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A TWO-DIMENSIONAL MATHEMATICAL MODELING OF DARRIEUS WIND TURBINE

AINAKUL YERSHINA, RUSTEM MANATBAYEV, ASSYLBEK TULEPBERGENOV, KALASOV NURDAULET, UALIYEV ZHOMART

Abstract: *The use of natural eco-friendly energy, such as wind, due to the use of wind energy devices [1-3]. This paper is a theoretical study of the interaction of the rotating turbine Darrieus-type H-rotor wind flow. The paper considers the interaction of a uniform wind flow with a rotating wind turbine Darrieus-type H-rotor, based on the Navier-Stokes equations, which is supplemented by members which take into account the lift and the drag force. The numerical solution of the equations in the variables u-v-p. The results obtained and the development of his methods of analysis will be useful for the design work for the creation of industrial designs windmill carousel.*

Key words: *modern wind turbine, vertical-axis, straight blade, lift force, drag force, flow, Navier-Stokes, numerical model, calculation.*

1. Introduction

Wind - an environmentally friendly source of energy! In all the developed countries of the world wind as an energy source, is beginning to play a significant role in their energy mix. Continuously expanding manufacturing and designing efficient wind turbines [1,3]. The territory of the Republic of Kazakhstan has enormous wind energy potential, which, of course, suggest a bright future for the application of wind power units in the country.

Realizing the importance of the wind industry in the Republic of many scientific groups and individuals with more enthusiasm (and in many cases their own) are studying the problems of wind power and achieve certain results (Almaty Institute of Power Engineering and Telecommunications, Kazakh National University named after al-Farabi, Kazakh Scientific Research Institute of Power Engineering, Farm Equipment, The Society Joint " Kazakh Scientific Research Institute «hydro unit» " JSC Asia Energy project installation and many others).

Cost-effectiveness of modern wind turbines is determined by the utilization of wind energy per unit area of the surface swept by the propeller in the airflow. Therefore, their design refers to the category of the most high-tech industries, based on current knowledge of aerodynamics, the theory of machines, materials, etc.

Average wind-energy unit (WPU), Darrieus , invented by French engineer (naturalist) Darrie in 1904, consists of a blade -type aircraft wings with symmetrical about the chord profile (max) , connecting vertical shaft rotation with working wings (also symmetric profile) extending parallel to the axis rotation. The plane of the wing span (max) is horizontal and lies in the plane of rotation . The main distinguishing feature: Darrieus wind turbine works not responding to the changing direction of the wind.

2. Description of the force computation methods

The purpose of this paper is to examine the theoretical 2D simulation of wind turbine carousel allows you to determine the optimum parameters of the turbine (see Fig. 1). We consider the development of a mathematical model to study the interaction of wind turbines carousel "Darrieus" with fixed air flow [1-4].

Under the action of the wind flow on the blades appear lift and drag forces. The lifting force acts on the blade positively spinning rotor. And the power of resistance - negative. For efficient

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operation of the unit value of these forces depend on the parameter Z, called "rapidity of the unit."

Parameter rapidity of the wind wheel is defined as follows

$$Z = \frac{W_*}{u_\infty}$$

where , u_∞ - flow rate , $W_* = \omega r_o$ - linear speed of the blade , ω - angular speed of rotation of the rotor , r_o - radius , the distance from the center of rotation of the shaft to the blade.

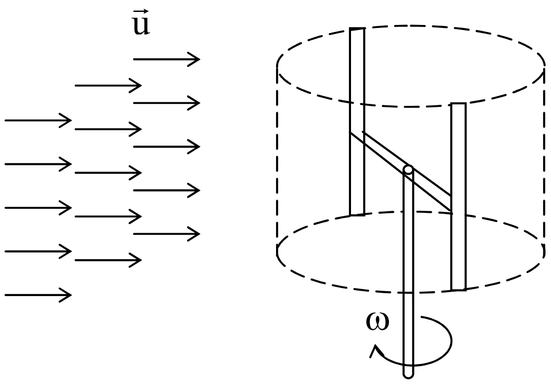

Fig. 1. *Schematic of the wind turbine carousel*

As the aerodynamic resistance forces wind flow projections are taken averaged resistance forces and lift the blade (in the opposite direction) on the axis Ox and Oy in the form of point sources R_x, R_y , and enter the right side of the equations of motion.

