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Содержание

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Some generalization of notion of algebraic independence

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Let T be a complete theory T, A be a set of some $|A|^+$-saturated model M and p be a non-algebraic one-type over A. Let $\alpha \in p(M)$, i.e. α lies in the set of all realizations of p in M. An 2-A-formula $\phi(x,y)$ is called to be *p-preserving*, if $\phi(M,\alpha) \subset p(M)$. We call *algebraic closure of* α *in* p the set $alg_p(\alpha) := \{\beta \in p(M) | \beta \in alg(A\alpha)\}$. We call *quasi-neighborhood of* α *in* p the set

$$QV_p(\alpha) := \{\beta | \text{there is p-preserving 2-A-formula } \phi(x,y), \text{ such that } \beta \in \phi(M,\alpha)\}.$$

It follows from definition and properties of algebraic closure $alg_p(\alpha) \subseteq QV_p(\alpha)$ and $alg_p(\alpha) = alg(A\alpha) \cap p(M)$.

We say, that p-preserving $2-A$-formula $\phi(x,y)$ is called to be $V-p$*-preserving* (($p \iff p$)-*preserving*), if $\psi(x,y) := \phi(y,x)$ is p-preserving too. For example, $x = y$ is $V-p$-preserving for any non-algebraic one-type. Define *algebraic neighborhood of* α *in* p as the next set $alg_{V,p}(\alpha) := \{\beta \in alg_p(\alpha) | \alpha \in alg_p(\beta)\}$.

We define *neighborhood of* α *in* p as the set $V_p(\alpha) :=$
$\{\beta | \text{there exists V-p-preserving 2-A-formula } \phi(x,y), \text{ such that } \beta \in \phi(M,\alpha)\}$.

Notice that $alg_{V,p}(\alpha) \subseteq alg_p(\alpha)$, $alg_{V,p}(\alpha) \subseteq V_p(\alpha) \subseteq QV_p(\alpha)$.

There are complete theories distinguished these notions. For strongly minimal theories these four notions are coincided. For complete theories admitting exchange principle for algebraic closure in any one-type the notions algebraic closure in type and algebraic neighborhood in type are coincided, for example for ω-stable theories of finite rank of Morley, geometrical theories.

We say that $2-A$-formula $\phi(x,y)$ is $(p \to q)$*-preserving*, if $\phi(M,\alpha) \subset q(M)$, and is $(p \iff q)$*-preserving*, if $\phi(M,\alpha) \subset q(M)$ and $\phi(\beta, M) \subset p(M)$, for some (equivalently, any) $\alpha \in p(M, \beta \in q(M))$.

Let $\alpha \in p(M)$, $p \in S_1(A)$. Then define *quasi-neighborhood of* α *in* q and *quasi-neighborhood of* α *over the set* A as two next sets: $QV_q(\alpha) := \{\beta | \text{there is } (p \to q) - \text{preserving 2-A-formula } \phi(x,y), \beta \in \phi(M,\alpha)\}$, $QV(\alpha) := \cup_{q \in S_1(A)} QV_q(\alpha)$.

Notice that we can define *algebraic neighborhood of* α *over the set* A as the set

$$alg(A\alpha) \setminus alg(A) = \cup_{q \in S_1(A)} alg_q(A) =: alg(A\alpha|A).$$

Define *neighborhood of* α *in* q and *neighborhood of* α *over the set* A as the sets $V_q(\alpha) := \{\beta | \text{there is } (p \iff q) - \text{preserving 2-A-formula } \phi(x,y), \beta \in \phi(M,\alpha)\}$, $V(\alpha) := \cup_{q \in S_1(A)} V_q(\alpha)$.

Notice that for every complete theory for every set A for every two one-types p, q over A, for any $\alpha \in p(M)$, $alg_q(\alpha) = alg_{V,q}(\alpha)$ iff T satisfies exchange principle for algebraic closure.

Definition of V-independent tuple. For sequence $< \alpha_1, \alpha_2, ..., \alpha_n >$, for $1 \leq k < j \leq n$ denote by $p_{k,j} := tp(\alpha_j | A\alpha_, ..., \alpha_k)$.

We say that sequence of different elements $< \alpha_1, \alpha_2, ..., \alpha_n >$ of model M is V-independent, if for every $1 \leq i < n$, for every $i < j \leq n$ the following holds:

$$(i)\ \alpha_j \notin V_{p_{i-1,j}}(\alpha_i);\ (ii)\ \alpha_i \notin V_{p_{i-1,i}}(\alpha_j).$$

Notice that the condition (i) is equivalent to condition (ii) for small theories having the property: RK-relation (Rudin-Keisler) on the set of types is relation of equivalence, and in this situation we have the partition of $M \setminus alg(A)$ by relation of equivalence (in general, non-definable) $x \in V(y)$, here $V(y)$ over set A.

In our report we define (V, n)-gon and present the theorem that for arbitrary complete theory the existence of $(V, 3)$-gon implies for any $n(3 < n < \omega)$ the existence of (V, n)-gon.

