6th International IFS and Contemporary Mathematics Conference

June, 07-10, 2019 Mersin Turkey

CONFERENCE PROCEEDING BOOK

EDITOR

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ISBN 978-605-68670-2-6

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PREFACE

We are very pleased to introduce the abstracts of the 6th International IFS and Contemporary Mathematics Conference (IFSCOM2019).
As previous conferences, the theme was the link between the Mathematics by many valued logics and its applications.

In this context, there is a need to discuss the relationships and interactions between many valued logics and contemporary mathematics.

Finally,in the previous conference, it made suceessful activities to communicate with scientists working in similar fields and relations between the different disciplines.

This conference has papers in different areas; multi-valued logic, geometry, algebra, applied mathematics, theory of fuzzy sets, intuitionistic fuzzy set theory, mathematical physics, mathematics applications, etc.

Thank you to all paticipants scientists offering the most significant contribution to this conference.

Thank you to Scientific Committee Members, Referee Committee Members, Local Committee Members, University Administrators, Mersin University Mathematic Department.

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JUNE, 07-10, 2019, MERSIN, TURKEY
pp: 42-49
ISBN:978-605-68670-2-6

SURFACE TO EXACT SOLUTION OF NONLINEAR SCHRODINGER EQUATION

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ABSTRACT. Heisenberg ferromagnetic equation is considered in (1+1)-, (2+1)-dimensions. Surface with appropriate coefficients of the first fundamental form is found for regular onesolitonic solution of the nonlinear Schrodinger equation with gravity which is Lakshmanan equivalence to Heisenberg ferromagnetic equation.

1. INTRODUCTION

Nonlinear models describing different physical phenomena can be solved by inverse scattering method [1]-[6]. One of the well-known nonlinear models is Heisenberg ferromagnetic model

$$\mathbf{S}_t = \mathbf{S} \times \mathbf{S}_{xx}, \tag{1.1}$$

where $\times$ is vector product, $\mathbf{S} = (S_1, S_2, S_3), \mathbf{S} = S_1^2 + S_2^2 + S_3^2 = 1$.

Lakshmanan established, that the model (1) at $\mathbf{S}^2 = +1$ is equivalent in the geometrical sense to nonlinear Schrodinger equation which is crucial for physical applications

$$i\psi_t + \psi_{xx} + 2\beta|\psi|^2\psi = 0, \tag{1.2}$$

where $\beta = +1$, ψ is complex function. This equivalence is called by Lakshmanan equivalence. We note, that Lakshmanan equivalence is valid both for integrable and for nonintegrable nonlinear differential equations, and by definition its applicability domain is limited by establishing an equivalence between a spin system and nonlinear differential equation, for example Schrodinger type. Moreover, for integrable nonlinear differential equation Lakshmanan equivalence does not imply knowledge of Lax representation of considered nonlinear differential equations.

Now some generalizations of the model (1) in (2+1)-dimensions are known. For example, in [5] a generalized Heisenberg Ferromagnetic model is considered

$$\mathbf{S}_t = (\mathbf{S} \times \mathbf{S}_y + u\mathbf{S})_x, \tag{1.3}$$

$$u_x = -(\mathbf{S}, (\mathbf{S}_x \times \mathbf{S}_y)), \tag{1.4}$$

Date: xxxx a, 2019, accepted yyyy b, 2019.
2000 *Mathematics Subject Classification.* Primary 35J10, 35Q51; Secondary 34A34, 37K40.
Key words and phrases. nonlinear equation, immersion, surface, solitonic solution.

where $\mathbf{S}$ is spin vector, $S_1^2+S_2^2+S_3^2=1$, $\times$ is vector product, u is a scalar function. We identify the spin vector $\mathbf{S}$ and vector $\mathbf{r}_x$ according to [2]

$$\mathbf{S}\equiv\mathbf{r}_x.$$

Then (3a), (3b) take the form

$$\mathbf{r}_{xt}=(\mathbf{r}_x\times\mathbf{r}_{xy}+u\mathbf{r}_x)_x$$

$$u_x=-(\mathbf{r}_x,(\mathbf{r}_{xx}\times\mathbf{r}_{xy})).$$

Surface corresponding to onesolitonic solution of the model (1) is found, and the result is formulated and proved in the theorem [5] below. At first we present the one-soliton solution of the equation (3a), (3b) [2],

