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Transport Properties of Inertial Confinement Fusion Plasmas

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In this paper the transport properties of non-isothermal dense deuterium-tritium plasmas were studied. Based on the effective interaction potentials between particles, the Coulomb logarithm for a two-temperature non-isothermal dense plasma was obtained. These potentials take into consideration long-range multi-particle screening effects and short-range quantum-mechanical effects in two-temperature plasmas. Transport processes in such plasmas were studied using the Coulomb logarithm. The obtained results were compared with the theoretical works of other authors and with the results of molecular dynamics simulations.

1 Introduction

Recently, much attention has been paid to the studies of high-energy-density matter and, as a consequence, matter at high pressures and temperatures. Investigations in the field of inertial confinement fusion (ICF) on heavy ion beams take a special place among the works on various aspects of this problem. The reason for this special interest is reduction in resources of fossil fuels and the fact that nuclear fusion becomes one of the possible solutions of energy problems. Accelerators that can be used to fulfill this task operate in laboratories all over the world. They are well known as a major instrument in the experimental studies of nuclear physics, elementary particle physics and physics of dense plasmas [1-3]. Absence of new theoretical and experimental data on the transport properties of deuterium-tritium (DT) plasma requires an adequate qualitative description of the interaction of heavy ions with dense plasma in a wide range of parameters. Understanding and controlling the high-pressure behaviors of DT fuel are of crucial interest for the success of ignition experiments. Accurate knowledge of the transport coefficients in a dense DT plasma is important for correct description of processes occurring in ICF. This problem was the subject of many theoretical and experimental studies [4-7]. In [8] the transport properties of dense plasmas such as diffusion and viscosity were studied using orbital-free molecular dynamics (OFMD). Kress et al. [9] obtained viscosities and mutual diffusions of DT plasmas using both finite-temperature Kohn-Sham density-functional theory molecular dynamics (QMD) and OFMD.

One of the most promising approaches to the study of transport properties of dense DT plasma is a pair-collision approximation. To study the transport properties of dense DT plasma one can use two approaches. One of them is to calculate the transport coefficients determined on the basis of scattering cross sections [10, 11]. The second approach is base on the solution of the kinetic equations [12–14]. The collision integral in this equation contains a logarithmically divergent integral over impact parameters, which is replaced by the Coulomb logarithm.

In this paper the model previously proposed in [15–17] for the description of dense plasmas properties on the basis of effective interaction potentials [18, 19] is extended to compute ionic transport properties and thermal conductivity for deuterium and deuterium-tritium ICF plasmas. Below we present a brief description of the model and the results of calculation of plasma transport properties. To show the correctness of the model, its results are compared with the results of QMD and OFMD simulations.

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2 Coulomb logarithm on the basis of the effective potential

Transport properties are obtained on the basis of Coulomb logarithm using the effective potentials for ICF plasma. The Coulomb logarithm based on the effective interaction potential of particles is determined by the scattering angle of pair Coulomb collisions. Introducing the center of mass in the collision process, the Coulomb logarithm is written as [15, 20, 21]:

$$\lambda_{ei} = \frac{1}{b_\perp^2} \int_0^{b_{\max}} \sin^2\left(\frac{\theta_c}{2}\right) b\, db\,, \tag{1}$$

The center-of-mass scattering angle θ_c can be obtained from the formula [20]:

$$\theta_c = \pi - 2b \int_{r_0}^{\infty} \frac{dr}{r^2} \left(1 - \frac{\Phi_{\alpha\beta}(r)}{E_c} - \frac{b^2}{r^2}\right)^{1/2}, \tag{2}$$

here $E_c = \frac{1}{2} m_{\alpha\beta} v^2$ is the energy of the center of mass, $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of the particles of kinds α and β (ion or electron); $b_\perp = Z_\alpha Z_\beta e^2/(m_{\alpha\beta} v^2)$, $b_{\min} = \max b_\perp, \lambda\!\!\!^{-}_{\alpha\beta}$ describes the minimum impact parameter, where $\lambda\!\!\!^{-}_{\alpha\beta} = \hbar/\sqrt{2\pi m_{ab} k_B T}$ is the thermal de Broglie wave length.

