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ИНСТИТУТ
ИНФОРМАЦИОННЫХ И
ВЫЧИСЛИТЕЛЬНЫХ
ТЕХНОЛОГИЙ КН МОН РК

ӘЛ-ФАРАБИ атындағы
ҚАЗАҚ ҰЛТТЫҚ УНИВЕРСИТЕТІ

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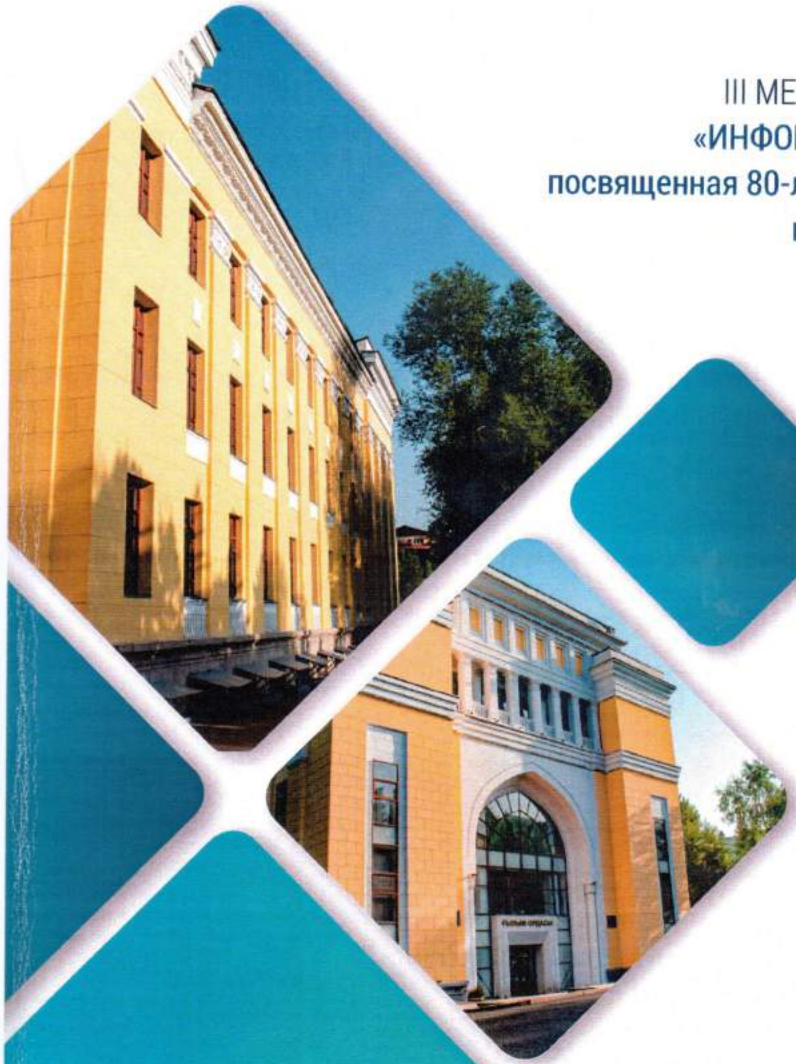
МАТЕРИАЛЫ

III МЕЖДУНАРОДНОЙ НАУЧНОЙ КОНФЕРЕНЦИИ
«ИНФОРМАТИКА И ПРИКЛАДНАЯ МАТЕМАТИКА»
посвященная 80-летнему юбилею профессора Бияшева Р.Г.
и 70-летию профессора Айдарханова М.Б.



(ЧАСТЬ I)

26-29 сентября 2018 года

Алматы

Институт информационных и вычислительных технологий
МОН РК

Казахский Национальный Университет имени аль-Фараби

Университет Туран

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МАТЕРИАЛЫ

III Международной научной конференции
«Информатика и прикладная математика»,
посвященная 80-летнему юбилею
профессора Бияшева Р.Г.
и 70-летию профессора Айдарханова М.Б.

26-29 сентября 2018 года, Алматы, Казахстан

Часть 1

Алматы 2018

УДК 004(063)
ББК 32.973
И74

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И 74 **Информатика и прикладная математика:** Мат. III Межд. науч. конф. (26-29 сентября 2018 г). Часть 1. – Алматы, 2018. – 346 с.

ISBN 978-601-332-165-3

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УДК 004(063)
ББК 32.973

ISBN 978-601-332-165-3

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ON THE COEFFICIENT INVERSE PROBLEM OF HEAT CONDUCTION IN A DEGENERATING DOMAIN

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Abstract. *In the paper we consider a coefficient inverse problem for the heat equation in a degenerating angular domain. It has been shown that the inverse problem for the homogeneous heat equation with homogeneous boundary conditions has a nontrivial solution up to a constant factor consistent with the integral condition. Moreover, the solution of the considered inverse problem is found in explicit form.*

Key woirds: *Coefficient inverse problem Heat equation Degenerating domain.*

1. Introduction

The inverse problems of this kind were investigated in the papers [1], [2] (see also literature from these works). In that papers it is assumed that the movable boundaries move according to the law obeying Holder class and the domain does not degenerate and the time interval is limited. There uniqueness and existence of the solution of the inverse problem where the required coefficient is a continuous function are established and numerical solutions are obtained.

The peculiarity of our study is that we consider the inverse problem for the heat equation in the degenerating angular domain. For the sake of simplicity and for the purpose of showing the effect of the degeneration of the domain, we consider the problem, where, firstly, the moving part of the boundary changes linearly; secondly, the boundary value problem is completely homogeneous; thirdly, the time interval is semi-bounded. It is known that when a domain degenerates at some points, the methods of separation of variables and integral transformations are generally not applicable to this type of problems. In this paper, to prove the existence of a non-trivial solution for the original problem we use the methods and results of our earlier works [3]–[6] where solutions are found with help of theory of thermal potentials and the Volterra integral equation of the second kind.