$$S_3(x,y,t)=1-\frac{2\eta^2}{\eta^2+\xi^2}sech^2(\chi_{1R}),$$

$$S^+(x,y,t)=\frac{2\eta}{\eta^2+\xi^2}[i\xi-\eta th(\chi_{1R})]sech(\chi_{1R}),$$

$$\chi_1=\chi_{1R}+i\chi_{1I},\quad \lambda_1=\eta+i\xi,$$

$$m_1=m_{1R}(\rho)+im_{1I}(\rho),\quad m_j(y,t)=m_j(\rho),$$

$$\chi_{1R}=\eta x+m_{1R}(\rho)+c_{1R},\quad \rho=y+i\lambda_j t,$$

$$\chi_{1I}=\xi x+m_{1I}(\rho)+c_{1I}, c=ln(2\eta/\lambda_1^*),$$

$$m_{1R}(\rho)=Re[m_1(\rho)],\quad m_{1I}(\rho)=Im[m_1(\rho)],$$

which we use in the following theorem.

Theorem 1.1. *Main Theorem. One-soliton solution of the spin system (3a)-(3b) can be represented as components of the vector* $\mathbf{r}_x$, *where* $r_1=\frac{2\eta}{(\eta^2+\xi^2)ch\chi_{1R}}+c_1, r_2=\frac{2\xi}{\eta^2+\xi^2}arctg(sh\chi_{1R})+c_2, r_3=x-\frac{2\eta}{\eta^2+\xi^2}th\chi_{1R}+c_3$, *where* c_1,c_2,c_3 *are constants. Solution of the form* $\mathbf{r}_x$ *corresponds to the surface with the following coefficients of the first and second fundamental forms*

$$E=1,\quad G=\frac{4m_{1Ry}^2}{(\eta^2+\xi^2)ch^2\chi_{1R}},$$

$$F=\frac{2\eta m_{1Ry}}{(\eta^2+\xi^2)ch^2\chi_{1R}},\quad L=\frac{4\eta^3\xi m_{1Ry}}{\sqrt{g}(\eta^2+\xi^2)^2ch^4\chi_{1R}},$$

$$M=\frac{4\eta^2\xi m_{1Ry}^2}{\sqrt{g}(\eta^2+\xi^2)^2ch^4\chi_{1R}},\quad N=\frac{4\eta\xi m_{1Ry}^3}{\sqrt{g}(\eta^2+\xi^2)^2ch^4\chi_{1R}}.$$

2. Soliton surface

In this work we consider soliton immersion in Fokas-Gelfand sense [3]. In the modern literature the notion immersion is widely expanded and related not only to the soliton theory. It is a transition from sophisticated origin problem to some simple problem.

According to the work of Fokas-Gelfand [3] we present the description of the soliton immersion. In (1+1)-dimension the nonlinear differential equations are given in the form of zero curvature condition

$$U_t-V_x+[U,V]=0,$$

where $[U,V]=UV-VU$, the matrix U is prescribed, and matrix V is expressed in the terms of elements of matrix U. Such nonlinear differential equations are compatibility condition of the linear systems

$$\phi_x=U\phi, \phi_t=V\phi.$$

In this case there exists a surface with immersion function $P(x,t)$ defined by formulas $\frac{\partial P}{\partial x}=\phi^{-1}X\phi, \frac{\partial P}{\partial t}=\phi^{-1}Y\phi$. The surface defined by $P(x,t)$ is identical to the surface in three-dimensional space defined by coordinates $x_j=P_j(x,t), \quad j=1,2,3$. Frame on the surface is given by triple [3]

$$\frac{\partial P}{\partial x}=\phi^{-1}X\phi, \quad \frac{\partial P}{\partial t}=\phi^{-1}Y\phi, \quad N=\phi^{-1}J\phi,$$

where $J=\frac{[X,Y]}{|[X,Y]|}, \quad \mid X\mid=\sqrt{<X,X>}$. Here by definition

$$<X,Y>=-\frac{1}{2}tr(XY),$$

where X,Y are some matrixes. The first and second fundamental forms in the Fokas-Gelfand sense are given as

$$I=<X,X>dx^2+2<X,Y>dxdt+<Y,Y>dt^2,$$

$$II=<\frac{\partial X}{\partial x}+[X,U],J>dx^2+2<\frac{\partial X}{\partial t}+[X,V],J>dxdt+ \tag{2.1}$$
$$+<\frac{\partial Y}{\partial t}+[Y,V],J>dt^2.$$