The following dimensionless variables are used: the coupling parameter

$$\Gamma_{ee} = \frac{e^2}{a k_B T_e}, \qquad \Gamma_{ii} = \frac{Z_i^2 e^2}{a k_B T_i} \left(\frac{n_i}{n_e}\right)^{1/3}, \qquad \Gamma_{ei} = \frac{Z_i e^2}{a k_B T_{ei}}, \tag{3}$$

where *e* is the electron charge; the average distance between particles is equal to $a = (3/4\pi n_e)^{1/3}$, k_B is the Boltzmann constant. In formula (2) $\Phi_{\alpha\beta}(r)$ is the interaction potential and r_0 is the distance of the closest approach for a given impact parameter b:

$$1 - \frac{\Phi_{\alpha\beta}(r_0)}{E_c} - \frac{b^2}{r_0^2} = 0. \tag{4}$$

It is known that in order to correctly describe static and dynamic properties of plasmas the collective screening effect is to be taken into account. In this work the dense plasma, for which quantum effects at short distances must be taken into account is considered. Further, the effective interaction potential including both charge screening at large distances and quantum effects at short distances will be used [16, 17]:

$$\begin{aligned}\Phi_{\alpha\beta}(r) = {} & \frac{Z_\alpha Z_\beta}{r} \frac{1}{\gamma^2\sqrt{1-\left(2k_D/\lambda\!\!\!^{-}_{ee}\gamma^2\right)^2}} \left(\left(\frac{1/\lambda\!\!\!^{-2}_{ee} - B^2}{1 - B^2 \lambda\!\!\!^{-2}_{\alpha\beta}}\right)\right) \exp(-Br) - \left(\left(\frac{1/\lambda\!\!\!^{-2}_{ee} - A^2}{1 - A^2 \lambda\!\!\!^{-2}_{\alpha\beta}}\right)\right) \exp(-Ar) \\ & - \frac{Z_\alpha Z_\beta e^2}{r} \frac{(1-\delta_{\alpha\beta})}{1 + C_{\alpha\beta}} \exp\left(-r/\lambda\!\!\!^{-}_{\alpha\beta}\right),\end{aligned} \tag{5}$$

here

$$A^2 = \frac{\gamma^2}{2}\left(1 + \sqrt{1 - \left(\frac{2k_D}{\lambda\!\!\!^{-}_{ee}\gamma^2}\right)^2}\right), \; B^2 = \frac{\gamma^2}{2}\left(1 - \sqrt{1 - \left(\frac{2k_D}{\lambda\!\!\!^{-}_{ee}\gamma^2}\right)^2}\right), \; C_{\alpha\beta} = \frac{k_D^2 \lambda\!\!\!^{-2}_{\alpha\beta} - k_i^2 \lambda\!\!\!^{-2}_{ee}}{\lambda\!\!\!^{-2}_{ee}/\lambda\!\!\!^{-2}_{\alpha\beta} - 1},$$

where $2k_D/(\lambda\!\!\!^{-}_{ee}\gamma^2) < 1$, $k_D^2 = k_e^2 + k_i^2$ is the screening parameter, taking into account the contribution of electrons and ions, $\gamma^2 = k_i^2 + 1/\lambda\!\!\!^{-2}_{ee}$. In non-isothermal plasmas a characteristic electron-ion temperature T_{ei} appears [22, 23]. In [22] it was shown that for a correct description of plasma properties the electron-ion temperature may be expressed in the form: $T_{ei} = \sqrt{T_e T_i}$. These effective potentials can be used for nonisothermal and isothermal plasmas.

In the study of the transport properties of a dense two-temperature plasma an important role is played by the Coulomb logarithm. This paper presents the results of investigation of the transport properties of dense plasma on the basis of Coulomb logarithm using the effective potential (5).

