VIII International Conference

Plasma Physics
and
Plasma Technology

Contributed papers

Volume I

Minsk, Belarus,
September 14 – 18, 2015

Institue of Physics
National Academy of Sciences of Belarus

STRAGGLING IN DEGENERATE TWO-COMPONENT PLASMAS

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The laser-induced or heavy-ion-induced implosion of fusion fuel pellets starts from normal solid-state conditions and leads to the extreme conditions of inertially confined plasmas in a distinct domain of warm dense matter (WDM) of high densities and temperatures. Usually, the plasma stopping power is defined as the magnitude of the mean energy loss per unit path length: $S = -\Delta E / \Delta x$. Clearly, the treatment of the stopping power of these manmade plasmas require a quantum-mechanical formulation in all ranges of plasma coupling and degeneracy. The quantum-mechanical description of the energy loss in a way that can be immediately applied to plasmas under various conditions was obtained and discussed in detail in /1, 2, 3/ to name a few. The polarizational contribution to the stopping power S of an electron one-component plasma relates it to the system loss function $L(k,\omega) = -\mathrm{Im}\,\varepsilon^{-1}(k,\omega)/\omega$, where $\varepsilon^{-1}(k,\omega)$ is the plasma inverse dielectric function:

$$S(v) = \frac{2(Z_p e)^2}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_0^{kv} \omega^2 L(k,\omega) d\omega, \tag{1}$$

where v, $Z_p e$ are the projectile velocity and charge. A contemporary discussion of the topic can be found in /4/. The stopping power is also an important tool of plasma diagnostics. Particularly, the "minus first" (projectile) velocity power moment of the stopping power /5/ or the stopping high-velocity asymptotic form /6/,

$$S(v \to \infty) = \left(\frac{Z_p e \omega_p}{v} \right)^2 Ln \frac{2 m_e v^2}{\hbar \omega_p}, \tag{2}$$

where ω_p is the plasma frequency. Usually, the interaction of target electrons with the plasma ions is neglected in the stopping power. Nevertheless, the target plasma electron-ion static structure factor influences the plasma polarizational stopping power. This effect has been studied using the Feynman-like form for the plasma loss function /7/:

$$\frac{L(k,\omega)}{\pi C_0(k)} = \frac{\omega_2^2(k)-\omega_1^2(k)}{\omega_2^2(k)}\delta(\omega) + \frac{\omega_1^2(k)}{2\omega_2^2(k)}\ \delta(\omega-\omega_2(k)) + \delta(\omega+\omega_2(k)) \ , \quad (3)$$

stemming from the canonical solution of the moment problem of reconstruction of the system inverse dielectric function $\varepsilon^{-1}(k,\omega)$ /8/. Here, the characteristic frequencies

$$\omega_j(k) = \sqrt{\frac{C_{2j}(k)}{C_{2j-2}(k)}}, \quad j = 1,\ 2,$$

and

$$C_\nu(k) = \pi^{-1}\int_{-\infty}^{\infty}\omega^\nu L(k,\omega)d\omega, \quad \nu = 0,\ 1,\ ...$$

are the power frequency moments of the loss function. Due to the parity of the latter, all odd-order frequency moments vanish. The even-order frequency moments are determined by the static characteristics of the system. After a straightforward calculation one obtains:

$$C_0(k) = (1-\varepsilon^{-1}(k,0)), \quad C_2(k) = \omega_p^2$$

$$C_4(k) = \omega_p^4(1+K(k)+U(k)+H),$$

with $K(k) = \ \langle v_e^2\rangle k^2 + \hbar^2 k^4/(2m)^2 \ /\ \omega_p^2$, $\langle v_e^2\rangle$ being the average squared characteristic velocity of the plasma electrons. The last two terms in the fourth moment stem from the interaction contribution to the system Hamiltonian and can be expressed in terms of the partial structure factors $S_{ab}(k),\ a,b = e,i$ and plasma is modelled as a H-like system with $n_e = Zn_i$ /9/. The Nevanlinna formula of the theory of moments expresses the dielectric function which satisfies the known sum rules $C_{2\nu}\ {}_{\nu=0}^{2}$,

$$\varepsilon^{-1}(k,z) = 1 + \frac{\omega_p^2(z+q)}{z(z^2-\omega_2^2)+q(z^2-\omega_1^2)}, \quad (4)$$

in terms of a function $q = q(k,z)$, which is analytic in the upper complex half plane $\operatorname{Im} z > 0$ and possesses there a positive imaginary part. It must also satisfy the limiting condition: $(q(k,z)/z) \to 0$ as $z \to \infty$ for $\operatorname{Im} z > 0$. In an electron liquid this Nevanlinna parameter function plays the role of the dynamic local-field correction (LFC) $G(k,\omega)$. In particular, the Ichimaru visco-elastic model expression for $G(k,\omega)$ is equivalent to the Nevanlinna function approximated as i/τ_m, τ_m being the effective relaxation time of the Ichimaru model /9/. In a multi-component system the Nevanlinna parameter function stands for the species' dynamic LFC's. In general, we do not have enough phenomenological conditions to determine that function $q(k,\omega)$ which would lead to the exact expression for the loss function. One might benefit /10/ from the Perel'-Eliashberg /11/ expression for the high-frequency asymptotic form of the imaginary part of the dielectric function of the system considered here, $\mathrm{Im}\,\varepsilon(k,\omega \gg (\beta\hbar)^{-1}) \approx (4/3)^{1/4} r_s^{3/4} / 3(\omega_p/\omega)^{9/2}$, where $r_s = (4\pi n/3)^{-1/3} me^2/\hbar^2$ is the Brueckner parameter. This result also implies that higher even-order frequency moments, $C_{2l}(k)$, $l \geq 3$, diverge. The model (3) corresponds to the limiting case with $q(k,\omega) = 0$. It was shown in /7/ using (3) that in a hydrogen-like two-component plasma, (2) is substituted by

