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СЕКЦИЯ №2. ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И ДИНАМИЧЕСКИЕ СИСТЕМЫ

TO DEFINE THE RIGHT-HAND SIDE OF HIGH ORDER NON-HOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS ON ADDITIONAL INFORMATION

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The problem of reconstructing of solutions and special right sides for inhomogeneous linear differential equations of higher orders with the redefined two- point boundary conditions is solved. It is possible to perform a full description of all possible available right sides.

1. Statement of the problem and its consideration.
Let's consider the following problem arising in various sections of mathematics:

$$y^{(n)}(t)+p_1(t)y^{(n-1)}(t)+...+p_1(t)y(t)=\mu(t)+b(t)\,v(t)\,,\ 0\le t\le 1 \quad (1)$$

at conditions

$$U_j(y)=\sum_{\nu=1}^{n-1}\left[\alpha_{j\nu}y^{(\nu)}(0)+\beta_{j\nu}y^{(\nu)}(1)\right]=b_j\,,\ j=\overline{1,n} \quad (2)$$

$$V_i(y)=\sum_{\nu=1}^{n-1}\left[\gamma_{i\nu}y^{(\nu)}(0)+\theta_{i\nu}y^{(\nu)}(1)\right]=a_i\,,\ i=\overline{1,n} \quad (3)$$

where $p_k(t), k=1,n;$ $\mu(t)$ is given piecewise continuous functions, $v(t)$ is required function.
We assume, that
1) The matrix rank $(\alpha_{j\nu}, \beta_{j\nu})$ is equal to n,
2)Homogeneous problem $\frac{dy}{dt}=A(t)y(t)$,

$$y^{(n)}(t)+p_1(t)y^{(n-1)}(t)+...+p_n(t)y(t)=0,$$

$$U_j(y)=0\,,\ j=\overline{1,n}$$

has only trivial solution.

The considered problem (1) - (3) can be interpreted. At first, as a problem of definition of the right - hand side of the non-homogeneous equation (1) on additional information. Then it represents a so-called inverse problem, the review on which is available in S.I.Kabanihin's monography [1].

Secondly, as the redefined boundary problem for system of the ordinary differential equations. In that case it is important to know enough general conditions of solvability of the problem (1) - (3). The theory of the redefined tasks is shown in P.I.Dudnikov, S.N.Samborsky's work [2].

In the third, as an initial step by working out of algorithms of the solution of optimal control problem for system of the ordinary differential equations at the set

$$p_k(t),\ k=\overline{1,n};\ b(t), \mu(t),\ \alpha_{j\nu},\quad \beta_{j\nu},\quad \gamma_{j\nu}(t),\quad \theta_{j\nu}(t),$$

$$b_j,\ a_i,\ j=\overline{1,n},\ i=\overline{1,m},\ \nu=\overline{1,n-1}.$$

The similar approach is in details developed in a cycle of works on S.A.Ajsagaliev's works on optimal control [3,4,5].

Analogues of problem (1) - (3) for the abstract operator equations and the equations with partial derivatives can be found in works [6,7].

2.Additional lemmas and the basic result of the work
Let's enter the following determinants:

$$\Delta=\det\left\|U_j(y_k)\right\|_{nxk}, \quad (4)$$

$$g(x,t)=\pm\frac{1}{2}\frac{\begin{vmatrix} y_1(t) & y_2(t) & \dots & y_n(t) \\ y_1'(t) & y_2'(t) & \dots & y_1'(t) \\ \dots & \dots & \dots & \dots \\ y_1^{(n-2)}(t) & y_2^{(n-2)}(t) & \dots & y_n^{(n-2)}(t) \\ y_1(x) & y_2(x) & \dots & y_n(x) \end{vmatrix}}{\begin{vmatrix} y_1(t) & y_2(t) & \dots & y_n'(t) \\ y_1'(t) & y_2'(t) & \dots & y_n'(t) \\ \dots & \dots & \dots & \dots \\ y_1^{(n-1)}(t) & y_2^{(n-1)}(t) & & y_n^{(n-1)}(t) \end{vmatrix}}, \qquad (5)$$

if $x>t$, we take a sign plus, and if $x<t$, we take a sign a minus.