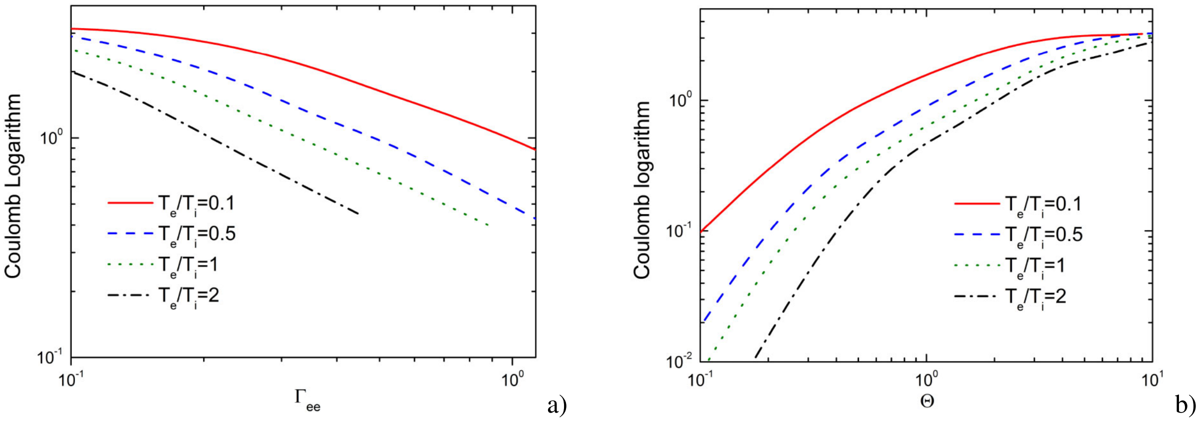

Fig. 1 The Coulomb logarithm of the two-temperature DT plasma as a function of (a) the coupling parameter (Γ) and (b) the degeneracy parameter (Θ).

Fig.1 shows the calculated values of the Coulomb logarithm as a function of the coupling parameter Γ and the degeneracy parameter $\Theta = k_B T/E_F$ (E_F is the Fermi energy) for different ratios of temperatures of electrons and ions in a dense two-temperature DT plasma. As weakly coupled plasma is considered, in Fig. 1a) the values of the Coulomb logarithm are shown for $\Gamma_{ee} < 1$ and $\Gamma_{ii} < 1$. The values of the Coulomb logarithm decrease with increase in the ratio $T_e < T_i$. At the given temperature of electrons T_e the lower value of the Coulomb logarithm at higher values of T_e/T_i is the result of stronger screening by the ion component of the plasma. Fig. 1b) shows that an increase in the degeneracy parameter Θ leads to the increase in the values of the Coulomb logarithm. At constant density we have lower values of the screening length at higher temperatures. This leads to higher values of the Coulomb logarithm.

3 Transport properties of dense ICF plasmas

Transport phenomena in a dense plasma are of significant interest in various fields of science and technology (plasma physics, ICF, physics of warm dense matter, etc.) [24, 25]. In particular, intensive studies of inertial confinement fusion require more reliable information about transport coefficients, i.e. coefficients of thermal conductivity, diffusion and viscosity. Let us consider the dense DT plasma particles interacting through the effective potential (5).

The diffusion coefficient, viscosity, and thermal conductivity of plasma are connected with the effective collision frequency by the equations:

$$D = \frac{k_B T}{m_e \nu_{eff}}, \tag{6}$$

$$\eta = \frac{5}{4}\sqrt{\frac{m}{\pi}}\frac{(k_B T)^{5/2}}{e^4 \lambda}, \tag{7}$$

$$k = \frac{5 n_e k_B^2 T}{m_e \nu_{eff}}, \tag{8}$$

where e is the electron charge, m_e is the mass of the electron, n is the density of plasma particles, and

$$\nu_{eff} = (4/3)\sqrt{2\pi}e^4\lambda/\sqrt{m_e}(k_B T)^{3/2} \tag{9}$$

is the effective collision frequency directly proportional to the Coulomb logarithm. The diffusion coefficient D and viscosity η are reduced to a dimensionless form: $D^* = D/\omega_p a^2$ and $\eta^* = \eta/n_i M \omega_p a^2$, $k^* = k/(m_e \omega_p/a)$, where $\omega_p = (4\pi\, n_i/M)^{1/2} Z e$ is the plasma frequency for ions mass M. For the DT mixture considered in this work we use $M = (2+3)/2 = 2.5\, amu$ [26].