We also note works [7] and [8] devoted to the study of the existence of nontrivial solutions for partial differential equations, including for degenerating equations. In the paper [10] a theorem on the unique solvability of the non-homogeneous boundary value problem in weighted Holder spaces was obtained. We also note publications [11]–[15] of other authors that are close by category of this item of work.

The paper is divided as follows. In Section 2, we give statement of the problem. In Section 3, we give auxiliary inverse problem in infinite domain. In Section 4, we present equivalent form of auxiliary problem. Section 5 is devoted to existence of the nontrivial solution (up to a constant factor). Nontrivial solution of equivalent form of auxiliary inverse problem is described in Sections 5 and 6. Sections 7 and 8 are devoted to the mathematical justification of the solution of the auxiliary inverse problem obtained in sections 5 and 6. In Section 9 we establish the order of singularity of the solution of the original inverse problem for small values of independent variable t.

Finally, conclusions are made in Section 10.

2. Statement of the problem

In the domain $G_T = \{(x,t) \mid 0 < x < t,\ 0 < t < T,\}, T < +\infty$, we consider an inverse problem of finding a coefficient $\lambda(t)$ and a function $u(x,t)$ for following heat equation:

$$u_t(x,t) = u_{xx}(x,t) - \lambda(t)u(x,t), \tag{1}$$

with homogeneous boundary conditions

$$u(x,t)\,|_{x=0} = 0, \quad u(x,t)\,|_{x=t} = 0, 0 < t < T, \tag{2}$$

suspect to the overspecification

$$\int_0^t u(x,t)dx = E(t), \quad E(t) \geq \delta > 0, 0 < t < T, \tag{3}$$

where $E(t) \in L_\infty(0,T)$ is a given function.

3. The auxiliary problem

In accordance to the problem (1)–(3) we will set an auxiliary inverse problem in the domain $G_\infty = \{(x,t) \mid 0 < x < t,\ t > 0\}$:

$$u_t(x,t) = u_{xx}(x,t) - \lambda(t)u(x,t), \tag{4}$$

with homogeneous boundary conditions

$$u(x,t)|_{x=0} = 0, \quad u(x,t)|_{x=t} = 0, t > 0, \tag{5}$$

suspect to the overspecification

$$\int_0^t u(x,t)dx = \widetilde{E}(t),\ t > 0, \tag{6}$$

$$\widetilde{E}(t) = \begin{cases} E(t), & 0 < t < T, \\ E_1(t), & T \le t < \infty, \end{cases} \tag{7}$$

where $E_1(t) \ge \delta > 0$ -- an arbitrary bounded function.

Remark 1. Solving in G_∞ the problem (4)–(7) and restricting down its solution to the domain G_T, we can find a solution $\{u(x,t), \lambda(t); (x,t) \in G_T\}$ of the original inverse problem (1)–(3).

4. Equivalent problem

In the problem (4)–(6) we replace the required function by the following transformation

$$w(x,t) = e^{\int_0^t \lambda(s)ds} u(x,t) = \hat{\lambda}(t)u(x,t). \tag{8}$$

Then the inverse problem (4)–(6) reduces to a problem for a homogeneous heat equation:

$$w_t(x,t) = w_{xx}(x,t),\ \{x,t\} \in G_\infty, \tag{9}$$

with homogeneous boundary conditions

$$w(x,t)|_{x=0} = 0, \quad w(x,t)|_{x=t} = 0, t > 0, \tag{10}$$

subject to the overspecification

$$\int_0^t w(x,t)dx = \hat{\lambda}(t)\widetilde{E}(t),\ \widetilde{E}(t) \ge \delta > 0, t > 0. \tag{11}$$

5. On a nontrivial solution of the homogeneous boundary value problem (9)–(10)

It follows from our previous results [3]–[6] that a homogeneous boundary value problem (9)–(10) along with a trivial solution has a nontrivial solution up to a constant factor defined by formulas:

$$w(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x}{(t-\tau)^{3/2}} \exp\left\{-\frac{x^2}{4(t-\tau)}\right\} \nu(\tau)d\tau +$$

$$+\frac{1}{4\sqrt{\pi}} \int_0^t \frac{x-\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\} \varphi(\tau)d\tau, \tag{12}$$

$$\nu(t) = \frac{1}{2\sqrt{\pi}} \int_0^t \frac{\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} \varphi(\tau)d\tau, \tag{13}$$

where function $\varphi(t)$ is defined according to the formula:

$$\varphi(t) = C\varphi_0(t),\ C = const \neq 0, \tag{14}$$

$$\varphi_0(t) = \frac{1}{\sqrt{t}} \exp\left\{-\frac{t}{4}\right\} + \frac{\sqrt{\pi}}{2}\left[1 + erf\left(\frac{\sqrt{t}}{2}\right)\right], \tag{15}$$

moreover, the function $\varphi(t)$ belongs to the following class:

$$\theta(t)\varphi(t) \in L_\infty(R_+), \tag{16}$$

where

$$\theta(t) = \begin{cases} \sqrt{t}\exp\left\{\frac{t}{4}\right\}, & if \quad 0 < t \leq T, \\ 1, & if \quad T < t < +\infty. \end{cases} \tag{17}$$

