

МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РЕСПУБЛИКИ КАЗАХСТАН  
КОМИТЕТ НАУКИ

ИНСТИТУТ МАТЕМАТИКИ И МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ  
КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ ИМЕНИ А.ЛЪ-ФАРАБИ

УНИВЕРСИТЕТ ИМЕНИ СУЛЕЙМАНА ДЕМИРЕЛЯ

**МЕЖДУНАРОДНАЯ НАУЧНАЯ КОНФЕРЕНЦИЯ**

**«АЛГЕБРА, АНАЛИЗ, ДИФФЕРЕНЦИАЛЬНЫЕ  
УРАВНЕНИЯ И ИХ ПРИЛОЖЕНИЯ»**

*посвящается 60-летию академика НАН РК Аскара Серкүлсөвичи Держумадильдыса*  
Алматы, 8–9 апреля 2016 года

**ТЕЗИСЫ ДОКЛАДОВ**

Алматы – 2016

Modelling the thermal processes in the electric arc of high-current breaking device, process electric contact devices in the related fields of constructing the plasmatrons leads to the study of the boundary value problems for the heat equation in non-cylindrical domains [2, 3].

In work [4] the exact solutions to the boundary value problems of non-stationary heat conduction in the degenerating domain with the uniformly moving boundary are constructed, classes of uniqueness for solutions to the given problems are established.

The special interest is the case of the degenerating domain when the boundary of the domain moves along the automodeling law. Research of the homogeneous boundary value problem and its reduction to a singular Volterra integral equation of the second kind and finding explicitly the eigenfunction to this equation in this domain determines the contents of this work.

#### 1. Statement of the problem

We consider the first boundary value problem of heat conduction in the degenerating domain (domain with moving boundary, the boundary of the domain is moving with variable velocity): In the domain

$$G = \{(x, t) : t > 0, 0 < x < \sqrt{t}\}$$

to find a solution to the heat equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \quad (1)$$

that satisfies the boundary conditions:

$$u(x, t)|_{x=0} = 0, \quad u(x, t)|_{x=\sqrt{t}} = 0. \quad (2)$$

#### 2. Reducing the problem to an integral equation

The solution of the problem (1) – (2) we look for as the sum of the heat potentials of the double layer [5].

It is known that any solution of heat equation (1) can be represented by this form [5, p. 476–480].

As a result we obtain an integral equation:

$$\varphi(t) - \int_0^t k\left(\frac{\tau}{t}\right) \frac{\varphi(\tau)}{t} d\tau = 0, \quad (3)$$

where

$$k\left(\frac{\tau}{t}\right) = \frac{1}{2a\sqrt{\pi}} \left\{ \frac{1 + \sqrt{\tau/t}}{(1 - \tau/t)^{3/2}} \exp\left(-\frac{(1 + \sqrt{\tau/t})^2}{4a^2(1 - \tau/t)}\right) + \frac{1 - \sqrt{\tau/t}}{(1 - \tau/t)^{3/2}} \exp\left(-\frac{(1 - \sqrt{\tau/t})^2}{4a^2(1 - \tau/t)}\right) \right\}.$$

#### 3. Solving the integral equation

It can be shown that, integral equation (3) has eigenfunction

$$\varphi(t) = t^{-1/2}. \quad (4)$$

Shall show later that equation (3) has no other eigenfunctions.

#### 4. Conclusion

Stated problem (1) – (2) is reduced to singular homogeneous Volterra integral equation of the second kind (3). Exact solution (4) to the obtained integral equation is found.

#### Литература

1. *Kharin S.N.* The analytical solution of the two-phase Stefan problem with boundary flux condition // Математический журнал. - 2014. - Т. 14. №1(51). - С. 55-76
2. *Клим Е.И.* Решение одного класса сингулярных интегральных уравнений с линейными интегралами // ДАН СССР. - 1957. - Т. 113. - С. 24-27
3. *Харин С.Н.* Тепловые процессы в электрических контактах и связанных сингулярных интегральных уравнений. Диссертация на соискание ученой степени к.ф.м.н. - Алма-Ата: ИММ акад. Наук КазССР, 1970. - 13 с.
4. *Амангайычева М.М., Жендигей М.Т., Костакова М.Т., Рамазанов М.И.* About Dirichlet boundary value problem for the heat equation in the infinite angular domain // Boundary Value Problems. - 2014. - 2014:213. - P. 1-21. doi:10.1186/s13661-014-0213-4
5. *Тулочнов А.Н., Самарский А.А.* Уравнения математической физики. - Изд. 4-е. - М.: Наука, 1966. - 724 с.