To determine the mass of these forces is first determined by the angle of attack α and attack speed $\vec{W}$, and the coefficients of lift and drag forces depends on the angle of attack (Fig. 2).

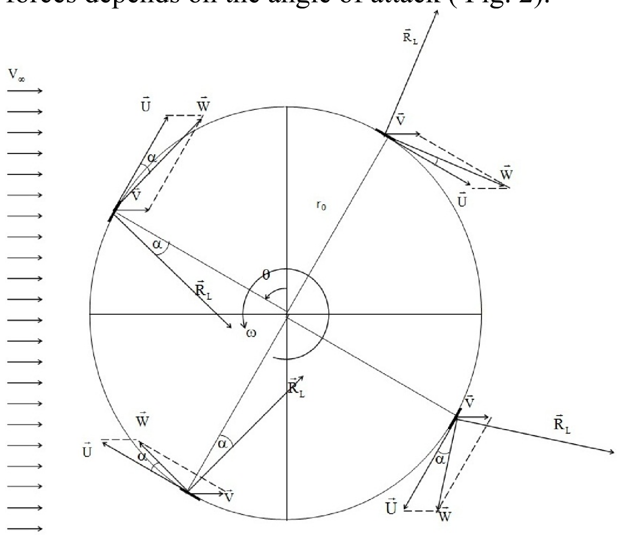

Fig. 2. *Rotating counterclockwise by a working wind turbine blades*

$$\left|\vec{W}\right| = \sqrt{\left(u\sin\theta - v\cos\theta\right)^2 + \left(r_0\omega + u\cos\theta + v\sin\theta\right)^2}, \qquad (1)$$

$$\mathrm{tg}\alpha = \frac{u\sin\theta - v\cos\theta}{r_0\omega + u\cos\theta + v\sin\theta}.$$

Introducing the local number of rapidity $Z = \frac{r_o\omega}{u_\infty}$, obtain,

$$\alpha = \mathrm{arctg}\left(\frac{u\sin\theta - v\cos\theta}{Z + u\cos\theta + v\sin\theta}\right) \qquad (2)$$

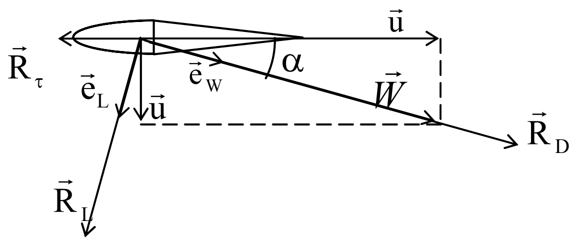

Fig. 3. *Schematic view of aerodynamic forces*

Determining the aerodynamic forces to introduce the unit vectors as shown in Fig.3: $\vec{e}_w$ - resistance unit vector directed along the relative velocity vector, and $\vec{e}_L$ - a unit vector directed perpendicular to lift it.

Elementary components of the aerodynamic forces:

a) lift pover of the profile:

$$\vec{R}_L = C_L(\alpha)\rho\frac{W^2}{2}h\vec{e}_L, \qquad (3)$$

where $C_L(\alpha)$ – lift coefficient, h - length of chord,

b) the resistance force

$$\vec{R}_D = C_D(\alpha)\rho\frac{W^2}{2}h\vec{e}_W, \qquad (4)$$

$C_D(\alpha)$ – coefficient of drag force. Then calculated - R_x, R_y

$$R_x = \text{пр}_{Ox}\left(\vec{R}_L\right) + \text{пр}_{Ox}\left(\vec{R}_D\right)$$
$$R_y = \text{пр}_{Oy}\left(\vec{R}_L\right) + \text{пр}_{Oy}\left(\vec{R}_D\right) \qquad (5)$$

A mathematical model of the impact of fixed air flow at a rate of "u" in the rotating wind turbine "Darrieus" is described by non-stationary Navier-Stokes and continuity equations and has the form:

$$\begin{cases}\frac{\partial\vec{V}}{\partial t}+(\vec{V}\nabla)\vec{V}=-\nabla p+\nu\nabla^2\vec{V}+\vec{R}, \\ \nabla\vec{V}=0.\end{cases} \quad (6)$$

The two-dimensional model of the task.

$$\begin{cases}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+R_x, \\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+R_y, \\ \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.\end{cases} \quad (7)$$

With the boundary conditions

$$\begin{aligned}&u(x,0)=u(x,1)=u(0,y)=u_\infty, \\ &\frac{\partial u(1,y)}{\partial x}=0; \\ &v(x,0)=v(x,1)=v(0,y)=0, \\ &\frac{\partial v(1,y)}{\partial x}=0.\end{aligned} \quad (8)$$

As the aerodynamic resistance forces wind flow projections are taken averaged resistance forces and lift the blade (in the opposite direction) on the axis Ox and Oy in the form of point sources R_x , R_y and enter the right side of the equations of motion.

