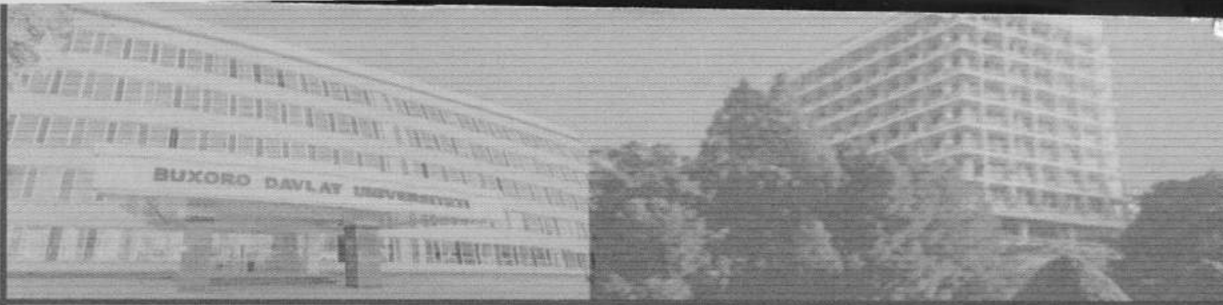

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БУХАРСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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Transactions of the conference

Volume №2

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Konferentsiya maqolalari

To'plam №2

АКТУАЛЬНЫЕ ПРОБЛЕМЫ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ - АЛЬ-ХОРЕЗМИ 2016

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МИНИСТЕРСТВО ВЫСШЕГО И СРЕДНЕГО СПЕЦИАЛЬНОГО ОБРАЗОВАНИЯ РЕСПУБЛИКИ УЗБЕКИСТАН

КОМИТЕТ ПО КООРДИНАЦИИ И РАЗВИТИЮ НАУКИ И ТЕХНОЛОГИЙ ПРИ КАБИНЕТЕ МИНИСТРОВ РУз

НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ УЗБЕКИСТАНА
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9-10 november, 2016

ТРУДЫ

МЕЖДУНАРОДНОЙ КОНФЕРЕНЦИИ "АКТУАЛЬНЫЕ ПРОБЛЕМЫ ПРИКЛАДНОЙ МАТЕМАТИКИ И ИНФОРМАЦИОННЫХ ТЕХНОЛОГИЙ – АЛЬ-ХОРЕЗМИ 2016"

9-10 ноября 2016 года

"Актуальные проблемы прикладной математики и информационных технологий – аль-Хорезми 2016": Труды международной конференции (9-10 ноября 2016 г.,Ташкент).

Главные редакторы академик А.С. Садуллаев, академик Ш.А. Аюпов. – Ташкент, НУУз им. Мирзо Улугбека, 2016 г. -444 с.

Сборник содержит труды участников международной конференции "Актуальные проблемы прикладной математики и информационных технологий – аль-Хорезми 2016" и предназначен для студентов, магистров, докторантов, преподавателей ВУЗов, специалистов в области математики, прикладной математики и информационных технологий.

СЕКЦИЯ №5. ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И ДИНАМИЧЕСКИЕ СИСТЕМЫ. НЕКОРРЕКТНЫЕ И ОБРАТНЫЕ ЗАДАЧИ.

где $C = const$.

Применяя метод регуляризации решением характеристического уравнения аналогично как в [
можем переписать (11) в виде:

$$\mathbb{L}\varphi_1 \equiv \varphi_1(t) - \mathbb{M}\varphi_1 = \frac{C}{\sqrt{t}},$$