As it is shown in the work [3] the immersion function P can be defined as

$$P=\gamma_0\phi^{-1}\phi_\lambda+\phi^{-1}M_1\phi=\sum_{j=1}^{3}P_jf_j,$$

where M_1 is matrix function defined by λ,x,t. $f_j=-\frac{i}{2}\sigma_j$ is corresponding algebra basis, σ_j are Pauli matrixes and $[f_i,f_j]=f_k$. In this case, X,Y can be written

$$X=\gamma_0U_\lambda+M_{1x}+[M_1,U], Y=\gamma_0V_\lambda+M_{1t}+[M_1,V].$$

Let the matrixes X,Y,J have the forms

$$X=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix}, \quad Y=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{pmatrix}, \quad J=\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22}\end{pmatrix}. \tag{2.2}$$

In this case elements of the matrix J are expressed through elements of the matrix X and Y in correspondence to the following formulas

$$c_{11}=\frac{a_{12}b_{21}-b_{12}a_{21}}{\mid[X,Y]\mid}, \quad c_{21}=\frac{a_{21}(b_{11}-b_{22})+b_{21}(a_{22}-a_{11})}{\mid[X,Y]\mid},$$

$$c_{12}=\frac{b_{12}(a_{11}-a_{22})+a_{12}(b_{22}-b_{11})}{\mid[X,Y]\mid}, \quad c_{22}=\frac{a_{21}b_{12}-b_{21}a_{12}}{\mid[X,Y]\mid}. \tag{2.3}$$

Then the first fundamental form (4) of the surface is $I=Edx^2+2Fdxdt+Gdt^2$, where

$$E=-\frac{1}{2}(a_{11}^2+2a_{12}a_{21}+a_{22}^2), \ F=-\frac{1}{2}(a_{11}b_{11}+a_{12}b_{21}+a_{21}b_{12}+a_{22}b_{22}), \tag{2.4}$$

$$G=-\frac{1}{2}(b_{11}^2+2b_{12}b_{21}+b_{22}^2). \tag{2.5}$$

As example of the soliton equation leading to the immersion we consider nonlinear Schrodinger equation (2). In this case the matrixes U, V take the forms [5]

$$U = \frac{\lambda\sigma_3}{2i} + U_0, \; U_0 = i\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix},$$

$$V = \frac{i\lambda^2}{2}\sigma_3 + i|q|^2\sigma_3 - i\lambda\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix} + \begin{pmatrix} 0 & \bar{q}_x \\ -q_x & 0 \end{pmatrix}. \tag{2.6}$$

The lemma is valid.

Lemma 2.1. *The second fundamental form in the Fokas-Gelfand sense corresponding to the regular onesolitonic solution q of the nonlinear Schrodinger equation has the form*

$$II = Ldx^2 + 2Mdxdt + Ndt^2, \tag{2.7}$$

where

$$L = -\frac{1}{2}\{a_{11x}c_{11} + a_{12x}c_{21} + a_{21x}c_{12} + a_{22x}c_{22} - \lambda i(a_{21}c_{12} - a_{12}c_{21}) + \tag{2.8}$$

$$+iq(a_{12}c_{11} + a_{22}c_{12} - a_{11}c_{12} - a_{12}c_{22}) + i\bar{q}(a_{21}c_{22} + a_{11}c_{21} - a_{22}c_{21} - a_{21}c_{11})\},$$

$$M = -\frac{1}{2}\{a_{11t}c_{11} + a_{12t}c_{21} + a_{21t}c_{12} + a_{22t}c_{22} + i(\lambda^2 + 2|q|^2)(a_{21}c_{12} - a_{12}c_{21}) +$$

$$+(q_x + \lambda iq)(a_{11}c_{12} + a_{12}c_{22} - a_{12}c_{11} - a_{22}c_{12}) +$$

$$+(\bar{q}_x - \lambda i\bar{q})(a_{11}c_{21} + a_{21}c_{22} - a_{21}c_{11} - a_{22}c_{21})\},$$

$$N = -\frac{1}{2}\{b_{11t}c_{11} + b_{12t}c_{21} + b_{21t}c_{12} + b_{22t}c_{22} + i(\lambda^2 + 2|q|^2)(b_{21}c_{12} - b_{12}c_{21}) +$$

$$+(q_x + \lambda iq)(b_{11}c_{12} + b_{12}c_{22} - b_{12}c_{11} - b_{22}c_{12}) +$$

$$+(\bar{q}_x - \lambda i\bar{q})(b_{11}c_{21} + b_{21}c_{22} - b_{21}c_{11} - b_{22}c_{21})\}.$$