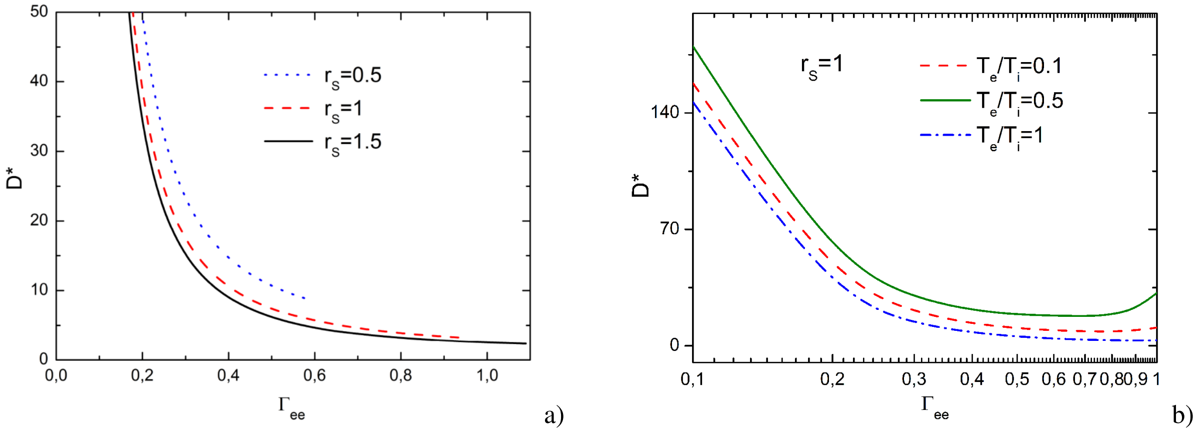

Fig. 2 The reduced diffusion coefficients as a function of the coupling parameter (Γ), $T_e = T_i$: (a) for different values of density parameter (r_S), (b) for different ratios of temperatures of electrons and ions at $r_S = 1$.

Figs. 2a), 2b) and 3a), 3b) show the results for the diffusion, thermal conductivity and viscosity coefficients of the dense plasma as a function of the coupling parameter at $r_S = 0.5$, $r_S = 1$ and $r_S = 1.5$. In order to get a more detailed physical description of the processes, the obtained numerical results are calculated using the effective interaction potential (5) taking into account both quantum and collective effects. From Figs. 2a) and 3a), 3b) one can see that at lower values of the coupling parameter the thermal conductivity, viscosity, and diffusion coefficients have higher values. For larger densities these transport coefficients have lower values.

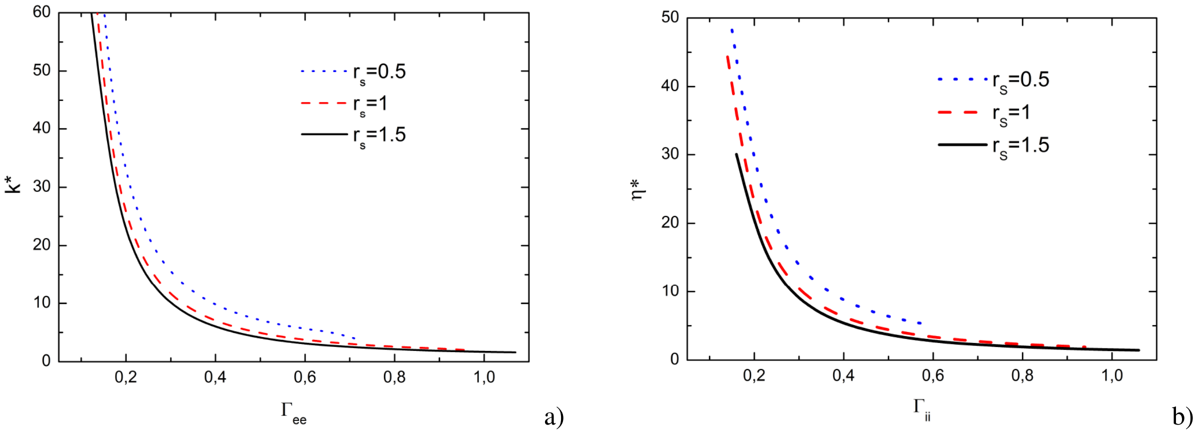

Fig. 3 The reduced thermal conductivity (a) and reduced viscosity (b) coefficients as a function of the coupling parameter (Γ) for different values of density parameter (r_S), $T_e = T_i$.