где

$$\mathbb{M}\varphi_1 = \left(\frac{m}{2a\sqrt{\pi}} + \frac{ka}{\sqrt{\pi}}\right)\int_0^t \frac{1}{\sqrt{t-\tau}} e^{-\frac{m^2 t\tau}{a^2(t-\tau)}}\varphi_1(\tau)d\tau + \left(\frac{ka}{\sqrt{\pi}} - \frac{m}{2a\sqrt{\pi}}\right)\int_0^t \frac{1}{\sqrt{t-\tau}}\varphi_1(\tau)d\tau +$$

$$+\left(\frac{a}{2m}\right)^2\left(\frac{m}{2a\sqrt{\pi}} + \frac{ka}{\sqrt{\pi}}\right)\int_0^t \left\{\int_\xi^t \frac{1}{\tau^{3/2}\sqrt{t}\sqrt{\tau-\xi}} r\left[\left(\frac{a}{2m}\right)^2\left(\frac{1}{t} - \frac{1}{\tau}\right)\right] e^{-\frac{m^2\tau\xi}{a^2(\tau-\xi)}} d\tau\right\}\varphi_1(\xi)d\xi +$$

$$+\left(\frac{a}{2m}\right)^2\left(\frac{ka}{\sqrt{\pi}} - \frac{m}{2a\sqrt{\pi}}\right)\int_0^t \left\{\int_\xi^t \frac{1}{\tau^{3/2}\sqrt{t}\sqrt{\tau-\xi}} r\left[\left(\frac{a}{2m}\right)^2\left(\frac{1}{t} - \frac{1}{\tau}\right)\right] d\tau\right\}\varphi_1(\xi)d\xi.$$

Интегральное уравнение (12) является уравнением Вольтерра со слабой особенностью, поэтому
единственное решение

$$\varphi_1(t) = \mathbb{L}^{-1}\left[\frac{C}{\sqrt{t}}\right].$$

Используя соотношения (6) и (3) заключаем, что однородная задача (1)-(2) имеет нетривиа
решение с точностью до постоянного множителя в классе существенно ограниченных функций с
который определяется следующим образом:

$$(x + t^{1/2})^{-1}\, u(x,t) \in L_\infty(G), \text{ т.е. } u(x,t) \in L_\infty(G;\, (x + t^{1/2})^{-1}).$$

Таким образом справедливы:

Теорема 1. *Граничная задача (1)-(2) имеет наряду с тривиальным решением и нетривиа
решение $u(x,t) = C\,\tilde{u}(x,t)$, где $\tilde{u}(x,t) \in L_\infty(G;\, (x+\sqrt{t})^{-1})$, и $C = const$.*

Теорема 2. *В классе функций $L_\infty\left(G;\, [x^{1+\alpha} + t^{(1+\alpha)/2}]^{-1}\right)$ граничная задача (1)-(2) имеет т
тривиальное решение $u(x,t) \equiv 0$.*

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ESTIMATES OF SOLUTIONS OF FREE THERMAL CONVECTION PROBLEM

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Consider the container Ω, filled with incompressible viscous fluid with initial time temperature.

Let the container surface $\partial\Omega \equiv S$ set for $t > 0$ temperature distribution and inside $\Omega \subset R^3$ are
sources with data from specific heat flow. Observed that the temperature of the liquid in the containe
change. In the case of uneven liquid warmth density will change. This process is called free convection [1

1.Formulation of task and purpose of the work
This mathematical task will be solved as follows [1]. Search for vector $v\,(x,t)$ (speed motion of liquid) and
temperature $\omega\,(x,t)$ that satisfy system of equations:

$$\frac{\partial \vec{v}\,(x,t)}{\partial t} - v\Delta\vec{v} + \sum_{k=1}^{3} v_k \vec{v}_{x_k} + \vec{\zeta}\omega\,(x,t) = \vec{f} - grad\, p\,(x,t)\,,$$

$$\operatorname{div} \vec{v}(x,t) = 0, \tag{2}$$

$$\frac{\partial \omega (x,t)}{\partial t} - \Delta\omega + (\vec{v}\nabla\omega) = g(x,t), \tag{3}$$

boundary conditions

$$v(x,t)|_{x\in S} = 0,\ \omega(x,t)|_{x\in S} = 0,\ x\in S, \tag{4}$$

and the initial condition

$$\vec{v}(x,t)|_{t=0} = v_0(x),\ \omega(x,t)|_{t=0} = \omega_0(x). \tag{5}$$

In equation (1) vector $\vec{\zeta}$ (volumetric expansion) is an arbitrary specified vector, $\vec{f}(x,t)$, $g(x,t)$, $\vec{v}_0$, $\omega_0(x)$ - set smooth (smoothness will show below) features v - kinematic coefficient of viscosity.

