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Optimal Control of the Rotor System Motion

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Abstract. Among the problems of the rotor machines dynamics the special attention is given to the problems of creation of the automatic balancing devices (ABD) in form of a hollow rotor, filled by a liquid, and the liquid-solidbody ABD. The theoretical and experimental works on research of the ABD on the base of a hollow rotor filled partially with a liquid and of the liquid-solidbody ABD are not enough. Therefore development of the methods of research of dynamics of the rotor machines with the ABD and such machines designs is an actual, new and perspective problem. In the present work the mathematical model of the rotor system with the ABD taking into account of the engine characteristics is offered. Let's consider the model of the rotor with electric drive with one disk, set up at the flexible shaft without skew. The shaft is lean on two bearings (fig. 1).

Introduction

The problems of research of dynamics of the rotor machines with the ABD and the questions of the rotor systems fluctuations control were considered also in the works [1]-[3]. One of the ways of the rotor machines vibration reduction is an optimal control of their movement [4]. In the present work the mathematical model of the rotor system with the ABD taking into account of the engine characteristics is offered. Let's consider the model of the rotor with electric drive with one disk, set up at the flexible shaft without skew. The shaft is lean on two bearings (fig. 1).

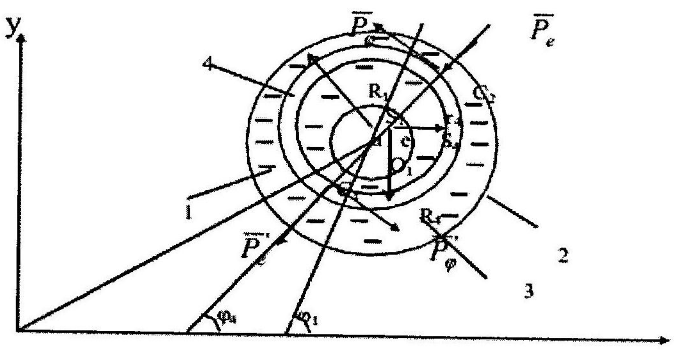

Figure 1. The rotor system with auto-balancing device

Derive the equation of motion of the system

Let's suppose that the shaft mass is small in comparison with the mass m_1, the disk is made as the closed axisymmetric cavity, filled by a liquid, where also axisymmetric float with the mass m_2 is placed. The float has possibility of free motion and hasn't eccentricity.

Using Lagrange's equations of the second type we can obtain the necessary system of the equations of motion of the oscillatory system with five freedoms.

1. $(m_1+m_4)\ddot{x}+k\dot{x}+cx=m_1a(\ddot{\varphi}_1\sin\varphi_1+\dot{\varphi}_1^2\cos\varphi_1)-m_4[(\ddot{e}-e\dot{\varphi}_4^2)\cos\varphi_4-$
$-(2\dot{e}\dot{\varphi}_4+e\ddot{\varphi}_4)\sin\varphi_4]-B(\dot{\varphi}_4-2\dot{\varphi}_1)e\sin\varphi_4-A\dot{e}\cos\varphi_4.$

2. $(m_1+m_4)\ddot{y}+k\dot{y}+cy=m_1a(\dot{\varphi}_1^2\sin\varphi_1-\ddot{\varphi}_1\cos\varphi_1)-m_4[(\ddot{e}-e\dot{\varphi}_4^2)\sin\varphi_4+$
$+(2\dot{e}\dot{\varphi}_4+e\ddot{\varphi}_4)\cos\varphi_4]-B(\dot{\varphi}_4-2\dot{\varphi}_1)e\cos\varphi_4-A\dot{e}\sin\varphi_4.$

3. 3/ $m_4\ddot{e}+m_4\ddot{x}\cos\varphi_4+m_4\ddot{y}\sin\varphi_4+m_4e\dot{\varphi}_4^2=A\dot{e}.$ (1)

4. $(m_4e^2+J_{S_4})\ddot{\varphi}_4+2e\dot{e}\dot{\varphi}_4m_4+m_4e(\ddot{y}\cos\varphi_4-\ddot{x}\sin\varphi_4)=$
$=[B(\dot{\varphi}_4-2\dot{\varphi}_1)e(e+R_4)].$

