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Simulation of Movement of Drill Rods at Large Deformations

Lelya Khajiyeva[a], Almatbek Kydyrbekuly[b], Almaz Sergaliyev[c], Aliya Umbetkulova[d]

Al-Farabi Kazakh National University,al-Farabi avn.71, 050040 Almaty, Kazakhstan

[a]khadle@mail.ru, [b]almatbek@list.ru, [c]almaz.ss@gmail.com, [d]aliya1988kz@mail.ru

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Abstract. The purpose of the paper is modeling of nonlinear vibrations and stability of movement of boring columns at finite deformations. Movement of boring columns for shallow drilling (up to 500 m) applied in oil-gas extractive industry is considered.

Nonlinear models of movement of acompressed-torsioned drill rod within the nonlinear theory of finite deformations of V.V. Novozhilov are constructed. A method for its analysis and criterion of dynamic stability are offered. The numerical analysis of its elastic dislocations and instability zones of the basic resonance is carried out, which confirm the efficiency of the offered nonlinear dynamic model of rod elements and techniques for their calculation.

Introduction

The paper is dedicated to applied problems of dynamic stability of frameworks. We consider stability of movement of drill rods, which are used in petroleum industry. We know from drilling practice that up to 30 % of boreholes are rejected. Major factors for a borehole rejection are its curvature and drill rods breakage. Curvature intensity is determined by the action of many factors, which can be divided into three groups: geological, technological and technical.

Some researchers believe that geological conditions of rock bedding and distinction in their hardness and drilling area principal cause of borehole curvature.

Under another concept, a principal cause of borehole curvature is instability of the rod rectilinear form. This can be caused by various factors, such as dynamic cross influences; large inertial forces that arise at drilling; initial curvature of the rod; stress concentrators and other factors. Absence of a uniform model for a drill rod movement, which would describe a borehole curvature in view of all factors, draws attention of many researchers to the present problem.

The basic purpose of the present paper is simulation of stability of drill rods movement at various complicating factors, in view of finite strains in particular. They can arise under the action of large variable axial forces and twisting moments.

Stable movement of a drill rod will be understood as its movement in the absence of movement resonant modes, which are most dangerous for normal work of the drilling rig.

The solution for the present problem assumes definition of an optimum operating mode of a drill rod with maintenance of trouble-free operation of boring machines.

In connection with an object in view the following problems are solved in the paper:

1) Development of a nonlinear model of a drill rod movement in view of finite deformations.

2) Development of a technique for definition of instability zones of resonant vibrations of a drill rod with Floquet theory application.

3) Development of a design procedure for instability of resonant vibrations of a drill rod with a method of partial discretization.

4) The numerical analysis of stability of drill rod movement with the specified techniques application.

Nonlinear Model of Movement of a Drill Rod for Shallow Drilling

Movement of a drill rod for shallow drilling (up to 500 m), which is used in oil and gas industry (Fig. 1), is considered.

Linear models of drill rods movement are known in literature. One of their restrictions is the admission of deformation smallness.

In the present work deformations finiteness of a drill rod is allowed. It can be caused by changeability of axial forces N(t) and twisting moments M(t):

$$N(x,t) = N_0(x) + N_t(x)\Phi_N(t) \ , \tag{1}$$

$$M(x,t) = M_0(x) + M_t(x)\Phi_M(t) \, , \tag{2}$$

where $N_0(x)$ - longitudinal force caused by a construction body weight mgx and constant in time pressing force N_1:

$$N_0(x) \ = \ N_1 + \text{mgx}, \tag{3}$$

g – gravity acceleration, x - distance from the top end of a drill rod, $\Phi(t)$ - periodic function of time describing a loading mode. The elementary variant of function Φ(t) corresponds to harmonious influence

$$\Phi_N(t) = \cos\theta t \, . \tag{4}$$

Similarly for twisting moment M (t): $M_0(x)$ - nominal moment, constant in time; M_t defines contribution of a variable component; $\Phi_M(t)$ - periodic function.

