

Trends in Mathematics
Research Perspectives

Vladimir V. Mityushev
Michael V. Ruzhansky
Editors

Current Trends in Analysis and Its Applications

Proceedings of the 9th ISAAC
Congress, Kraków 2013



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Geometrical Features of the Soliton Solution

Zhanat Zhunussova

Abstract It is well known, that integrable equations are solvable by the inverse scattering method (Ablowitz and Clarkson in *Solitons, Non-linear Evolution Equations and Inverse Scattering*, 1992). Investigating of the integrable spin equations in $(1+1)$, $(2+1)$ dimensions are topical both from the mathematical and physical points of view (Lakshmanan and Myrzakulov in *J. Math. Phys.* 39:3765–3771, 1998; Gardner et al. in *Phys. Rev. Lett.* 19(19):1095–1097, 1967). Integrable equations admit different kinds of physically interesting solutions as solitons, vortices, monopoles etc. We consider an integrable spin M-I equation (Myrzakulov and Vijayalakshmi in *Phys. Lett. A* 233:391–396, 1997). There is a corresponding Lax representation. And the equation allows an infinite number of integrals of motion. We construct a surface corresponding to soliton solution of the equation. Further, we investigate some geometrical features of the surface.

Keywords Surface · Soliton · Nonlinear equation

1 Introduction

We consider the connection between the surface and the soliton equation M-I which has the form [2],

$$\mathbf{S}_t = (\mathbf{S} \times \mathbf{S}_y + u\mathbf{S})_x, \quad (1.1)$$

$$u_x = -(\mathbf{S} \cdot (\mathbf{S}_x \times \mathbf{S}_y)), \quad (1.2)$$

where \mathbf{S} is spin vector, $S_1^2 + S_2^2 + S_3^2 = 1$, \times is vector product, u is a scalar function. We identify the spin vector \mathbf{S} and vector \mathbf{r}_x according to [2]

$$\mathbf{S} = \mathbf{r}_x. \quad (1.3)$$

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Then (1.1), (1.2) take the form

$$\mathbf{r}_{xt} = (\mathbf{r}_x \times \mathbf{r}_{xy} + u\mathbf{r}_x)_x, \quad (1.4)$$

$$u_x = -(\mathbf{r}_x, (\mathbf{r}_{xx} \times \mathbf{r}_{xy})). \quad (1.5)$$

If we integrate (1.4) by x , then it takes the form

$$\mathbf{r}_t = \mathbf{r}_x \times \mathbf{r}_{xy} + u\mathbf{r}_x. \quad (1.6)$$

Taking into account Gauss-Weingarten equation and $E = \mathbf{r}_x^2 = 1$ the system is defined as

$$\mathbf{r}_t = \left(u + \frac{MF}{\sqrt{\Lambda}}\right)\mathbf{r}_x - \frac{M}{\sqrt{\Lambda}}\mathbf{r}_y + \Gamma_{12}^2\sqrt{\Lambda}\mathbf{n}, \quad (1.7)$$

$$u_x = \sqrt{\Lambda}(L\Gamma_{12}^2 - M\Gamma_{11}^2), \quad (1.8)$$

where

$$\Gamma_{11}^2 = \frac{2EF_x - EE_t - FE_x}{2\Lambda}, \quad (1.9)$$

$$\Gamma_{12}^2 = \frac{EG_x - FE_t}{2\Lambda}, \quad (1.10)$$

$\Lambda = EG - F^2$. $M=1$ equation is integrable equation and has soliton solutions.

2 Construction of Surface Corresponding to Soliton Solution

Here we present the one-soliton solution of (1.1), (1.2) [2],

$$S_3(x, y, t) = 1 - \frac{2\eta^2}{\eta^2 + \xi^2} \operatorname{sech}^2(\chi_{1R}), \quad (2.1)$$

$$S^+(x, y, t) = \frac{2\eta}{\eta^2 + \xi^2} [i\xi - \eta \operatorname{th}(\chi_{1R})] \operatorname{sech}(\chi_{1R}), \quad (2.2)$$