The numerical calculation of the Navie-Stokes equations is made in terms of velocity - pressure [4-8]. For easy calculation of the pressure field is used the idea of " artificial compressibility ", which is physically correct description of the interaction of the air flow with a rotating turbine (air - compressed gas). The essence of the idea is to introduce the continuity equation in the form of an additional term

$$\frac{\partial}{\partial t}\left(p+\frac{V^2}{2}\right).$$

The result is a modified system of equations of the form

$$\begin{cases}\frac{\partial\vec{V}}{\partial t}+(\vec{V}\nabla)\vec{V}=-\nabla p+\nu\nabla^2\vec{V}+\vec{R}, \\ \frac{\partial(p+V^2/2)}{\partial t}+\nabla\vec{V}=0,\text{èëè} \\ \frac{\partial p}{\partial t}+\nabla\vec{V}=0,\end{cases} \quad (9)$$

for solutions that use different versions of the method of splitting. The basic idea of splitting is based on the method of particle-in-cell PIC. According to this scheme, first calculates the speed $\tilde{\tilde{V}}$ of the intermediate field equation

$$\frac{\tilde{\tilde{\vec{V}}}-\vec{V}^n}{\tau}+(\vec{V}^n\nabla)\vec{V}^n=\nu\nabla^2\vec{V}^n$$

(the value $\vec{V}^n$ is assumed known). Then, this field tinker with the pressure gradient

$$\vec{V}^{n+1}=\tilde{\tilde{\vec{V}}}-\tau\nabla p,$$

where p - stationary solution

$$\frac{\partial p}{\partial t}+\nabla\tilde{\tilde{\vec{V}}}=\tau\nabla^2 p.$$

As a result of these steps is satisfied both equations (6).

In this problem, use the explicit scheme of splitting by physical factors. We introduce the following notation:

$$\begin{aligned}&\nabla\times\vec{V}=\vec{\omega}, \\ &\nabla\vec{V}=D, \\ &\nabla\tilde{\tilde{\vec{V}}}=\tilde{D}.\end{aligned} \quad (10)$$

It is believed that at a time $t_n=n\tau$ (τ - time step, n - number of steps) are known, the velocity field $\vec{V}$ and pressure p. Then, detection circuit unknown functions in time $t_n=(n+1)\tau$ can be represented as a three-stage splitting scheme:

$$1):\begin{cases}\frac{\tilde{u}-u^n}{\tau}=-\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)- \\ \qquad -\nu\left(\frac{\partial^2 v}{\partial y\partial x}-\frac{\partial^2 u}{\partial y^2}\right)+R_x \\ \frac{\tilde{v}-v^n}{\tau}=-\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)- \\ \qquad -\nu\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right)+R_y\end{cases}; \quad (11)$$

$$2):\quad \nabla^2 p=-\frac{1}{\tau}\left(\frac{\partial\tilde{u}}{\partial x}+\frac{\partial\tilde{v}}{\partial y}\right); \quad (12)$$

Equation (12) is obtained by taking the divergence of both sides of (13) with the continuity equation (conditions solenoidality) $\operatorname{div}\vec{V}^{n+1}=0$

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$$3):\qquad \begin{cases} \dfrac{u^{n+1}-\widetilde{u}}{\tau} = -\dfrac{\partial p}{\partial x} \\ \dfrac{v^{n+1}-\widetilde{v}}{\tau} = -\dfrac{\partial p}{\partial y} \end{cases}. \qquad (13)$$

3. Results

The computational cycle is as follows:

1) a known at the initial time of the velocity field of (11) is an intermediate velocity field, thereby determining the right side of equation (12),

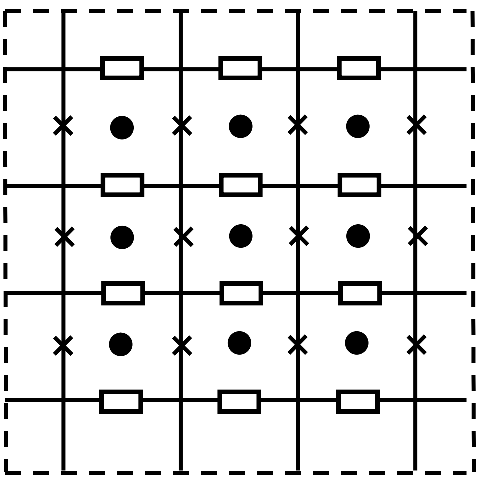
2) solved the Poisson equation (12) for the pressure;

3) corrects the final (at this time layer) the field rate (13). The cycle is repeated until the execution of a test set or a specific point in time.

