Інститут математики НАН України

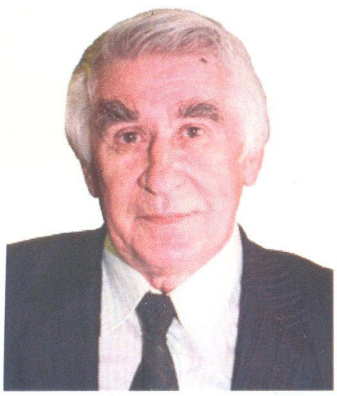
Міжнародна математична конференція

«Боголюбовські читання DIF-2013. Диференціальні рівняння, теорія функцій та їх застосування»

з нагоди 75-річчя з дня народження академіка А. М. Самойленка

23 – 30 червня 2013 р.
Севастополь, УКРАЇНА

Тези доповідей



Institute of Mathematics of NAS of Ukraine

International mathematical conference

«Bogolyubov readings DIF-2013. Differential equations, theory of functions and their applications»

on the occasion of the 75th anniversary of academician A. M. Samoilenko

June 23 – 30, 2013
Sevastopol, UKRAINE

ABSTRACTS

4. X. J. Liu, R. Lin, B. Jin and Y. B. Zeng. A generalized dressing approach for solving the extended KP and the extended mKP hierarchy. J. Math. Phys. 2009, 50, P. 053506-1 053506-12.
5. O. I. Chvartatskyi and Yu. M. Sydorenko. Matrix generalizations of integrable systems with Lax integro-differential representations. J. Phys.: Conf. Ser.. 2013, 411, P. 012010-1 – 012010-11, http://arxiv.org/abs/1212.3444
6. L. P. Nizhnik. Integration of multidimensional nonlinear equations by the inverse problem method. Dokl. Akad. Nauk SSSR. 1980, 254 (2), P. 332–335
7. O. Chvartatskyi and Yu. Sydorenko. A new bidirectional generalization of (2+1)-dimensional matrix k-constrained KP hierarchy. submitted to J. Math. Phys. 2013.

BOUNDARY-VALUE PROBLEM WITH INITIAL JUMP FOR A SINGULARLY PERTURBED INTEGRAL-DIFFERENTIAL EQUATIONS

M. K. Dauylbayev
Al-Farabi Kazakh National university, Almaty, Kazakhstan
dmk57@mail.ru

Let's consider in the interval $[0,1]$ the third order linear integral-differential equation with the small parameter $\varepsilon > 0$ at the highest derivative:

$$L_\varepsilon y(t,\varepsilon) \equiv \varepsilon^2 y''' + \varepsilon A_0(t) y'' + A_1(t) y' + A_2(t) y = F(t) + \int_0^1 \sum_{i=0}^{1} H_i(t,x) y^{(i)}(x,\varepsilon)dx \qquad (1)$$

with boundary conditions:

$$h_1 y(t) \equiv y(0,\varepsilon) = \alpha, \qquad h_2 y(t) \equiv y'(0,\varepsilon) = \beta, \qquad h_3 y(t) \equiv y(1,\varepsilon) = \gamma. \qquad (2)$$

We suppose that the roots of the additional characteristic equation $\mu^2 + A_0(t)\cdot\mu + A_1(t) = 0$ satisfy the conditions $\mu_1(t) < -\gamma_1 < 0$, $\mu_2(t) > \gamma_2 > 0$.

A similar problem for differential equations considered in [1]. For the solution of the integral-differential boundary-value problem (1), (2) under certain specific conditions one has the following asymptotic as $\varepsilon \to 0$ estimates:

$$|y^{(i)}(t,\varepsilon)| \le C\Big(|\alpha| + \varepsilon|\beta| + |\gamma| \max_{0\le t\le 1}|H_1(t,1)| + \max_{0\le t\le 1}|F(t)|\Big) +$$
$$+\frac{C}{\varepsilon^{i-1}} e^{-\gamma_1 \frac{t}{\varepsilon}} \Big(|\alpha| + |\beta| + |\gamma| \max_{0\le t\le 1}|H_1(t,1)| + \max_{0\le t\le 1}|F(t)|\Big) +$$
$$+\frac{C}{\varepsilon^{i}} e^{-\gamma_2 \frac{1-t}{\varepsilon}} \Big(|\alpha| + \varepsilon|\beta| + |\gamma| \max_{0\le t\le 1}|H_1(t,1)| + \max_{0\le t\le 1}|F(t)|\Big), \qquad i = 0,1,2,$$

where $C > 0$ is a constant independent on ε.