The nonlinear model of a drill rod rotation in view of finite strains is constructed according to Novozhilov V.V. nonlinear theory of deformations, where components of deformations for the general three-dimensional case of deformation are defined as:

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial x}\right)^2\right] = e_{xx} + 0,5\left[e_{xx}^2 + \left(0,5e_{xy} + \omega_z\right)^2 + \left(0,5e_{xz} - \omega_y\right)^2\right],$$

$$\varepsilon_{xy} = \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} + \frac{\partial U}{\partial x}\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\frac{\partial V}{\partial y} + \frac{\partial W}{\partial x}\frac{\partial W}{\partial y} = e_{xy} + e_{xx}\left(0,5e_{xy} - \omega_z\right) + e_{yy}\left(0,5e_{xy} + \omega_z\right) + \tag{5}$$

$$+\left(0,5e_{xz} - \omega_y\right)\left(0,5e_{yz} + \omega_x\right).$$

For the second Novozhilov V.V. system of simplifications, when not only unit elongations e_{ii} and shears e_{ij}, but also corners of turn ω_i are small in comparison with a unity, in Eq.5 we neglect corners of turn in degrees above the second one:

$$\begin{aligned} \varepsilon_{xx} &\approx e_{xx} + \frac{1}{2}\left(\omega_y^2 + \omega_z^2\right), & \varepsilon_{xy} &\approx e_{xy} - \omega_x\omega_y, \\ \varepsilon_{yy} &\approx e_{yy} + \frac{1}{2}\left(\omega_x^2 + \omega_z^2\right), & \varepsilon_{xz} &\approx e_{xz} - \omega_x\omega_z, \\ \varepsilon_{zz} &\approx e_{zz} + \frac{1}{2}\left(\omega_x^2 + \omega_y^2\right), & \varepsilon_{yz} &\approx e_{yz} - \omega_y\omega_z. \end{aligned} \tag{6}$$

The strain-energy function of volumetric deformation [1] is received:

$$\begin{aligned} \Phi = G\Bigg[&\left(1 + \frac{\nu}{1-2\nu}\right)(\alpha + U_x\alpha_x + \alpha_x^2 + V_y\alpha_y + \alpha_y^2 + W_z\alpha_z + \alpha_z^2) + \\ &+ \frac{2\nu}{1-2\nu}((W_z + 0,5\alpha_z)(U_x + V_y + 0,5\alpha_x + 0,5\alpha_y) + (U_x + 0,5\alpha_x)\times(V_y + 0,5\alpha_y)) + \\ &+ \frac{1}{2}\left((U_y\beta_x + V_x\beta_y + W_xW_y\right)^2 + (V_z\beta_y + W_y\beta_z + U_yU_z)^2 + (W_x\beta_z + U_z\beta_x + V_xV_z)^2)\Bigg], \end{aligned} \tag{7}$$

where indexes at components of elastic transfer $U(x,y,z,t)$, $V(x,y,z,t)$ also $W(x,y,z,t)$ mean differentiation of these functions on the specified variables, and the following designations are entered:

$$\alpha = U_x^2 + V_y^2 + W_z^2; \quad \alpha_x = U_x^2 + V_x^2 + W_x^2; \quad \alpha_y = U_y^2 + V_y^2 + W_y^2; \tag{8}$$

$$\alpha_z = U_z^2 + V_z^2 + W_z^2; \quad \beta_x = 1 + U_x; \quad \beta_y = 1 + V_y; \quad \beta_z = 1 + W_z;$$

According to the deformations circuit (Fig. 2) and expression (Eq.8) a nonlinear dynamic model of a drill rod rotation is constructed:

$$EJ_V \frac{\partial^2}{\partial_{x^2}}\left[\frac{\partial^2 V}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial V}{\partial x}\right)^2\right)\right]+\frac{\partial^2}{\partial_{x^2}}\left[M(x,t)\frac{\partial U}{\partial \mathrm{x}}\right]+\frac{\partial}{\partial x}\left[\mathrm{N(x,t)}\frac{\partial \mathrm{V}}{\partial \mathrm{x}}\right]+K_1 V=-\frac{rF}{\mathrm{g}}\frac{\partial^2 V}{\partial t^2}$$

$$EJ_U \frac{\partial^2}{\partial_{x^2}}\left[\frac{\partial^2 U}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial U}{\partial x}\right)^2\right)\right]+\frac{\partial^2}{\partial_{x^2}}\left[M(x,t)\frac{\partial V}{\partial \mathrm{x}}\right]+\frac{\partial}{\partial x}\left[\mathrm{N(x,t)}\frac{\partial U}{\partial \mathrm{x}}\right]+K_1 U=-\frac{rF}{\mathrm{g}}\frac{\partial^2 U}{\partial t^2} \tag{9}$$

where $K_1 = \gamma F\,\omega^2/g$, U and V are motions of elastic line of a column in planes XOY and XOZ, accordingly. The boundary conditions in case of hinge leaning columns are set as equality to zero of motions and the bending moment on the ends:

$$V = EJ_V\frac{\partial^2 V}{\partial x^2}=0,\quad U = EJ_U\frac{\partial^2 U}{\partial x^2}=0,\qquad (x=0,\ x=l) \tag{10}$$

Simulation of Stability of a Drill Rod Movement

It has been noted above that steady movement of a drill rod will be understood as its movement in the absence of resonant modes of vibrations.

