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System Design and Analysis

Mechanical Engineering and Control Automation

Animation of Motion of Mechanisms and Robot Manipulators in the Maple System with the Construction of Diagrams of Internal Forces on the Links

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ABSTRACT

This paper presents the technique of analytical determination of internal forces in the links of planar mechanisms and robot manipulators with statically indeterminate structures taking into account the distributed dynamical loads, a dead weight and active external loads. By the described technique the Maple18 programs are created. Animations of the motion of mechanisms with the construction on links the intensity of lateral and axial distributed inertial loads, the bending moments, lateral and axial forces, depending on kinematic characteristics of links are obtained.

CCS Concepts

• Applied computing → Physical sciences and engineering → Engineering → Computer-aided design

Keywords

Mechanisms, Robot manipulators, Distributed inertial forces, Internal forces, Dynamic equilibrium, Link compliance, Kinematic parameters, Statically indeterminate mechanisms, CAD, Animation

1. INTRODUCTION

There are a variety of graph-analytical and numerical calculation methods on strength and stiffness of rod robotic systems and mechanisms, in which the distributed inertia forces of difficult character aren't considered [1-3]. The distributed inertia forces of complex nature appear in links of rod mechanisms within the motion process. The intensity of distribution of inertia forces along the link depends on the mass distribution along the link and the fast-changing kinematic characteristics of the mechanism.

Therefore, relations between the intensity of distributed inertia forces and link weight with geometrical, physical and kinematic characteristics are determined in this work.

As internal loads of each continual link are all defined by a set of internal loads in its separate sections, and by the approximation matrices, so the task was to calculate the internal loads in finite number of sections of elements.

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As a result, we refer to discrete model of elastic calculation of links of rod mechanisms. In the work of elastic calculation of planar rod mechanisms for each instantaneous position of the mechanisms they are brought to link structures, which degree of freedom is equal to zero based on D'Alembert's principle. For definition of internal loads in links of designed scheme of mechanism, the structure is divided into elements, and both pin and rigid nodes. This is the first time the elements are divided into three types of beam. Discrete models of these beams having constant sections which are under the action of lateral and axial distributed trapezoidal loads are constructed. These constructed discrete models allow to determine the quantity of the independent dynamic equilibrium equations, components of a vector of forces in calculated sections and to construct discrete model of all structure.

The dynamic equilibrium equations for discrete model of an element of the link with constant sections under the action of lateral and axial inertial loads of a trapezoidal shape are also received in this work as well as the equilibrium equations of pin and rigid nodes expressed through required parameters of internal forces.

If we integrate the equations of dynamic equilibrium of elements and nodes into a single system, we will receive the equations of dynamic equilibrium of all discrete model of system. A sort of systems of equations is sufficient for definition of internal forces in links of mechanisms, which structure is statically definable. The vector of forces and vector of loads in calculated sections of discrete models of mechanisms are formed from vectors of forces and vectors of loads in calculated sections of their separate elements, respectively.

For mechanisms having statically indeterminate links, the number of equilibrium equations is less than the number of unknowns by the number of degree of statically indetermination of construction. Hence, for definition of internal forces of statically indeterminate mechanisms the flexibility matrices were constructed optional for entire discrete model of rod mechanism. On the given algorithm the programs in the MAPLE18 system were created and animations of the motion of mechanisms with the construction on links the intensity of lateral and axial distributed inertial loads, the bending moments, cross and axial forces, depending on kinematic characteristics of links are obtained.

2. INERTIA FORCES AND APPROXIMATION MATRIX

Considering the plane-parallel motion of the kth link of mechanism with constant sections with respect to OXY fixed coordinate system, the following laws of distribution of the lateral and axial inertia forces along a link, that arise from self-mass of a link are defined [4]:

$$\begin{cases} q_k(x_k') = a_{kq} + b_{kq} x_k'; \\ n_k(x_k') = a_{kn} + b_{kn} x_k'. \end{cases} \quad (1)$$

where:

$a_{kq} = -\gamma_k A_k \cos\theta_k - \frac{\gamma_k A_k}{g} w_{kp}^{y_k}$; $\quad b_{kq} = -\frac{\gamma_k A_k}{g} \varepsilon_k$;

$a_{kn} = -\gamma_k A_k \sin\theta_k - \frac{\gamma_k A_k}{g} w_{kp}^{x_k}$; $b_{kn} = \frac{\gamma_k A_k}{g} w_k^2$; θ_k is an angle that determines the position of kth link with respect to fixed OXY coordinate systems; ω_k, ε_k are the angular velocity and angular acceleration of the kth link, respectively; $w_{kp}^{x_k}, w_{kp}^{y_k}$ are the components of P_k point acceleration of kth link directed along the axis of link and perpendicular; γ_k is a specific weight of material of the kth link, A_k is a cross-sectional area of kth link; g is an acceleration of gravity.