Our goal is to prove a clear task (1)-(5) $C^{2+\alpha,\,1+\frac{\alpha}{2}}(Q_T)$, $0<\alpha<1$.

If we discard in equations (1), (3) nonlinear components, the task (1)-(5) splits into two separate linear tasks: the first initial-value problem for linearized non-stationary equation of Navier-Stokes and first initial-value problem for the thermal conductivity equation.

2. Linear tasks

For completeness, here are the specified tasks of hydrodynamics and thermal conductivity with the known in $C^{2+\alpha,\,1+\frac{\alpha}{2}}(Q_T)$ results: for initial-boundary problem of non- stationary Navier-Stokes system in a cylindrical field $Q_T = \Omega\times[0,\mathrm{T}]$:

$$\begin{gathered}\vec{v}_t - \eta\Delta\vec{v} + \nabla p(x,t) = \vec{f}(x,t),\\ \operatorname{div}\vec{v}(x,t) = 0\\ \vec{v}(x,t)|_{x\in S} = 0,\ \vec{v}(x,0) = \vec{v}_0(x).\end{gathered} \tag{6}$$

For this problem in [2, 5] Solonnikov V.A. proved the following

Theorem 1. Let $S\in C^{(2+\alpha)}$, $0<\alpha<1$. In all $\vec{f}(x,t)\in \overset{0\alpha}{J}(Q_T)$, $\vec{v}_0(x)\in C^{2+\alpha}(\overline{\Omega})$, satisfying the conditions of reconciliation

$$\begin{aligned}&\vec{v}_0(x)|_{x\in S} = 0,\\ &Pv\vec{v}_0(x) + \vec{f}(x,0)\Big|_{x\in S} = 0,\end{aligned} \tag{7}$$

where P - is orthogonal to the $L_2(\Omega)$ projector on $\overset{0}{J}(\Omega)$ there exists a unique solution of problem (6) for which the estimate $\vec{v}(x,t)\in C^{2+\alpha,1+\frac{\alpha}{2}}\left(\vec{Q}_T\right)$, $\nabla p(x,t)\in C^{\alpha,\frac{\alpha}{2}}(Q_T)$

$$|\vec{v}|_{Q_T}^{(2+\alpha)} + |\nabla p|_{Q_T}^{(\alpha)} \le C\left(\left|\vec{f}\right|_{Q_T}^{(\alpha)} + |\vec{v}|_{\Omega}^{(2+\alpha)}\right) \tag{8}$$

For the problem of the heat equation

$$\left(\frac{\partial}{\partial t} - \Delta\right)u(x,t) = g(x,t),\ u(x,t)|_{x\in S} = 0,\ u(x,t)|_{t=0} = u_0(x,t) \tag{9}$$

[3] has next.

Theorem 2. Let $S\in C^{(2)}$. If $g(x,t)\in C^{\alpha,\frac{\alpha}{2}}\left(\vec{Q}_T\right)$, $u_0\in C^{2+\alpha}(\Omega)$ and approval satisfies the following condition

$$\vec{u}_0(x) + \vec{f}(x,0)\Big|_{x\in S} = 0,$$

then (9) is uniquely solvable $C^{2+\alpha,\,1+\frac{\alpha}{2}}\left(\vec{Q}_T\right)$ and its solution satisfies the estimate

$$|u|_{Q_T}^{(2+\alpha)} \le C\left[|g|_{Q_T}^{(\alpha)} + |u_0|_{\Omega}^{(2+\alpha)}\right]. \tag{10}$$