Substituting $\nu(t)$ (13) in (12), we obtain

$$w(x,t) = w_+(x,t) + w_-(x,t), \tag{18}$$

where

$$w_+(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x+\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x+\tau)^2}{4(t-\tau)}\right\} \varphi(\tau)d\tau, \tag{19}$$

$$w_-(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x-\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\} \varphi(\tau)d\tau. \quad (20)$$

6. The solution of the inverse problem (9)–(11)

From (14) and (18)–(20) we obtain for the solution $w(x,t) = Cw_0(x,t)$ of the homogeneous boundary value problem (9)–(10) the following representation:

$$w_0(x,t) = w_{0+}(x,t) + w_{0-}(x,t), \quad (21)$$

where

$$w_{0+}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x+\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x+\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau)d\tau, \quad (22)$$

$$w_{0-}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x-\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x-\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau)d\tau. \quad (23)$$

Further using the representation (21)–(23) for the integral condition (11), we get:

$$\int_0^t w_0(x,t)dx = \int_0^t w_{0+}(x,t)dx + \int_0^t w_{0-}(x,t)dx = \hat{\lambda}(t)\tilde{E}(t). \quad (24)$$

By a commutativity property in the integrals of the formula (24), in the sense of the Dirichlet formula, we have:

$$\int_0^t w_{0\pm}(x,t)dx = \frac{1}{4\sqrt{\pi}} \int_0^t \varphi_0(\tau)d\tau \int_0^t \frac{x\pm\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\} dx. \quad (25)$$

Let's calculate the interior integrals from (25). We get

$$\frac{1}{4\sqrt{\pi}} \int_0^t \frac{x\pm\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\} dx = \left\| y = \frac{(x\pm\tau)^2}{4(t-\tau)} \right\| =$$

$$= \frac{1}{2\sqrt{\pi(t-\tau)}} \int_{\frac{\tau^2}{4(t-\tau)}}^{\frac{(t\pm\tau)^2}{4(t-\tau)}} \exp\{-y\}dy =$$

$$= \frac{1}{2\sqrt{\pi(t-\tau)}} \left(\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} - \exp\left\{-\frac{(t\pm\tau)^2}{4(t-\tau)}\right\}\right). \quad (26)$$

Then from (11), (24)–(26) we obtain

$$\int_0^t w_0(x,t)dx = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}[2\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} -$$

$$-\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)]\varphi_0(\tau)d\tau = \hat{\lambda}_0(t)\widetilde{E}(t). \quad (27)$$

From ratios (8), (11), (27) and $w(x,t) = Cw_0(x,t)$ we find the required coefficient

$$\lambda(t) = \frac{d\ln(\hat{\lambda}(t))}{dt} = \frac{(\hat{\lambda}(t))'}{\hat{\lambda}(t)} = \lambda_0(t), \quad (28)$$

where we have used the equality

$$\left(\frac{\int_0^t w(x,t)dx}{\widetilde{E}(t)}\right)' : \frac{\int_0^t w(x,t)dx}{\widetilde{E}(t)} = \left(\frac{\int_0^t w_0(x,t)dx}{\widetilde{E}(t)}\right)' : \frac{\int_0^t w_0(x,t)dx}{\widetilde{E}(t)}.$$

Thus, the following Theorem 1 is proved.

Theorem 1. *The inverse problem (1)–(3) has the following solution* $\{u(x,t),\ \lambda(t)\}$*: the coefficient* $\lambda(t) = \lambda_0(t)$ *is determined uniquely by the formula (28) by restricting it down to a finite interval* $(0,T)$ *and the solution* $u(x,t)$ *is found by means of the restriction of the function:*

$$u(x,t) = Cu_0(x,t) = C[\hat{\lambda}_0(t)]^{-1}w_0(x,t), \quad (29)$$

on the bounded triangle G_T *where* $w_0(x,t)$ *is defined by formulas (21)–(23).*

Remark 2. Sections 7 and 8 are devoted to the mathematical justification and identification of the features of the solution of the boundary value problem (9)–(10). It will be shown that this solution has a singularity of order $t^{-1/2}$ at small values of t. Since the domain G_T is determined by the relations $0 < x < t, 0 < t < T$, the small value of the variable t provides a small value of the variable x.

According to formulas (21)–(23), (15) the solution $w_0(x,t)$ is a nonnegative function. It should be noted that the function $\widetilde{E}(t)$ from (11) also is a nonnegative function, since the integral (24) is nonnegative and the coefficient $\hat{\lambda}_0(t)$ (8) is nonnegative function.

7. Estimate of the integral from (27)

In this section, we will establish boundedness of the integral from (27), considering that function $\varphi_0(t)$ (15) belongs to the class (16)–(17). The following theorem is true.

Theorem 2. *The integral in (27) is bounded function on semi-axis* R_+.

The proof of Theorem 2 will follow from the statements of the following Lemmas 1–4.

Lemma 1. *Let* $0 < t < T$ *and* $C = \|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. *Then the following estimate holds*

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le C\sqrt{\pi}. \tag{30}$$

Proof. We obtain

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}-\frac{\tau}{4}\right\}\sqrt{\tau}\exp\left\{\frac{\tau}{4}\right\}\varphi_0(\tau)d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau = C\sqrt{\pi}.$$

Lemma 1 is proved.

