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ABSTRACTS

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Inverse Spectral Problems and Related Topics

(Dedicated to the 70th anniversary of an outstanding expert in differential operators and spectral problems, Professor Muhtarbay Otelbaev)

Necessary minimum conditions and Fourier method for numerical solution of linear inverse problems

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Two different linear inverse problems are considered. First is an application of the quasisolution and Fourier method to a mixed initial-boundary problem for Laplace's equation (Problem I).

Second is a source identification problem for heat transfer equation in the case of given final observation data (Problem II). In the considered case the source density has the form $f(x)g(t)$, where the function $g(t)$ is given, and the function $f(x)$ is to be determined.

Because both of the considered problems are classical ones, there exists multiple references. We mention here only the closest to the considered topic ([1])-([10]). The necessary condition of the minimum of the residual functional, which expresses unknown function through the solution to corresponding adjoint problem, is used ([2],[6]-[9]). Then, both problems are reduced to a system consisting of mutually dependent direct and adjoint problems as follows:

Problem I. Residual functional is defined as

$$J[r] = \frac{1}{2}\int_0^b (u_x(0,y;r) - g(y))^2 dy + \frac{1}{2}\beta\int_0^b r^2(s)ds \to \min, \quad (1)$$

coupled system of direct and adjoint problem is the following

$$\left\{ \begin{array}{l} u_{xx} + u_{yy} = 0, \\ v_{xx} + y_{yy} = 0, (x,y) \in \Omega = (0,a) \times (0,b) \end{array} \right. \quad (2)$$

$$\begin{array}{l} v(x,0) = 0,\ v(x,b) = 0, \\ u(x,0) = 0,\ u(x,b) = 0, \\ v(a,y) = 0,\ u(0,y) = 0, \\ \beta \cdot u(a,y) - v_x(a,y) = 0, \\ u_x(0,y;r) - v(0,y) = g(y). \end{array} \quad (3)$$

Problem II. Corresponding residual functional equals to

$$J[f] = \int_0^1 (u(x,T) - u_1(x))^2 dx + \beta\int_0^1 f^2(x)dx \to \min, \quad (4)$$

and necessary minimum conditions yields to the following system

$$\left\{ \begin{array}{l} u_t = \Delta u - \frac{g(t)}{2\beta}\int_0^T v(x,s)g(s)ds, \\ v_t = -\Delta v, \quad x \in \Omega, t \in (0,T) \\ u(x,0) = 0, \quad v(x,T) = 2(u(x,T) - u_1(x)), \quad x \in \Omega, \\ \nabla u(x,t) = 0, \nabla v(x,t) = 0, x \in \Gamma_1 \\ \nabla u(1,t) + \mu u(1,t) = 0, \nabla v(x,t) + \mu v(x,t) = 0, x \in \Gamma_2, \quad \partial\Omega = \Gamma_1 \cap \Gamma_2 \end{array} \right. \quad (5)$$

The explicit formulas of the solutions are obtained by Fourier method when Ω is a rectangular. Then the series are calculated numerically. Computational experiments are performed for different type of syntetic data, and admissible parameter ranges are established.

Numerical simulations have shown that the use of Fourier method is an efficient way to solve the Problem I. We have a choice either to use it directly ([1]) or to apply it with a quasisolution approach. It turns out that the use of small $\beta > 0$ leads to better results than those when we apply Fourier method directly by taking $\beta = 0$. In addition, the case $\beta > 0$ lets us consider more number of harmonics. We have determined more admissible values of main parameters such as a harmonic number and a regularization parameter. It turns out that the most useful range for β is above $10^{-15} - 10^{-16}$, and one for M is $15 - 18$.

For Problem II the accuracy of the solution strongly depends on the behavior of the function $g(t)$. In several cases a recovery of the unknown solution have been obtained with machine accuracy, even if a regularization parameter is equal to zero. Discontinuous and oscillating functions are recovered with sufficiently accuracy as well.

Short-Bio Balgaisha Mukanova graduated from the Mathematics and Mechanics Department of Navosibirsk State University, then graduate school at the Institute of Theoretical and Applied Mechanics in Novosibirsk, USSR. She received a candidate degree (analog of PhD in USSR) in Mechanics of Fluids, Gaz and Plazma (1985); then Physics and Mathematics Doctor degree at Kazakh National University (2010). Since 2002 she has been working at the Department of Mathematical and Computer Modeling in the Kazakh National University. Her research interests include numerical methods of the solution to ill-posed problems.

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