3. Formulation of a nonlinear problem and its study

The system (1) - (5) in the form of a differential equation. Let $\vec{f}(x,t) \in \dot{J}(Q_T)$, P - orthogonal (in $L_2(\Omega)$) the projector on $\dot{J}(\Omega)$. We project both sides of equation (1) in the space $\dot{J}(\Omega)$, to exclude grad$p(x,t)$. Obtained from the equation (3) can be written as a single equation for the vector $U \equiv \begin{pmatrix} \vec{v} \\ \omega \end{pmatrix}$ with four components:

$$\frac{dU}{dt} + A_0U + K(U,U) = F$$

where $A_0U = \begin{pmatrix} vP\Delta\vec{v} - \vec{\xi}\omega \\ \Delta\omega \end{pmatrix}$, $F = \begin{pmatrix} \vec{f} \\ g \end{pmatrix}$,

$$K(U,U) = \begin{pmatrix} P\sum_{k=1}^{3} v_k \vec{v}_{x_k} \\ \vec{v}\nabla\omega \end{pmatrix}$$

. Equation (2) do not write, as we will always work in the space of solenoidal vectors $\vec{v}(x,t)$, t. E. Such that $\vec{v}(x) \in \dot{J}(\Omega)$.

The initial conditions (5) are convenient to reduce to an uniform, while the left side of (6) there will be new linear terms. Let $U' = \begin{pmatrix} \vec{v}' \\ \omega' \end{pmatrix}$, where $\vec{v}'$(x, t) and ω'(x, t) any function satisfying the conditions (4) and (5) (which can be constructed, for example, solving the problem (5) - (7) and (9)).

Then $U - U' = \vec{v} = \begin{pmatrix} \omega \\ \theta \end{pmatrix}$ we obtain the equation

$$\frac{dv}{dt} + A_0v + 2K(U',v) + K(v,v) = F_1 \quad (12)$$

where

$$K(\vec{v},U') = \frac{1}{2}\begin{pmatrix} P\sum_{k=1}^{3}\left(v_k\vec{v}_{x_k} + v'_k\vec{v}_{x_k}\right) \\ \vec{v}\mathrm{grad}\omega' + \mathrm{grad}\omega\vec{v}' \end{pmatrix},$$

$$F_1 = F - \frac{dU'}{dt} - A_0U' - K(U,U') = K(U',U')|_{t=0} - K(U,U') \equiv \begin{pmatrix} \vec{f}_1 \\ g_1 \end{pmatrix}.$$

In addition, the vector function $\vec{v}$(x, t) satisfies the zero initial condition

$$\vec{v}(x,t)|_{t=0} = 0 \quad (13)$$

and its components

$$\vec{\omega}(x,t)|_{x\in S} = 0,\ \vec{\theta}(x,t)\Big|_{x\in S} = 0 \quad (14)$$

Let $S \in C^{3+\alpha}$. For the existence of the solution of (12) - (14) in the class $C^{2+\alpha,1+\alpha/2}(Q_T)$ requires that the vectors $\vec{f}(x,t) \in j^{\alpha}(Q_T)$, $\vec{v}_0$(x) $\in C^{2+\alpha}\left(\bar{\Omega}\right)$ and the function $g(x,t) \in C^{\alpha,\alpha/2}(Q_T)$, satisfy the following conditions

$$\vec{v}_0(x)|_{x\in S} = 0,\ \omega(x)|_{x\in S} = 0 \quad (1$$

$$P\Delta\vec{v}_0 - P\sum_{k=1}^{3} v_{0k}\vec{v}_{0x_k} + \vec{\xi}(x,0)\,\omega_0(x)\Big|_{x\in S} - \vec{f}(x,0)\Big|_{x\in S} = 0,$$

$$\vec{v}_0(x)\nabla\omega_0(x) + g(x,0)|_{x\in S} = 0,$$

in other words, we assume that the vectors $\vec{f}(x,t)$, $\vec{v}_0(x)$ satisfy the conditions of Theorem $g(x,t)$, $\omega_0(x)$ - - Theorem 2.