We have calculated diffusion, thermal conductivity and viscosity coefficients for different values of temperatures and densities on the basis of the Coulomb logarithm using the effective potential (5). Figs. 4a) and 4b) show the diffusion coefficient and viscosity for the DT dense plasma calculated by the Coulomb logarithm as a function of coupling parameter (Γ) at plasma density ρ=6.135 g/cm^3, ρ=13.45 g/cm^3, ρ=26.3 g/cm^3, ρ=108 g/cm^3, respectively. It is seen that the values of diffusion coefficient and viscosity increase as the temperature rises.

Figs. 5a) and 5b), show the dependence of thermal conductivity of deuterium plasma on temperature for different values of density ρ=43.105 g/cm^3 and ρ=199.561 g/cm^3. The red solid lines represent the thermal conductivity obtained using the effective interaction potential (5), while the black solid triangles represent the QMD results [27]. The blue dash-dotted lines represent the common Coulomb logarithm $\lambda = \ln \Lambda$. In [27] the QMD calculations of deuterium thermal conductivity were made for a wide range of densities and temperatures. Hu et al. [27] used the following function to descibe the results of the QMD calculations of deuterium thermal conductivity on ICF implosions:

$$k_{QMD} = \frac{20\,(2/\pi)^{3/2}\,k_B^{7/2}T^{5/2}}{\sqrt{m_e}Z_{eff}\,e^4}\,\frac{0.095\,(Z_{eff}+0.24)}{1+0.24\,Z_{eff}}\,\frac{1}{ln\Lambda_{QMD}}. \tag{10}$$

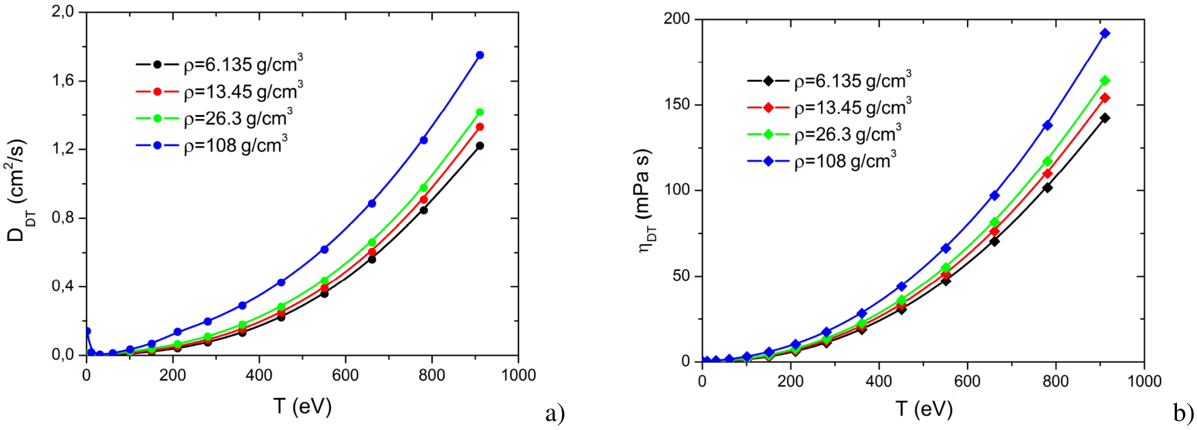

Fig. 4 The transport coefficients of dense DT plasma as a function of temperature for different values of density: (a) diffusion coefficients, (b) The viscosity.

From Figs. 5a) and 5b) is is seen that the thermal conductivity of deuterium plasma increases as temperature increases. Note that at high densities and relatively low temperatures the result obtained using the effective potential is close to the result obtained using quantum molecular dynamics.