Proof. We substitute the matrixes (6), (10) to (5). After some algebra we get (11), (12). The lemma is proved. □

3. Surface to a regular onesolitonic solution

We consider a particular case at $\gamma_0 = 1, \quad M_1 = 0$. In this case we get

$$X = U_\lambda = \frac{1}{2i}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; Y = V_\lambda = -i\begin{pmatrix} -\lambda & \bar{q} \\ q & \lambda \end{pmatrix}, \tag{3.1}$$

$$J = \begin{pmatrix} 0 & -\frac{\bar{q}}{\sqrt{q\bar{q}}} \\ \frac{q}{\sqrt{q\bar{q}}} & 0 \end{pmatrix},$$

and $P = \phi^{-1}\phi_\lambda$. In order to calculate the explicit expressions for immersion function P we consider the regular onesolitonic solution of the nonlinear Schrodinger equation which has the form [4]

$$q(x,t) = 2\eta\frac{exp(-2i\xi x - 4i(\xi^2 - \eta^2)t - i\delta)}{ch[2\eta(x + 4\xi t - x_0)]}, \tag{3.2}$$

where $x_0 = \frac{1}{2\eta}ln|\frac{m_{02}}{m_{01}}|$, $\delta = argm_{02} - argm_{01}$, $\xi = Re\lambda$, $\eta = Im\lambda$.

Theorem 3.1. *Regular onesolitonic solution of the nonlinear Schrodinger equation corresponds to the surface in Fokas-Gelfand sense with the coefficients of the first fundamental form*

$$E = \frac{64\eta^2(\xi^2+\eta^2)}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t-x_0)]}, \tag{3.3}$$

$$F = \frac{128\eta^2\xi(\xi^2+\eta^2)}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t-x_0)]}, \tag{3.4}$$

$$G = \frac{256\eta^2(\xi^2+\eta^2)^2}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t-x_0)]}, \tag{3.5}$$

where $\lambda_1 = const.$

Proof. Solution of the linear system we find in the form

$$\psi = \phi e^{-(\frac{\lambda\sigma_3}{2i}x + \frac{i\lambda^2}{2}\sigma_3 t)}. \tag{3.6}$$

Taking into account (16) and applying (10), we get

$$\psi_x = (\frac{\lambda\sigma_3}{2i} + U_0)\psi - \psi\frac{\lambda\sigma_3}{2i} = \frac{\lambda\sigma_3}{2i}\psi - \psi\frac{\lambda\sigma_3}{2i} + U_0\psi = [\frac{\lambda\sigma_3}{2i}, \psi] + U_0\psi. \tag{3.7}$$

We take

$$\psi = I - \frac{\tilde{A}}{\lambda-\lambda_1^*}, \text{ where } \tilde{A} = \begin{pmatrix} \tilde{a} & \tilde{b} \\ \tilde{c} & \tilde{d} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \lambda_1^* - const. \tag{3.8}$$

We substitute (18) to (17)

$$\psi_x = U_0 - \frac{U_0\tilde{A}}{\lambda-\lambda_1^*} - \frac{1}{2i}[\sigma_3, \tilde{A}] - \frac{\lambda_1^*}{2i(\lambda-\lambda_1^*)}[\sigma_3, \tilde{A}]. \tag{3.9}$$

On the other hand, from (18) it follows that

$$\psi_x = -\frac{\tilde{A}_x}{\lambda-\lambda_1^*}. \tag{3.10}$$

From (19) and (20) we get

$$-\frac{\tilde{A}_x}{\lambda-\lambda_1^*} = U_0 - \frac{U_0\tilde{A}}{\lambda-\lambda_1^*} - \frac{1}{2i}[\sigma_3, \tilde{A}] - \frac{\lambda_1^*}{2i(\lambda-\lambda_1^*)}[\sigma_3, \tilde{A}]. \tag{3.11}$$

Thus

$$\tilde{A}_x = U_0\tilde{A} + \frac{\lambda_1^*}{2i}[\sigma_3, \tilde{A}], U_0 = \frac{1}{2i}[\sigma_3, A]. \tag{3.12}$$

We note, that

$$[\sigma_3, \tilde{A}] = \sigma_3\tilde{A} - \tilde{A}\sigma_3 = 2\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}. \tag{3.13}$$

