МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РЕСПУБЛИКИ КАЗАХСТАН
КОМИТЕТ НАУКИ

ИНСТИТУТ МАТЕМАТИКИ И МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ

КАЗАХСКИЙ НАЦИОНАЛЬНЫЙ УНИВЕРСИТЕТ ИМЕНИ АЛЬ-ФАРАБИ

УНИВЕРСИТЕТ ИМЕНИ СУЛЕЙМАНА ДЕМИРЕЛЯ

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«АЛГЕБРА, АНАЛИЗ, ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И ИХ ПРИЛОЖЕНИЯ»

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Modelling the thermal processes in the electric arc of high-current breaking device, process electric contactdevices in the related fields of constructing the plasmatrons leads to the study of the boundary value problems for the heat equation in non-cylindrical domains [2, 3].

In work [4] the exact solutions to the boundary value problems of non-stationary heat conduction in the degenerating domain with the uniformly moving boundary are constructed, classes of uniqueness for solutions to the given problems are established.

The special interest is the case of the degenerating domain when the boundary of the domain moves along the automodeling law. Research of the homogeneous boundary value problem and its reduction to a singular Volterra integral equation of the second kind and finding explicitly the eigenfunction to this equation in this domain determines the contents of this work.

1. Statement of the problem

We consider the first boundary value problem of heat conduction in the degenerating domain (domain with moving boundary, the boundary of the domain is moving with variable velocity): In the domain

$$G = \left\{(x;\, t): \quad t > 0, \quad 0 < x < \sqrt{t}\right\}$$

to find a solution to the heat equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \tag{1}$$

that satisfies the boundary conditions:

$$u(x,\, t)|_{x=0} = 0, \quad u(x,\, t)|_{x=\sqrt{t}} = 0. \tag{2}$$

2. Reducing the problem to an integral equation

The solution of the problem (1) – (2) we look for as the sum of the heat potentials of the double layer [5].

It is known that any solution of heat equation (1) can be represented by this form [5, p. 476–480].

As a result we obtain an integral equation:

$$\varphi(t) - \int\limits_0^t k\left(\frac{\tau}{t}\right) \frac{\varphi(\tau)}{t}\, d\tau = 0, \tag{3}$$

where

$$k\left(\frac{\tau}{t}\right) = \frac{1}{2a\sqrt{\pi}} \left\{ \frac{1+\sqrt{\tau/t}}{(1-\tau/t)^{3/2}} \exp\left(-\frac{\left(1+\sqrt{\tau/t}\right)^2}{4a^2(1-\tau/t)} \right) + \right.$$
$$\left. + \frac{1-\sqrt{\tau/t}}{(1-\tau/t)^{3/2}} \exp\left(-\frac{\left(1-\sqrt{\tau/t}\right)^2}{4a^2(1-\tau/t)} \right) \right\}.$$

3. Solving the integral equation

It can be shown that, integral equation (3) has eigenfunction

$$\varphi(t) = t^{-1/2}. \tag{4}$$

Shall show later that equation (3) has no other eigenfunctions.

4. Conclusion

Stated problem (1) – (2) is reduced to singular homogeneous Volterra integral equation of the second kind (3). Exact solution (4) to the obtained integral equation is found.

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Numerical simulation of two-phase flow in channel using volume-of-fluid method

Introduction. In this work we present a volume-of-fluid (VOF) method for the solution of the 2D incompressible Navier-Stokes equations with interfaces and surface effects [1-3]. We will more particularly investigate the accuracy of the numerical representation of the surface tension and of the associated pressure jump.

Model. We numerically solve the Navier-Stokes equations for 2D incompressible two-phase flow in the channel:

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) = -\nabla p + \nabla \cdot (2\mu E), \tag{1}$$

$$E = \frac{1}{2}(\nabla\vec{u} + \nabla\vec{u}^T), \qquad (2)$$

$$\nabla \cdot \vec{u} = 0, \qquad (3)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{u}) = 0, \qquad (4)$$

$$\rho = F\rho_1 + (1-F)\rho_2, \qquad (5)$$

$$\mu = F\mu_1 + (1-F)\mu_2, \qquad (6)$$

where F is the parameter that identify a given fluid i (i=1 or 2) is present at a particular location **x**:

$$F(x) = \begin{cases} 1, & \text{if } x \text{ is in fluid } i \\ 0, & \text{if } x \text{ is not in fluid } i \end{cases} \qquad (7)$$

If we substitute the equation (5) into the equation (4), we have that:

$$\frac{\partial F}{\partial t} + \vec{u} \cdot \nabla F = 0. \qquad (8)$$

In order to find the shape and location of the interface between the two phases, we use the volume-of-fluid method and advect this interface using equation (8). More information on how to find and construct the interface between the two phases is available in the literatures [1-3]. Equations (1, 3 and 8) numerically solved using the projection method on the staggered grid [1] and the following boundary conditions were used:

1) Inlet boundary condition:

$$u_{in} = U, \qquad (9)$$

$$v_{in} = 0. \qquad (10)$$

2) At the walls of the channel:

$$u_w = \lambda\frac{\partial u}{\partial \vec{n}}, \qquad (11)$$

$$v_w = 0, \qquad (12)$$

where $\lambda \geq 0$ is the slip length and $\vec{n}$ is the normal vector to the wall [1].

3) At the interface between the two phases - S:

$$[\vec{u}]_S = 0, \qquad (13)$$

$$-[-p + 2\mu\vec{n} \cdot E \cdot \vec{n}]_S = \sigma k, \qquad (14)$$

$$k = -\nabla \cdot \vec{n}, \qquad (15)$$

$$-[2\mu\vec{t} \cdot E \cdot \vec{n}]_S = \vec{t} \cdot \nabla_S \sigma, \qquad (16)$$

where σ is the surface tension, k is the curvature, $\vec{n}$ is the normal vector to the interface S and $\vec{t}$ is the tangent vector to the interface S. In order to solve the equations (1, 3 and 8) we need set outlet boundary conditions too. The outflow conditions are not known a priori. But nevertheless, we need to prescribe suitable conditions to make the problem determinate. An analysis of outflow boundary conditions is given in the literature [4].

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Using neural networks for development of a stock exchange robot

There is a framework for analysis exchange rates and construction of measures of competitiveness [1]. This framework is based in part on the model imperfect substitutes of Armington (1969), and on its empirical implementation by McGuirk (1987) and Wickham (1987). It involves calculation of CPI-based real effective exchange rates (REERs) for almost 140 countries. Trade weights are constructed using data on trade flows in manufacturing and primary (non-oil) commodities. It also uses unit labor costs in manufacturing