We construct a vector $\vec{v}'(x,t) \in C^{2+\alpha,1+\alpha/2}(Q_T)$ and function ω(x, t) $\in C^{2+\alpha,1+\alpha/2}(Q_T)$. Then we are going to solve the (6) and (9) problems.

It is easy to check that all the conditions of Theorem 1 and 2 are made and the vector $\vec{v}'(x,t)$ $C^{2+\alpha,1+\alpha/2}(Q_T)$, function $\omega'(x,t) \in C^{2+\alpha,1+\alpha/2}(Q_T)$, $0 < \alpha < 1$.

In addition to vector $\vec{v}'(x,t)$ and function $\omega'(x,t)$ estimates

$$|\vec{v}'|_{Q_T}^{(2+\alpha)} \le c\left[\left(\vec{f}\Big|_{Q_T}^{(\alpha)} + |\vec{v}_0|_{\Omega}^{(2+\alpha)}\right) + \left(|\vec{v}_0|_{\Omega}^{(1+\alpha)}\right)^2 + \left(|\omega_0|^{(1+\alpha)}\right)^2\right], \quad ($$

$$|\omega'|_{Q_T}^{(2+\alpha)} \le C\left[|g|_{Q_T}^{(\alpha)} + |\omega_0|_{\Omega}^{(2+\alpha)} + |\vec{v}_0|_{\Omega}^{(1+\alpha)}\,|\omega_0|_{\Omega}^{(1+\alpha)}\right]. \tag{17}$$

The vector should be a solution of problem (12) - (14), in which

$$F_1 = F - \frac{du'}{dt} - A_0 u' - K(u', u') = K(u', u')|_{t=0} - K(u', u') \tag{18}$$

it belongs to the class and satisfies $\overset{0\alpha}{J}(Q_T) \times \overset{0\alpha}{J}(Q_T)$ and satisfies the condition

$$F_1(x,t)|_{t=0} = 0. \tag{19}$$

Consider the problem (14) - (15) and following the linearized problem

$$\frac{dV}{dt} + A_0 V + 2K(U', V) = F_1(x,t), \tag{20}$$

$$V(x,t)|_{t=0} = 0 \tag{21}$$

with the right side $F_1(x,t)$, in which the vector $F_1(x,t)$satisfies the condition (19).

Operators A_0 and $K(u, ...)$ define on the set $D^{2+\alpha}(Q_T)$ solenoidal vectors $C^{2+\alpha,1+\alpha/2}(Q_T) \times C^{2+\alpha,1+\alpha/2}(Q_T)$, satisfying the zero boundary conditions (16) and the initial conditions

$$u(x,t)|_{t=0} = 0,\ u_t(x,t)|_{t=0} = 0.$$

To study the problem (20) - (21) we need to get some estimates. We have the following

Lemma: If $\vec{v}, \omega \in C^{1+\alpha,1+\alpha/2}(Q_T)$ $\operatorname{div}\vec{v} = 0$, then

$$|K(\vec{v},\vec{\omega})|_{Q_T}^{(\alpha)} \le C\left(\sup_{\bar{Q}_T}|\vec{v}|\cdot|\omega|_{Q_T}^{(1+\alpha)} + \sup_{\bar{Q}_T}|\omega|\cdot|\vec{v}|_{Q_T}^{(1+\alpha)}\right) \tag{22}$$

This inequality is easily proved if we use well-known evaluation [5]: for any $\vec{v}(x,t) \in C^{1+\alpha,\frac{1+\alpha}{2}}Q_T$ is appropriate evaluations such as

$$|\vec{v}|_{Q_T}^{(\gamma)} \le C\left(|\vec{v}|^{(1+\alpha)}\right)^{\frac{\gamma}{1+\alpha}} \cdot \left(|\vec{v}|^{(1+\alpha)}\right)^{\frac{\gamma}{1+\alpha}} \left(\sup|\vec{v}(x,t)|\right)^{\frac{1+\alpha-\gamma}{1+\alpha}}, \gamma < 1+\alpha; \tag{23}$$