Let us consider a particular case of model, described by the Eq.9, - a case of a flat bend of the bar rotating at speed ω under the influence of variable longitudinal force $N(t)=N_0+N_t\Phi(t)$. We believe

$$U(x,t)=\sum_{k=1}^{\infty} f_k(t)\sin\frac{k\pi x}{l}$$

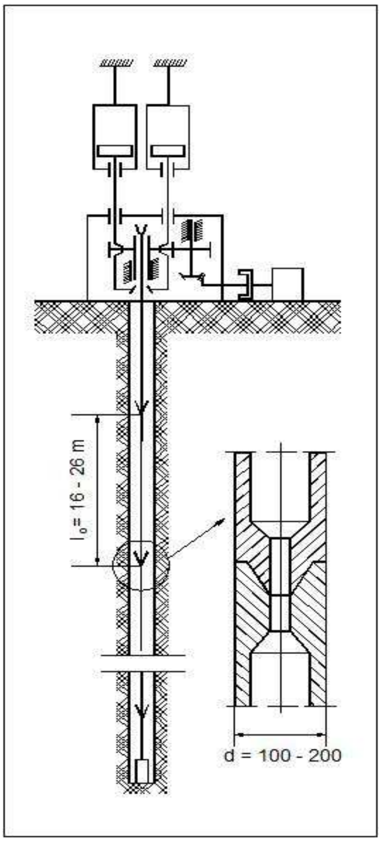


Fig.1 - The kinematic circuit of the boring machine

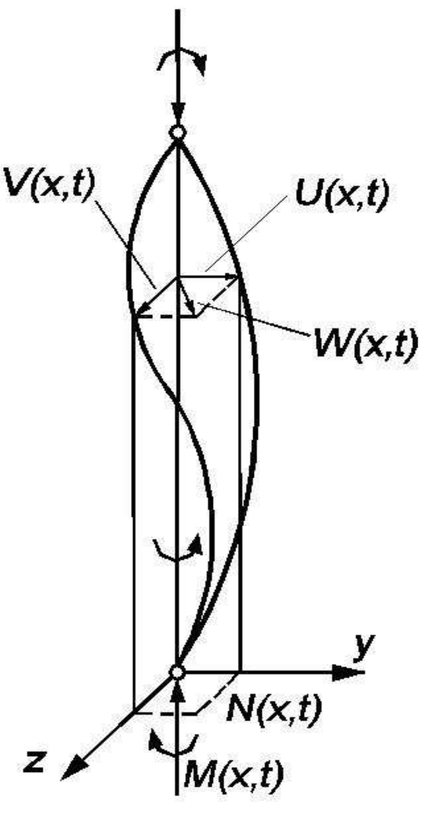


Fig.2 - The form of the curved axis of a rod

It satisfies boundary conditions, described by Eq.10. When applying a Bubnov-Galyorkin method of variables division, we have a nonlinear parametric equation:

$$\ddot{f} + C_k^2(1-2\nu\cos\Omega t)f + \alpha f^3 = 0, \quad (11)$$

where $C_k = \frac{k^2\pi^2}{l^2}\sqrt{\frac{EI}{m}\left(1-\frac{N_o}{N_k}\right)} = \omega_o$, $N_k = \frac{k^2\pi^2 EI}{l^2}$, $\nu_k = \frac{N_t}{2(N_k - N_0)}$, $\alpha = \frac{3Ek^4\pi^4}{8\rho l^4}$. (12)

Stability of the basic resonance is investigated:

$$f_0 = r_1\cos(\Omega t - \varphi_1). \quad (13)$$

A small increment $f = f_0 + \delta f$ is set. An equation of the perturbed state of Hill type is received:

$$\frac{d^2\delta f}{dt^2} + \delta f\left[C_k^2 + 1,5\alpha r_1^2 - 2C_k^2\nu\cos\Omega t + 1,5\alpha r_1^2\cos 2\varphi_1\cos 2\Omega t + 1,5\alpha r_1^2\sin 2\varphi_1\sin 2\Omega t\right] = 0. \quad (14)$$

Behavior character of the solution Eq.14 allows to judge stability or instability of the basic resonance by Lyapunov.