It follows that as $\varepsilon \to 0$ the values $y'(0,\varepsilon)$, $y''(0,\varepsilon)$ have the order of greatness

$$y''(0,\varepsilon) = O\Big(\frac{1}{\varepsilon}\Big), \qquad y'(1,\varepsilon) = O\Big(\frac{1}{\varepsilon}\Big), \qquad y''(1,\varepsilon) = O\Big(\frac{1}{\varepsilon^2}\Big), \qquad \varepsilon \to 0.$$

Thus, it follows, that the solution of the boundary problem has an initial jumps at both ends of the interval, and the various orders. Namely, at $t = 0$ the initial jump has first order and at $t = 1$ the initial jump has zero order.

1. Kasymov K. A., Zhakipbekova D. A., Nurgabyl D. N. Presentation of the boundary-value problem for a linear differential equation with a small parameter at the highest derivatives. Vestnik KazNU. Ser. mathem., mech., computer science, 2001, № 3, pp. 73–78.

On solutions of the SU(2)-reductions of the Yang–Mills equations

V. S. Dryuma
Institute of Mathematics and Computer Science, 5, Academiei str., MD-2028 Chisinau, Republic of Moldova
valdryum@gmail.com; valery@dryuma.com

New particular solutions of the nonlinear equation

$$\frac{\partial^2}{\partial x \partial y} f(x,y) - f(x,y)^3 = 0, \tag{1}$$

and its two-component generalization

$$\frac{\partial^2}{\partial x \partial y} f(x,y) - f(x,y)\, h(x,y)^2 = 0, \quad \frac{\partial^2}{\partial x \partial y} h(x,y) - h(x,y)\, f(x,y)^2 = 0, \tag{2}$$

which are known examples of reduction of the full SU(2) Yang-Mills system of equations

$$\partial_\mu \partial_\mu \vec{A}_\nu - \partial_\nu \partial_\mu \vec{A}_\mu + [\vec{A}_\mu, (\partial_\nu \vec{A}_\mu - \partial_\mu \vec{A}_\nu + [\vec{A}_\mu, \vec{A}_\nu])] - \partial_\mu [\vec{A}_\mu, \vec{A}_\nu] = 0 \tag{3}$$

are obtained.

The system of the equations

$$2\frac{\partial}{\partial x} f(\vec{x},t) \frac{\partial}{\partial t} f(\vec{x},t) + 2 f(\vec{x},t) \frac{\partial^2}{\partial t \partial x} f(\vec{x},t) + f(\vec{x},t)^2 \frac{\partial}{\partial x} f(\vec{x},t) = 0,$$

$$2\frac{\partial}{\partial y} f(\vec{x},t) \frac{\partial}{\partial t} f(\vec{x},t) + 2 f(\vec{x},t) \frac{\partial^2}{\partial t \partial y} f(\vec{x},t) + f(\vec{x},t)^2 \frac{\partial}{\partial y} f(\vec{x},t) = 0,$$

$$2\frac{\partial}{\partial z} f(\vec{x},t) \frac{\partial}{\partial t} f(\vec{x},t) + 2 f(\vec{x},t) \frac{\partial^2}{\partial t \partial z} f(\vec{x},t) + f(\vec{x},t)^2 \frac{\partial}{\partial z} f(\vec{x},t) = 0,$$

which is an example of a new multidimensional reduction is considered.

Particular solutions of this system of equations are constructed and their properties are discussed.

For deriving solutions of considered equations is applied the method of parametric presentation of the functions and their particular derivatives which was developed by author [1]. As a result of application of this method, the equation can be transformed into another equation, with the help of which you can obtain non-trivial solutions of the original equation.

For example, the equation (1) can be transformed to the equation

$$-\theta(\xi,t)^3 + \xi^4 \frac{\partial^2}{\partial t \partial \xi} \theta(\xi,t) - \xi^3 \frac{\partial}{\partial t} \theta(\xi,t) = 0. \tag{4}$$

The simplest solution of the equation (4) has the form $\theta(\xi,t) = \frac{\xi^{3/2}\sqrt{c_2}}{\sqrt{(2+\xi\, C_1\, c_2)(-2\, c_2 t + C_1)}}$. As a result of a reverse transformation, it turns out nontrivial solution of the original equation (1).

1. Dryuma V. On the equations determining the Ricci flows on manifolds. International Journal of Geometric Methods in Modern Physics, Vol. 10, No. 4 1320009 (9 pages), World Scientific Publishing Company (2013).