5. $(m_1a^2+J_{S_1})\ddot{\varphi}_1+K_{\varphi 1}\dot{\varphi}_1+m_1a(\ddot{y}\cos\varphi_1+\ddot{x}\sin\varphi_1)=$
$=B(\dot{\varphi}_4-2\dot{\varphi}_1)e(e+R_4)+M_Д-M_C$

The summands $A\dot{e}$, $B(\dot{\varphi}_4-2\dot{\varphi}_1)e$ are correspondingly the radial and tangent component of the forces of viscosity by the float and liquid interaction. In fact they are the integral characteristics, i.e. they depend both on the float geometrical dimensions and the liquid properties. Choice of the type of the engine driving moments M_D and M_C, and also knowledge of their changes range is one of the main goals of our research. Now the shaft-rotor system is substituted by its analogue – the system of horizontal pendulums (fig. 2).

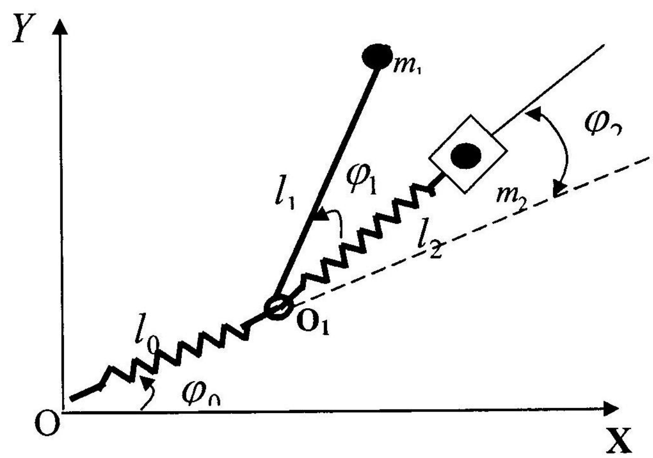

Figure 2. Choice of the two-mass system generalized coordinates

The spring OO_1 can rotate in a horizontal plane around the point O and can be distorted by the linear law. The point M_1 with the mass m_1 can rotate in a horizontal plane around the point O_1. The point M_2 with the mass m_2 can rotate around the point in horizontal plane and at the same time the spring MO_1 is linearly distorted in the radial direction. Let's choose the immovable coordinate system with the origin in the point $O(0,0)$. Then with a help of the Lagrange's equations of the second type for this system we can obtain the necessary system of the equations of the vibratory system with five freedoms motion

$$m_1\ddot{\ell}_0-m_2\ell_1\ddot{\varphi}_1\sin\varphi_1-m_2\ell_1\dot{\varphi}_1^2\cos\varphi_1+m_2\ddot{\ell}_0-m_2\dot{\ell}_2\dot{\varphi}_2\sin\varphi_2-$$
$$-m_2\ell_2\ddot{\varphi}_2\sin\varphi_2-m_2\ell_2\dot{\varphi}_2^2\cos\varphi_2+m_2\ddot{\ell}_2\cos\varphi_2-m_2\dot{\ell}_2\dot{\varphi}_2\sin\varphi_2=$$
$$=m_1\ell_0\dot{\varphi}_0^2+m_1\ell_1\dot{\varphi}_0\dot{\varphi}_1\cos\varphi_1+m_2\ell_2\dot{\varphi}_2\sin\varphi_2-\kappa_0(\ell_0-\bar{\ell}_0);$$