UDC 532.5

Kudaikulov A.A.

Al-Farabi Kazakh National University (Kazakhstan, Almaty)

e-mail: aziz.kudaikulov@gmail.com

#### Numerical simulation of two-phase flow in channel using volume-of-fluid method

**Introduction.** In this work we present a volume-of-fluid (VOF) method for the solution of the 2D incompressible Navier-Stokes equations with interfaces and surface effects [1-3]. We will more particularly investigate the accuracy of the numerical representation of the surface tension and of the associated pressure jump.

**Model.** We numerically solve the Navier-Stokes equations for 2D incompressible two-phase flow in the channel:

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot (2\mu E). \quad (1)$$

$$k = -\nabla \cdot \vec{n}, \quad (15)$$

$$- [2\vec{t} \cdot E \cdot \vec{n}]_S = \vec{t} \cdot \nabla_S \sigma, \quad (16)$$

where  $\sigma$  is the surface tension,  $k$  is the curvature,  $\vec{n}$  is the normal vector to the interface  $S$  and  $\vec{t}$  is the tangent vector to the interface  $S$ . In order to solve the equations (1, 3 and 8) we need set outlet boundary conditions too. The outflow conditions are not known a priori. But nevertheless, we need to prescribe suitable conditions to make the problem determinate. An analysis of outflow boundary conditions is given in the literature [4].

### References

1. *Grejar Tryggvason, Ruben Scardovelli, Stephane Zaleski* Direct Numerical Simulations Of Gas-Liquid Multiphase Flows. - New York: Cambridge University Press, 2011. - 337 P.
2. *James Edward Palkod Jr., Elbridge Gerry Puckett* Second-order accurate volume-of-fluid algorithms for tracking material interfaces // Journal of Computational Physics. - 2004. - V. 199, №2. - p. 465-502
3. *Denis Gueyffier, Jie Li, Ali Nadim, Ruben Scardovelli, Stephane Zaleski* Volume-of-Fluid Interface Tracking with Smoothed Surface Stress Methods for Three-Dimensional Flows // Journal of Computational Physics. - 1999. - V. 152, №2. - p. 423-456
4. *B. Christer, V. Johansson* Boundary Conditions for Open Boundaries for the Incompressible Navier-Stokes Equation // Journal of Computational Physics. - 1993. - V. 105, №2. - p. 233-251

UDC 519.8

Sarsenova Zh.N., Kulpeshov B.Sh.  
International Information Technology University (Kazakhstan, Almaty)  
e-mail: zhibeksarsenova@gmail.com

International Information Technology University, Institute of Mathematics and Mathematical Modeling (Kazakhstan, Almaty)  
e-mail: b.kulpeshov@itu.kz

### Using neural networks for development of a stock exchange robot

There is a framework for analysis exchange rates and construction of measures of competitiveness [1]. This framework is based in part on the model imperfect substitutes of Arrington (1969), and on its empirical implementation by McGuirk (1987) and Wickham (1987). It involves calculation of CPI-based real effective exchange rates (REERs) for almost 140 countries. Trade weights are constructed using data on trade flows in manufacturing and primary (non-oil) commodities. It also uses unit labor costs in manufacturing

$$E_i = \frac{1}{2}(\nabla \vec{u} + \nabla \vec{u}^T), \quad (2)$$

$$\nabla \cdot \vec{u} = 0, \quad (3)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0, \quad (4)$$

$$\rho = F \rho_1 + (1 - F) \rho_2, \quad (5)$$

$$\mu = F \mu_1 + (1 - F) \mu_2, \quad (6)$$

where  $F$  is the parameter that identify a given fluid  $i$  ( $i=1$  or  $2$ ) is present at a particular location  $\mathbf{x}$ :

$$F(x) = \begin{cases} 1, & \text{if } x \text{ is in fluid } i \\ 0, & \text{if } x \text{ is not in fluid } i \end{cases} \quad (7)$$

