Resonant Fluctuations of Nonlinear System with Nonlinear-Viscous Resistance

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In the paper resonant fluctuations of geometrically nonlinear systems with nonlinear-viscous resistance, which can be caused by final deflections of mechanical system from its unperturbed movement, and also by its high speed of movement or use of highly damping materials, accordingly, are investigated. It is known, that at studying of below resonance modes of movement of system the dissipative forces, as a rule, are not considered. However, in the resonant phenomena they play an essential role.

Resonant fluctuations at the basic frequency in nonlinear systems of the kind are considered:

$$\ddot{x} + K_1\dot{x} + K_2\dot{x}^2 + \alpha_1 x + \alpha_3 x^3 = F\cos\Omega t \quad (1)$$

Nonlinear systems of type (1) find wide application at modelling of movement of separate and also connected elements of constructions and machines. They model movement of mechanical systems both with one degree of freedom, and elastic systems with the distributed parameters, which have an infinite number of degrees of freedom. For the latter multidimensional equations of movement can be resulted by well-known methods of mechanics for deformable mediums (methods of separation of variables, for example, Bubnov-Galyorkin direct method) to the equations of type (1).

Considering resonance of system (1) on the basic frequency, it is possible to approximate its solution with a simple harmonic with frequency of fluctuations equal to frequency of perturbing force [1]:

$$x(t) = r_1\cos(\Omega\, t - \varphi_1), \quad (2)$$

using thus a method of harmonious balance for reception of amplitude-frequency characteristics and phase angle φ_1. It is known, that the difference of phases between proper fluctuations and external influence thus can render essential influence on change of amplitude and fluctuations frequency.

According to the method of harmonious balance, unknown factors of decomposition of the solution in Fourier series, in particular, the amplitude and phase of the harmonious solution (1), are received from equations by equating factors at identical harmonics in the left part of the equation (1) and perturbing force $F\cos\Omega t$:

$$\begin{aligned} &\left(-r_1\Omega^2+\alpha_1 r_1+0,75\alpha_3 r_1^3\right)\cos\phi_1+K_1 r_1\Omega\sin\phi_1=F, \\ &\left(-r_1\Omega^2+\alpha_1 r_1+0,75\alpha_3 r_1^3\right)\sin\phi_1-K_1 r_1\Omega\cos\phi_1=0. \end{aligned} \tag{3}$$

Amplitude-frequency characteristics (AFC) of the basic resonance of system (1) and phase angle φ_1 [2] are received:

$$r_1^2\left[(-\Omega^2+\alpha_1+0,75\alpha_3 r_1^2)^2+K_1^2\Omega^2\right]=F^2, \tag{4}$$

$$tg\phi_1=\frac{K_1\Omega}{\alpha_1+0,75\alpha_3 r_1^2-\Omega^2} \tag{5}$$

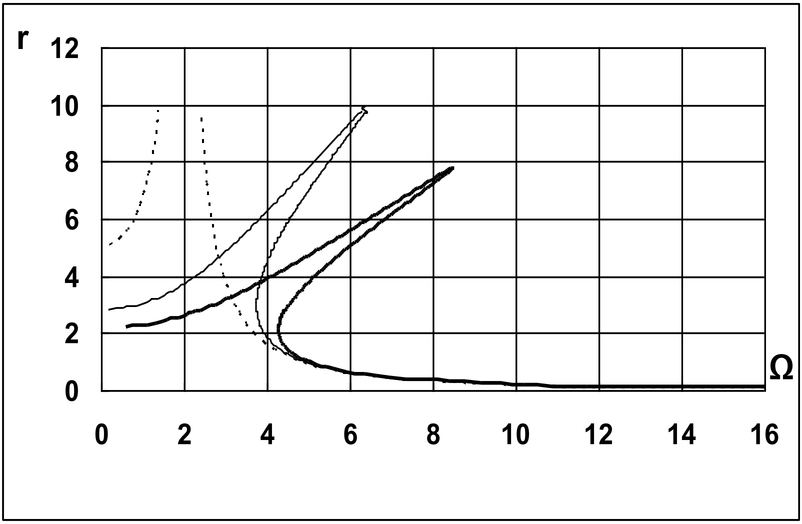
With a view of building a nonlinear system from undesirable resonant modes of movement influence of its parameters on the resonance at basic frequency is investigated and the numerical analysis of resonant curves is carried out. From formulas (4)-(5) it is obvious, that nonlinear-viscous resistance does not influence the resonance at the basic frequency. Its influence will affect the resonance at the maximum frequencies, which are multiple to the basic frequency of fluctuations equal to frequency of perturbing force Ω .

Influence of a linear component of resistance forces is investigated. The limiting case, when $K_1=0$ (fig. 1, dashed line) is considered. It is established, that the increase in resistance brings to reduction of amplitude of harmonious fluctuations.

Influence of components of the rigid characteristic of system (1) on the resonant phenomena in it is considered. It is established, that strengthening of the linear component characteristic does not influence amplitudes of resonant fluctuations, and leads to insignificant increase in resonant frequencies. Strengthening of rigidity characteristic, that is nonlinear component of restoring force (fig. 1), is accompanied by reduction of amplitude of resonant fluctuations and increase in resonant frequencies in comparison with a linear case (fig. 1, dashed line).

Influence of perturbing force on system resonance at the basic frequency is investigated. From the numerical analysis of system (1) it follows, that the increase in amplitude of perturbing force leads to increase in amplitude of resonant fluctuations of system.

From AFC analysis of systems with nonlinear-viscous resistance and rigid characteristic it follows, that the resonance in such a system at the basic frequency passes at bigger frequencies, than for a linear case. All the curves have an outstanding strong nonlinear character, shown in elongation of curves to the right. Ambiguous definition of amplitudes of fluctuations for the set frequencies at AFC is characterized by instability of resonance at the given frequency, which is proved by the borders of areas of instability of resonance.



- - - - at $\alpha_3 = 0;\ K_1 = 0,3;\ K_3 = 0,5;\ \alpha_1 = 4;\ F = 20$

____ at $\alpha_3 = 0,5;\ K_1 = 0,3;\ K_3 = 0,5;\ \alpha_1 = 4;\ F = 20$

____ at $\alpha_3 = 1,5;\ K_1 = 0,3;\ K_3 = 0,5;\ \alpha_1 = 4;\ F = 20$

Fig. 1 – Influence of the nonlinear component of rigid characteristic on resonance at the basic frequency

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