Then we substitute (23) to (7) and get

$$U_0 = \frac{1}{i}\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}. \tag{3.14}$$

Substituting (23) to (22), we get

$$\begin{pmatrix} \tilde{a}_x & \tilde{b}_x \\ \tilde{c}_x & \tilde{d}_x \end{pmatrix} = \frac{1}{i}\begin{pmatrix} \tilde{b}\tilde{c} & \tilde{b}\tilde{d} \\ -\tilde{c}\tilde{a} & -\tilde{c}\tilde{b} \end{pmatrix} + \frac{\lambda_1^*}{i}\begin{pmatrix} 0 & \tilde{b} \\ -\tilde{c} & 0 \end{pmatrix}, \tag{3.15}$$

From (10) and (24) we get

$$(3.16)\qquad i\begin{pmatrix} 0 & \bar{q} \\ q & 0 \end{pmatrix} = \frac{1}{i}\begin{pmatrix} 0 & b \\ -c & 0 \end{pmatrix} \Rightarrow \begin{cases} i\bar{q} = \frac{1}{i}b \\ iq = -\frac{1}{i}c \end{cases} \Rightarrow \begin{cases} b = -\bar{q} \\ c = q. \end{cases}$$

Therefore, we have found the matrix $\tilde{A}$ in the explicit form with components (25). Using (14) we get

$$(3.17)\qquad \tilde{a} = i2\eta th[2\eta(x + 4\xi t - x_0)] + c_1.$$

From (25) it follows that $\tilde{a} = -\frac{i\tilde{c}_x}{c} - \lambda_1^* \Rightarrow \tilde{a} = -\frac{1}{i}\int \bar{q}q dx$. Using (14) we get

$$(3.18)\qquad \tilde{a}_x = \frac{1}{i}\tilde{b}\tilde{c} \Rightarrow \tilde{a}_x = \frac{1}{i}(-\bar{q})q.$$

Then

$$(3.19)\qquad \tilde{a} = -\frac{iq_x}{q} - \lambda_1^*.$$

Consequently, taking into account (25), (26), we get

$$(3.20)\qquad \tilde{d} = \frac{i\tilde{b}_x}{\tilde{b}} - \lambda_1^* \Rightarrow \tilde{d} = \frac{i(-\bar{q})_x}{(-\bar{q})} - \lambda_1^* \Rightarrow \tilde{d} = \frac{i\bar{q}_x}{\bar{q}} - \lambda_1^*.$$

From (25), (26) it follows that

$$(3.21)\qquad \tilde{d}_x = -\frac{1}{i}\tilde{c}\tilde{b}.$$

Moreover, in view of (23), (31), we have

$$(3.22)\qquad \tilde{d} = \frac{1}{i}\int q\bar{q}dx.$$

Taking into account (22), we get (28) in the form

$$(3.23)\qquad \tilde{d} = -\tilde{a}.$$

Therefore,

$$(3.24)\qquad \tilde{a} = -i2\eta th[2\eta(x + 4\xi t - x_0)] + c_1.$$

Thus, the matrix $\tilde{A}$ for regular onesolitonic solution (14) of the nonlinear Schrodinger equation takes the form

$$(3.25)\qquad \tilde{A} = \begin{pmatrix} i2\eta th[2\eta(x + 4\xi t - x_0)] + c_1 & -2\eta\frac{exp\{i(2\xi x + 4(\xi^2-\eta^2)t+\delta)\}}{ch[2\eta(x-x_0+4\xi t)]} \\ 2\eta\frac{exp\{-i(2\xi x + 4(\xi^2-\eta^2)t+\delta)\}}{ch[2\eta(x-x_0+4\xi t)]} & -i2\eta th[2\eta(x + 4\xi t - x_0)] + c_1 \end{pmatrix}.$$

Then we take $\phi = I - \frac{A}{(\lambda-\lambda_1)^2}$, where λ_1 is constant, then from (13), we get

$$(3.26)\qquad P = \phi^{-1}\phi_\lambda = (I + \frac{\tilde{A}}{\lambda - \lambda_1})\frac{\tilde{A}}{(\lambda - \bar{\lambda_1})^2}.$$