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On simultaneous omitting and realizing countable families of non-principal types

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Let $\{p_n(\bar{x}_n) | n \in \omega\}$ and $\{q_n(\bar{y}) | n \in \omega\}$ be two families of complete non-isolated types over an empty set in a small theory T, such that for every natural number $n \in \omega$ there is a model $\mathfrak{M}_n \models T$ which realizes p_i and omits q_i for each $i \leq n$.

Question. Is there a countable model of the theory T which realizes every p_i and at the same time omits every q_i?

We give a criteria for existence of such a countable model, but this is not a complete answer to the question.

A complete countable theory T is said to be *small* if $|\bigcup\limits_{n<\omega} S_n(T)| = \omega$, where $S_n(T)$ is the set of all n-types over $\emptyset$. Notice that for every countable model $\mathfrak{M} = \langle M, \Sigma \rangle$ of a small theory T, for every finite $A \subset M$ the set of all 1-types over A is at most countable ($|S_1(A)| \leq \omega$), and there is a countable saturated model $\mathfrak{N} = \langle N, \Sigma \rangle$, such that $\mathfrak{M}$ is an elementary substructure of $\mathfrak{N}$. Each type $q_i(\bar{y}_i)$ can be represented as a set of formulas $\{H_{i,m} | m \in \omega\}$ such that $T \vdash \forall y_i (H_{i,m+1}(\bar{y}_i) \to H_{i,m}(\bar{y}_i))$ (a strictly decreasing family).

Let $p_n(\bar{x}_n)$ be a non-principal type over a finite subset A of some model of T. Denote by T_0 the logical closure of $T \cup \bigcup\limits_{n\in\omega} p_n(\bar{c}_n)$ in the signature $\Sigma(C) := \Sigma \cup \{c_n | n \in \omega\}$. Denote by $T_{0,n}$ the logical closure of $T_0 \cup \bigcup\limits_{j\leq n} p_j(\bar{c}_j)$ in the signature $\Sigma(C_n) := \Sigma \cup \{c_j | j \leq n\}$.

Theorem 1. *If for some $i, n, m \in \omega$ we have $T_0 \vdash \forall \bar{y}_i (\phi(\bar{y}_i, c_1, c_2, ..., c_n) \to H_{i,m}(\bar{y}_i))$, then $T_{0,n} \vdash \forall \bar{y}_i (\phi(\bar{y}_i, c_1, c_2, ..., c_n) \to H_{i,m}(\bar{y}_i))$.*

Let $\phi(\bar{y}_i, c_1, c_2, ..., c_n)$ ($\phi(\bar{y}_i, \bar{c}_n)$) be a formula such for every $H_{i,m}(\bar{y}_i) \in q_i(\bar{y}_i)$ the following holds: $T_0 \vdash \forall \bar{y}_i (\phi(\bar{y}_i, \bar{c}_n) \to H_{i,m}(\bar{y}_i))$. Then by Theorem 1 we have that $T_{0,n} \vdash \forall \bar{y}_i (\phi(\bar{y}_i, \bar{c}_n) \to H_{i,m}(\bar{y}_i))$. Since $T_{0,n}$ has infinitely many models of T omitting $q_i(\bar{y}_i)$, $T_{0,n} \cup \{\neg \exists \bar{y}_i \phi(\bar{y}_i, \bar{c}_n)\}$ has to be consistent and consequently, $T_0 \cup \{\neg \exists \bar{y}_i \phi(\bar{y}_i, \bar{c}_n)\}$ has to be consistent. Moreover, for any $k \in \omega$ such that $i \leq n + k$, we have $\mathfrak{M}_{n+k} \models \neg \exists \bar{y}_i \phi(\bar{y}_i, \bar{c}_n)$.

Let us extend the theory T_0. Take T_1 to be a logical closure of the set $T_0 \cup \{\neg \exists \bar{y}_i \phi(\bar{y}_i, \bar{c}_n)$ formula of $\Sigma(C_n) | \exists i \in \omega, \forall m \in \omega, T_0 \vdash \phi(\bar{y}_i, \bar{c}_n) \to H_{i,m}(\bar{y}_i)\}$. Notice that T_1 is consistent, because any finite subset of T_1 has infinite number of models of $\Sigma(C_n)$ for appropriate $n \in \omega$. Suppose T_1 is not complete. For every $n, i \in \omega$ we consider the following set of one-$\Sigma(C_n)$-formulas

$$\Gamma_{n,i} := \{\phi(\bar{y}_i, \bar{c}_n) | \exists \bar{y}_i (\phi(\bar{y}_i, \bar{c}_n)) \in T_{1,n},\ \forall m \in \omega, T_1 \cup \{\forall \bar{y}_i (\phi(\bar{y}_i, \bar{c}_n) \to H_{i,m}(\bar{y}_i))\} \text{ is consistent}\}.$$

For each $n, i, l, m \in \omega$ and for l-th formula $\phi_l \in \Gamma_{n,i}$ denote the next $\Sigma(C_n)$-sentence: $S_{n,i,l,m}(\bar{c}_n) := \forall \bar{y}_i (\phi_l(\bar{y}_i, \bar{c}_n) \to H_{i,m}(\bar{y}_i))$.