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Algorithms Development of the Attitude Determination and Control of the Low-Orbit Small Satellites

In many cases for the execution of different problems which are assigned within the mission of small satellite, it is necessary to provide the required accuracy of its orientation although the small satellite is more influenced by the external disturbances due to its small mass. The main sources of the disturbances for the attitude of small satellites are the torques of external forces, however there are no less impact of the disturbances caused by the faults of sensors and actuators of small satellite and disturbances caused by the various uncertainties. In this connection for the solution of the assigned to the small satellite tasks, it is necessary to develop the algorithms of high-accuracy attitude determination and algorithms of attitude control that are stable to different disturbances. There are many different theories and techniques of attitude determination and control for small satellites that are developed by many authors [1-7]. However, not all these techniques can be used with respect to any satellite due to the fact that the all satellites are unique by its content of components of control system. In this connection, this theme of research is relevant during the last few decades.

6.1. INTRODUCTION

The purpose of work is the development of algorithms for precise attitude determination of the small satellite in conditions of inaccessibility of full vector of measurements and development of algorithms of attitude control of small satellite taking into account the external disturbances, disturbances caused by the faults of actuators and uncertainties of the inertia moments of the small satellite.

The work is dedicated to the development of algorithms of attitude determination and control of small satellite during orbital orientation mode in case of incomplete vector of measurements and existence of faults and uncertainties:

- the non-linear control for the maintenance of orbital orientation of small satellite in case of incomplete vector of measurements is developed;
- the robust linear control for the maintenance of orbital orientation of small satellite taking into account the parametric uncertainties of inertia moments of small satellite and faults of actuators (reaction wheels) is developed;
- the linear attitude control that is tolerant to the faults of actuators (reaction wheels) of small satellite is developed.

Orbital orientation mode here is considered as the control mode of a small satellite when the axes of the body coordinate system superposed with the axes of the orbital coordinate system.

For the development of non-linear attitude control law for small satellite and determination of its feedback coefficients the Lyapunov function method is used. Tuning of input data of Kalman filter algorithm is conducted during the numerical analysis of convergence and accuracy of Kalman filter using the different input data.

The development of linear robust control is conducted on the basis of H_∞-control theory taking into account additional conditions on the pole placement.

Small satellite the prototype of which is nanosatellite on the basis of CubeSat3U platform with the mass of 4.2 kg and inertia moments $J = [0.04088;\ 0.04088;\ 0.1116]\ kg \cdot m^2$ is considered in this work. The orbit of considered satellite is sun-synchronous with the height of 560 km.

The three-axis gyro sensor, three-axis magnetic sensor, two-axis sun sensors and three reaction wheels are the main components of attitude determination and control system of the considered small satellite.

6.2. MATHEMATICAL MODEL OF ROTATIONAL MOTION OF SATELLITE

For the description of satellite motion there are used several coordinate systems: fixed inertial coordinate system $O\mathrm{x}_{\mathrm{i}}y_iz_i$ with the origin at the Earth center of mass; body coordinate system $\mathrm{Cx_b y_b z_b}$ with the origin at the satellite center of mass, axes of given coordinate system coincides with the principal central axes of inertia of satellite; orbital coordinate system $\mathrm{Cx_o y_o z_o}$ with the origin at the satellite center of mass, the direction of $\mathrm{Cx_o}$ axis coincides with the direction of satellite motion, $\mathrm{Cz_o}$ axis is directed to the Earth center from the satellite center of mass, $\mathrm{Cy_o}$ axis complements the system to right-handed system. The description of orientation of the axes of the body coordinate system related to the axes of other coordinate systems is performed using quaternions.