$$S_{TCP}(v \to \infty) = \left(\frac{Z_p e \omega_p}{v}\right)^2 Ln \frac{2m_e v^2}{\hbar\omega_p \sqrt{1+H}}. \tag{5}$$

The second quantity of interest to characterize the slowing down process is the energy-loss straggling $\Omega^2(v)$, which describes the statistical fluctuations of the energy loss of the particle and is defined /2/ as the square of the standard deviation of the energy-loss distribution per unit pathlength, i.e.,

$$\Omega^2(v) = \frac{\left\langle \Delta E^{\,2} \right\rangle - \left\langle \Delta E \right\rangle^2}{\Delta x}.$$

The polarizational contribution to the straggling is also defined by the system loss function:

$$\Omega^2(v) = \frac{2(Z_p e)^2 \hbar}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_0^{kv} \omega^3 L(k,\omega) \coth\frac{\beta\hbar\omega}{2} d\omega, \tag{6}$$

where β^{-1} is the target plasma temperature. Now, the same procedure which led to (5) for the straggling asymptotic form give the following expression:

$$\Omega^2_{TCP}(v \to \infty) = \frac{(Z_p e \omega_p)^2 \hbar}{v^2} \int_{\omega_p\sqrt{1+H/v}}^{2mv/\hbar} \omega_2(k) \coth\left(\frac{\beta\hbar\omega_2(k)}{2}\right)\frac{dk}{k}.$$

It is well-known that in classical systems $(\beta \to 0)$, $\Omega^2(v) = 2S(v)/\beta$. Similarly, when $\beta \to 0$ and $\hbar \to 0$,

$$\Omega^2(v \to \infty) = \frac{2S(v \to \infty)}{\beta}.$$

We have calculated both the stopping and straggling and their asymptotic forms. The NPF was chosen to satisfy the Perel'-Eliashberg asymptotic form.

$$\mathrm{Im}\,\varepsilon(k,\omega \gg (\beta\hbar)^{-1}) \approx (4/3)^{1/4} r_s^{3/4} / 3(\omega_p/\omega)^{9/2},$$

The target plasma static structure factors were calculated in the hyper-netted approximation. The results are presented in Figs. 1-2, where the stopping power and straggling, and their asymptotic forms for very fast projectiles are displayed for $r_s = 2.5256$ and two different values of the coupling parameter $\Gamma = \beta e^2 \sqrt[3]{4\pi n_e / 3}$. These numerical data tentatively confirm the above analytic results, which might serve for the WDM diagnostics.

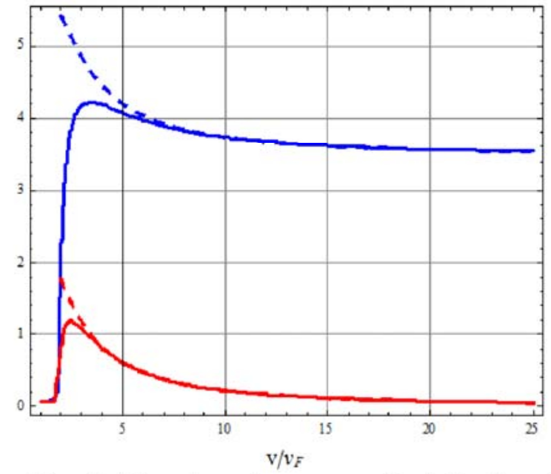

Fig. 1. The stopping power (red line) and straggling (blue line), and their asymptotic forms (dashed lines of respective colors) for $\Gamma = 1.077$

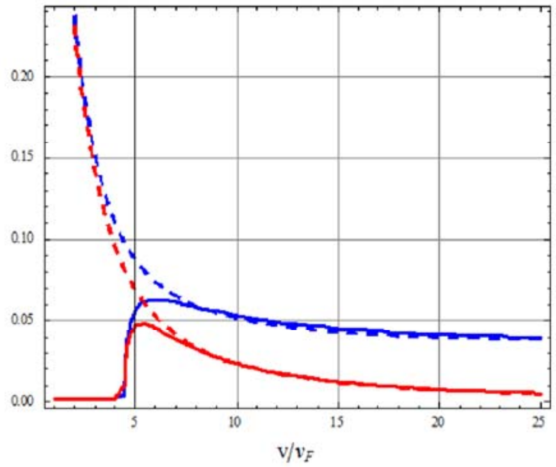

Fig. 2. As in Fig. 1, but for $\Gamma = 0.11$

Acknowledgements. This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan (grant No 3119/GF4)

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