on Δ. Note that Δ must be a subset of $\Upsilon \times \Upsilon$. We interlink the concept of [2] and [3], to find the orbital regular graphs of a finite solvable group.

Keywords: Solvable group, orbital, orbital graph, orbital regular graph, regular action.
2020 Mathematics Subject Classification Numbers: 05C20, 05C25

References

[1] F. M. Goodman, *Algebra Abstract and Concrete Streesing Symmetry*, Prentice Hall, 2^{nd} edition, 2002.

[2] P. Sole, The edge-forwarding index of orbital regular graphs, *Discrete Math.*, 130, 171-176, (1994).

[3] S. M. S. Omer, N. H. Sarmin, and A. Erfanian, The orbit graph for some finite solvable groups, *AIP Conference Proceedings*, 863, (2014).