$$
\begin{aligned}
&m_1\ell_0^2\ddot{\varphi}_0 + m_1\ell_0\dot{\ell}_0\dot{\varphi}_0 + m_1\ell_0\ell_1\dot{\varphi}_1\cos\varphi_1 + m_1\ell_0\ell_1\ddot{\varphi}_1\cos\varphi_1 - \\
&- m_1\ell_0\ell_1\dot{\varphi}_1^2\sin\varphi_1 + 2m_2\ell_0\dot{\ell}_0\dot{\varphi}_0 + m_2\ell_0^2\ddot{\varphi}_0 + m_2\dot{\ell}_0\ell_2\dot{\varphi}_2\cos\varphi_2 + \\
&+ m_2\ell_0\dot{\ell}_2\dot{\varphi}_2\cos\varphi_2 + m_2\ell_0\ell_2\ddot{\varphi}_2\cos\varphi_2 - m_2\ell_0\ell_2\dot{\varphi}_2^2\sin\varphi_2 + \\
&+ m_2\dot{\ell}_0\ell_2\sin\varphi_2 + m_2\ell_0\ddot{\ell}_2\sin\varphi_2 + m_2\ell_0\dot{\ell}_2\dot{\varphi}_2\cos\varphi_2 = 0 \\
&m_1\ell_1^2\ddot{\varphi}_1 + 2m_1\ell_0\dot{\ell}_1\dot{\varphi}_1 + m_1\dot{\ell}_0\dot{\ell}_1\dot{\varphi}_0\cos\varphi_1 + m_1\ell_0\dot{\ell}_1\dot{\varphi}_0\cos\varphi_1 + \\
&+ m_1\ell_0\dot{\ell}_1\ddot{\varphi}_0\cos\varphi_1 - m_1\ddot{\ell}_0\ell_1\sin\varphi_1 - m_1\dot{\ell}_0\dot{\ell}_1\sin\varphi_1 - m_1\ell_0\dot{\ell}_1\dot{\varphi}_1\cos\varphi_1 = \\
&= -m_1\ell_0\ell_1\varphi_0\dot{\varphi}_1\sin\varphi_1 - m_1\dot{\ell}_0\ell_1\dot{\varphi}_1\cos\varphi_1 - \varphi_1 + (M_D - M_C) \\
&\qquad m_2\ddot{\ell}_2 + m_2\dot{\ell}_0\dot{\varphi}_0\sin\varphi_2 + m_2\ell_0\ddot{\varphi}_0\sin\varphi_2 + m_2\ell_0\dot{\varphi}_0\dot{\varphi}_2\cos\varphi_2 + \\
&+ m_2\ddot{\ell}_0\cos\varphi_2 - m_1\dot{\ell}_0\dot{\varphi}_2\sin\varphi_2 = m_2\ell_2\dot{\varphi}_2^2 + m_2\ell_0\dot{\varphi}_0\dot{\varphi}_2\cos\varphi_2 - \\
&- m_1\dot{\ell}_0\dot{\varphi}_2\sin\varphi_2 - \kappa\left(\ell_2 - \bar{\ell}_2\right) - \theta\dot{\ell}_2 \\
&m_2\ell_2^2\ddot{\varphi}_2 + 2m_2\ell_2\dot{\ell}_2\dot{\varphi}_2 + m_2\dot{\ell}_0\ell_2\dot{\varphi}_0\cos\varphi_2 + m_2\ell_0\dot{\ell}_2\dot{\varphi}_0\cos\varphi_2 + \\
&+ m_2\ell_0\ell_2\ddot{\varphi}_0\cos\varphi_2 - m_2\ell_0\ell_2\dot{\varphi}_0\dot{\varphi}_2\sin\varphi_2 - m_2\ddot{\ell}_0\ell_2\sin\varphi_2 - \\
&- m_2\dot{\ell}_0\dot{\ell}_2\sin\varphi_2 - m_2\dot{\ell}_0\ell_2\dot{\varphi}_2\cos\varphi_2 = m_2\ell_0\ell_2\dot{\varphi}_0\dot{\varphi}_2\sin\varphi_2 + \\
&+ m_2\ell_0\dot{\ell}_2\dot{\varphi}_0\cos\varphi_2 - m_2\dot{\ell}_0\ell_2\dot{\varphi}_2\cos\varphi_2 - m_2\dot{\ell}_0\dot{\ell}_2\sin\varphi_2 - \theta\ell_2(\dot{\varphi}_1 - \dot{\varphi}_2) \\
&m_2\ell_2^2\ddot{\varphi}_2 + 2m_2\ell_2\dot{\ell}_2\dot{\varphi}_2 + m_2\dot{\ell}_0\ell_2\dot{\varphi}_0\cos\varphi_2 + m_2\ell_0\dot{\ell}_2\dot{\varphi}_0\cos\varphi_2 + \\
&+ m_2\ell_0\ell_2\ddot{\varphi}_0\cos\varphi_2 - m_2\ell_0\ell_2\dot{\varphi}_0\dot{\varphi}_2\sin\varphi_2 - m_2\ddot{\ell}_0\ell_2\sin\varphi_2 - \\
&- m_2\dot{\ell}_0\dot{\ell}_2\sin\varphi_2 - m_2\dot{\ell}_0\ell_2\dot{\varphi}_2\cos\varphi_2 = m_2\ell_0\ell_2\dot{\varphi}_0\dot{\varphi}_2\sin\varphi_2 + \qquad (2) \\
&+ m_2\ell_0\dot{\ell}_2\dot{\varphi}_0\cos\varphi_2 - m_2\dot{\ell}_0\ell_2\dot{\varphi}_2\cos\varphi_2 - m_2\dot{\ell}_0\dot{\ell}_2\sin\varphi_2 - \theta\ell_2(\dot{\varphi}_1 - \dot{\varphi}_2)
\end{aligned}
$$