If we substitute the equation (5) into the equation (4), we have that:

$$\frac{\partial F}{\partial t} + \vec{u} \cdot \nabla F = 0. \quad (8)$$

In order to find the shape and location of the interface between the two phases, we use the volume-of-fluid method and advect this interface using equation (8). More information on how to find and construct the interface between the two phases is available in the literatures [1-3]. Equations (1, 3 and 8) numerically solved using the projection method on the staggered grid [1] and the following boundary conditions were used:

1) Inlet boundary condition:

$$u_{in} = U, \quad (9)$$

$$v_{in} = 0, \quad (10)$$

$$u_w = \lambda \frac{\partial u}{\partial \vec{n}}, \quad (11)$$

$$v_w = 0, \quad (12)$$

where  $\lambda \geq 0$  is the slip length and  $\vec{n}$  is the normal vector to the wall [1].

3) At the interface between the two phases -  $S$ :

$$[\vec{u}]_S = 0, \quad (13)$$

$$- [ -p + 2\mu \vec{n} \cdot E \cdot \vec{n} ]_S = \sigma k, \quad (14)$$

- Ергалиев М.Г.* Об одном свойстве решения задачи Дирихле для уравнения теплопроводности в вырождающейся неограниченной области ..... 212
- Жанабеков Ж.Ж., Жанабеков А.Ж., Нарбаева С.М.* О приближенном решении нелинейных задач теории фильтрации ..... 213
- Жилисбаева К.С., Тулекенова Д.Т., Утегенова Н.Д.* Моделирование движения космического аппарата с помощью формата TLE ..... 217
- Захарьянова Г.К.* Волновые процессы в орбитальной среде при действии импульсных источников ..... 219
- Касенов С.Е., Нурсеитов Д.Б., Нурсеитова А.Т.* Численное моделирование задач продолжения для уравнения Гельмгольца ..... 221
- Кенжебаев К.К., Сартабаев Ж.А.* Исследование периодического решения квазилинейной системы обыкновенных дифференциальных уравнений на основе одной матричной функции ..... 224
- Моисеева Е.С., Найманова А.Ж.* Модификация  $k - \omega$  модели турбулентности применительно к расчету сверхзвукового течения многокомпонентной газовой смеси ..... 227
- Оразов Е.Т.* Теоретико-игровое моделирование трансграничного водораздела ..... 229
- Орумбаева Н.Т.* О полупериодической краевой задаче для системы гиперболических уравнений ..... 233
- Отенов Н.О., Саидүллин А. А., Кучаев Т. А., Тулепбаев К. М.* Отличия алгоритмов схватки в казакша курес и борьбе Хагак ..... 235
- Шаган Н.Ш.* Математическое моделирование сверхзвукового течения в плоском канале с поперечным вдувом струи ..... 237
- Alexeyeva L.A., Abmetzhanova M.M.* Stationary boundary problems of oscillations of thermoelastic rod and their solutions ..... 240
- Aubakirov A.B.* 3D and 2D impedance operators for electromagnetic waves scattering problem ..... 244
- Baikonys A.* Mathematical Modeling in Information and Communication Technology ..... 244
- Kavokin A.A., Kulakhmetova A.T., Shpadi Yu.R.* Numerical approximation of boundary conditions in the electrical contacts problem ..... 244
- Костякова М.Т.* On an integral equation of the Dirichlet problem for the heat conduction equation in the degenerating domain ..... 243
- Kudaikulov A.A.* Numerical simulation of two-phase flow in channel using volume-of-fluid method ..... 245
- Narvanova Zh.N., Kulpeshov B.Sh.* Using neural networks for development of a stock exchange robot ..... 247
- Yamakhanova K.R., Bekova G.T., Kershilik A., Shegai Zh.* Exact solutions of the two-component Schrödinger-Maxwell-Bloch equation in  $(3+1)$ -dimensional ..... 250