OPTIMAL SPATIAL REORIENTATION OF A SPACECRAFT UNDER ELLIPSOIDAL CONSTRAINT ON CONTROLS

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The problem of optimal spatial turn of a spacecraft (SC) with fixed endpoints of trajectories is considered. We use the SC's mathematical model with a quaternion representation for angular coordinates [1, 2]:

$$\begin{aligned}
&J_1\dot{\omega}_1(t) = (J_2 - J_3)\omega_2(t)\omega_3(t) + M_1(t),\\
&J_2\dot{\omega}_2(t) = (J_3 - J_1)\omega_1(t)\omega_3(t) + M_2(t),\\
&J_3\dot{\omega}_3(t) = (J_1 - J_2)\omega_1(t)\omega_2(t) + M_3(t),\\
&\dot{\lambda}_0(t) = 0.5\,[-\omega_1(t)\lambda_1(t) - \omega_2(t)\lambda_2(t) - \omega_3(t)\lambda_3(t)],\\
&\dot{\lambda}_1(t) = 0.5\,[\omega_1(t)\lambda_0(t) + \omega_3(t)\lambda_2(t) - \omega_2(t)\lambda_3(t)],\\
&\dot{\lambda}_2(t) = 0.5\,[\omega_2(t)\lambda_0(t) - \omega_3(t)\lambda_1(t) + \omega_1(t)\lambda_3(t)],\\
&\dot{\lambda}_3(t) = 0.5\,[\omega_3(t)\lambda_0(t) + \omega_2(t)\lambda_1(t) - \omega_1(t)\lambda_2(t)],
\end{aligned} \tag{1}$$

where $\omega(t) = (\omega_1(t),\ \omega_2(t),\ \omega_3(t))$ is a vector of a SC angular velocity; $\Lambda(t) = (\lambda_0,\ \lambda_1,\ \lambda_2,\ \lambda_3)$ is a quaternion (SC angular coordinates); $J_1,\ J_2,\ J_3$ are a SC's central principal moments of inertia.

An admissible control (moment of external forces) $M(t) = M_1(t),\ M_2(t),\ M_3(t))$ must satisfy the constraint of an ellipsoid form:

$$0.5\,[M_1^2(t)/J_1 + M_2^2(t)/J_2 + M_3^2(t)/J_3] \le d_0. \tag{2}$$

It is required to determine the optimal control $M(t)$, satisfying the limitation (2), which transfers the system (1) from given initial state $\omega(t_0) = \omega_0$, $\Lambda(t_0) = \Lambda_0$ to a desired final state $\omega(T) = \omega_T$, $\Lambda(T) = \Lambda_T$ within a time interval $[t_0,\ T]$, and minimizes the objective functional

$$I(M) = \int_{t_0}^{T} \sqrt{M_1^2(t)/J_1 + M_2^2(t)/J_2 + M_3^2(t)/J_3}\,dt \to \inf_M. \tag{3}$$

The problem (1)–(3) is solved using the quasilinearization procedure [3] and the method suggested in [4] for solving a linear quadratic problem with constraints on control values. The proposed algorithm is represented in a form convenient for computer-aided implementation.

References

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