$\sum\limits_{i=1}^{3}\sup_{\bar{Q}_T}|\vec{v}_{x_i}| \le C\left(|\vec{v}|_{Q_T}^{(1+\alpha)}\right)^{\frac{1}{1+\alpha}}\left(\sup_{\bar{Q}_T}|\vec{v}|\right)^{\frac{\alpha}{1+\alpha}}$ and for $\vec{v}(x,t) \in C^{2+\alpha,1+\alpha/2}(Q_T)$

$$|\vec{v}|^{(1+\alpha)} \le C\left(|\vec{v}|_{Q_T}^{(2+\alpha)}\right)^{\frac{1+\alpha}{2+\alpha}}\left(\sup_{\bar{Q}_T}|\vec{v}|\right)^{\frac{\alpha}{2+\alpha}} \tag{24}$$

holds if we use the below given evaluations for the problem (21) - (22):

$$|K(u', V)|_{Q_T}^{(\alpha)} \le \varepsilon \sup_{Q_T}|u'|\,|V|_{Q_T}^{(2+\alpha)} + C_\varepsilon\left(|u'|_{Q_T}^{(1+\alpha)}\right)\cdot \sup_{Q_T}|V| \tag{25}$$

$$|K(U,U)|_{Q_T}^{(\alpha)} \le C\,\min(1,T)\left(|U|_{Q_T}^{(2+\alpha)}\right) \tag{26}$$

are right for $V(x,t)$, $\vec{v}(x,t) \in D_0^{2+\alpha}(Q_T)$, $\vec{u}(x,t) \in C^{1+\alpha,\frac{1+\alpha}{2}}(Q_T) \cap J^\alpha(Q_T)$

These estimates allow us to prove the following theorem.

Theorem 3. Let $S \in C^{(3+\alpha)}$, $u'(x,t) \in C^{2+\alpha,1+\frac{\alpha}{2}}(Q_T)$, $0 < \alpha < 1$. Then for any $G(x,t) \in \overset{0\alpha}{J}(Q_T) \times \overset{0\alpha}{J}(Q_T)$ problem (20)-(21) on the right part of $G(x,t)$ has a unique solution $V(x,t) \in D_0^{2+\alpha}(Q_T)$, for which we have the evaluation

$$|V|_{Q_T}^{(2+\alpha)} \le C(T)\,|F_1|_{Q_T}^{(\alpha)}. \tag{27}$$

Proof. In case $u' = 0$ is proved the theorem follows from the estimate (25) - (26).

We solve the problem (20) - (21) the method of successive approximations, shallow $V^{(0)} = 0$, a $V^{(n+1)}, n \geq 0$ defining of tasks:

$$\frac{dV^{(n+1)}}{dt} + A_0 V^{(n+1)} = -2K\left(u', V^{(n)}\right) + F_1, \quad V^{(n+1)}\big|_{t=0} = 0 \tag{28}$$

The convergence of $V^{(n)}$ is proved as follows. For the difference of $Z^{(n+1)} = V^{(n+1)} - V^{(n)}$ we can obtain the problem

$$\frac{dZ^{(n+1)}}{dt} + A_0 Z^{(n+1)} = -2K\left(u', Z^{(n)}\right), \quad Z^{(n+1)}\big|_{t=0} = 0$$

for solution of inequality (25) and evaluation (22) of Lemma is equal to inequality

$$\left|Z^{n+1}\right|^{(2+\alpha)}_{Q_T} \leq C\left|K\left(u', Z^{(n)}\right)\right|^{(\alpha)}_{Q_T} \leq C(T)\left|F\right|^{(\alpha)}_{Q_T}$$

and from this ratio, related to inequality we gain the

$$\left|Z^{n+1}\right|^{(2+\alpha)}_{Q_T} \leq \varepsilon\left|Z^n\right|^{(2+\alpha)}_{Q_T} + C_\varepsilon m\int_0^t \left|Z^n\right|^{(2+\alpha)}_{Q_T} d\tau, \; m > 0 \tag{25'}$$