Here $y_k(x), k=1,n$ is any fundamental system of solutions of the homogeneous equation

$$y^{(n)}(t)+p_1(t)y^{(n-1)}(t)+\dots+p_n(t)y(t)=0.$$

$$H(x,t)=\begin{vmatrix} y_1(t) & y_2(t) & \dots & y_n(t) & g(x,t) \\ U_1(y_1) & U_1(y_2) & \dots & U_1(y_n) & U_1(g) \\ U_2(y_1) & U_2(y_2) & \dots & U_2(y_n) & U_2(g) \\ \dots & \dots & \dots & \dots & \dots \\ U_n(y_1) & U_n(y_2) & \dots & U_n(y_n) & U_n(g) \end{vmatrix} \qquad (6)$$

Lemma 1. Let are set $p_k(t)\in C^k[0.1], k=\overline{1,n}$, $b(t), \mu(t), v(t)\in C[0,1]$ and numbers $\alpha_{j\nu}$, $\beta_{j\nu}$. Then the solution of the equation (1), at $b=0, b_2=0, \dots, b_n=0$ satisfying to condition (2) has the from

$$y(t)=\int_0^1\frac{H(t,\tau)}{\Delta}[\mu(\tau)+b(\tau)v(\tau)]d\tau, \qquad (7)$$

$$if\ \Delta\neq 0.$$

Proof of the lemma. The conclusion of the formula (7) can be found in M.A.Najmark's book [8].

Lemma 2. Let be $p_k(t)\in C^k[0.1], k=\overline{1,n}$, $b(t), \mu(t), v(t)\in C[0,1]$ and numbers $\alpha_{j\nu}$, $\beta_{j\nu}$ $\gamma_{j\nu}$, $\theta_{j\nu}$, a_j.

Then

$$\int_0^1 V_{i,t}(H(t,\tau))b(t)v(t)d\tau=\xi_i,\quad i=\overline{1,m} \qquad (8)$$

Where

$$\xi_i=\Delta\bullet a_i-\int_0^1 V_{i,t}(H(t,\tau))\mu(t)d\tau. \qquad (9)$$

Proof of the lemma 2. From the condition (3) we obtain

$$V_i(y)=a_i,\quad i=\overline{1,m}$$

Then from the lemma 1 follows

$$\frac{1}{\Delta}\int_0^1 V_{i,t}(H(t,\tau))[\mu(\tau)+b(t)v(t)]d\tau=a_i,\ i=\overline{1,m}. \qquad (10)$$

Whence

$$\int_0^1 V_{i,t}(H(t,\tau))b(t)v(t)d\tau=a_i\Delta-\int_0^1 V_{i,t}(H(t,\tau))\mu(t)d\tau=\xi_i,\ i=\overline{1,m}. \qquad (11)$$

The lemma 2 is completely proved. Now we formulate the basic result of the work.

Theorem. At any $w(t)\in L_2(0,1)$ the set

$$T=\left\{v(t)=w(t)+\overline{b(t)}\overline{V}A^{-1}(\vec{\xi}-\vec{\eta})\right\} \quad (12)$$

gives the full description of such functions v(t) that there are solutions of the equation (1) with conditions (2) and (3) where

$$\vec{V}=\begin{pmatrix} V_{1,t}(H(t,\tau)) \\ V_{2,t}(H(t,\tau)) \\ \cdots\cdots\cdots \\ V_{m,t}(H(t,\tau)) \end{pmatrix}, \quad (13)$$

$$A=\left\|\langle V_{i,t}(H(t,\tau))b(\tau),b(\tau)V(H(t,\tau))\right\|_{ixs}, \quad (14)$$

$$\vec{\xi}=(\xi_1,\xi_2,\dots,\xi_m)^T, \quad (15)$$

$$\vec{\eta}=(V_1(z_1),V_2(z_1),\dots,V_m(z_1))^T \quad (16)$$

The converse statement is also right.