Gravity-gradient torque and residual magnetic torque are considered as the main external disturbances acting on the satellite. Sun pressure and aerodynamic disturbances are not considered because of the small middle cross-section of satellite. Dynamic Euler equations are used as the equations of dynamics of the satellite and kinematic equations in quaternions are used as the equations of kinematics [8], [9]:

$$\overrightarrow{\dot{\omega}_{\mathrm{bi}}^{b}} = J^{-1}\left[-\overrightarrow{\omega_{bi}^{b}} \times (J\overrightarrow{\omega_{bi}^{b}} + \overrightarrow{h_a^b}) + \overrightarrow{M_c^b} + \overrightarrow{M_e^b}\right], \quad (6.1)$$

where $J = \{J_x, J_y, J_z\}$ - diagonal (3x3) matrix of small satellite inertia tensor; $\overrightarrow{\omega_{bi}^{b}}$ - angular velocity of small satellite in the body coordinate system; $\overrightarrow{h_a^b}$ - angular moment of reaction wheels; $\overrightarrow{M_e^b}$ - moment of external forces in the projections to the body coordinate system; $\overrightarrow{M_c^b}$ - control moment of reaction wheels.

$$\overrightarrow{\omega_{\mathrm{bo}}^{\mathrm{b}}} = 2\overrightarrow{Q_{bo}}^{*} \otimes \overrightarrow{\dot{Q}_{bo}} \tag{6.2}$$

where $\vec{Q}_{bo}$ - quaternion that sets the current angular position of the small satellite in the orbital coordinate system; $\overrightarrow{Q_{bo}}^{*}$ - quaternion that is inverse to $\vec{Q}_{bo}$, $\overrightarrow{Q_{bo}}^{*} = q_0^{\mathrm{bo}} - \overrightarrow{q^{bo}}$; $\otimes$ - the operation of multiplication of quaternions.

$$\overrightarrow{\omega_{bi}^{b}} = \overrightarrow{\omega_{bo}^{b}} + \overrightarrow{\omega_{oi}^{b}} = \overrightarrow{\omega_{bo}^{b}} + R_b^o \overrightarrow{\omega_{oi}^{o}}, \tag{6.3}$$

where R_b^o is the direction cosine matrix represents the rotation between the orbital and body coordinate system; $\overrightarrow{\omega_{oi}^{o}} = [0 \; -\omega_0 \;\; 0]^T$ is the angular velocity of the orbital coordinate system relative to an inertial coordinate system.

6.3. THE NON-LINEAR CONTROL FOR THE MAINTENANCE OF ORBITAL ORIENTATION OF SMALL SATELLITE IN CASE OF INCOMPLETE VECTOR OF MEASUREMENTS

In this chapter let us consider the problem of development of the algorithm for determination of the angular position and angular velocity of the small satellite on the basis of measurements of sun and magnetic sensors and develop attitude control law providing the stability of its orbital orientation.

Nonlinear control law is derived in this work with feedback on the angular position and angular velocity of the satellite:

$$\overrightarrow{M_c^b} = -K_\omega \vec{\omega} - K_Q \vec{\mathrm{q}} - \omega_o \overrightarrow{a_2} \times \vec{h} \tag{6.4}$$

where $\vec{\omega} = \overrightarrow{\omega^b_{\mathrm{bo}}}$, $\vec{h} = \overrightarrow{h^b_a}$, $\overrightarrow{Q_{bo}} = \vec{Q}$, and

$\overrightarrow{a_2} = \left[2(q_1 q_2 + q_0 q_3) \quad q_0^2 - q_1^2 + q_2^2 - q_3^2 \quad 2(q_2 q_3 - q_0 q_1)\right]^{\mathrm{T}}$.

Stability investigation of attitude determination and control system with the control law in the form (6.4) is carried out using the Lyapunov functions method.

According to Lyapunov's theorem for the asymptotic stability of the attitude determination and control system of the small satellite it is sufficient that the time derivative of the Lyapunov function $\mathrm{V}(\vec{\tilde{\omega}}, \vec{\tilde{\mathrm{Q}}})$ ($\vec{\tilde{\omega}} = [0\ 0\ 0], \vec{\tilde{\mathrm{Q}}} = [0\ 0\ 0\ 1]$) be the negative definite function, i.e. for all $\vec{\tilde{\omega}}, \vec{\tilde{\mathrm{Q}}} \neq 0$ $\dot{\mathrm{V}}(\vec{\tilde{\omega}}, \vec{\tilde{\mathrm{Q}}}) \leq 0$ [10].