According to Floquet theory [2] we set solution Eq.14 as follows:

$$\delta f = e^{\mu t}P(t). \quad (15)$$

The type of expression $P(t)$ defines a fluctuations instability zone.

Having set $P(t) = b_1\cos(\Omega t - \psi_1)$, the border of the first instability zone of the basic resonance is received:

$$\Delta(\mu = 0) = \Omega^4 + \frac{27}{16}\alpha^2 r_4 + (3c_k^2\alpha - 3\Omega^2\alpha) - 2\Omega^2 c_k^2 + c_k^4 = 0. \quad (16)$$

Steel and dural dill rods are considered. The following is established:

1) The first and second forms of vibrations influence the instability zone width (Fig. 3). In case of the second form it is wider. Higher forms of vibrations do not influence their instability zones ($N_0 = 500\,N$, $N_t = 2195,5\ kN$, $d = 120mm$, $D = 200\,mm$, $L = 300\,m$ $E_{st} = 2,1\cdot 10^{11}\,Pa$).

2) In case of increase in length of a bar the instability zone (Fig. 4) increases ($N_0 = 500\,N$, $N_t = 2195,5\ kN$, $d = 120\,mm$, $D = 200\,mm$, $E_{st} = 2,1\cdot 10^{11}\,Pa$).

3) In all the considered cases instability zones for the dural bar ($E_{dur.} = 0,7\cdot 10^{11}\,Pa$, $\rho = 2700\,kg/m^3$) are bigger than for the steel $E_{st} = 2,1\cdot 10^{11}\,Pa$, $\rho = 7800\,kg/m^3$ on (Fig. 5).

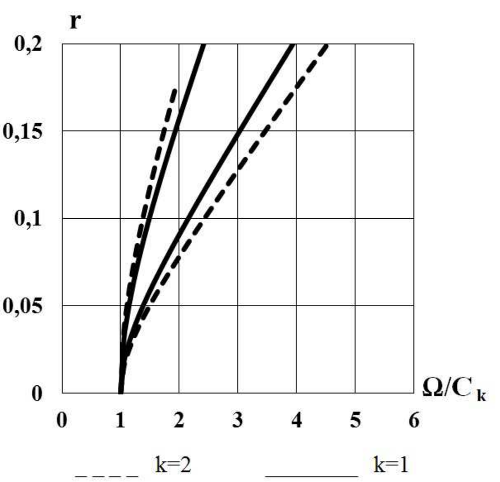


Fig.3 - Zones of instability of 1 and 2 forms vibrations

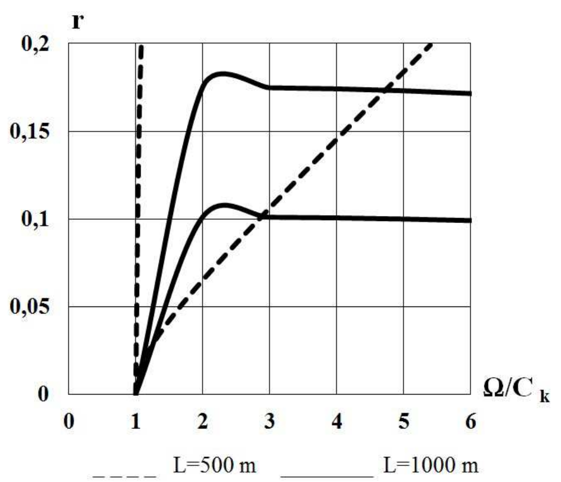


Fig.4 - Influence of length on to a zone of stability

Analysis of Stability of a Drill Rod Movement with a Method of Partial Discretization