It uses a "checkerboard" grid, ie coordinates of the grid functions are separated in space.

It uses the "chess" grid, ie staggered coordinates of grid functions in the space (see. Figure 4).

This enables clearly interpret each cell as an element of protection which is characterized calculated pressure at its center $p_{i,j}$ and divergence D (according to the sign of the value D represents the occurrence of the source or drain in a given volume). Knowledge of the normal component of the velocity vector on the boundary of the cell allows you to calculate the flux of momentum (momentum per unit mass) over that boundary.



▭ – $v_{i,j-1/2}, v_{i,j+1/2}$

× – $v_{i,j-1/2}, v_{i,j+1/2}$

• – $p_{i,j}$

Figure 4. The "chess" grid

For this problem in a Cartesian coordinate system and an irregular grid two-dimensional finite difference scheme is as follows:

$$\begin{aligned}
&\frac{\widetilde{u}_{i+1/2,j} - u^{n}{}_{i+1/2,j}}{\tau} = \\
&\frac{\left(u^n_{i,j}\right)^2 - \left(u^n_{i+1,j}\right)^2}{\left(x_{i+1}-x_i\right)} + \\
&+ \frac{(uv)^n_{i+1/2,j-1/2} - (uv)^n_{i+1/2,j+1/2}}{\left(y_{j+1/2}-y_{j-1/2}\right)} - \\
&- \frac{\nu}{\left(y_{j+1/2}-y_{j-1/2}\right)} \times \\
&\times \left[\left(\frac{v^n_{i+1,j+1/2} - v^n_{i,j+1/2}}{\left(x_{i+1}-x_i\right)} - \right.\right. \\
&\left. - \frac{u^n_{i+1/2,j+1} - u^n_{i+1/2,j}}{\left(y_{j+1}-y_j\right)}\right) - \\
&- \left(\frac{v^n_{i+1,j-1/2} - v^n_{i,j-1/2}}{\left(x_{i+1}-x_i\right)} - \right. \\
&\left.\left. - \frac{u^n_{i+1/2,j} - u^n_{i+1/2,j-1}}{\left(y_j - y_{j-1}\right)}\right)\right]; \qquad (14) \\
&\frac{\widetilde{v}_{i,j+1/2} - v^{n}{}_{i,j+1/2}}{\tau} = \\
&\frac{(uv)^n_{i-1/2,j+1/2} - (uv)^n_{i+1/2,j+1/2}}{\left(x_{i+1/2}-x_{i-1/2}\right)} + \\
&+ \frac{\left(v^n_{i,j}\right)^2 - \left(v^n_{i,j+1}\right)^2}{\left(y_{j+1}-y_j\right)} + \\
&+ \frac{\nu}{\left(x_{i+1/2}-x_{i-1/2}\right)} \times \\
&\times \left[\left(\frac{v^n_{i+1,j+1/2} - v^n_{i,j+1/2}}{\left(x_{i+1}-x_i\right)} - \right.\right. \\
&\left. - \frac{u^n_{i+1/2,j+1} - u^n_{i+1/2,j}}{\left(y_{j+1}-y_j\right)}\right) - \\
&- \left(\frac{v^n_{i,j+1/2} - v^n_{i-1,j+1/2}}{\left(x_i - x_{i-1}\right)} - \right. \\
&\left.\left. - \frac{u^n_{i-1/2,j+1} - u^n_{i-1/2,j}}{\left(y_{j+1}-y_j\right)}\right)\right];
\end{aligned}$$

$$\widetilde{D}_{i,j}=\frac{\widetilde{u}_{i,j}-\widetilde{u}_{i-1,j}}{(x_i-x_{i-1})}+\frac{\widetilde{v}_{i,j}-\widetilde{v}_{i,j-1}}{(y_j-y_{j-1})};$$