It is obvious that the system of the equations (2) is identical to the system (1), with the only difference that in the first one torsion is taken into account by the summand $\chi\varphi_1$. In this case external impact is applied on the bob M_1, i.e. the moment of the external driving forces (M_D –M_C) operates on the point M_1 (not on the shaft as in the preceding case), therefore, naturally, torsion is taken into account. In this case, described by the system (2), the same problems are considered. The mechanical model difference from the shaft-rotor system consists in the fact that in our opinion this mechanical model is more evident from the point of view of physics and also the processes in the pendulum systems are well-studied enough.

Let's formulate the stated problem.

1. It is necessary to define a range of the changes of the device's external driving moments difference (M_D - M_C) by the preset other parameters of the system (1) from the condition of dimensional restrictions: the shaft should make the oscillations in the limited space, i.e. $x^2 + y^2 \le \Gamma$, *where* Γ characterizes the horizontal dimension of a vertical housing.

2. Let all the values of the parameters, belonging to the system (1), are given. Let a range of the moments difference changes is known too on the grounds of the results of the first stage

$$M_{min} \le M_B - M_C \le M_{max},$$

but there is no the explicit dependence of (M_D - M_C) on the time.

It is necessary to define dependence of (M_D - M_C) on the time from a condition of some functional minimization. Specific choosing of the last ones is given below. Every time the functional is chosen so that economical mode of operation or increase of the shaft-rotor system lifetime would be taken into account.

1. Algorithm of the acceleration problem solving.

1. We assign the initial data for $l_0, l_2, \varphi_0, \varphi_1, \varphi_2$, and also the law of external moments $M_D - M_C = f(t)$.
2. Let $t = t_0$ (the initial moment of time).
3. Let's substitute in the Lagrange equation, corresponding to $\varphi_1(t)$, the initial data instead of the unknowns $l_0(t)$, $l_2(t)$, $\varphi_0(t)$, $\varphi_2(t)$.
4. We find the obtained equation solution, i.e. we define φ_1 at the moment $t + \Delta t$.
5. Let's substitute the initial data instead of the unknowns $l_2(t)$, $\varphi_0(t)$, $\varphi_2(t)$ and the found value $\varphi_1(t + \Delta t)$ instead of φ_1 in the Lagrange equation, corresponding to $l_0(t)$.
6. Let's find l_0 at the moment $t + \Delta t$ from the obtained equation.
7. Let's substitute the initial data instead of $l_2(t)$, $\varphi_2(t)$ and the just found values instead of φ_1 and l_0 in the Lagrange equation, corresponding to $\varphi_0(t)$.
8. After the obtained equation solving, let's find $\varphi_0(t + \Delta t)$.
9. Let's substitute the initial data instead of $\varphi_2(t)$ and the just found values instead of φ_1, l_0, φ_0 in the Lagrange equation, corresponding to $l_2(t)$.
10. Let's find $l_2(t + \Delta t)$.
11. It remained to find $\varphi_2(t + \Delta t)$.
12. Repeat the steps in cycle, substituting $t_0 + \Delta t$ instead of t_0.
13. The process is repeated in the range of accelerations, changing slowly enough.