Lemma 2. *Let* $T < t < \infty$ *and* $C = \|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. *Then the following estimate is true*

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le 2C. \tag{31}$$

Proof. We have

$$\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le$$

$$\le C\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}d\tau = \left\|z=\sqrt{t-\tau}\right\| =$$

$$=\frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\int_0^{\sqrt{t}} \exp\left\{-\frac{z^2}{4}-\frac{t^2}{4z^2}\right\}dz \le$$

$$\le \frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\int_0^\infty \exp\left\{-\frac{z^2}{4}-\frac{t^2}{4z^2}\right\}dz =$$

$$= \frac{2C}{\sqrt{\pi}}\exp\left\{\frac{t}{2}\right\}\frac{1}{2}\cdot\frac{\sqrt{\pi}}{\sqrt{\frac{1}{4}}}\exp\left\{-2\sqrt{\frac{1}{4}\cdot\frac{t^2}{4}}\right\} = 2C.$$

Here we used a well-known equality ([9], formula 3.325):

$$\int_0^\infty \exp\left\{-\mu x^2-\frac{\eta}{x^2}\right\}dx = \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt{\mu}}\exp\left\{-2\sqrt{\mu\eta}\right\} \qquad (32)$$

Lemma 2 is proved.

Lemma 3. *Let* $0<t<T$ *and* $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. *Then the following estimate take place*

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le C\sqrt{\pi}. \qquad (33)$$

Proof. We have

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le$$

$$\le \frac{C}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau = C\sqrt{\pi}.$$

Lemma 3 is proved.

Lemma 4. *Let* $T<t<\infty$ *and* $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. *Then the following estimate is correct*

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le 2C. \qquad (34)$$

Proof. We have

$$\frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}(\exp\left\{-\frac{t\tau}{t-\tau}\right\}+1)\varphi_0(\tau)d\tau \le$$

$$\le \frac{C}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}d\tau = \left\|z=\frac{\sqrt{t-\tau}}{2}\right\| =$$

$$= \frac{4C}{\sqrt{\pi}} \int_0^{\frac{\sqrt{t}}{2}} \exp\{-z^2\} dz = 2C \cdot erf\left(\frac{\sqrt{t}}{2}\right) \le 2C.$$

Lemma 4 is proved.

From estimates (30)–(34) established in Lemmas 1–4 we obtain the assertion of Theorem 2. Theorem 2 is proved.

8. Estimate of the solution (21)–(23)

In this section, we establish boundedness of the solution (21)–(23) of the boundary value problem (9)–(10), considering that function $\varphi_0(t)$ (15) belongs to the class (16)–(17). The following theorem is true.

Theorem 3. *The solution (21)–(23) of the problem (9)–(10) is limited, excluding the set* $\{x,t: 0<x<t, 0<t<\varepsilon, \varepsilon>0 - small\ \ number\}$ *near the point* $\{x=0, t=0\}$, *where the solution has a singularity of order* $t^{-1/2}$.

The proof of Theorem 3 will follow from statements of the following Lemmas 5–6.

Lemma 5. *Let* $0<t<T$ *and* $C = \|\theta(t)\varphi_0(t)\|_{L_\infty}(0<t<T)$. *Then the following estimate holds*

$$w_{0\pm}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x\pm\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau) d\tau \le C\frac{\sqrt{\pi}}{4}. \qquad (35)$$

Proof.

$$w_{0\pm}(x,t) = \frac{1}{4\sqrt{\pi}} \int_0^t \frac{x\pm\tau}{(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau) d\tau \le$$

$$\le \frac{C}{4\sqrt{\pi}} \int_0^t \frac{|x\pm\tau|}{\sqrt{\tau}(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)} - \frac{\tau}{4}\right\} d\tau = CI_{1\pm}(x,t).$$

We transform the kernel in the last integral. Using the ratios:

$$\frac{|x\pm\tau|}{\sqrt{\tau}(t-\tau)^{3/2}} \le \frac{|x\pm t|+(t-\tau)}{\sqrt{\tau}(t-\tau)^{3/2}} = \frac{|x\pm t|}{\sqrt{\tau}(t-\tau)^{3/2}} + \frac{1}{\sqrt{\tau(t-\tau)}},$$

$$-\frac{(x\pm\tau)^2}{4(t-\tau)} = -\frac{[x\pm t\mp(t-\tau)]^2}{4(t-\tau)} = -\frac{(x\pm t)^2}{4(t-\tau)} + \frac{\pm 2x+t}{4} + \frac{\tau}{4},$$

we have

$$I_{1\pm}(x,t) \le \frac{1}{4\sqrt{\pi}} \exp\left\{\frac{\pm 2x+t}{4}\right\} \int_0^t \frac{|x\pm t|}{\sqrt{\tau}(t-\tau)^{3/2}} \exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}\right\} d\tau +$$

$$+\frac{1}{4\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau = \pm I_{1\pm}^1(x,t)+I_{1\pm}^2(x,t). \quad (36)$$

Firstly, we show boundedness of the first integral (36). For this we introduce the following substitutions $2z_\pm = |x\pm t|(t-\tau)^{-1/2}, z_{1\pm}^2 = z_\pm^2-(x\pm t)^2(4t)^{-1}$. Then we obtain

$$I_{1\pm}^1(x,t)=\frac{2\exp\{-\frac{x^2}{4t}\}}{\sqrt{\pi t}}\int_0^\infty \exp\{-z_{1\pm}^2\}dz_{1\pm} = \frac{\exp\{-\frac{x^2}{4t}\}}{\sqrt{t}}. \quad (37)$$

For the second integral $I_{1\pm}^2(x,t)$ in formula (36) we have:

$$I_{1\pm}^2(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau \le$$

$$\le \frac{1}{4\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau = \frac{\sqrt{\pi}}{4}.$$

Lemma 5 is proved.