$$D^{n+1}_{i,j}=\frac{u^{n+1}_{i,j}-u^{n+1}_{i-1,j}}{(x_i-x_{i-1})}+\frac{v^{n+1}_{i,j}-v^{n+1}_{i,j-1}}{(y_j-y_{j-1})}.$$

$$\frac{p_{i+1,j}-p_{i,j}}{(x_{i+1}-x_i)^2}-\frac{p_{i,j}-p_{i-1,j}}{(x_i-x_{i-1})^2}+$$
$$+\frac{p_{i,j+1}-p_{i,j}}{(y_{j+1}-y_j)^2}-\frac{p_{i,j}-p_{i,j-1}}{(y_j-y_{j-1})^2}=\frac{\widetilde{D}_{i,j}}{\tau};$$
$$D^{n+1}_{i,j}=0.$$

$$u^{n+1}_{i+1/2,j}=\widetilde{u}_{i+1/2,j}-\frac{\tau}{(x_{i+1}-x_i)}(p_{i+1,j}-p_{i,j});$$

$$v^{n+1}_{i,j+1/2}=\widetilde{v}_{i,j+1/2}-\frac{\tau}{(y_{j+1}-y_j)}(p_{i,j+1}-p_{i,j}).$$

For numerical implementation of the scheme we will present values of velocities through i,j. For this purpose we replace $i+1/2\to i, j+1/2\to j, i-1/2\to$ $i-1, j-1/2\to j-1$. Formulas (14) will take the following form:

$$\frac{\widetilde{u}_{i,j}-u^n{}_{i,j}}{\tau}=$$
$$\frac{\left(\frac{u^n_{i,j}+u^n_{i-1,j}}{2}\right)^2-\left(\frac{u^n_{i,j}+u^n_{i+1,j}}{2}\right)^2}{(x_{i+1}-x_i)}+$$
$$+\frac{(uv)^n_{i,j-1}-(uv)^n_{i,j}}{\left(\frac{y_{j+1}-y_{j-1}}{2}\right)}-$$
$$-\frac{\nu}{\left(\frac{y_{j+1}-y_{j-1}}{2}\right)}\times$$
$$\times\left[\left(\frac{v^n_{i+1,j}-v^n_{i,j}}{(x_{i+1}-x_i)}-\frac{u^n_{i,j+1}-u^n_{i,j}}{(y_{j+1}-y_j)}\right)-\right.$$
$$\left.-\frac{v^n_{i+1,j-1}-v^n_{i,j-1}}{(x_{i+1}-x_i)}-\frac{u^n_{i,j}-u^n_{i,j-1}}{(y_j-y_{j-1})}\right];$$

$$\frac{\widetilde{v}_{i,j}-v^n{}_{i,j}}{\tau}=\frac{(uv)^n_{i-1,j}-(uv)^n_{i,j}}{\left(\frac{x_{i+1}-x_{i-1}}{2}\right)}+$$
$$+\frac{\left(\frac{v^n_{i,j}+v^n_{i,j-1}}{2}\right)^2-\left(\frac{v^n_{i,j}+v^n_{i,j+1}}{2}\right)^2}{(y_{j+1}-y_j)}+$$
$$+\frac{\nu}{\left(\frac{x_{i+1}-x_{i-1}}{2}\right)}\times$$
$$\times\left[\left(\frac{v^n_{i+1,j}-v^n_{i,j}}{(x_{i+1}-x_i)}-\frac{u^n_{i,j+1}-u^n_{i,j}}{(y_{j+1}-y_j)}\right)-\right.$$
$$\left.-\left(\frac{v^n_{i,j}-v^n_{i-1,j}}{(x_i-x_{i-1})}-\frac{u^n_{i-1,j+1}-u^n_{i-1,j}}{(y_{j+1}-y_j)}\right)\right];$$

$$\frac{p_{i+1,j}-p_{i,j}}{(x_{i+1}-x_i)^2}-\frac{p_{i,j}-p_{i-1,j}}{(x_i-x_{i-1})^2}+$$
$$+\frac{p_{i,j+1}-p_{i,j}}{(y_{j+1}-y_j)^2}-\frac{p_{i,j}-p_{i,j-1}}{(y_j-y_{j-1})^2}=\frac{\widetilde{D}_{i,j}}{\tau};$$
$$D^{n+1}_{i,j}=0.$$

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$$u_{i,j}^{n+1} = \tilde{u}_{i,j} - \frac{\tau}{\left(x_{i+1} - x_i\right)}\left(p_{i+1,j} - p_{i,j}\right);$$

$$v_{i,j}^{n+1} = \tilde{v}_{i,j} - \frac{\tau}{\left(y_{j+1} - y_j\right)}\left(p_{i,j+1} - p_{i,j}\right).$$

The results of the computation of this problem is shown graphically in Fig. 5-8. As can be seen from Figures 5-8, the greater the value of the angular velocity of the wind turbine, the stronger occurs a curvature of the stream line.

