ROBIN-TYPE PROBLEM FOR SECOND ORDER ELLIPTIC SYSTEMS ON PLANE WITH SINGULAR LINES

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In present work Robin-type problem for the class of second order elliptic systems on plane with singular lines is solved.

Let $0 < \varphi_0 \leq 2\pi$, $G = \left\{ z = re^{i\varphi} : \ 0 \leq r < \infty, \ 0 \leq \varphi \leq \varphi_0 \right\}$.

We consider the equation

$$\frac{\partial^2 V}{\partial \bar{z} \partial z} + \frac{a(\varphi)}{2\bar{z}} \frac{\partial V}{\partial \bar{z}} + \frac{b(\varphi)}{2\bar{z}} \frac{\partial V}{\partial z} + \frac{c(\varphi) r^{\alpha-2} v}{4 \prod_{j=1}^{m} (y - k_j x)^{\alpha_j}} + \frac{d(\varphi) \bar{V}}{4r^2} = \frac{h(\varphi) r^{\mu+\alpha-2} v}{4 \prod_{j=1}^{m} (y - k_j x)^{\alpha_j}}, \qquad (1)$$

where

$$z = x + iy,$$

$$\frac{\partial}{\partial z} = \frac{1}{2}\Big(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\Big) = \frac{e^{-i\varphi}}{2}\Big(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \varphi}\Big),$$

$$\frac{\partial}{\partial \bar{z}} = \frac{e^{i\varphi}}{2}\Big(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \varphi}\Big),$$

$$\frac{\partial^2}{\partial \bar{z} \partial z} = \frac{\partial}{\partial \bar{z}}\Big(\frac{\partial}{\partial z}\Big) = \frac{1}{4}\Big(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{1}{r}\frac{\partial}{\partial r}\Big);$$

$$a(\varphi), \quad b(\varphi), \quad c(\varphi), \quad d(\varphi), \quad h(\varphi) \in C[0, \varphi_0], \quad \varphi \in [0, \varphi_0],$$

$$0 < \varphi_j < \varphi_0, \quad k_j = \tan \varphi_j, \quad 0 < \alpha_j < 1, \quad (j = \overline{1, m}), \quad \alpha = \sum_{j=1}^{m} \alpha_j.$$

We obtained the solution of equation (1) in class

$$W_p^2(G) \cap C^2(G), \qquad p > 1, \qquad (2)$$

where $W_p^2(G)$ is a S.L.Sobolev's space [1].

In this work the following problem for equation (1) is solved :

Problem **R.** Find the solution of equation (1) from class (2) satisfies the following conditions

$$V(r, 0) = \beta_1 r^{\mu}, \qquad \frac{\partial V}{\partial \varphi}(r, 0) = \beta_2 r^{\mu}, \qquad (3)$$

where β_1, β_2, μ – given real numbers.

References

[1] Vekya I.N. (1959) Generalized analitic funcion. - Moskow.