For the element under the action of lateral and axial trapezoidal distributed loads, the approximation matrix [5] connecting internal loads in any section of the element with values of internal loads in designed sections has an appearance:

$$[H_k(x_k')] = \begin{bmatrix} h_{11}(x_k') & h_{12}(x_k') & h_{13}(x_k') & h_{14}(x_k') & 0 & 0 & 0 \\ h_{21}(x_k') & h_{22}(x_k') & h_{23}(x_k') & h_{24}(x_k') & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{35}(x_k') & h_{36}(x_k') & h_{37}(x_k') \end{bmatrix}, \quad (2)$$

3. DISCRETE MODELS OF CALCULATION OF ELASTICITY OF ELEMENTS AND MECHANISMS IN GENERAL

To determine the static indeterminacy of the system, the formula is used [3]:

$$k = 3K - Ш. \quad (3)$$

where K is a number of the closed contours, $Ш$ is a number of simple (single) nodes, k is a degree of redundancy of designed model of mechanism. The designed model of fourth class mechanism in Fig. 1 is statically determinate, if the rods of basic links 2 and 5 are assumed to be hinged together. In this case the number of closed contours is $K = 4$, the number of simple (single) nodes are $Ш = 12$, then, $k = 0$, i.e. the structural model of the system is statically determinate.

If the rods of basic links 2 and 5 are assumed to be connected rigidly, so the number of single nodes are $Ш = 6$ and $k = 6$, i.e. the structural model of the system will be six times statically indeterminate.

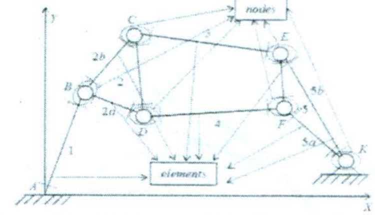

Figure 1. The designed model of fourth class mechanism and its division into elements and nodes for elastic calculation.

A discrete model for elastic calculation of six times statically indeterminate fourth class mechanism is constructed in Fig. 2 with the sought parameters [6].

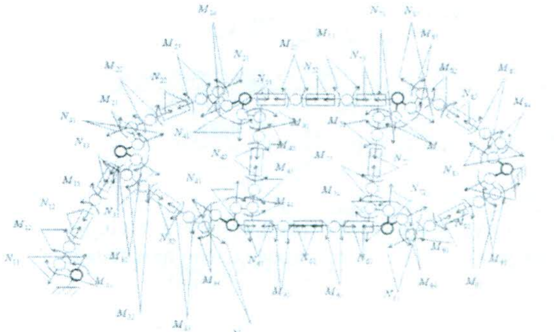

Figure 2. The discrete model of the fourth class mechanism with constant sections of links and statically indeterminate structure.

3.1 Dynamic Equilibrium Equations of Discrete Models of Elements and Nodes

Obtained through required parameters a system of dynamic equilibrium equations of discrete models of elements is as follows [7]:

$$[A_k]\{S_k\} = \{F_k\}, \quad (4)$$

where

$$\{S_k\} = \{M_{k1}, M_{k2}, M_{k3}, M_{k4}, N_{k1}, N_{k2}, N_{k3}\}^T,$$

$$\{F_k\} = \left\{b_{kq}, -a_{kq}\frac{l_k^2}{2} - b_{kq}\frac{l_k^3}{6}, -b_{kn}, -a_{kn} l_k - b_{kn}\frac{l_k^2}{2}\right\}^T,$$

$$[A_k] = \begin{bmatrix} -\frac{27}{l_k^3} & \frac{81}{l_k^3} & -\frac{81}{l_k^3} & \frac{27}{l_k^3} & 0 & 0 & 0 \\ -\frac{9}{2} & 9 & -\frac{9}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{l_k^2} & -\frac{8}{l_k^2} & \frac{4}{l_k^2} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

A revolute kinematic pair is formed with the help of three elements of mechanism: j, k and i. The elements i and k are connected rigidly in Fig. 3. The sections of all three elements are constant over their entire length. Obtained through required parameters a system of dynamic equilibrium equations for this node of elements is as follows:

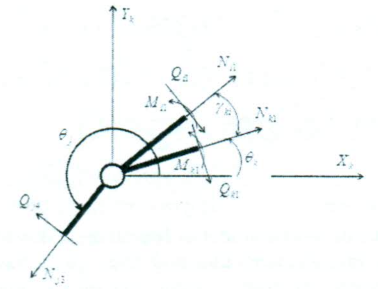

Figure 3. Rigid node of mechanism with constant sections of elements.