It well corresponds physics of the phenomenon of interaction of the wind turbine of rotary type with a stationary airflow.

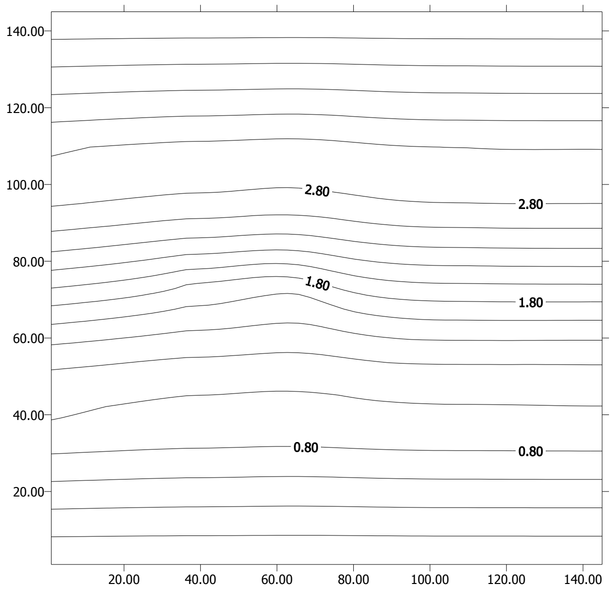

Fig. 5. Interaction of the wind turbine with the air flow (ω=3)

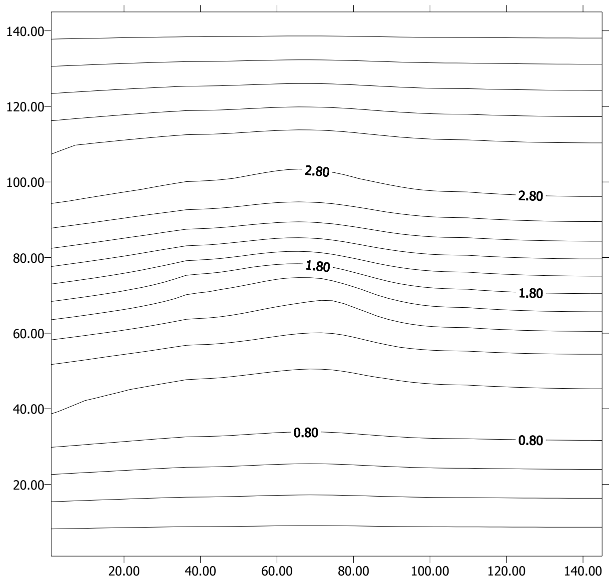

Fig. 6. Interaction of the wind turbine with the air flow (ω=4)

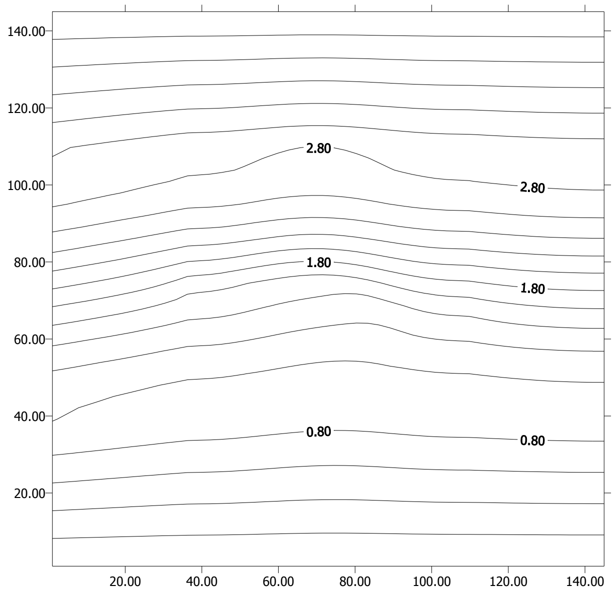

Fig. 7. Interaction of the wind turbine with the air flow (ω=5)