2. Determination of the range of the driving moment changing.

The driving moment is considered as the constant one, i.e. in algorithm of the preceding item $f(t)$ are taken as the constants.

Algorithm of the maximum driving moment calculation.

1. Let's all the parameters of the shaft-rotor system with auto-balancing are given.
2. Let's take the small value $M_D = f = const$, not dependent on time.
3. By algorithm of the preceding item we shall find the magnitude $l_0(t)$ – the shaft amplitude and $\dot{\varphi}_1(t)$ – the rotor angular velocity till the moment $t = T$. At that the following is possible:

3.1. $l_0(T) \approx \Gamma$ (where Γ is the dimensional restriction of a design);

3.2. by $t > T/2$ the angular velocity $\dot{\varphi}_1(t)$ of the rotor is practically constant though the amplitude $l_0(t)$ is much less than Γ;

3.3. T is large enough.

4. If the item 3.2 is hold, i.e. by the given M_D the rotor rotation is stabilized and the resonance condition is not expected, we may increase M_D on some value and then it is necessary to return to the step 3 and repeat the calculating process.
5. If the item 3.1 or 3.3 is hold, the calculating process comes to an end, because that value of M_D has been found, by which the near-resonance conditions are begun.

The cases are possible when the initial power of the engine is large and it is necessary to decrease the value of M_D.

3. Optimization of the parameters of the shaft-rotor system with auto-balancing.

As we realize only numerical experiments, which don't demand large financial expenses, we can find more acceptable set of the initial parameters of the shaft-rotor system with ABD at the base of the preceding item algorithm. For that the variations in the parameters' space are necessary. Of course, the difficulties appear with increase of the parameters' space dimension.

Let's present algorithm of several parameters optimization.

1. Let the initial values of the parameters of the shaft-rotor system with ABD are given.
2. Let's take some value of the driving moment $M_D = f$, not dependent on time.
3. The shaft amplitude $l_0(t)$ and the rotor angular velocity $\dot{\varphi}_1(t)$ calculation is realized at the time interval from 0 to T. If $l_0(t_1) \approx \Gamma$ by $\exists\, t_1 < T$, it is necessary to go to the step 4. If there is no necessity in this, it means that the initial parameters are acceptable.
4. One of the parameters is changed (for example, the float geometrical characteristic) move up in.
5. Then we again return to the step 3. At the same time, if the condition $l_0(t_2) \approx \Gamma$ appears earlier than it was (i.e. $t_1 > t_2$), it is necessary to decrease the value of the varied parameter and go to the step 3. If the last one results in a condition $l_0(t_3) \approx \Gamma$ and $t_3 < t_1$, it is necessary to vary the value of the other parameter (for example, properties of material, out of which the float is made).

6. If all the efforts of the parameters varying don't lead to desired result, it is necessary to decrease the value of M_D from the step 2.

4. Optimal control of the shaft-rotor system with ABD

Before finding of the optimal law of the driving moment on time, it is necessary to choose the criterion, by which preference will be given to any dependence against another one.

4.1 The criterion of the shaft amplitude minimum can be written in the form

$$J(f(t)) = \int_0^T (l_0^2(t))\, dt \to \min \text{ by all } f(t): M_{min} \le f(t) \le M_{max}.$$

At the same time $l_0(t)$ is found from the system (1.11) by the different available f(t) by algorithm of the item 3 and every time the integral J(f) is calculated. Then their values are compared. The preference is given to that function f(t), which corresponds with the smallest value of the functional J(f) calculated values. It should be noted that it is necessary to look through all the possible functions f(t) and to discard that values, which don't minimize the functional J(f). It is complicated problem. Such problem is solved in the theory of optimal control with a help of maximum principle. In our case the function has to be maximized will be linear. It achieves the maximum value only on the ends of the closed interval $[M_{min}, M_{max}]$. Therefore we are interested only with those functions f(t), which can assume two values: $f(t) \equiv M_{max}$ by some t, and by another t – the value $f(t)=M_{min}$. Let's notice that if $M_{min} = 0$, the problem solving is possible by $f \equiv 0$, because at this case there are no external impacts and the system is in rest and therefore the shaft amplitude equals to zero, which means equality to zero of the functional J(f). That is why only the case $M_{min} > 0$ is interesting.

