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PHYSICO-TECHNICAL FACULTY

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NUMERICAL METHODS IN THERMAL PHYSICS

TEXTBOOK FOR STUDENTS

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The textbook is devoted to the application of numerical methods in Thermal Physics, it describes the methods of obtaining of different finite-difference schemes used for the numerical solution of differential equations describing hydroaero-dynamic and thermal processes and phenomena.

The textbook is mainly intended for the undergraduate students of thermal physics department of the technical physics faculty, but it can be also useful for the students of other departments, post-graduate and PhD students.

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