In our case the Lyapunov function is derived in the form:

$$\mathrm{V}(\vec{\omega}, \vec{\mathrm{Q}}) = \frac{1}{2} \vec{\omega}^T J \vec{\omega} - \frac{1}{2} \omega_0^2 \overrightarrow{a_2}^T J \overrightarrow{a_2} + 2(1 - q_0). \qquad (6.5)$$

It is apparent that Lyapunov function $\mathrm{V}(\vec{\omega}, \vec{\mathrm{Q}}) > 0$ and becomes zero only at the given angular position: $\vec{\tilde{\omega}} = [0\ 0\ 0], \vec{\tilde{\mathrm{Q}}} = [0\ 0\ 0\ 1]$.

The time derivative of the Lyapunov function (6.5) with account of the equations of motion (6.1), (6.2) and (6.4) has the form:

$$\dot{\mathrm{V}}(\vec{\omega}, \vec{\mathrm{Q}}) = -\overrightarrow{\omega^T} \mathrm{K}_\omega \vec{\omega} - \overrightarrow{\omega^T} K_q \vec{q} + \overrightarrow{\omega^T} \vec{q} = -\mathrm{K}_\omega \overrightarrow{\omega^T} \vec{\omega} - \overrightarrow{\omega^T} \vec{q} (K_q - 1). \qquad (6.6)$$

Assuming that $K_q = 1$ (6.6) will has the form:

$$\dot{\mathrm{V}}(\vec{\omega}, \vec{\mathrm{Q}}) = -\mathrm{K}_\omega \overrightarrow{\omega^T} \vec{\omega} \qquad (6.7)$$

The obtained function will be negative at $\mathrm{K}_\omega > 0$.

Thus, in result of stability investigation of small satellite attitude determination and control system it is found that at $K_q = 1$ and $\mathrm{K}_\omega > 0$ control law (6.4) provides the stability of satellite orbital orientation.

The value of the coefficient K_ω is determined using the theory of optimal synthesis of linear-quadratic regulator.

The simulation results of small satellite angular motion under the action of non-linear control are given in the pictures below. Initial angular position and angular velocity of the small satellite are accepted as: $\overrightarrow{Q_{bo}} = [0.9169; 0.1179; -0.2339; 0.301]\ ^\circ$, $\overrightarrow{\omega_{bi}^{b}} = [0.09; -0.01; 0.03]$ rad/sec. The required angular position and angular velocity of small satellite are accepted as: $\overrightarrow{Q_{ob}} = [1.0; 0.0; 0.0]\ ^\circ$, $\overrightarrow{\omega_{bi}^{b}} = [0.0; 0.0; 0.0]$ rad/sec.

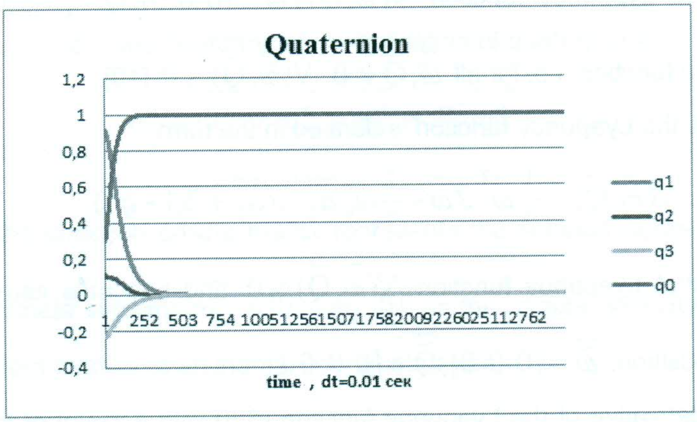

Fig. 6.1. Angular position obtained as a result of using the non-linear control with state feedback

