On the orbital evolution of radiating binary systems

A. A. Bekov, and S. B. Momynov

Citation: AIP Conference Proceedings **1959**, 040005 (2018); doi: 10.1063/1.5034608
View online: https://doi.org/10.1063/1.5034608
View Table of Contents: http://aip.scitation.org/toc/apc/1959/1
Published by the American Institute of Physics

On the Orbital Evolution of Radiating Binary Systems

A.A. Bekov[1,a)] and S.B.Momynov[2,b)]

[1]*JSC "National Center for Space Research and Technology", Shevchenko St., 15, Almaty, 050010, Kazakhstan*
[2]*K.I. Satpayev Kazakh National Research Technical University, Satpaev St., 22a, Almaty, 050013, Kazakhstan*

[a)]Corresponding author: bekov@mail.ru
[b)]s.momynov@gmail.com

Abstract. The evolution of dynamic parameters of radiating binary systems with variable mass is studied. As a dynamic model, the problem of two gravitating and radiating bodies is considered, taking into account the gravitational attraction and the light pressure of the interacting bodies with the additional assumption of isotropic variability of their masses. The problem combines the Gylden-Meshchersky problem, acquiring a new physical meaning, and the two-body photogravitational Radzievsky problem. The evolving orbit is presented, unlike Kepler, with varying orbital elements - parameter and eccentricity, defines by the parameter $\mu(t)$, area integral C and quasi-integral energy $h(t)$. Adiabatic invariants of the problem, which are of interest for the slow evolution of orbits, are determined. The general course of evolution of orbits of binary systems with radiation are determined by the change of the parameter $\mu(t)$ and the total energy of the system.

INTRODUCTION

When studying the dynamics of gravitating systems, taking into account evolutionary factors (secular mass loss, dissipation, accretion, and other non-gravitational effects), it is necessary to choose the initial unperturbed orbit, which is different from Keplerian and takes into account, in the first approximation, the main evolutionary tendencies of the dynamic system. The work is devoted to research in this direction. The two-body problem, which takes into account the gravitational attraction and the light pressure of interacting bodies, is considered. The problem of two gravitating and radiating bodies was first formulated and studied by V.V. Radzievsky [1]. We consider the problem of two gravitating and radiating bodies with the additional assumption of isotropic variability of the masses of interacting bodies. An example of application of the nonstationary photogravitational two-body problem in the theory of space flight with a solar sail taking into account the variability of the gravitational parameter in the problem shows that this problem needs further investigation [2, 3]. It is expected that the results of the research will provide additional information on the properties of motion of binary systems with radiation and will find application in various problems of dynamical astronomy in which it is necessary to take into account the dependence of the gravitational parameter of celestial bodies on time in the process of evolution.

RESULTS OF THE RESEARCH

We consider the problem of two gravitating and radiating bodies, first formulated and studied by V.V. Radzievsky [1], with the additional assumption of isotropic variability of the masses of interacting bodies.

The equations of the relative motion in the problem under consideration have the form:

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3}, \tag{1}$$

where

$$\mu = G(q_1 m_1 + q_2 m_2) \tag{2}$$

The Eighth Polyakhov's Reading
AIP Conf. Proc. 1959, 040005-1–040005-4; https://doi.org/10.1063/1.5034608
Published by AIP Publishing. 978-0-7354-1660-4/$30.00

is a variable factor, in which: G is the gravitational constant, m_1 and m_2 are the masses of the bodies, q_1 and q_2 are the coefficients of reduction of the bodies m_1 and m_2. In the general case, the masses m_1 and m_2 are definite given functions of time t. The problem (1) unites the Gylden-Meshchersky problem ($\mu = \mu(t)$), introducing a new physical meaning into it, and the photogravitational problem of the two bodies of Radzievsky ($\mu = const$) [1-5].

The equations of motion (1) multiplied vectorwise by $\vec{r}$ and scalarly by the velocity $\dot{\vec{r}}$ are given, respectively, by the area integral $\vec{C} = const$:

$$\vec{r} \times \dot{\vec{r}} = \vec{C}, \quad \vec{C} \cdot \vec{r} = \vec{C} \cdot \dot{\vec{r}} \equiv 0 \tag{3}$$

and the quasi-integral of energy h:

$$\dot{\vec{r}}^2 - 2\frac{\mu}{r} \equiv h = h_0 - 2\int \frac{d\mu}{r}. \tag{4}$$