$$\chi_1 = \chi_{1R} + i\chi_{1I}, \quad \lambda_1 = \eta + i\xi, \quad (2.3)$$

$$m_1 = m_{1R}(\rho) + im_{1I}(\rho), \quad m_j(y, t) = m_j(\rho), \quad (2.4)$$

$$\chi_{1R} = \eta x + m_{1R}(\rho) + c_{1R}, \quad \rho = y + i\lambda_1 t, \quad (2.5)$$

$$\chi_{1I} = \xi x + m_{1I}(\rho) + c_{1I}, \quad c = \ln(2\eta/\lambda_1^*), \quad (2.6)$$

$$m_{1R}(\rho) = \operatorname{Re}[m_1(\rho)], \quad m_{1I}(\rho) = \operatorname{Im}[m_1(\rho)], \quad (2.7)$$

which we use in the following theorem.

Theorem 2.1 (Main Theorem) *One-soliton solution (2.1)–(2.7) of the spin system M-I can be represented as components of the vector r_x , where*

$$r_1 = \frac{2\eta}{(\eta^2 + \xi^2)ch\chi_{1R}} + c_1, \quad (2.8)$$

$$r_2 = \frac{2\xi}{\eta^2 + \xi^2} \operatorname{arctg}(sh\chi_{1R}) + c_2, \quad (2.9)$$

$$r_3 = x - \frac{2\eta}{\eta^2 + \xi^2} th\chi_{1R} + c_3, \quad (2.10)$$

c_1, c_2, c_3 are constants. Solution of the form (2.8)–(2.10) corresponds to the surface with the following coefficients of the first and second fundamental forms

$$E = 1, \quad G = \frac{4m_{1Ry}^2}{(\eta^2 + \xi^2)ch^2\chi_{1R}}, \quad (2.11)$$

$$F = \frac{2\eta m_{1Ry}}{(\eta^2 + \xi^2)ch^2\chi_{1R}}, \quad L = \frac{4\eta^3 \xi m_{1Ry}}{\sqrt{g}(\eta^2 + \xi^2)^2 ch^4\chi_{1R}}, \quad (2.12)$$

$$M = \frac{4\eta^2 \xi m_{1Ry}^2}{\sqrt{g}(\eta^2 + \xi^2)^2 ch^4\chi_{1R}}, \quad N = \frac{4\eta \xi m_{1Ry}^3}{\sqrt{g}(\eta^2 + \xi^2)^2 ch^4\chi_{1R}}, \quad (2.13)$$

Proof From (1.3) we have

$$(S_1, S_2, S_3) = (r_{1x}, r_{2x}, r_{3x}), \quad (2.14)$$

i.e.

$$r_{1x} = S_1, \quad r_{2x} = S_2, \quad r_{3x} = S_3. \quad (2.15)$$

Hence

$$r_1 = \int S_1 dx + c_1, \quad (2.16)$$

$$r_2 = \int S_2 dx + c_2, \quad (2.17)$$

$$r_3 = \int S_3 dx + c_3, \quad (2.18)$$

where c_1, c_2, c_3 are constants of integration. Note

$$S^+ = S_1 + iS_2 = r_x^+, \quad (2.19)$$

then

$$r^+ = r_1 + ir_2 = \int S^+ dx + c^+, \quad (2.20)$$

where c^+ is constant of integration. Substituting (2.1) in (2.18), we have

$$\begin{aligned} r_3 &= \int S_3 dx + c_3 = \int \left[1 - \frac{2\eta^2}{\eta^2 + \xi^2} \operatorname{sech}^2(\chi_{1R}) \right] dx + c_3 \\ &= x - \frac{2\eta}{(\eta^2 + \xi^2)} \operatorname{th}(\chi_{1R}) + c_3^*, \end{aligned} \quad (2.21)$$

where $c_3^* = c_3 + c_3'$, $c_3 = c_3^*$, then

$$r_3 = x - \frac{2\eta}{(\eta^2 + \xi^2)} \operatorname{th}(\chi_{1R}) + c_3. \quad (2.22)$$

Substituting (2.2) into (2.20) we have

$$\begin{aligned} r^+ &= r_1 + ir_2 = \int S^+ dx + c^+ \\ &= \int \frac{2\eta}{\eta^2 + \xi^2} [i\xi - \eta \operatorname{th}(\chi_{1R})] \operatorname{sech}(\chi_{1R}) dx + c^+ \\ &= \frac{2i\xi}{\eta^2 + \xi^2} \operatorname{arctg}(\operatorname{sh} \chi_{1R}) + \frac{2\eta}{\eta^2 + \xi^2} \frac{1}{\operatorname{ch} \chi_{1R}} + c'' + c^+ + c'''. \end{aligned} \quad (2.23)$$