Source: own elaboration

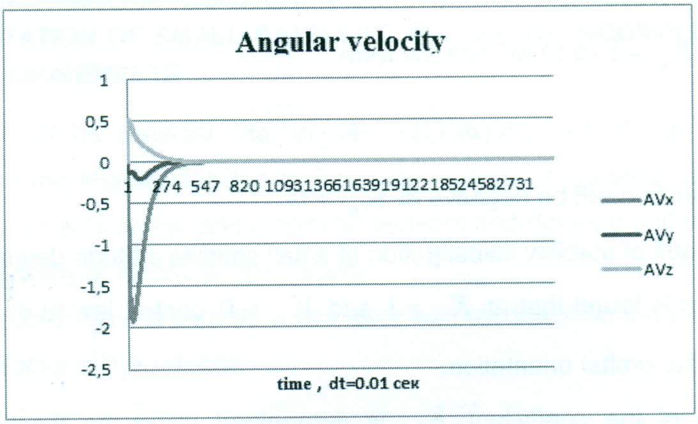

Fig. 6.2. Angular position obtained as a result of using the non-linear control with state feedback

Source: own elaboration

Fig. 6.1 – 6.2 shows the variation of the angular position and angular velocity of satellite with time. Here the feedback coefficient K_ω is determined as result of using the theory of synthesis of linear H_2 - control, $K_\omega = 0.4347$.

As it can be seen from Fig. 6.1 – 6.2, the developed nonlinear attitude control provides the stability of small satellite orbital orientation and sufficiently high quality of transient characteristics of the considered attitude determination and control system.

For accurate determination of angular velocity and the angular position of the small satellite in the absence of measurements of the angular velocity sensor there is developed an algorithm on the basis of Kalman filter which operates by the principle prediction-correction.

Predicted values of angular velocity and angular position of small satellite are determined at the first stage of algorithm in the process of solving the linearized equations of motion with the initial conditions determined as the output parameters of the Kalman filter, obtained at the previous time step:

$$\dot{\vec{\mathrm{x}}} = \mathrm{F}\vec{\delta}, \tag{6.8}$$

where

$$\vec{\mathrm{x}} = [\overrightarrow{\hat{\omega}_{k+1}^-}, \overrightarrow{\hat{Q}_{k+1}^-}],$$

$$\mathrm{F} = \begin{bmatrix} [J\vec{\hat{\omega}}\times] - [\vec{\hat{\omega}}\times]J + [\vec{h}\times] - K_\omega & K_q - 2K_\omega[\overrightarrow{\omega_{oi}^b}\times] + 2[\vec{h}\times][\overrightarrow{\omega_{oi}^b}\times] \\ \frac{1}{2}I_{3x3} & -\left[\vec{\hat{\omega}}\times\right] \end{bmatrix}.$$

At the first stage it is also determined the current deviation of the estimated state vector from its true value which is characterized by a covariance matrix P:

$$\mathrm{P}_{k+1}^- = \Phi_k P_k^+ \Phi_k^T + N, \tag{6.9}$$

where $\Phi_k = e^{\mathrm{F}\Delta\mathrm{T}} = 1 + F\Delta T + (F\Delta T)^2/2! + \ldots$, N - matrix of system noise.

For obtaining the predicted values of orientation parameters of satellite and covariance matrix it is required their corrected values be known at the current step. This fact is one of the main problems of implementation of the Kalman filter. The choice of initial orientation parameters of small satellite and the covariance matrix affects the convergence and accuracy of the Kalman filter. For the solution of this problem in each case, there are used different techniques of determination of initial input parameters of Kalman filter – parameters of orientation and covariance matrix.

In this work for determination of initial values of the orientation parameters there is proposed the algorithm of rough estimate of the small satellite orientation parameters on the basis of sun sensor measurements: suppose that on the basis of measurements of sun sensors we can obtain the angular position of satellite relative to the sun, given by the matrix R_b^s. Also on board of the satellite there is a model of the motion of the sun, which allows to determine the position of the sun relative to the orbital coordinate system defined by the matrix R_o^s. Then, the angular position of satellite relative to the orbital coordinate system, characterized by a matrix R_o^b, we can determine on the basis of matrix expression:

$$R_o^b = R_b^s \cdot R_o^{s^{-1}}. \tag{6.10}$$

The obtained matrix R_o^b can be converted into a quaternion $\overrightarrow{Q_{ob}}$. After conducting the rough estimate of the angular position of the satellite, its angular velocity is determined with help of kinematic equations (6.2), where $\overrightarrow{\dot{Q}_{bo}}$ is calculated on the basis of the difference of measurements $\overrightarrow{Q_{ob}}$ on time step t_k and t_{k-1}.