Integration with respect to μ is performed in the range from μ_0 to μ, corresponding to changes in time from zero to the current time t. As a consequence of (3) and the equations of motion (1), it is easy to obtain another quasi-integral, the Laplace vector $\vec{f}$ (such as $\vec{f} \cdot \vec{C} \equiv 0$):

$$\dot{\vec{r}} \times \vec{C} - \frac{\mu\vec{r}}{r} \equiv \vec{f} = \vec{f_0} - \int \frac{\vec{r} d\mu}{r}, \tag{5}$$

the square of which, as follows from the identities (5) and (4), has the same form as in the unperturbed Kepler problem, but with variables μ, f, h:

$$\vec{f}^2 = f^2 = \mu^2 + hC^2. \tag{6}$$

These are the initial relations required for further analysis. The relations obtained are exact and acceptable for any $\mu(t)$ [5]. The relations similar to (3) - (5) were derived in [6] for the case $\mu(t) = 1 + \sigma\varphi(t)$, where σ is a small quantity, $\varphi(t)$ is a monotone differentiable function of time t.

Since the characteristic dynamic variable in this problem is the parameter $\mu(t)$, differentiation and integration with respect to time can be replaced everywhere with respect to μ, which considerably simplifies the computation.Then, denoting the derivatives of $d/d\mu$ by a prime and differentiating (4), (5), and (6), we find, respectively:

$$h' = -\frac{2}{r}, \tag{7}$$

$$\vec{f}' = -\frac{\vec{r}}{r}, \tag{8}$$

$$\vec{f} \cdot \vec{f}' \equiv f \cdot f' = \mu + \frac{1}{2}h' \cdot C^2 = \mu - \frac{C^2}{r}. \tag{9}$$

On the other hand, according to (8),

$$f \cdot f' = \vec{f} \cdot \vec{f}' = -\vec{f}\frac{\vec{r}}{r} = -f\cos\varphi, \tag{10}$$

where the angle $\varphi = \widehat{\vec{r} \cdot \vec{f}}$ is called the true anomaly. Comparing (9) and (10), we arrive at the trajectory form of problem (1) [5]:

$$r = \frac{C^2}{\mu(1 + \frac{f}{\mu}\cos\varphi)}. \tag{11}$$

Taking (6) into account, we obtain an evolving orbit

$$r = \frac{C^2}{\mu(1 + e\cos\varphi)} \tag{12}$$

with variable orbit elements

$$p = \frac{C^2}{\mu}, \quad e = \sqrt{1 + \frac{hC^2}{\mu^2}}, \quad \omega = \theta - \varphi, \tag{13}$$

where ω is the argument of periapsis, calculated from some fixed direction, θ is the polar angle. Solutions of the type (12) in the case of the Gylden-Meshchersky problem can be found in the investigations of various authors (see, for example, [7]). Using the results of investigations of the Gylden-Meshchersky problem, we can immediately indicate integrable cases of problem (1): these are cases of $\mu(t) = G(q_1 m_1 + q_2 m_2)$ variation according to the laws of I.V. Meshchersky or B.E. Gelfgat [8]. From the results of the study of the photogravitational two-body problem [1], it follows that the trajectory of motion (12), depending on the signs of μ and h, can be any osculating conic section.

Define the elements of the orbit (12): a - the semi-major axis, e - eccentricity, T - period, in terms of characteristics of motion (C is a constant area and $E = \frac{h}{2}$ is the total energy). We have

$$\begin{aligned} a &= -\frac{1}{2E} G(q_1 m_1 + q_2 m_2), \\ e^2 &= 1 + \frac{2EC^2}{G^2 (q_1 m_1 + q_2 m_2)^2}, \\ T &= \frac{2\pi}{C} a^2 \sqrt{1 - e^2}. \end{aligned} \tag{14}$$

Using (14), we obtain

$$\begin{aligned} \frac{\delta a}{a} &= \frac{\delta(q_1 m_1 + q_2 m_2)}{(q_1 m_1 + q_2 m_2)} - \frac{\delta E}{E}, \\ \frac{\delta T}{T} &= \frac{\delta(q_1 m_1 + q_2 m_2)}{(q_1 m_1 + q_2 m_2)} - \frac{3}{2} \frac{\delta E}{E}, \\ \frac{e \delta e}{1 - e^2} &= \frac{\delta(q_1 m_1 + q_2 m_2)}{(q_1 m_1 + q_2 m_2)} - \frac{1}{2} \frac{\delta E}{E}. \end{aligned} \tag{15}$$

For isotropic mass loss we have $\delta E = -\delta\mu / r$. In the case of a slow change in the parameter μ, assuming that, on average, per revolution $< 1/r >= 1/a$, and using (14), we obtain

$$\delta E / E = 2 \delta\mu / \mu, \tag{16}$$

from which we get the adiabatic invariant [9] of the form

$$E \sim G^2 (q_1 m_1 + q_2 m_2)^2. \tag{17}$$

Therefore system (15) takes the form

$$\begin{aligned} \frac{\delta a}{a} &= -\frac{\delta(q_1 m_1 + q_2 m_2)}{(q_1 m_1 + q_2 m_2)}, \\ \frac{\delta T}{T} &= -2 \frac{\delta(q_1 m_1 + q_2 m_2)}{(q_1 m_1 + q_2 m_2)}, \\ \frac{e \delta e}{1 - e^2} &= 0, \end{aligned} \tag{18}$$

from which the Jeans invariants follow [10]:

$$\mu a = const, \ \mu^2 T = const, \ e = const, \tag{19}$$

generalized to the case $\mu(t)$ defined by the formula (2). The invariants (19) are of interest for studying the slow evolution of orbits; the general evolution of the orbits of binary systems with radiation is determined by equations (15).

