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THE NONCOMMUTATIVE $H_p^{(r,s)}(\mathcal{A};\ell_\infty)$ AND $H_p(\mathcal{A};\ell_1)$ SPACES

Let $\mathcal{M}$ be a finite von Neumann algebra equipped with a normal faithful tracial state τ. Let $\mathcal{D}$ be a von Neumann subalgebra of $\mathcal{M}$, and let $\Phi : \mathcal{M} \to \mathcal{D}$ be the unique normal faithful conditional expectation such that $\tau \circ \Phi = \tau$. A finite subdiagonal algebra of $\mathcal{M}$ with respect to Φ is a w^*-closed subalgebra $\mathcal{A}$ of $\mathcal{M}$ satisfying the following conditions:

– $\mathcal{A} + J(\mathcal{A})$ is w^*-dense in $\mathcal{M}$;

– Φ is multiplicative on $\mathcal{A}$, i. e., $\Phi(ab) = \Phi(a)\Phi(b)$ for all $a, b \in \mathcal{A}$;

– $\mathcal{A} \cap J(\mathcal{A}) = \mathcal{D}$, where $J(\mathcal{A})$ is the family of all adjoint elements of the element of $\mathcal{A}$, i. e., $J(\mathcal{A}) = \{a^* : a \in \mathcal{A}\}$.

The algebra $\mathcal{D}$ is called the diagonal of $\mathcal{A}$. It's proved by Exel [1] that a finite subdiagonal algebra $\mathcal{A}$ is automatically maximal. Given $0 < p \leqslant \infty$ we denote by $L_p(\mathcal{M})$ the usual noncommutatve L_p-spaces associated with $(\mathcal{M}, \tau)$. Recall that $L_\infty(\mathcal{M}) = \mathcal{M}$, equipped with the operator norm (see [2]). The norm of $L_p(\mathcal{M})$ will be denoted by $\|\cdot\|_p$. For $p < \infty$ we define $H_p(\mathcal{A})$ to be closure of $\mathcal{A}$ in $L_p(\mathcal{M})$, and for $p = \infty$ we simply set $H_\infty(\mathcal{A}) = \mathcal{A}$ for convenience. These are so called Hardy spaces associated with $\mathcal{A}$. They are noncommutative extensions of the classical Hardy space on the torus $\mathbb{T}$. This noncommutative Hardy spaces have received a lot of attention since Arveson's pioneer work. For references see [6], [3], [4], [5], [9], [2].

The theory of vector-valued noncommutative L_p-spaces are introduced by Pisier in [10] for the case $\mathcal{M}$ is hyperfinite. Junge introduced these spaces for general setting in [11](see also [12]).

In this paper we define the spaces $H_p^{(r,s)}(\mathcal{A};\ell_\infty)$ and $H_p(\mathcal{A};\ell_1)$ by a similar way. First we proved that both two spaces are Banach spaces. Then shown basic properties of $H_p^{(r,s)}(\mathcal{A},\ell_\infty)$ and $H_p(\mathcal{A};\ell_1)$. For $0 < p, r, s \leqslant \infty$, we extended the conditional expectation Φ to a contractive projection from $H_p^{(r,s)}(\mathcal{A};\ell_\infty)$ onto $L_p^{(r,s)}(\mathcal{D};\ell_\infty)$ and from $H_p(\mathcal{A};\ell_1)$ onto $L_p(\mathcal{D};\ell_1)$, respectively. Next part is devoted to dual of $H_p(\mathcal{A};\ell_1)$, predual of $H_p(\mathcal{A};\ell_\infty)$. Then proved Interpolation properties of $H_p(\mathcal{A};\ell_1)$ and $H_p(\mathcal{A};\ell_\infty)$. We will use this ℓ_∞- and ℓ_1-valued version of the noncommutative Hardy space $H_p(\mathcal{A})$ to study noncommutative version of Hardy martingales, operator space analytic UMD property and operator space analytic convexity. For references see [14], [10], [15].

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