We denote $c_1 = c''$, $c_2 = c^+ + c'''$, then

$$r^+ = \frac{2\eta}{(\eta^2 + \xi^2) \operatorname{ch} \chi_{1R}} + c_1 + i \left(\frac{2\xi}{\eta^2 + \xi^2} \operatorname{arctg}(\operatorname{sh} \chi_{1R}) + c_2 \right), \quad (2.24)$$

i.e. we have obtained

$$r_1 = \frac{2\eta}{(\eta^2 + \xi^2) \operatorname{ch} \chi_{1R}} + c_1, \quad r_2 = \frac{2\xi}{\eta^2 + \xi^2} \operatorname{arctg}(\operatorname{sh} \chi_{1R}) + c_2. \quad (2.25)$$

Thus, (2.22), (2.25) give us (2.8)–(2.10).

We proceed to prove the second part of the theorem. From (2.22) and (2.25) we have

$$r_{1x} = -\frac{2\eta^2 \operatorname{sh} \chi_{1R}}{(\eta^2 + \xi^2) \operatorname{ch}^2 \chi_{1R}}, \quad r_{2x} = \frac{2\eta\xi}{(\eta^2 + \xi^2) \operatorname{ch} \chi_{1R}}, \quad (2.26)$$

$$r_{3x} = 1 - \frac{2\eta^2}{(\eta^2 + \xi^2) \operatorname{ch}^2 \chi_{1R}}, \quad r_{1y} = -\frac{2\eta \operatorname{sh} \chi_{1R} m_{1Ry}}{(\eta^2 + \xi^2) \operatorname{ch}^2 \chi_{1R}}, \quad (2.27)$$

$$r_{2y} = \frac{2\xi m_{1Ry}}{(\eta^2 + \xi^2) \operatorname{ch} \chi_{1Ry}}, \quad r_{3y} = -\frac{2\eta m_{1Ry}}{(\eta^2 + \xi^2) \operatorname{ch}^2 \chi_{1Ry}}. \quad (2.28)$$

Then we can calculate

$$\begin{aligned} E = \mathbf{r}_x^2 &= r_{1x}^2 + r_{2x}^2 + r_{3x}^2 \\ &= \frac{4\eta^4 sh^2 \chi_{1R}}{(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} + \frac{4\eta^2 \xi^2}{(\eta^2 + \xi^2)^2 ch^2 \chi_{1R}} \\ &\quad + 1 - \frac{4\eta^2}{(\eta^2 + \xi^2) ch^2 \chi_{1R}} + \frac{4\eta^4}{(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} = 1. \end{aligned} \quad (2.29)$$

Similarly, using (2.25) and (2.28) we obtain

$$G = \mathbf{r}_y^2 = r_{1y}^2 + r_{2y}^2 + r_{3y}^2 = \frac{4m_{1Ry}^2}{(\eta^2 + \xi^2) ch^2 \chi_{1R}} = 1, \quad (2.30)$$

$$F = (\mathbf{r}_x, \mathbf{r}_y) = r_{1x}r_{1y} + r_{2x}r_{2y} + r_{3x}r_{3y} = \frac{2\eta m_{1Ry}}{(\eta^2 + \xi^2) ch^2 \chi_{1R}}. \quad (2.31)$$

Formulas (2.29)–(2.31) give us the first three equations (2.11)–(2.13). Using (2.29)–(2.31) we compute

$$\Lambda = EG - F^2 = \frac{4m_{1Ry}^2(\eta^2 sh^2 \chi_{1R} + \xi^2 ch^2 \chi_{1R})}{(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}}. \quad (2.32)$$