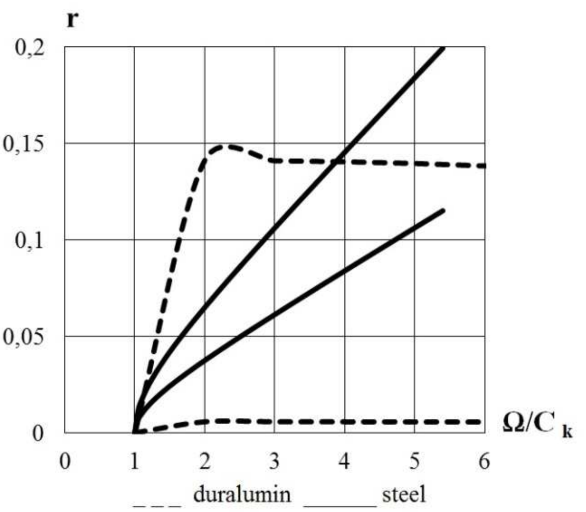
Another way for the behavior analysis the solution of Eq.14 is a method of its partial discretization [3]. It allows receiving of an analytical solution of the Hill type equation, which characterizes behavior of small indignation δf in time *t*. For this purpose the second component of Eq.14 is discrete in a class of generalized functions:

$$\frac{d^2\delta f}{dt^2}+\frac{1}{2}\sum_{i=1}^{n}(t_i+t_{i-1})[(C_k^2+1,5\alpha r_1^2-2C_k^2\nu\cos\Omega t_i+1,5\alpha r_1^2\cos 2\varphi_1\cos 2\Omega t_i+$$
$$+1,5\alpha r_1^2\sin 2\varphi_1\sin 2\Omega t_i)\times\delta f(t_i)\delta(t-t_i)-(C_k^2+1,5\alpha r_1^2-2C_k^2\nu\cos\Omega t_{i-1}+ \quad (17)$$
$$1,5\alpha r_1^2\cos 2\varphi_1\cos 2\Omega t_{i-1}+1,5\alpha r_1^2\sin 2\varphi_1\sin 2\Omega t_{i-1})\times\delta f(t_{i-1})\delta(t-t_{i-1})]=0 ,$$

where $i=1,n$ - the number of arguments t discrete; $\delta(t-t_k)$ - delta-function of Dirac.
The solution for the Eq.17 is not difficult.

Setting entry conditions: $\delta f(0)=\delta f_0$; $\delta\dot{f}(0)=\delta\dot{f}_0$, the solution Eq.17 is defined (Fig 6).

Setting *t* discretely, we receive a recurrent formula of calculation of unknown $\delta f(t)$ at the *k*-th step of partitioning of argument *t*. Analyzing behavior character $\delta f(t)$, it is possible to judge stability of the researched state. Reduction of size $\delta f(t)$, in due course *t* (exhaustive process) illustrates that $\delta f(t)\to 0$, i.e. the researched state stability. If the oscillating process is increasing, then the state is unstable.



$N_0=500\,N,\ N_t=2195,5\,kN,\ d=120\,mm,$
$L=500\,m,\ D=200\ mm,\ \Omega=0,5;\ r=0,05$

Fig.5 – Zones of instability of 1-st form of vibrations with a method of partial discretization

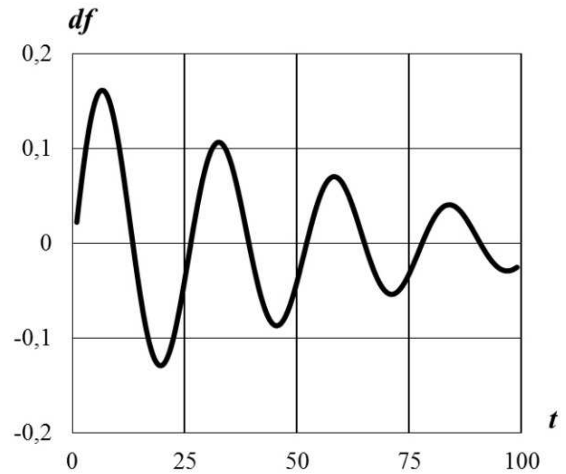


Fig.6 – Analysis of stability of a drill rod movement dural and steel chisel bars

Summary

In this paper we consider the nonlinear model of the drill rod's motion, which based on the theory of finite deformations by of V.V. Novozhilov. As a result of researches it is found that the elongation of the rod leads to increasing of the main resonance in the instability zone. Besides, the zone of instability dural drill rod is greater than for steel. Using well-known the Floquet method and proposed here method of a partial sampling are in good consistent with the obtained results. Proposed here the quasi-analytic method of the discretization partial is simple and can be applied in the research of nonlinear models.

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