As it is seen from (9), the predicted value of the covariance matrix depends on its corrected value obtained in the previous time step and the matrix of system noise. The determination of initial covariance matrix P and matrix of system noise N, that contributes the obtaining of the least time of convergence of Kalman filter and higher accuracy of determination of satellite orientation parameters, is conducted as a result of several "runs" of Kalman filter at the different initial values of matrices P, N and analysis of the obtained results.

At the second stage of the algorithm after receiving the measurement vector $\overrightarrow{z_{k+1}}$ from magnetic and sun sensors it is calculated the corrected values of satellite orientation parameters $\overrightarrow{\hat{\omega}_{k+1}^{-}}, \overrightarrow{\hat{Q}_{k+1}^{-}}$ and error covariance matrix P_{k+1}^{+} by formulas that are known from Kalman filter algorithm.

The simulation results of the algorithm of determining the satellite orientation parameters are given in the figures below.

Initial angular position and angular velocity of the small satellite are obtained as a result of rough estimation based on the sun sensor readings:

$$\begin{aligned} &\overrightarrow{Q_{bo}(t_0)} = [0.9109; 0.1578; -0.2678; 0.2712]^{\circ} \\ &\overrightarrow{\omega_{bi}^{b}} = [0.0899; 0.01003; 0.03005] \text{ rad/sec} \end{aligned} , \tag{6.11}$$

The initial error covariance matrix is defined in the course of several numerical experiments of determination of satellite orientation parameters, as a matrix, at which there is achieved the highest accuracy of determination of orientation parameters:

$$P(t_0) = \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6.12}$$

Results of comparison of the estimated orientation parameters (red dot line) of satellite with their true values (blue bold line) are given in Fig. 6.3 – 6.4. It can be seen from the figures that angular velocity and angular position of small satellite estimated by means of a Kalman filter have a small deviation from their true values.

Results of small satellite angular motion modeling under the action of nonlinear control (6.4) with the feedback angular velocity and angular position obtained by using algorithm on the base of Kalman filter are shown at the Fig. 6.5 – 6.6.