Lemma 6. *Let* $T<t<\infty$ *and* $C=\|\theta(t)\varphi_0(t)\|_{L_\infty}(T<t<\infty)$. *Then the following estimate is correct*

$$w_{0\pm}(x,t)=\frac{1}{4\sqrt{\pi}}\int_0^t \frac{x\pm\tau}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau \le C. \quad (38)$$

Proof. As in proof of Lemma 5, using similar transformations of independent variables, we obtain

$$w_{0\pm}(x,t)\le \frac{C}{4\sqrt{\pi}}\int_0^t \frac{|x\pm\tau|}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm\tau)^2}{4(t-\tau)}\right\}d\tau \le$$

$$=\frac{C}{4\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^t \frac{|x\pm t|}{(t-\tau)^{3/2}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}-\frac{t-\tau}{4}\right\}d\tau +$$

$$+\frac{C}{4\sqrt{\pi}}\exp\left\{\frac{x\pm t}{2}\right\}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{(x\pm t)^2}{4(t-\tau)}-\frac{t-\tau}{4}\right\}d\tau =$$

$$=C\left[I_{2\pm}^1(x,t)+I_{2\pm}^2(x,t)\right] \quad (39)$$

Using the substitution $2z_{\pm} = |x \pm t|(t-\tau)^{-1/2}$, for the first integral we get:

$$I^1_{2\pm}(x,t) = \frac{1}{\sqrt{\pi}}\exp\left\{\frac{x \pm t}{2}\right\}\int_{\frac{|x\pm t|}{\sqrt{t}}}^{\infty}\exp\left\{-z^2 - \frac{(x \pm t)^2}{16z^2}\right\}dz \le$$

$$\le \frac{1}{\sqrt{\pi}}\exp\left\{\frac{x \pm t}{2}\right\}\int_0^{\infty}\exp\left\{-z^2 - \frac{(x \pm t)^2}{16z^2}\right\}dz = \frac{1}{2}, \qquad (40)$$

here we used a well-known equality (32).

For the second integral $I^2_{2\pm}$ we have

$$I^2_{2\pm}(x,t) = \frac{1}{4\sqrt{\pi}}\exp\left\{\frac{x \pm t}{2}\right\}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{(x \pm t)^2}{4(t-\tau)} - \frac{t-\tau}{4}\right\}d\tau =$$

$$= \left\| z_{\pm} = \frac{2\sqrt{t-\tau}}{|x \pm t|} \right\| =$$

$$= \frac{|x \pm t|}{4\sqrt{\pi}}\exp\left\{\frac{x \pm t}{2}\right\}\int_0^{\frac{2\sqrt{t}}{|x \pm t|}}\exp\left\{-\frac{(x \pm t)^2}{16}z_{\pm}^2 - \frac{1}{z_{\pm}^2}\right\}dz_{\pm} \le$$

$$\le \frac{|x \pm t|}{4\sqrt{\pi}}\exp\left\{\frac{x \pm t}{2}\right\}\int_0^{\infty}\exp\left\{-\frac{(x \pm t)^2}{16}z_{\pm}^2 - \frac{1}{z_{\pm}^2}\right\}dz_{\pm} = \frac{1}{2}, \qquad (41)$$

where in (41) the known equality (32) was used.

Lemma 6 is proved.

From estimates (35) and (38) established in Lemmas 5–6 we obtain assertion of theorem 3. The peculiarity of the solution at the point $\{x=0, t=0\}$ follows from the estimate (37) from the proof of Lemma 5. Theorem 3 is proved.

9. Asymptotic of the integral in (27) for small values of the independent variable t

In Theorem 2 we have established boundedness of the integral in the left-hand side of formula (27) at $t \in R_+$. In this section, we want to answer the question: what is asymptotic behavior of this integral for small values of the independent variable t? This asymptotic is important for determining the classes of functions for the required solutions $\{u(x,t),(x,t) \in G_T; \lambda(t), t \in (0,T)\}$ in the inverse problem (1)–(3).

The following theorem holds.

Theorem 4. *For small values of the variable t, the solution of the inverse problem (1)–(3) $\{u(x,t),\lambda(t)\}$ has the order of singularity equal to t^{-1}.*

To prove Theorem 4 we will first show the justice of the following lemma.

Lemma 7. *The integral in formula (27) for small values of the variable t decreases with the order equal to $t^{1/2}$.*

Proof. For this purpose, we will consider each summand in the integral in (27) separately. For the first term we have:

$$J_1(t)=\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}\varphi_0(\tau)d\tau. \tag{42}$$

We divide the integral (42) into three integrals taking into account the formula (15):

$$J_{11}(t)=\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}-\frac{\tau}{4}\right\}d\tau, \tag{43}$$

$$J_{12}(t)=\frac{1}{2}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}d\tau, \tag{44}$$

$$J_{13}(t)=\frac{1}{2}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\}erf\left(\frac{\sqrt{\tau}}{2}\right)d\tau. \tag{45}$$

Let's calculate the integral (43).

$$J_{11}(t)=\frac{1}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}\exp\left\{-\frac{t\tau}{4(t-\tau)}\right\}d\tau,$$

where, after replacing the variable $z=\frac{t}{2\sqrt{t-\tau}}$, we have:

$$J_{11}(t)=\frac{\sqrt{t}\exp\left\{\frac{t}{4}\right\}}{\sqrt{\pi}}\int_{\frac{\sqrt{t}}{2}}^{\infty}\frac{1}{z\sqrt{z^2-\frac{t}{4}}}\exp\left\{-z^2\right\}dz=$$

$$=\frac{4\exp\left\{\frac{t}{4}\right\}}{\sqrt{\pi t}}\int_{\frac{\sqrt{t}}{2}}^{\infty}\left(\frac{z}{\sqrt{z^2-\frac{t}{4}}}-\frac{\sqrt{z^2-\frac{t}{4}}}{z}\right)\exp\{-z^2\}dz=$$