A solution of the Gylden-Meshchersky problem in the form of an evolving orbit is given in [11]

$$r = \frac{p}{1 + e \cos\varphi}, \tag{20}$$

which coincides in the form with Keplerian, but has a variable parameter p and eccentricity e:

$$p = \frac{C^2}{\mu}, \ \ \mu = \frac{\alpha\varphi + \beta}{\delta t + \gamma}, \ \ e = \frac{k}{\alpha\varphi + \beta}, \tag{21}$$

where $\alpha, \beta, \delta, \gamma, k$ are arbitrary constants. The solution of (21) generalizes the case of integrability of the problem for the first Meshchersky law [4] for changing mass of the bodies: the law of mass change (21) includes the first law of the Meshchersky mass change of the bodies, and the eccentricity e of the orbit is a variable quantity, a function of the polar angle. The special cases of the solution (21) are invariants

$$p\mu = C^2, \;\; \mu e = const, \;\; \frac{p}{e} = const. \tag{22}$$

The second invariant of the form (22) is also indicated in the work [12], and was obtained by L. Chiara, V.V. Radzievsky, and L.P. Surkova in different ways [12]. The invariants of the form (22) are presented in [5] for various found parametric solutions of the Gylden-Meshchersky problem. Thus, the obtained solutions of the problem in the form of an evolving orbit with a variable parameter and eccentricity of the orbit will make it possible to reveal new properties of motion in the problem of two and many gravitating and radiating bodies.

CONCLUSION

The paper considers the problem of two gravitating and radiating bodies under the assumption of isotropic variability of the masses of interacting bodies. The moment integral, the energy quasi-integral and the quasi-integral - the Laplace vector are derived for any law of variation of the gravitational parameter $\mu(t)$, and with their help the evolving orbit of the binary system with radiation is obtained. The elements of the orbit: the semi-major axis, eccentricity and period are determined in terms of characteristics of the motion - the integral of the moment and the total energy. The variations of the orbital elements as functions of these motion characteristics are determined. For the case of a slow variation of the parameter $\mu(t)$, a generalization of the adiabatic Jeans invariants to the case of a binary system with radiation is obtained. The case of a slow evolution of the orbits of binary systems with radiation is discussed. The invariants for various found solutions of the problem are presented. The results of the work can be used in the analysis of the dynamical evolution of binary non-stationary gravitating systems with radiation.

REFERENCES

[1] V. V. Radzievsky, Astron. journal (in Russian) **28**, 363–372 (1951).

[2] E. N. Polyakhova, *Space flight with a solar sail (in Russian)* (M . : Science., 1986) p. 304.

[3] E. N. Polyakhova, Bulletin of Leningrad State University. Series I. Math., Mech., Astron. (in Russian) **3**, 83–89 (1986).

[4] I. V. Meshchersky, *Works on the mechanics of bodies of variable mass (in Russian)* (M: GITTL, 1949) p. 276.

[5] A. A. Bekov, *Dynamics of double gravity systems with variable masses (in Russian)* (Saarbrucken, Deutschland: LAP LAMBERT Academic Publishing. GmbH and Co. KG., 2015) p. 224.

[6] N. S. Samoylova-Yakhontova, Bull. Institute of Theor. astronomy AS USSR (in Russian) **8**, 396–401 (1962).

[7] J. Hadjidemetriou, Advances in Astronomy and Astrophysics **5**, 131–188 (1967).

[8] L. M. Berkovich, Reports of the Academy of Sciences of the USSR (in Russian) **250**, 1088–1091 (1980).

[9] Y. P. Dontsov, Y. A. Zavenyagin, and G. N. Tilinin, Cosmic Research (in Russian) **23**, 634–636 (1985).

[10] J. Jeans, *Astronomy and Cosmogony* (Cambridge, 1929) p. 290.

[11] A. A. Bekov, Reports of NAS RK (in Russian) **2**, 45–47 (2006).

[12] V. V. Radzievsky and L. P. Surkova, Astron. journal (in Russian) **50**, 1200–1210 (1973).