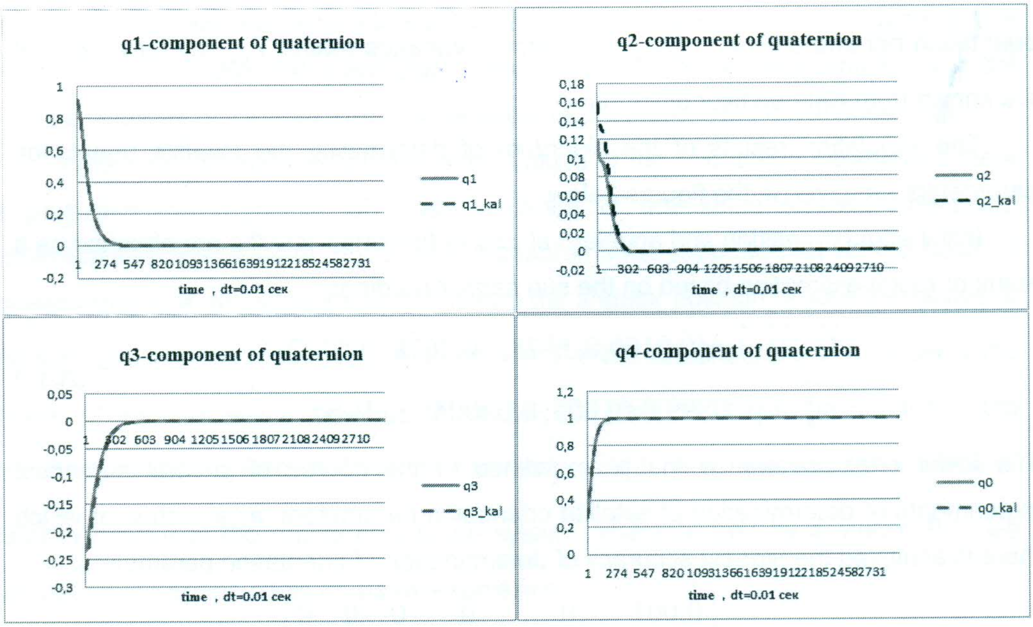

Fig. 6.3. The results of comparison of estimated angular position of satellite with its true value

Source: own elaboration

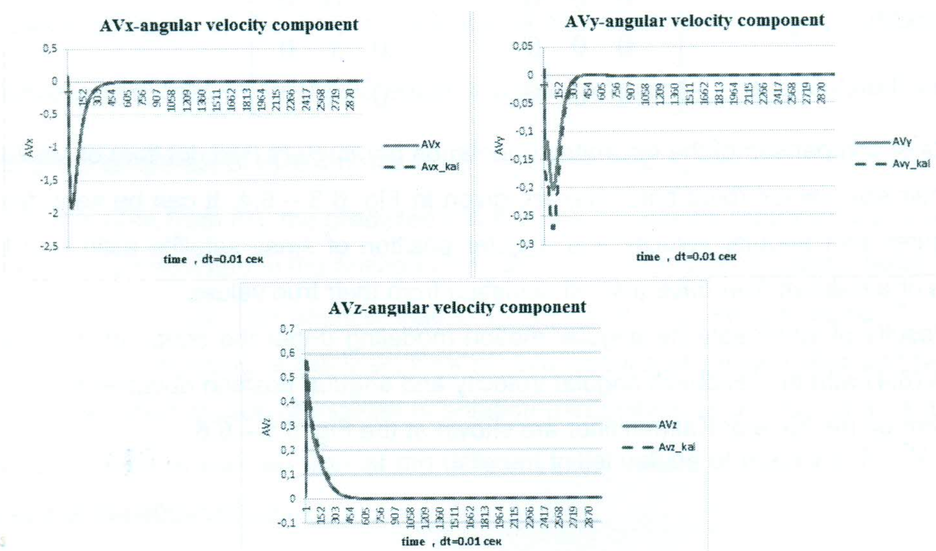

Fig. 6.4. The results of comparison of estimated angular velocity of satellite with its true value

Source: own elaboration

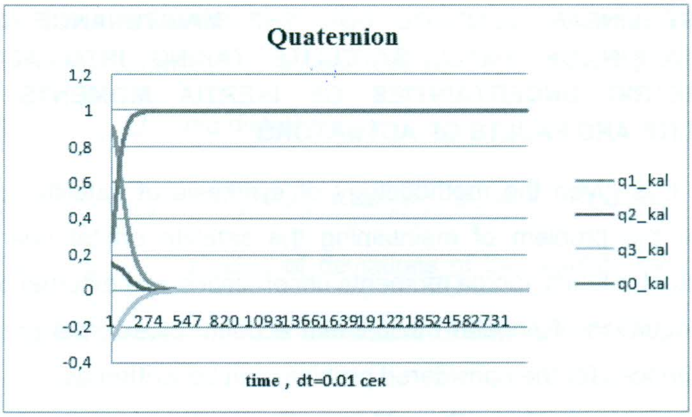

Fig. 6.5. Angular position obtained as a result of using the non-linear control with output feedback