$$=\frac{4\exp\left\{\frac{t}{4}\right\}}{\sqrt{\pi t}}\int_{\frac{\sqrt{t}}{2}}^{\infty}\frac{z}{\sqrt{z^2-\frac{t}{4}}}\exp\{-z^2\}dz-$$

$$-\frac{4\exp\left\{\frac{t}{4}\right\}}{\sqrt{\pi t}}\int_{\frac{\sqrt{t}}{2}}^{\infty}\frac{\sqrt{z^2-\frac{t}{4}}}{z}\exp\{-z^2\}dz=J_{11}^1(t)-J_{11}^2(t).$$

$$J_{11}^1(t)=\frac{2}{\sqrt{\pi t}}\int_{\frac{\sqrt{t}}{2}}^{\infty}\frac{1}{\sqrt{z^2-\frac{t}{4}}}\exp\left\{-\left(z^2-\frac{t}{4}\right)\right\}d\left(z^2-\frac{t}{4}\right)=\frac{2}{\sqrt{t}}.$$

In integral $J_{11}^2(t)$ we introduce a substitution in the following way $z_1=\sqrt{z^2-\frac{t}{4}}$, then:

$$J_{11}^2(t)=\frac{4}{\sqrt{\pi t}}\int_0^{\infty}\frac{z_1^2}{z_1^2+\frac{t}{4}}\exp\{-z_1^2\}dz_1=\frac{4}{\sqrt{\pi t}}\int_0^{\infty}\exp\{-z_1^2\}dz_1-$$

$$-\frac{\sqrt{t}}{\sqrt{\pi}}\int_0^{\infty}\frac{1}{z_1^2+\frac{t}{4}}\exp\{-z_1^2\}dz_1=\frac{2}{\sqrt{t}}-\frac{\sqrt{\pi}}{\sqrt{t}}\exp\left\{\frac{t}{4}\right\}erfc\left(\frac{\sqrt{t}}{2}\right),$$

where in the last integral we used a well-known equality from ([9], formula 3.466):

$$\int_0^{\infty}\frac{\exp\{-\mu^2x^2\}}{x^2+\beta^2}dx=erfc(\beta\mu)\frac{\pi}{2\beta}\exp\{\beta^2\mu^2\}. \tag{46}$$

Thus, for the integral (43) we have

$$J_{11}(t)=\frac{\sqrt{\pi}}{\sqrt{t}}\exp\left\{\frac{t}{4}\right\}erfc\left(\frac{\sqrt{t}}{2}\right). \tag{47}$$

For the integrals (44) and (45), using a substitute $z = 2\sqrt{t-\tau}$, we get:

$$J_{12}(t) = \frac{1}{2}\int_0^t \frac{1}{\sqrt{t-\tau}} \exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} d\tau =$$

$$= \frac{\exp\left\{\frac{t}{2}\right\}}{2}\int_0^{2\sqrt{t}} \exp\left\{-\frac{z^2}{16} - \frac{t^2}{z^2}\right\} dz, \tag{48}$$

$$J_{13}(t) = \frac{1}{2}\int_0^t \frac{1}{\sqrt{t-\tau}} \exp\left\{-\frac{\tau^2}{4(t-\tau)}\right\} erf\left(\frac{\sqrt{\tau}}{2}\right) d\tau =$$

$$= \frac{\exp\left\{\frac{t}{2}\right\}}{2}\int_0^{2\sqrt{t}} \exp\left\{-\frac{z^2}{16} - \frac{t^2}{z^2}\right\} erf\left(\frac{\sqrt{t-\frac{z^2}{4}}}{2}\right) dz. \tag{49}$$

The second therm in the integral (27) has the form:

$$J_2(t) = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}} \exp\left\{-\frac{(t+\tau)^2}{4(t-\tau)}\right\} \varphi_0(\tau) d\tau. \tag{50}$$

We divide the integral (50) into three integrals taking into account the formula (15):

$$J_{21}(t) = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}} \exp\left\{-\frac{(t+\tau)^2}{4(t-\tau)} - \frac{\tau}{4}\right\} d\tau, \tag{51}$$

$$J_{22}(t) = \frac{1}{4}\int_0^t \frac{1}{\sqrt{t-\tau}} \exp\left\{-\frac{(t+\tau)^2}{4(t-\tau)}\right\} d\tau, \tag{52}$$

$$J_{23}(t) = \frac{1}{4}\int_0^t \frac{1}{\sqrt{t-\tau}} \exp\left\{-\frac{(t+\tau)^2}{4(t-\tau)}\right\} erf\left(\frac{\sqrt{\tau}}{2}\right) d\tau. \tag{53}$$

Using the replacement $z = \frac{t}{\sqrt{t-\tau}}$, we compute the integral (51). We get

$$J_{21}(t) = \frac{\exp\left\{\frac{3t}{4}\right\}}{2\sqrt{\pi}} \int_0^t \frac{1}{\sqrt{\tau(t-\tau)}} \exp\left\{-\frac{t^2}{t-\tau}\right\} d\tau =$$

$$= \frac{\sqrt{t}\exp\left\{\frac{3t}{4}\right\}}{\sqrt{\pi}} \int_{\sqrt{t}}^{\infty} \frac{1}{z\sqrt{z^2-t}} \exp\{-z^2\} dz =$$

$$= \frac{\exp\left\{\frac{3t}{4}\right\}}{\sqrt{\pi t}} \int_{\sqrt{t}}^{\infty} \frac{z}{\sqrt{z^2-t}} \exp\{-z^2\} dz -$$

$$- \frac{\exp\left\{\frac{3t}{4}\right\}}{\sqrt{\pi t}} \int_{\sqrt{t}}^{\infty} \frac{\sqrt{z^2-t}}{z} \exp\{-z^2\} dz = J_{21}^1(t) - J_{21}^2(t),$$

$$J_{21}^1(t) = \frac{\exp\left\{-\frac{t}{4}\right\}}{\sqrt{\pi t}} \int_{\sqrt{t}}^{\infty} \exp\{-(z^2-t)\} d\left(\sqrt{z^2-t}\right) = \frac{\exp\left\{-\frac{t}{4}\right\}}{2\sqrt{t}}.$$