We calculate the components of the vector \mathbf{n}

$$\mathbf{n} = \frac{\mathbf{r}_x \times \mathbf{r}_y}{|\mathbf{r}_x \times \mathbf{r}_y|} = \frac{\mathbf{r}_x \times \mathbf{r}_y}{\sqrt{\Lambda}} = \frac{1}{\sqrt{\Lambda}}(n_1, n_2, n_3), \quad (2.33)$$

$$n_1 = \frac{1}{\sqrt{\Lambda}}(r_{2x}r_{3y} - r_{3x}r_{2y}) = -\frac{2\xi m_{1Ry}}{\sqrt{\Lambda}(\eta^2 + \xi^2) ch \chi_{1R}}. \quad (2.34)$$

Similarly, for the components

$$n_2 = \frac{1}{\sqrt{\Lambda}}(r_{3x}r_{1y} - r_{1x}r_{3y}) = -\frac{2\eta sh \chi_{1R} m_{1Ry}}{\sqrt{\Lambda}(\eta^2 + \xi^2) ch^2 \chi_{1R}}, \quad (2.35)$$

$$n_3 = \frac{1}{\sqrt{\Lambda}}(r_{1x}r_{2y} - r_{2x}r_{1y}) = 0. \quad (2.36)$$

Now, from (2.26), (2.28) we have

$$r_{1xx} = -\frac{2\eta^3 ch \chi_{1R}(ch^2 \chi_{1R} - 2sh^2 \chi_{1R})}{(\eta^2 + \xi^2) ch^4 \chi_{1R}} = -\frac{2\eta^3(1 - sh^2 \chi_{1R})}{(\eta^2 + \xi^2) ch^3 \chi_{1R}}, \quad (2.37)$$

$$r_{2cx} = -\frac{2\eta^2 \xi sh \chi_{1R}}{(\eta^2 + \xi^2) ch^2 \chi_{1R}}, \quad (2.38)$$

$$r_{3cx} = \frac{4\eta^3 sh \chi_{1R}}{(\eta^2 + \xi^2) ch^3 \chi_{1R}}. \quad (2.39)$$

Thus, using (2.34)–(2.39) we can compute

$$r_{3,33} = \frac{4\eta^3 \xi h \chi_{1R}}{(\eta^2 + \xi^2) ch^3 \chi_{1R}} \quad (2.40)$$

Taking into account, that $n_3 = 0$,

$$L = \frac{4\eta^3 \xi m_{1Ry}}{\sqrt{\Lambda}(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} \quad (2.41)$$

Similarly, we calculate

$$M = \frac{4\eta^2 \xi m_{1Ry}^2}{\sqrt{\Lambda}(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} \quad (2.42)$$

$$N = \frac{4\eta \xi m_{1Ry}^3}{\sqrt{\Lambda}(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} \quad (2.43)$$

The formulas (2.41)–(2.43) give us the last three equations (2.11)–(2.13). Using (2.11)–(2.13), for example, the Gaussian curvature can be calculated

$$\begin{aligned} K &= \frac{LN - M^2}{\Lambda} \\ &= \frac{1}{\Lambda} \left(\frac{4\eta^3 \xi m_{1Ry}}{\sqrt{\Lambda}(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} \frac{4\eta \xi m_{1Ry}^3}{\sqrt{\Lambda}(\eta^2 + \xi^2)^2 ch^4 \chi_{1R}} - \frac{16\eta^4 \xi^2 m_{1Ry}^4}{\Lambda(\eta^2 + \xi^2)^4 ch^8 \chi_{1R}} \right) \\ &= \frac{1}{\Lambda} \left(\frac{16\eta^4 \xi^2 m_{1Ry}^4}{\Lambda(\eta^2 + \xi^2)^4 ch^8 \chi_{1R}} - \frac{16\eta^4 \xi^2 m_{1Ry}^4}{\Lambda(\eta^2 + \xi^2)^4 ch^8 \chi_{1R}} \right) = 0. \end{aligned} \quad (2.44)$$

We see that for the surface Gaussian curvature is equal to zero. Theorem is proved. \square

3 Conclusion

Based on the results of work [2], where Gauss–Codazzi–Mainardi equation considered in multidimensional space, we have studied equation M-I and built the surface corresponding to soliton solution. Thus, this work fully reveals the meaning of the geometric approach [2] in (2 + 1)-dimensions.

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This book is a collection of papers from the 9th International ISAAC Congress held in 2013 in Kraków, Poland. The papers are devoted to recent results in mathematics, focused on analysis and a wide range of its applications. These include up-to-date findings of the following topics:

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