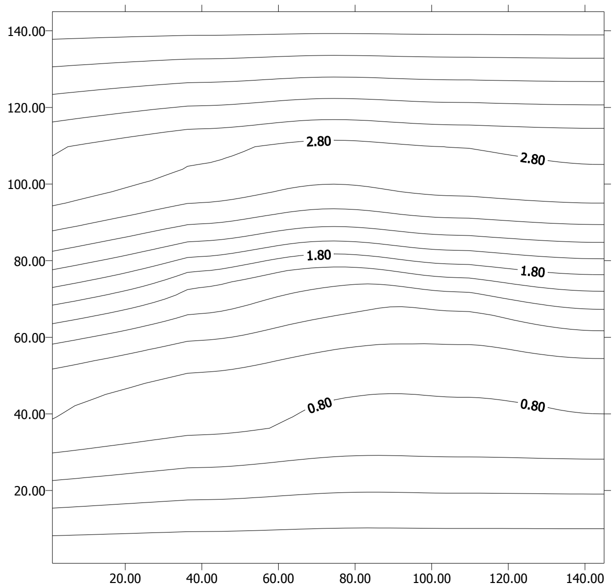

Fig. 8. Interaction of the wind turbine with the air flow (ω=6)

And in addition to numerical computations, developed this method of computation, the interaction of Darrieus wind turbine with a wind stream, and implemented a numerical research the flow around the individual elements of the wind turbine (airfoil blades and rotating working shaft) with the air flow.

The results of these computations are shown in Figures 9-11.

As can be seen from the figures 9-11, the flow pattern in the flow of these elements of the wind turbine, also in good agreement with the physics of the phenomenon.

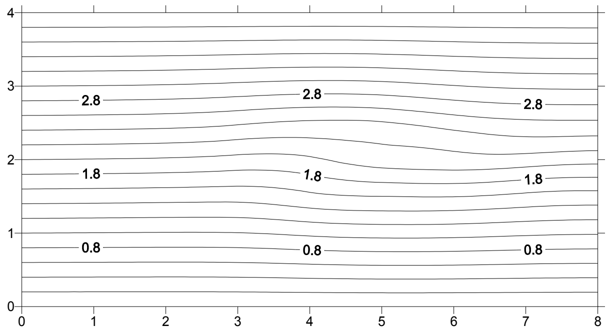

Fig. 9. Flow around thin airfoil NACA-0021 with angle 5 °

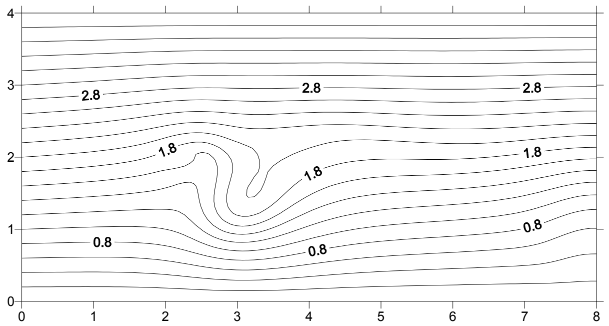

Fig. 10. Flow around thin airfoil NACA-0021 with angle 45 °

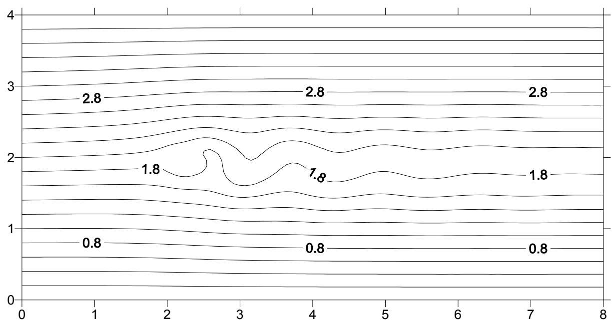

Fig. 11. Flow around a vertical rotating shaft

4. Conclusions

The paper considers two-dimensional mathematical model of steady operation of vertical axis Darrieus wind turbine, which is rotated by the action of the lift on the airfoil of working blades . The mathematical formulation of the problem and the method of computation of interaction of wind energy devices Darrieus with stationary air flow and received numerical results. According to the calculations, below the developed methodology to obtain reliable results, which is well-physics phenomena.

The results will be useful for engineers and designers in the design and manufacture of wind turbines with high technical and economic indicators.

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