On the other hand, we get

$$(3.27)\qquad P = \sum_{j=1}^{3} P_j f_j = -\frac{i}{2}\sum_{j=1}^{3} P_j\sigma_j = \begin{pmatrix} -\frac{i}{2}P_3 & -\frac{i}{2}P_1 - \frac{1}{2}P_2 \\ -\frac{i}{2}P_1 + \frac{1}{2}P_2 & \frac{i}{2}P_3 \end{pmatrix}.$$

From (36), (37) by (31) we get $P_3 = \frac{2i\tilde{a}}{(\lambda-\bar{\lambda}_1)^2}$. With help of (33) we find P_3 in the explicit form for regular onesolitonic solution of the nonlinear Schrodinger equation

$$P_3 = -\frac{4\eta}{(\lambda-\bar{\lambda})^2} th[2\eta(x+4\xi t - x_0)] + c_1. \tag{3.28}$$

From (36), (37) we get $P_2 = \frac{\tilde{c}-\tilde{b}}{(\lambda-\bar{\lambda}_1)^2}$. Thus

$$P_1 = \frac{i(\tilde{c}+\tilde{b})}{(\lambda-\bar{\lambda}_1)^2}, \quad P_2 = \frac{(\tilde{c}-\tilde{b})}{(\lambda-\bar{\lambda}_1)^2}, \quad P_3 = \frac{2i\tilde{a}}{(\lambda-\bar{\lambda}_1)^2}.$$

From (36), (14) using the well-known formulas

$$sh\zeta = \frac{e^{\zeta}-e^{-\zeta}}{2};\ ch\zeta = \frac{e^{\zeta}+e^{-\zeta}}{2};\ cos\zeta = \frac{e^{i\zeta}+e^{-i\zeta}}{2};\ sin\zeta = \frac{e^{i\zeta}-e^{-i\zeta}}{2i}, \tag{3.29}$$

where $\zeta = 2\eta(x - x_0 + 4\xi t)$, we obtain the values for the components P_1, P_2 of the matrix P

$$P_1 = \frac{4\eta sin(2\xi x + 4(\xi^2-\eta^2)t + \delta)}{(\lambda-\bar{\lambda})^2 ch[2\eta(x+4\xi t - x_0)]}, \tag{3.30}$$

$$P_2 = \frac{4\eta cos(2\xi x + 4(\xi^2-\eta^2)t + \delta)}{(\lambda-\bar{\lambda})^2 ch[2\eta(x+4\xi t - x_0)]}. \tag{3.31}$$

Then we calculate coefficients of the first fundamental form by formula

$$E = P_{1x}^2 + P_{2x}^2 + P_{3x}^2. \tag{3.32}$$

We calculate the derivatives P_{1x}, P_{2x}, P_{3x}. The square of the first derivatives is substituted to (41), then

$$E = \frac{64\eta^2(\xi^2+\eta^2)}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t - x_0)]}.$$

By the similar way, by the formulas

$$F = P_{1x}P_{1t} + P_{2x}P_{2t} + P_{3x}P_{3t}, \quad G = P_{1t}^2 + P_{2t}^2 + P_{3t}^2$$

we obtain the value

$$F = \frac{128\eta^2\xi(\xi^2+\eta^2)}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t - x_0)]}, \tag{3.33}$$

$$G = \frac{256\eta^2(\xi^2+\eta^2)^2}{(\lambda-\bar{\lambda})^4 ch^2[2\eta(x+4\xi t - x_0)]}. \tag{3.34}$$

Theorem is proved. □

The surface can be written in the form

$$P_3 = (P_1, P_2).$$

Then from (38), (40a), (40b) we get

$$P_3 = -\frac{(\lambda-\bar{\lambda})^2 sh[2\eta(x+4\xi t - x_0)] ch[2\eta(x+4\xi t - x_0)]}{4\eta sin(2\xi x + 4(\xi^2-\eta^2)t+\delta) cos(2\xi x + 4(\xi^2-\eta^2)t+\delta)} P_1 P_2.$$

Then we use possibilities of the editor Maple and construct the surface at some values of the parameters.

Remark 3.2. The short version of this paper was published in abstracts of the Third Dynamic Days in Central Asia, Nazarbayev University, Astana, Kazakhstan (September 02-05, 2016) [7].

4. Conclusion

Thus, we investigate Heisenberg ferromagnetic equation in (1+1)-, (2+1)-dimensions. As example, we have considered (1+1)-dimensional nonlinear Schrodinger equation. The first fundamental form with corresponding coefficients (15) is found for integrable surface corresponding to regular onesolitonic solution of the nonlinear Schrodinger equation with gravity.

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