So, the problem of the integral $J(f) = \int_0^t l_0^2(t)dt$ minimization is reduced to finding of the engine driving moment in form of the function

$$M_p - M_c = f(t) = \begin{cases} M_{\min}, & by\ somet\ \ from[0,T] \\ M_{\max}, & by\ othert\ from[0,T] \end{cases}$$

Thus, optimal control of the engine work consists of the fact that the engine works only in two modes: M_{max} – maximal allowable mode or M_{min} – minimal allowable mode. The problem will be solved when the moments of switching from one mode to another are found, i.e. it is necessary to find finite number of parameters:

$$0 < t_1 < t_2 < \ldots < t_n < T.$$

We again come to optimization of the parameters N, t_1, t_2, …, t_{N-1}, which need to be chosen in the best way.

Let's present algorithm of selection of the points of switching.

1. The parameters of the shaft-rotor system with ABD are given.
2. The range [M_{min}, M_{max}] of the engine driving moment changing is determined in accordance with parameters from the item 1^0.
3. Let N=1 (without switching).
4. $l_0(t)$ is calculated by $0 < t < T$ by algorithm from the item 1.3 by $f(t) = M_{min}$.
5. $J_1 = \int_0^T l_0^2(t)dt$ is calculated.
6. $l_0(t)$ is calculated by $0<t<T$ by algorithm from the item 1.3 by $f(t) = M_{max}$.
7. $J_2 = \int_0^T l_0^2(t)dt$ is calculated.
8. After comparison J_1 with J_2 the necessary f(t) is chosen.
9. Let N=2 (one switching).
10. $l_0(t)$ is calculated by $0 < t < T$ by algorithm from the item 1.3

$$\text{by } f(t) = \begin{cases} M_{\min}, & 0 < t < T/2 \\ M_{\max}, & T/2 \le t < T \end{cases}$$

and $J_3 = \int_0^T l_0^2(t)dt$ is calculated.

11. $l_0(t)$ is calculated by $0 < t < T$ by algorithm from the item 1.3

$$\text{by } f(t) = \begin{cases} M_{\max}, & 0 < t < T/2 \\ M_{\min,}, & T/2 \le t < T \end{cases}$$

and $J_4 = \int_0^T l_0^2(t)dt$ is calculated.

12. After comparison of J_1, J_2, J_3, J_4, the necessary f(t) is chosen.
13. Let N=3 (two switchings).
14. $l_0(t)$ is calculated by $0<t<T$ by algorithm from the item 1.3

$$\text{by } f(t) = \begin{cases} M_{\min}, & 0 < t < T/3 \\ M_{\max}, & T/3 < T < T \end{cases}$$

and $J_5 = \int_0^T l_0^2(t)\, dt$ is calculated.

15. J_6, J_7 are calculated at the same way.
16. After comparison of J_1, J_2, J_3, J_4, J_5, J_6, J_7, the necessary f(t) is chosen.
17. The calculating process is stopped, when f(t) ceases to change.

Similar algorithm is offered for the fastest achievement of the preset angular velocity of the rotor. It should be noted:

1. The problem of the fastest achievement of the preset angular velocity of the rotor helps to overpass resonance frequency of the shaft-rotor system with ABD in the best way, if the preset angular velocity correspond to over-resonance frequency. Similar problems were studied by many authors [4].

2. As the system of the differential equations (1) is the system, not solved for the highest derivatives, and hasn't canonical form

$$\dot{\vec{x}} = f(x, u, t),$$

direct application of the methods of optimal control is inconvenient. The problem of optimal control is reduced to the problem of parameter optimization. At the same time there is no necessity of the system of the equations (1) solving for the highest derivatives and their reducing to the canonical form.

3. Instead of algorithm from the item 1 it is possible to use any other method, available for solving of the Cauchy problem for the system (1). Simplicity of the given algorithm and its mechanical evidence make it very effective and convenient, especially in combination with the methods of Runge-Cutta type.

4. In contrast to maximum principle, where it is necessary to solve the boundary problems, here only the Cauchy problem for the system of the differential equations is used.

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