$$
\begin{cases}
-\frac{11\sin\theta_k}{2l_k}M_{k1}+\frac{9\sin\theta_k}{l_k}M_{k2}-\frac{9\sin\theta_k}{2l_k}M_{k3}+\frac{\sin\theta_k}{l_k}M_{k4}+ \\
+\cos\theta_k N_{k1}-\frac{11\sin(\theta_k+\gamma_{ki})}{2l_i}M_{i1}+\frac{9\sin(\theta_k+\gamma_{ki})}{l_i}M_{i2}-\frac{9\sin(\theta_k+\gamma_{ki})}{2l_i}M_{i3}+ \\
+\frac{\sin(\theta_k+\gamma_{ki})}{l_i}M_{i4}+\cos(\theta_k+\gamma_{ki})N_{i1}-\frac{\sin\theta_j}{l_j}M_{j1}+\frac{9\sin\theta_j}{2l_j}M_{j2}- \\
-\frac{9\sin\theta_j}{l_j}M_{j3}+\frac{11\sin\theta_j}{2l_j}M_{j4}+\cos\theta_j N_{j3}=0; \\
\frac{11\cos\theta_k}{2l_k}M_{k1}-\frac{9\cos\theta_k}{l_k}M_{k2}+\frac{9\cos\theta_k}{2l_k}M_{k3}-\frac{\cos\theta_k}{l_k}M_{k4}+\sin\theta_k N_{k1}+ \\
+\frac{11\cos(\theta_k+\gamma_{ki})}{2l_i}M_{ki}-\frac{9\cos(\theta_k+\gamma_{ki})}{l_i}M_{i2}+\frac{9\cos(\theta_k+\gamma_{ki})}{2l_i}M_{i3}- \\
-\frac{\cos(\theta_k+\gamma_{ki})}{l_i}M_{i4}+\sin(\theta_k+\gamma_{ki})N_{i1}+\frac{\cos\theta_j}{l_j}M_{j1}-\frac{9\cos\theta_j}{2l_j}M_{j2}+ \\
+\frac{9\cos\theta_j}{l_j}M_{j3}-\frac{11\cos\theta_j}{2l_j}M_{j4}+\sin\theta_j N_{j3}; \\
M_{i1}+M_{k1}=0.
\end{cases} \tag{5}
$$

3.2 The Matrix of Compliance of the Element with Constant Sections under the Distributed Cross and Axial Trapezoidal Loads

The matrix of compliance of the element is defined by equation [3]:

$$
[D_K]=\int_{l_k}[H_k(x_k^{'})]^T[D_{kx_k^{'}}][H_k(x_k^{'})]dx_k^{'}. \tag{6}
$$

Here $[D_{kx_k^{'}}]$ is a matrix of compliance of link section that is known from the course of strength of materials. For the rods working on bending and tension-compression the following equation is used:

$$
[D_{kx_k^{'}}]=\begin{bmatrix} \frac{1}{E_kI_k} & 0 & 0 \\ 0 & \frac{\mu}{G_kA_k} & 0 \\ 0 & 0 & \frac{1}{E_kA_k} \end{bmatrix}, \tag{7}
$$

where $\frac{1}{E_kI_k}$ is a bending compliance of the section; $\frac{\mu}{G_kA_k}$ is a shear compliance of the section; $\frac{1}{E_kA_k}$ is a tension-compression compliance of the section; $[H_k(x_k^{'})]$ is defined by Eq. (2).

Thus, the general view of the obtained matrix of the element's compliance considering the deformation of bending, tension-compression under the action of distributed lateral and axial trapezoidal loads has an appearance:

$$
[D_K]=\begin{bmatrix}
\frac{8l_k}{105E_kI_k} & \frac{33l_k}{560E_kI_k} & -\frac{3l_k}{140E_kI_k} & \frac{19l_k}{1680E_kI_k} & 0 & 0 & 0 \\
\frac{33l_k}{560E_kI_k} & \frac{27l_k}{70E_kI_k} & -\frac{27l_k}{560E_kI_k} & -\frac{3l_k}{140E_kI_k} & 0 & 0 & 0 \\
-\frac{3l_k}{140E_kI_k} & -\frac{27l_k}{560E_kI_k} & \frac{27l_k}{70E_kI_k} & \frac{33l_k}{560E_kI_k} & 0 & 0 & 0 \\
\frac{19l_k}{1680E_kI_k} & -\frac{3l_k}{140E_kI_k} & \frac{33l_k}{560E_kI_k} & \frac{8l_k}{105E_kI_k} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{2l_k}{15E_kA_k} & \frac{l_k}{15E_kA_k} & -\frac{l_k}{30E_kA_k} \\
0 & 0 & 0 & 0 & \frac{l_k}{15E_kA_k} & \frac{8l_k}{15E_kA_k} & \frac{l_k}{15E_kA_k} \\
0 & 0 & 0 & 0 & -\frac{l_k}{30E_kA_k} & \frac{l_k}{15E_kA_k} & \frac{2l_k}{15E_kA_k}
\end{bmatrix}. \tag{8}
$$