Source: own elaboration

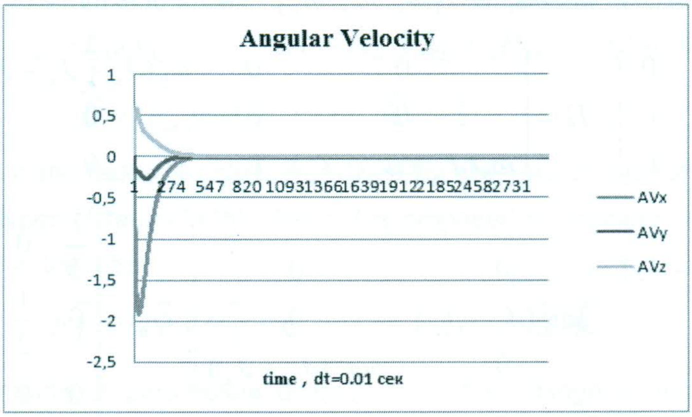

Fig. 6.6. Angular velocity obtained as a result of using the non-linear control with output feedback

Source: own elaboration

Comparing Fig. 6.2 and 6.6 it can be seen a slight deviation of the final angular velocity of the satellite, the value of which is of order 10^{-6}. But in general we can conclude that using of developed non-linear control and algorithm for determination of satellite orientation parameters solves the problem of maintaining the orbital orientation of satellite in condition of unavailability of angular velocity measurements.

6.4. ROBUST LINEAR CONTROL FOR THE MAINTENANCE OF ORBITAL ORIENTATION OF SMALL SATELLITE TAKING INTO ACCOUNT THE PARAMETRIC UNCERTAINTIES OF INERTIA MOMENTS OF SMALL SATELLITE AND FAULTS OF ACTUATORS

In this chapter it is given the methodology of synthesis of satellite robust attitude control to solve the problem of maintaining the satellite orbital orientation in the conditions of actuator faults, inertia moments uncertainties and external disturbances.

The linearized equation of angular motion with account of external gravitational and magnetic disturbances for the considered problem can be written as:

$$F\ddot{\vec{p}}+H\dot{\vec{p}}+Q\vec{p}=G_d\vec{w}+G_u\vec{u}\,, \tag{6.13}$$

where $p=[q_1,q_2,q_3]$, $\vec{u}=[-J_{rx}\dot{\omega}_{rx},-J_{ry}\dot{\omega}_{ry},-J_{rz}\dot{\omega}_{rz}]$ - vector of controlling torques of the reaction wheels; $\vec{w}=[m_x,m_y,m_z]$ - vector of external disturbances.

$$F=\begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix},\ H=\begin{bmatrix} 0 & 0 & -\omega_0(J_x+J_z-J_y) \\ 0 & 0 & 0 \\ \omega_0(J_x+J_z-J_y) & 0 & 0 \end{bmatrix},$$

$$Q=\begin{bmatrix} 4\omega_0^2(J_y-J_z) & 0 & 0 \\ 0 & 3\omega_0^2(J_z-J_x) & 0 \\ 0 & 0 & 3\omega_0^2(J_y-J_x) \end{bmatrix},\ G_u=\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix},$$

$$G_d=\begin{bmatrix} 0 & \frac{1}{2}B_z^o & -\frac{1}{2}B_y^o \\ -\frac{1}{2}B_z^o & 0 & \frac{1}{2}B_x^o \\ \frac{1}{2}B_y^o & -\frac{1}{2}B_x^o & 0 \end{bmatrix}, \tag{6.14}$$

where $\overrightarrow{B^o}=[B_x^o,B_y^o,B_z^o]$ - the magnetic induction vector of the magnetic field of the Earth in the orbital coordinate system.

To simulate the uncertainties of satellite moments of inertia we assume that the finite value of satellite moments of inertia can be considered as the sum of their nominal values $\widetilde{J}_x$, $\widetilde{J}_y$, $\widetilde{J}_z$ and disturbances:

$$J_x = \widetilde{J}_x + \Delta J_x \delta_x,\ J_y = \widetilde{J}_y + \Delta J_y \delta_y,\ J_z = \widetilde{J}_z + \Delta J_z \delta_z, \tag{6.15}$$

where ΔJ_x, ΔJ_y, ΔJ_z - the value of deviations of the satellite moment of inertia; $\delta_x, \delta_y, \delta_z$ - normalized parametric uncertainties of the satellite moments of inertia, $\delta_x, \delta_y, \delta_z \le 1$.