We sum this inequality in n , then the $\xi_N(t)\left(\xi_N(t) = \sum_{n=1}^N \left|Z^{(n)}\right|^{(\alpha)}_{Q_T}\right)$ for the function we get the evaluation

$$\xi_{N+1}(t) \leq \frac{1}{1-\varepsilon}\xi_1(t) + \frac{C_\varepsilon m}{1-\varepsilon}\int_0^t \xi_{N+1}(\tau)\,d\tau,$$

where $0 < \varepsilon < 1$. Consequently, the formula $\xi_N(t)$ is limited on any interval $[0, T]$ a constant, independent of N; series of $\sum_{n=1}^N \left|Z^{(n)}\right|^{(\alpha)}_{Q_T}$, and the sequence of V^n converges.

Just from the estimates (8) and (12) (See Theorems 1 and 2) that any solution of the problem (19) - (20) with the right side $F_1(x,t)$ satisfies the inequality

$$\left|V\right|^{(2+\alpha)}_{Q_t} \leq C\left[\left|F_1\right|^{(\alpha)}_{Q_t} + \varepsilon\left|V\right|^{(2+\alpha)}_{Q_t} + C_\varepsilon m\int_0^t \left|V\right|^{(2+\alpha)}_{Q_t} d\tau\right],$$

from which a sufficiently small ε is obtained (29); if $F_1(x,t) \equiv 0$ will also have a $V(x,t) = 0$ Thus, Theorem 3 is proved.

Theorem 4 (main). Let $S \in C^{(3+\alpha)}$. If inequality is done

$$C_1^2 C_2 \min(1,T)\left|F_1\right|^{(\alpha)}_{Q_T} \leq \frac{1}{4}, \tag{29}$$

where C_1 - Limited constant from (29), but C_2 - the type of inequality (28), then the problem (14) - (15) is uniquely solvable in the class at $D_0^{2+\alpha}(Q_T)$ and solutions for the evaluation

$$\left|\vec{v}\right|^{(2+\alpha)}_{Q_T} \leq \frac{2C_1\left|F_1\right|^{(\alpha)}_{Q_T}}{1+\sqrt{1-4C_1^2\min(1,T)\left|F_1\right|^{(\alpha)}_{Q_T}}}. \tag{30}$$

Proof. We solve the problem (14) - (15) by the method of successive approximations, shallow $\vec{v}^{(0)} = 0$, a $\vec{v}^{(n+1)}, n \geq 0$ defining of tasks

$$\frac{d\vec{v}^{(n+1)}}{dt} + A_0\vec{v}^{(n+1)} + K\left(u', \vec{v}^{n+1}\right) = F_1 - K\left(\vec{v}^{(n)}, \vec{v}^{(n)}\right), \; \vec{v}^{(n+1)}\Big|_{t=0} = 0, \; n = 0, 1, 2, ... \tag{31}$$

We will show the limitations of the rules $\left|v^{(n+1)}\right|^{(2+\alpha)}_{Q_T}$ by the conditions (31). From the inequalities such as (25), (26) and (27) comes,

$$\left|\vec{v}^{(n+1)}\right|_{Q_t}^{(2+\alpha)} \leq C_1 \left[|F_1|_{Q_t}^{(\alpha)} + C_2 \min(1,T) \left(\left|\vec{v}^{(n)}\right|_{Q_t}^{(2+\alpha)} \right)^2 \right] \tag{32}$$

As a result of the following statements: let $\alpha_n \geq 0$, $n = 0, 1, 2, ...$ satisfies the recurrence relation $\alpha_{n+1} \leq a\alpha_n^2 + b$. If $ab < \frac{1}{4}$, $\alpha_0 \leq \frac{2b}{1+\sqrt{1-4ab}}$, then $\alpha_0 \leq \frac{2b}{1+\sqrt{1-4ab}}$, $n = 0, 1, 2, ...$ Applying this assertion to the inequality, then $\left|v^{(n+1)}\right|_{Q_t}^{(2+\alpha)}$ estimated by the right side (30). Since $\left\{v^{(n)}\right\}$ is converged in $D^{2+\alpha}(Q_T)$ then the inequality (30) holds for the limit vector, which is considered the problem vector (12) - (13).