3.3 A Decision Equations of Determination of Internal Forces

Through combining the equilibrium equations of elements and nodes into a single system, the equilibrium equations of the discrete model of entire mechanism is obtained. These equations can be written in general form:

$$
[A]\{S\}=\{F\}. \tag{9}
$$

Such systems of equations are sufficient to determine the internal forces in the links of the mechanism of statically definable structure [8].

The matrix of equilibrium equations for the discrete model of mechanisms consists of matrices of equilibrium equations of their individual elements, as well as the equilibrium equations of their nodes. The matrix of dynamic equilibrium equations of discrete models of mechanisms is as follows:

$$
[A]=\begin{bmatrix} [A_1] & 0 & . & . & . & 0 \\ 0 & [A_2] & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & [A_n] \\ \multicolumn{6}{c}{\textit{Equilibrium equations of nodes}} \end{bmatrix}
$$

For discrete models of mechanisms the vector of force and the vector of loads in calculated sections are formed by vector of forces and loads in calculated sections of their individual elements. These vectors have the following vector form, respectively:

$$
\{F\}=\{\{F_1\},\{F_2\},...,\{F_n\}\}^T;\ \{S\}=\{\{S_1\},\{S_2\},...,\{S_n\}\}^T.
$$

For determination of internal forces of statically indeterminate mechanisms (in the way of determination of internal forces), it is necessary to build a compliance matrix for the entire discrete model of the rod mechanism. The matrix of compliance $[D]$ for the entire discrete model of the rod mechanism consists of the matrices of individual elements $[D_k]$. It is expressed in block-diagonal form:

$$\begin{bmatrix} [D_1] & 0 \; . \; . \; . & 0 \\ 0 & [D_2] \; . \; . \; . & 0 \\ . \; . \; . \; . \; . & . \; . \; . \; . \; . & . \; . \\ 0 & 0 \; . \; . \; . & [D_n] \end{bmatrix}$$

where n is a number of the elements of discrete model of mechanism. A formula below is used to determine the components of the vector of loads $\{S\}$:

$$\{S\}=[K][A]^T\left([A][K][A]^T\right)^{-1}\{F\}, \qquad (10)$$

$[K]=[D]^{-1}$ is taken into consideration.

Consider a fourth class mechanism with one drive link for determination of internal loads in links. According to the developed method, computer programs are created in the Maple18 system with motion animation of the mechanism, with the construction of regularities in the distribution of inertial forces and internal forces on the links of the mechanism. The results of the obtained inertial forces and internal forces for some positions of the mechanism are shown in Figures 4-8.

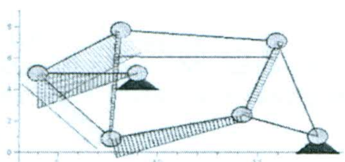

Figure 4. The diagram of axial distributed inertial forces arising from self-mass of the links of investigated

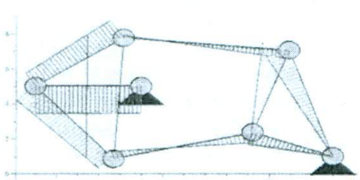

Figure 5. The diagram of lateral distributed inertial forces arising from self-mass of the links of investigated mechanism.

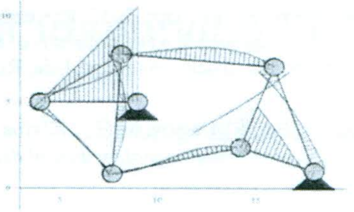

Figure 6. The diagram of bending moments arising from distributed inertial loads acting on the links of the statically definable mechanism.

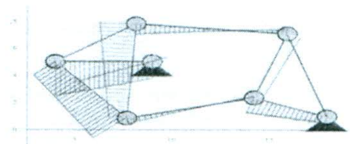

Figure 7. The diagram of axial forces arising from distributed inertial loads acting on the links of the statically definable mechanism.

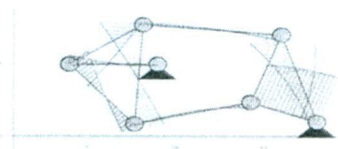

Figure 8. The diagram of lateral forces arising from distributed inertial loads acting on the links of the statically definable mechanism.

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