For the second integral after replacing $z_1 = \sqrt{z^2-t}$ we obtain

$$J_{21}^2(t) = \frac{\exp\left\{-\frac{t}{4}\right\}}{\sqrt{\pi t}} \int_0^{\infty} \frac{z_1^2}{z_1^2+t} \exp\{-z_1^2\} dz_1 = \frac{\exp\left\{-\frac{t}{4}\right\}}{\sqrt{\pi t}} \int_0^{\infty} \exp\{-z_1^2\} dz_1 -$$

$$- \frac{\sqrt{t}\exp\left\{-\frac{t}{4}\right\}}{\sqrt{\pi}} \int_0^{\infty} \frac{1}{z_1^2+t} \exp\{-z_1^2\} dz_1 = \frac{\exp\left\{-\frac{t}{4}\right\}}{2\sqrt{t}} - \frac{\sqrt{\pi}\exp\left\{\frac{3t}{4}\right\}}{2} erfc\left(\sqrt{t}\right).$$

Here we have used the equality (46).

Thus, finally for (51) we get

$$J_{21}(t) = \frac{\sqrt{\pi}\exp\left\{\frac{3t}{4}\right\}}{2} erfc\left(\sqrt{t}\right). \tag{54}$$

By replacing $z = \sqrt{t-\tau}$ for the integrals (52) and (53) respectively, we have:

$$J_{22}(t) = \frac{\exp\left\{\frac{3t}{4}\right\}}{4}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t^2}{t-\tau}+\frac{\tau}{4}\right\} =$$

$$= \frac{\exp\{t\}}{2}\int_0^{\sqrt{t}} \exp\left\{-\frac{z^2}{4}-\frac{t^2}{z^2}\right\}dz, \tag{55}$$

$$J_{23}(t) = \frac{\exp\{t\}}{2}\int_0^{\sqrt{t}} \exp\left\{-\frac{z^2}{4}-\frac{t^2}{z^2}\right\} erf\left(\frac{\sqrt{t-z^2}}{2}\right)dz. \tag{56}$$

The third term in the integral in (27) has the form:

$$J_3(t) = \frac{1}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{-\frac{t-\tau}{4}\right\}\varphi_0(\tau)d\tau. \tag{57}$$

We divide the integral (57) into the following three integrals taking into account the formula (15):

$$J_{31}(t) = \frac{\exp\left\{-\frac{t}{4}\right\}}{2\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{\tau(t-\tau)}}d\tau, \tag{58}$$

$$J_{32}(t) = \frac{\exp\left\{-\frac{t}{4}\right\}}{4}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{\frac{\tau}{4}\right\}d\tau, \tag{59}$$

$$J_{33}(t) = \frac{\exp\left\{-\frac{t}{4}\right\}}{4}\int_0^t \frac{1}{\sqrt{t-\tau}}\exp\left\{\frac{\tau}{4}\right\} erf\left(\frac{\sqrt{\tau}}{2}\right)d\tau. \tag{60}$$

After changing the variable $\tau = t\sin^2\theta$ in (58) we get:

$$J_{31}(t) = \frac{\sqrt{\pi}}{2}\exp\left\{-\frac{t}{4}\right\}. \tag{61}$$

By replacing $z = \sqrt{t-\tau}$ for (59) and (60) we obtain:

$$J_{32}(t) = \int_0^{\sqrt{t}} \exp\left\{-\frac{z^2}{4}\right\} d\left(\frac{z}{2}\right) = \frac{\sqrt{\pi}}{2} erf\left(\frac{\sqrt{t}}{2}\right), \tag{62}$$

$$J_{33}(t) = \int_0^{\frac{\sqrt{t}}{2}} \exp\left\{-z^2\right\} erf\left(\frac{\sqrt{t-4z^2}}{2}\right) dz. \tag{63}$$

Now we set asymptotic of the integral in (27) for small values of the variable $t > 0$. For the integrals $J_{11}(t)$ (43), $J_{21}(t)$ (51) and $J_{31}(t)$ (58) and their values (47), (54) and (61) respectively, we obtain the following asymptotic:

$$J_{11}(t) \approx \sqrt{\pi}\exp\left\{\frac{t}{4}\right\}(1 - \frac{\sqrt{t}}{\sqrt{\pi}}),$$

$$J_{21}(t) \approx \frac{\sqrt{\pi}}{2}\exp\left\{\frac{3t}{4}\right\}(1 - \frac{2\sqrt{t}}{\sqrt{\pi}}), \quad J_{31}(t) = \frac{\sqrt{\pi}}{2}\exp\left\{-\frac{t}{4}\right\}.$$

Hence we obtain

$$J_{11}(t) - J_{21}(t) - J_{31}(t) \approx \sinh\left\{\frac{t}{4}\right\}\left[\sqrt{\pi} - \sqrt{\pi}\exp\left\{\frac{t}{2}\right\} + 2\sqrt{t}\right] \approx t^{3/2}. \tag{64}$$