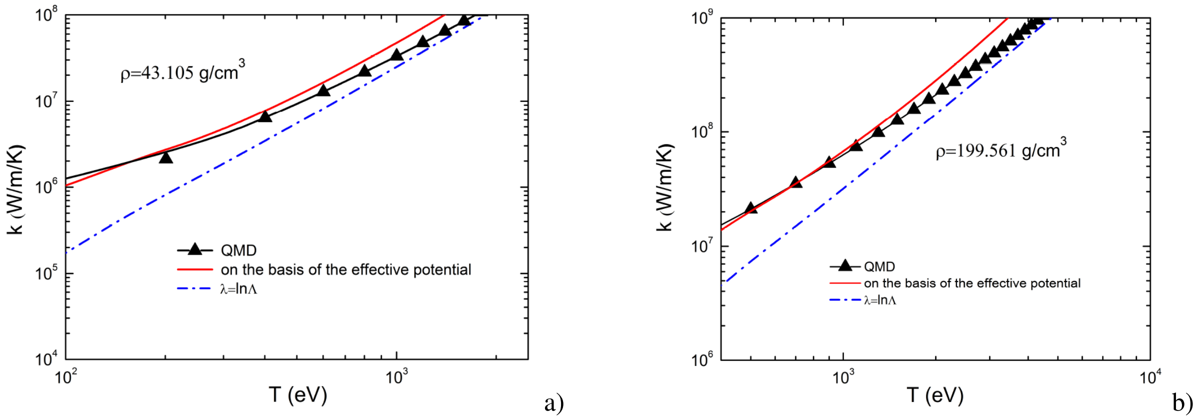

Fig. 5 The thermal conductivities of deuterium plasma for effective interaction potentials and QMD calculations as a function of temperature at ρ=43.105 g/cm^3 and ρ=199.561 g/cm^3.

Figs. 6a) and 6b) show the dependence of the diffusion and viscosity coefficients on the coupling parameter in comparison with the results obtained in hypernetted chain approximation (HNC) [28], by the method of molecular dynamics (MD) [28–30], in the framework of the kinetic theory [31, 32], and the theory of Landau-Spitzer [33]. Daligault-Baalrud theory is based on the binary scattering approximation with many-body correlations included through the use of an effective interaction potential [28–30,34]. Daligault-Baalrud's effective interaction potential was related to the potential of the mean force, which is the interaction potential between two particles when all surrounding particles are at fixed positions. Wallenborn-Baus applied the renormalized equilibrium kinetic theory, a general kinetic theory of phase-space correlation functions [32]. Figs. 6a) and 6b) show that the results obtained on the basis of effective potential (5) are in good agreement with the results of other works in the weakly coupled limit $\Gamma_{ee} << 1$, but this breaks down for $\Gamma_{ee} \geq 1$. The difference in the weakly-coupled case $\Gamma_{ee} \sim 1$ is caused by non-ideality and quantum effects.

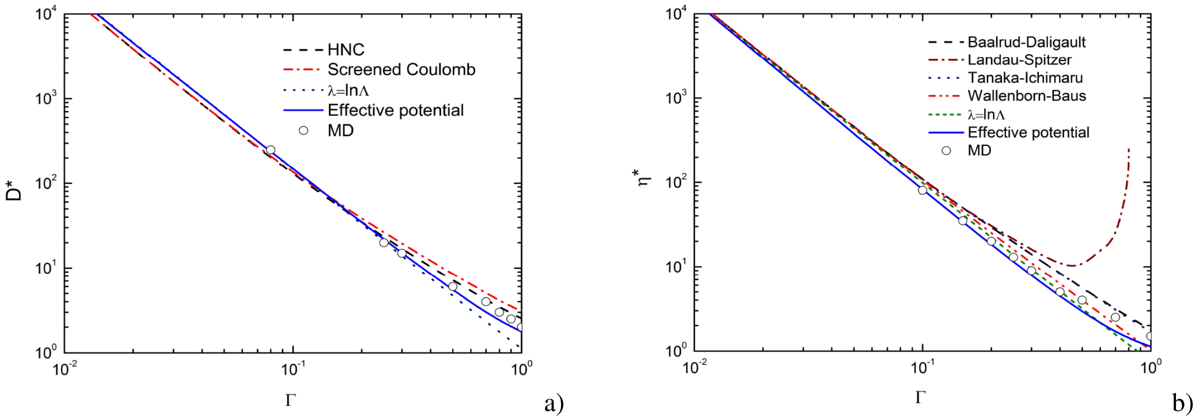

Fig. 6 The reduced diffusion (a) and viscosity (b) coefficients of dense plasma as a function of coupling parameter (Γ), $T_e = T_i$.