Proof of the theorem. The unknown function v(t), satisfying to the equation (8) we search in the form of

$$v(t)=\sum_{k=1}^{m}p_k\overrightarrow{b(\tau)}\overrightarrow{V_{k,t}(H(t,\tau))}$$

where p_k are some constants.

Then the system of expressions (8) is transformed to the form

$$A\bullet\vec{p}=\vec{\xi}, \quad (17)$$

Where $\vec{p}=(p_1,p_2,\dots,p_m)^T$.

By condition of the lemma 1 detA≠0, then

$$\vec{p}=A^{-1}\vec{\xi}. \quad (18)$$

The partial solution of non- homogeneous system have the form

$$v(t)=\overrightarrow{b(\tau)}\vec{V}A^{-1}\vec{\xi}. \quad (19)$$

Now, the common solution of homogeneous system of the equations

$$\int_0^1 V_{i,t}(H(t,\tau))b(t)u(t)d\tau=0,\quad i=\overline{1,m}$$

We search in the form

$$u(t)=w(t)+\sigma(t) \quad (20)$$

Where $w(t)$ is any vector function, and a $\sigma(t)$ *is* unknown vector function. From the equation (19) follows, that

$$\int_0^1 V_{i,t}(H(t,\tau))b(t)\sigma(t)d\tau=-\int_0^1 V_{i,t}(H(t,\tau))b(t)w(t)d\tau,\quad i=\overline{1,m} \quad (21)$$

It is easy to show, tha

$$tV_i(z_1(t))=\int_0^1 V_{i,t}(H(t,\tau))b(t)w(t)d\tau \quad (22)$$

Where $z_1(t)$ issolution of the problem.

$$z_1^{(n)}(t)+p_1(t)z_1^{(n-1)}(t)+\dots+p_n(t)z_1(t)=b(t)\,w(t),\ 0\le t\le 1 \quad (23)$$

$$U_j(z_1)=0,,\ j=\overline{1,n} \quad (24)$$

Then it is similar to how have solved the equation (8) of expressions (22) it is obtained

$$\sigma(t) = \overrightarrow{b(\tau)}\vec{V}A^{-1}\vec{\eta} \quad (25)$$

means $$u(t) = w(t) - \overrightarrow{b(\tau)}\vec{V}A^{-1}\vec{\eta} \quad (26)$$

Finally, we obtain, that

$$v(t) = w(t) - \overrightarrow{b(\tau)}\vec{V}A^{-1}(\vec{\xi} - \vec{\eta}).$$

The theorem is proved.

Requirements (2) are , problem (3) is redefined. Therefore $v(t)$ in the right part of the equation (1) cannot be any, that is gets out according to the theorem. The basic result of the work is formulated for homogeneous conditions (2) though it it is easy reformulate for any

$$b_1, b_2, \ldots, b_n.$$

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ON THE EXACT ESTIMATES OF THE BEST SPLINE APPROXIMATIONS OF FUNCTIONS WITH DERIVATIVES OF GENERALIZED FINITE VARIATION

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ABSTRACT. *The article is devoted to the exact (in the sense of the order of smallness) estimates of the best spline approximations of functions with derivative of generalized finite variation given on a finite segment of the straight line in uniform and integral metrics.*

1. DEFINITIONS AND NOTATIONS. Let N be the set of all natural numbers, $Z_+ = N \cup \{0\}$, $\Delta = [a,b]$ a finite segment of the straight line with the length $|\Delta| = b - a$, let $L_p(\Delta)$ be the space of all measurable by Lebesque real-valued on Δ functions f whose pth power is integrable. The space is equipped with the quasi-norm

$$\|f\|_{p,\Delta} := \left\{\int_\Delta |f(x)|^p \, dx\right\}^{1/p} \ (0 < p < \infty), \ \|f\|_{\infty,\Delta} = ess\sup\{|f(x)| : x \in \Delta\} \ (p = \infty).$$

Let $\Phi(u)$ be a continuous, increasing, convex to down function, defined on the interval $[0,\infty)$ and such that $\Phi(0) = 0$. For a function $f(x)$ defined and finite on a segment Δ the value