Further, for the integrals $J_{12}(t)$ (44), $J_{22}(t)$ (52) and $J_{32}(t)$ (59) and their values (48), (55) and (62) respectively we obtain the following asymptotic (recall that for small values of the variable t):

$$J_{12}(t) \approx \sqrt{t}\exp\left\{\frac{t}{2}\right\}, \quad J_{22}(t) \approx \frac{\sqrt{t}\exp\{t\}}{2}, \quad J_{32}(t) \approx \frac{\sqrt{t}}{2}.$$

From these formulas we have

$$J_{12}(t) - J_{22}(t) - J_{32}(t) \approx \sqrt{t}(\exp\left\{\frac{t}{2}\right\} - \exp\{t\} - \frac{1}{2}) \approx \sqrt{t}. \tag{65}$$

For the integrals $J_{13}(t)$ (45), $J_{23}(t)$ (53) and $J_{33}(t)$ (60) and their values (49), (56) and (63) respectively we get the following asymptotic:

$$J_{13}(t) \approx \sqrt{t}\exp\left\{\frac{t}{2}\right\}, \quad J_{23}(t) \approx \frac{\sqrt{t}\exp\{t\}}{2}, \quad J_{33}(t) \approx \frac{\sqrt{t}}{2}.$$

Taking these formulas into account, we obtain

$$J_{13}(t) - J_{23}(t) - J_{33}(t) \approx \sqrt{t}(\exp\left\{\frac{t}{2}\right\} - \frac{\exp\{t\}}{2} - \frac{1}{2}) \approx \sqrt{t}. \quad (66)$$

From the formulas (64), (65) and (66) it follows that for small values of the variable $t>0$ the integral in (27) has a decreasing order, equal to $\sqrt{t}$.

Lemma 7 is proved.

To the proof of Theorem 4. According to a property of the given function $\widetilde{E}(t)$ from the overspecification (7) and also from the statement of Lemma 7 and equality (27) we obtain that the function $\hat{\lambda}(t)$ should decrease at small values of the variable t with order equals to $t^{1/2}$. It follows that the order of singularity of the required coefficient $\lambda(t)$ at small values of the variable t equals t^{-1}. According to the ratio

$$u(x,t) = \frac{w(x,t)}{\hat{\lambda}(t)}$$

the solution of the boundary value problem (1)–(2) has the order of singularity equal $t^{-1/2}$. Indeed, it follows from the statement of the Theorem 3 that at small values of the variable t the solution $w(x,t)$ of the boundary value problem (9)-(10) has not a singularity.

Theorem 4 is completely proved.

10. Conclusion

In the paper we consider an inverse problem for the heat equation in a degenerating angular domain. We have shown that the inverse problem for the homogeneous heat equation with homogeneous boundary conditions has a nontrivial solution $\{u(x,t),\lambda(t)\}$ consistent with the integral condition. It was also proved that the found nontrivial solution has a singularity at the point $\{x=0,t=0\}$, which order equals to -1.

Supported by the grant projects AP05130928 (2018--2020), AP05132262 (2018--2020) and by the target program BR05236693 (2018--2020) from the Ministry of Science and Education of the Republic of Kazakhstan.

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A GREEDY ALGORITHM FOR ALLOCATING INDIVISIBLE JOBS IN A MULTIPROCESSOR SYSTEM

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Abstract. *We study the problem of distribution a multiprocessor system computing capacity over a fixed time period between jobs. Processors differ by speed; jobs differ by processing time and cost. Assuming that jobs are indivisible (preemptions are prohibited), the problem of maximizing the total cost of allocated jobs is equivalent to the multiple knapsack problem. In the report, a greedy allocation algorithm is proposed. The relative performance guarantee of the algorithm is 0.5, and its running-time is O(mn), where m is the number of jobs and n is the number of processors.*

1. Introduction. Assume that a system consisting of n processors provides paid services. There are m jobs, each of them can be included in a schedule for the planning period of duration T. Let $I = \{1, 2, \ldots, m\}$ be the set of jobs' numbers and $J = \{1, 2, \ldots, n\}$ be the set of processors' numbers.

Each job can be assigned to any processor. Processor j has the speed s_j (cycles per unit time) and capability $Q_j = T \cdot s_j$ within the planning period. Job i requires q_i processor cycles and has the cost c_i. If the job i is included in the schedule and will be, therefore, executed during the planning period, then the system obtains the payment c_i.

Jobs are indivisible in two senses: (a) in the planning period, either no part of a job is performed or this job terminates; (b) during the planning period, a job can use no more than one processor (scheduling without preemption). It follows from the jobs' indivisibility that any schedule can be identified with some partition $I = I_0 \cup I_1 \cup \ldots \cup I_n$, where $i \in I_j$ with $j \in J$ iff the job i is allocated to the processor j, and the jobs with numbers in I_0 are not included in the schedule.

Содержание

СЕКЦИЯ 1
Современные проблемы прикладной математики, информатики и теории управления

МАТЕРИАЛЫ
III Международной научной конференции
«Информатика и прикладная математика»,
посвященная 80-летнему юбилею
профессора Бияшева Р.Г.
и 70-летию профессора Айдарханова М.Б.

26-29 сентября 2018 года, Алматы, Казахстан

Часть 1

Под редакцией М.Н. Калимолдаева

Компьютерная верстка
А.А. Кулемзин

Подписано в печать 06.09.2018 г. Формат А4
Печать цифровая. Бумага офсетная. Усл. печ. л. 20,8.
Тираж 300 экз. Заказ № 006604.
Отпечатано в типографии НЦ ГНТЭ.
Алматы, ул. Богенбай батыра, 221