We calculated diffusion and viscosity of deuterium-tritium plasma for density ρ=5 g/cm^3 and temperatures ranging from 2 to 10 eV using the Coulomb Logarithm based on effective potentials taking into account quantum diffraction effects at short distances and screening at large distances. Figs. 7a) and 7b) show a comparison of the calculated data on diffusion and viscosity in a DT plasma with the theoretical results of other authors [9] such as finite-temperature Kohn-Sham density-functional theory molecular dynamics (QMD) and orbital-free molecular dynamics (OFMD). QMD simulations consider the electrons quantum mechanically through finite-temperature density-functional theory (FTDFT). OFMD simulations consider the kinetic energy of electrons semiclassically and therefore are able to reach higher temperatures. The obtained results are in good agreement with the results of QMD and OFMD simulations at higher temperatures, and therefore we conclude that our method can be used in this regime. At low temperatures below 3 eV the comparison with the QMD and OFMD results is not so good, because at these temperatures the effect of non-ideality becomes important. In comparison with the QMD results, our viscosities are not as good as for diffusion, where the temperature dependence is significantly different, whereas the results obtained for viscosities using the effective potentials agree with the results of OFMD simulations.

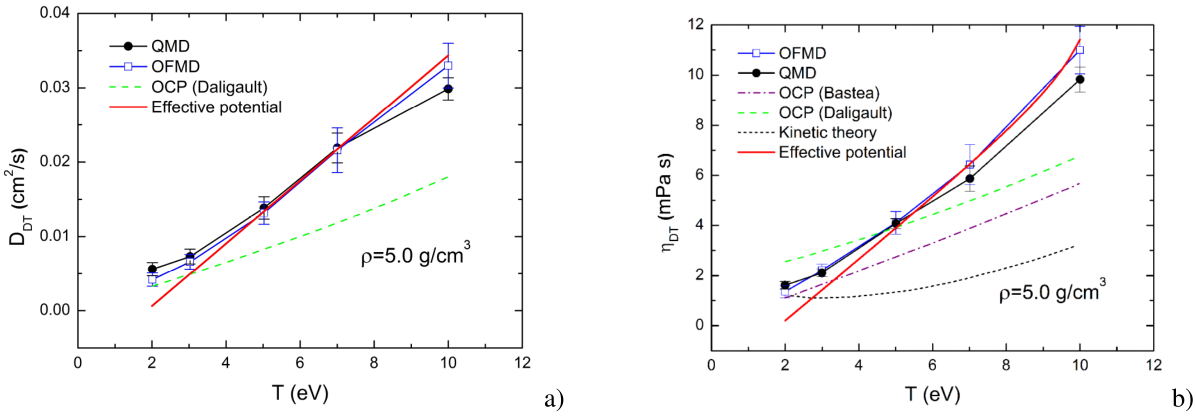

Fig. 7 The diffusion (a) and viscosity (b) coefficients for the DT plasma as function of temperature at density of ρ=5 g/cm^3.

4 Conclusion

The transport processes in dense DT plasmas were studied on the basis of two-temperature effective interaction potentials taking into account quantum diffraction effects at short distances and screening at large distances.

The results of application of the Coulomb logarithm and transport coefficients for different plasma parameters are consistent with the theoretical results of other authors, as well as with the results of the MD simulation. The obtained data show that the transport properties of dense plasma can be adequately expressed in terms of the Coulomb logarithm based on the effective potentials. Therefore, knowledge of the values of transport coefficients of heavy, charged particles in the plasma will help to more accurately calculate the design of a thermonuclear target.

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