In order to prove the convergence of $v^{(n)}$, we note that $Z^{(n+1)} = \vec{v}^{(n+1)} - \vec{v}^{(n)}$ is a solution of

$$\begin{gathered} \frac{dZ^{(n+1)}}{dt} + A_0 Z^{(n+1)} + 2K\left(u', Z^{(n+1)}\right) = -2K\left(U^{(n)} + U^{(n-1)}, Z^{(n)}\right) \\ Z^{(n+1)}\Big|_{t=0} = 0 \end{gathered} \tag{33}$$

If $\left|U^{(n)}\right|_{Q_T}^{(2+\alpha)}$ is already estimated, we can use for evaluation of $K\left(U^{(n)} + U^{(n-1)}, Z^n\right)$ inequality (25), so when $t \leq T$, it estimates similar (22)

$$\left|Z^{(n+1)}\right|_{Q_t}^{(2+\alpha)} \leq \varepsilon \left|Z^{(n)}\right|_{Q_t}^{(2+\alpha)} + C_\varepsilon \left|Z^{(n)}\right|_{Q_t}^{(\alpha)},$$

We have already seen that it implies the convergence of $\sum_{n=1}^{N} \left|Z^{(n)}\right|_{Q_t}^{(2+\alpha)}$, i.e. sequence $U^{(n)}$. And now, we will show the solution of problem $D_0^{2+\alpha}(Q_T)$ is right. Let U and U' - two solutions to the problem of the space $D_0^{2+\alpha}(Q_T)$, then their difference is a solution of problem

$$\frac{dZ}{dt} + A_0 Z + 2K(u', Z) + K(U, Z) + K(U', Z) = 0,$$

$$Z|_{t=0} = 0.$$

Consequently, we have to solve this problem $\left|Z^{(n+1)}\right|_{Q_t}^{(2+\alpha)} \leq \varepsilon |Z|_{Q_t}^{(2+\alpha)} + C_\varepsilon \int_0^t |Z|^{(2+\alpha)} dt$ and $Z = 0$. Theorem 4 is proved.

Note. Condition (29) ensures the solution of problem (12) - (13) with arbitrary $F_1(x,t)$ in cylinder Q_t, height T which depends on the rate of $F_1(x,t)$ and increases indefinitely when this rate tends to zero. Also for any $T > 0$ there exists $M = M(T)$, that the problem (14) - (15) is solvable in $C^{2+\alpha, 1+\frac{\alpha}{2}}(Q_T)$ at any $F_1(x,t) \in J^\alpha(Q_T)$, which is less than the norm of $M(t)$.

Based on this observation we can conclude that we have proved the classical solvability of the problem.

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В данной работе при составлении расчетной схемы с позиции теории оболочек, цистерны рассматриваются как составная конструкция, состоящая из цилиндрической оболочкой, связанной с сферической оболочкой (днище) и подкрепленной шпангоутом. Поперечные сечения шпангоутов рассматриваются по схеме кругового кольца, а стрингеры по схеме стержня.

Для моделирования процессов деформирования несущих элементов оболочечных конструкций- котла цистерны при статических и динамических нагружениях используются вариационные принципы Лагранжа и Гамильтона-Остроградского.

Разработка программных средств составляет одну из составных частей автоматизированной системы расчета оболочечных конструкций котла-цистерны на прочность и колебания. Так как точность расчетов и надежность проектных решений конкретных классов задач непосредственно зависят от адекватности выбираемых математических моделей и методов решения сформулированных краевых задач.

На основе вариационного принципа Гамильтона-Остроградского рассмотрены уточненной расчетной модели деформирования котла цистерны с учетом теории оболочек. Получена система дифференциальных уравнений для сферической и цилиндрической оболочек с граничными и начальными условиями для решения краевых задач котла цистерны.

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