**Abstracts of chapters**

**Part I. Minimization of functionals**

The minimization problems for the functionals are considered. These functionals depend from unknown function directly. We consider necessary conditions of minimum and additional problems.

**Chapter 1. Necessary conditions of extremum for functionals**

The easiest result of extremum theory is the equality to zero of derivative of the function at the point of extremum. The minimization problem for functionals includes the derivative of the given functional. It can be generalized to the minimization problems on a subspace or on an affine variety. The variational inequality is the necessary condition of minimum on a convex set. These results are the basis of optimization control theory.

**Chapter 2. Minimization of functionals. Addition**

We considered before necessary conditions of extremum for the functionals. Now we analyze additional problems. This is the sufficiently of extremum conditions, the solvability of extremum problems, the uniqueness of the solution, the well-posedness of the problem, the minimization of non-convex and non-smooth functionals, Ekeland principle, and approximate methods of theory extremum.

**Part II. Stationary systems**

The optimal control problems are considered. The minimized functional depends from unknown function that is the control non-directly. The functional depends from a state function that is the solution from a state equation. The considered system is stationary.

**Chapter 3. Linear systems**

Optimization problems for stationary systems are analyzed. The abstract linear systems and the systems described by linear elliptic equations are considered. At first, we determine necessary property of the considered systems. Then we prove the one-value solvability of the optimal control. We determine also necessary and sufficient conditions of optimality and additional properties of the optimal control problems.

**Chapter 4. Weakly nonlinear systems**

Optimal control problems for systems the systems described by abstract nonlinear equations with monotone operators and nonlinear elliptic equations are considered. The derivative of the functional depends from the derivative of control–state mapping here. It can be found with using of the inverse function theorem. We consider the case of the applicability of this result only.

**Chapter 5. Strongly nonlinear systems**

The optimal control problem for nonlinear elliptic equations are considered for the case of the non-applicability of the inverse function theorem. Then the dependence of the state function with respect to the control can be non-differentiable. We determine the extended operator derivative. The necessary conditions of optimality are obtained with using of the extended differentiability of control–state mapping.

**Chapter 6. Coefficients optimization problems**

Optimization problems for systems described by linear and nonlinear elliptic equations with coefficient control are considered. The boundary problem is solvable for the controls that belongs to the set of admissible controls, not for the control space. Gateaux derivative and extended one of control–state mapping are not applicable here. The necessary conditions of optimality are determined with using of the derivative with respect to the convex set and its extended analogue.

**Chapter 7. Systems with nonlinear control**

The general abstract control systems are considered. The differentiability of control–state mapping is determined here with using implicit function theorem. This result use the solvability of the linearized equation in the natural spaces. If this supposition get broken, then the dependence of state function from the control can be non-differentiable. However, we can determine its extended derivative. Optimization problems for the systems described by nonlinear elliptic equation with nonlinear control are considered as examples.

**Part III. Evolutional systems**

Optimal control problems for evolutional systems are considered.

**Chapter 8. Linear first order evolutional systems**

Optimal control problems for the abstract linear evolutional systems and linear parabolic equation are considered. We prove the one-value solvability of the optimal control; determine necessary and sufficient conditions of optimality and additional properties of the optimal control problems.

**Chapter 9. Nonlinear first order evolutional systems**

Optimal control problems for the nonlinear evolutional systems with monotone operators and nonlinear parabolic equation are considered. Necessary optimality conditions are obtained by the standard technique, if control–state mapping is Gateaux differentiable. We use extended derivative, if the dependence of the state function from the control is not Gateaux differentiable. Then we analyze the coefficient optimization problems.

**Chapter 10. Second order evolutional systems**

Optimization problems for linear and nonlinear abstract second order evolutional systems and hyperbolic equations. The necessary conditions of optimality are obtained with using the extended derivatives.

**Chapter 11. Navier – Stokes equations**

The optimization problems for the systems described by Navier – Stokes equations for the stationary and evolutional cases and with heat equations. At first, we analyze the properties of the considered boundary problems. Then we prove existence of the optimal controls. Finally, we determine the necessary conditions of optimality with using the extended differentiability of control–state mapping.

**Part IV. Addition**

**Chapter 12. Functors of the differentiation**

We interpret the operator of the differentiation as a functor. This result is applied for obtaining necessary conditions of optimality with using categories theory notions. The classical and extended differentiations are considered.