Then in the right side of (6.13) includes the additional term representing the disturbances due to the uncertainty of the moment of inertia of satellite:

$$F\vec{\ddot{p}} + H\vec{\dot{p}} + Q\vec{p} = G_d \vec{w} + \widetilde{G}_d \vec{\widetilde{w}} + G_u \vec{u}, \tag{6.16}$$

where $\mathrm{F} = F_0 + L_M \Delta_M P_M$, $H = H_0 + L_D \Delta_D P_D$, $Q = Q_0 + L_K \Delta_K P_K$, $\widetilde{G}_d = [L_M\ L_D\ L_K]$, $\vec{\widetilde{z}} = [P_M \ddot{p}, P_D \dot{p}, P_K p]$, $\vec{\widetilde{w}} = -\vec{\widetilde{z}}$.

To simulate the faults of actuators caused by changes of their effectiveness or various other types of faults in this thesis it is proposed to consider the equation of angular motion in the form:

$$F\vec{\ddot{p}} + H\vec{\dot{p}} + Q\vec{p} = G_d \vec{w} + G_u L_u (I + P_u \Delta_u)\vec{u} \tag{6.17}$$

where L_u - matrix of parameters characterizing the change in the efficiency of actuators, Δ_u - matrix representing normalized parametric uncertainties, P_u - matrix giving restrictions on Δ_u.

In this work for the synthesis of robust linear control taking into account the uncertainties of satellite moment of inertia and external disturbances it is proposed to use the theory of synthesis of H_∞-control. For the application of this theory equations (6.13), (6.16), (6.17) are reduced to:

$$\dot{\vec{x}} = A\vec{x} + B_1\vec{w} + B_2\vec{u},$$
$$\vec{z} = C_1\vec{x} + D_{11}\vec{w} + D_{12}\vec{u}, \quad (6.18)$$
$$\vec{y} = C_2\vec{x} + D_{21}\vec{w}.$$

And robust control is obtained as a function $\vec{u} = \mathrm{K}_\infty(\mathrm{s})\vec{y}$ that minimizes H_∞ - norm of transfer function $\|\mathrm{T}_{\mathrm{wz}}\|_\infty$ from $\vec{w}$ to $\vec{z}$, or provides the following:

$$\|\mathrm{T}_{\mathrm{wz}}\|_\infty < \gamma, \quad (6.19)$$

$$\mathrm{J}_\infty(K) = \|\mathrm{T}_{\mathrm{wz}}\|_\infty = \sup\left\{\frac{\|\vec{z}\|_2}{\|\vec{w}\|_2} : w \neq 0\right\}, \quad (6.20)$$

where $\gamma > 1, \gamma = \mathrm{const}$.

According to the theory of synthesis of H_∞ - control for the system (6.18) with the conditions (6.19) the H_∞ - controller can be determined in the form:

$$\mathrm{K}_\infty(s) = D_k + C_k(sI - A_k)^{-1}B_k. \quad (6.21)$$

if the following matrix inequality is satisfied for some $X_\infty > 0$:

$$\begin{bmatrix} A_{cl}^T X_\infty + X_\infty A_{cl} & X_\infty B_{cl} & C_{cl}^T \\ B_{cl}^T X_\infty & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0, \quad (6.22)$$

where

$$A_{cl} = \begin{pmatrix} A + B_2 D_k C_2 & B_2 C_k \\ B_k C_2 & A_k \end{pmatrix}, B_{cl} = \begin{pmatrix} B_1 + B_2 D_k D_{21} \\ B_k D_{21} \end{pmatrix}, \quad (6.23)$$
$$C_{cl} = (C_1 + D_{12} D_k C_2, D_{12} C_k), D_{cl} = D_{11} + D_{12} D_k D_{21}.$$

Thus, synthesis problem of robust control in the form of (6.21) is concentrated in determination of X_∞ and $\mathrm{K}_\infty(s)$ that satisfies to the inequality (6.22). To solve this