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Investigation of Coulomb Logarithm and Relaxation Processes in Dense Plasma on the Basis of Effective Potentials

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In this paper the relaxation properties of non-isothermal dense plasmas were studied. Based on the effective interaction potentials between particles, the Coulomb logarithm for two-temperature non-isothermal dense plasmas was obtained. These potentials take into consideration long-range multi-particle screening effects and short-range quantum-mechanical effects in two-temperature plasmas. The relaxation processes in such plasmas were studied using the Coulomb logarithm. The obtained results were compared with theoretical works of other authors and with the results of molecular dynamics simulation.

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1 Introduction

Intensive studies of the properties of dense non-ideal plasma were triggered by the idea of realization of inertial confinement fusion. It should be noted that it is especially important to study relaxation times of electrons and ions. In particular, during compression of a target by the flow of high-energy electrons the non-isothermal plasma with heated electrons and cold ions is created [1-2]. Non-isothermal plasma also appears during interaction of heavy ion beams with a target [3-4].

The temperature equalizes much faster within subsystems of electrons and ions than between electrons and ions. This is explained by a large mass difference between ions and electrons. Different methods are used to study relaxation processes in plasma, and among them there are the method of molecular dynamics (MD) [5-7] and quantum kinetic theory [8-13].

To determine the relaxation time of electrons and ions it is necessary to use the Coulomb logarithm. As it is known the Coulomb logarithm appears due to the long-range nature of the Coulomb potential. For convergence of theoretical results the so-called cutoff radius was introduced. The Coulomb logarithm expressed in terms of cutoff radius has the form [10]:

$$\Lambda = \ln \frac{b_{\max}}{b_{\min}}, \tag{1}$$

where $b_{\max}$, $b_{\min}$ are maximal and minimal impact parameters. As the minimal impact parameter the closest approach distance $b_c = Ze^2/k_BT$ or de Broglie thermal wave length $\lambda_{\alpha\beta} = \sqrt{2\pi\hbar^2/m_e k_B T_e}$ is taken. The thermal wave length is taken when it is necessary to take into account the quantum effects. Based on the numerical calculations in [11] the following formula for the Coulomb logarithm was suggested:

$$\Lambda_{GMS} = \frac{1}{2}\ln(1 + [\lambda_D^2 + R_{ion}^2]/[\Lambda^2/8\pi + b_C^2]). \tag{2}$$

where $R_{ion} = (3/4\pi n_p)^{1/3}$ is the distance of closest approach. This formula has a good agreement with the results obtained in the framework of T matrix theory.

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In papers [5-7] the relaxation time was calculated with high accuracy by MD simulation.

In the present work the relaxation time is studied using effective interaction potentials taking into account quantum effects at short distances and collective screening effect at large distances. To obtain the Coulomb logarithm the scattering angle was calculated in binary collision approximation. It is necessary to study the relaxation time for electron-ion temperature in dense semiclassical plasma by effective interaction potentials taking into account both quantum and collective effects in order to get a full physical description of processes.

In this work the dense two-temperature plasma in the temperature range $T = 10^4 \div 10^7$ K and in the range of density $n = n_e + n_i = 10^{20} \div 10^{24}\mathrm{cm}^{-3}$ is studied.

In section 1 the effective potentials used in this research are presented. In section 2 the values of Coulomb logarithm for different plasma parameters are given. In section three the relaxation time obtained in this work is compared with the results of other authors and with the MD data.

2 Effective interaction potentials of plasma particles

It is known that in order to correctly describe static and dynamic properties of plasmas the collective screening effect is to be taken into account. In this work the dense plasma is considered for which quantum effects must be taken into account at short distances. Further the effective interaction potential which takes into account both charge screening at large distance and quantum effects at short distance will be used:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta}{r\sqrt{1-(2\lambda_{\alpha\beta}/r_D)^2}}(\exp(-rB) - \exp(-rA)), \tag{3}$$

where $B^2 = \frac{1}{2\lambda_{\alpha\beta}^2}(1-\sqrt{1-(2\lambda_{\alpha\beta}/r_D)^2})$, $A^2 = \frac{1}{2\lambda_{\alpha\beta}^2}(1+\sqrt{1-(2\lambda_{\alpha\beta}/r_D)^2})$, r_D is the Debye radius, $\lambda_{\alpha\beta}$ is the thermal wave length of particles, and $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$. In non-isothermal plasma a characteristic electron-ion temperature T_{ei} appears [16,17]. In [17] it was shown that for a correct description of plasma properties the electron-ion temperature is to be taken in the form:

$$T_{ei} = \sqrt{T_e T_i}. \tag{4}$$

In particular, it was shown that electron-ion temperature in the form (4) gives a correct asymptotic of the static structure factor. Thus, we take $T_{\alpha\beta} = \sqrt{T_\alpha T_\beta}$.

The effective potential (3) has a finite value at zero distance. In the absence of screening the potential (3) transforms into the well known Deutsch potential [18]. On the basis of the Deutsch potential, for the first time, the thermal relaxation in a strongly coupled two-temperature plasma was investigated by Hansen and McDonald [19]. If we assume that the thermal wave length is equal to zero, the effective potential (3) turns into Debye potential.

It is should be noted that the effective potential (3) always has a real value even if $1-(2\lambda_{\alpha\beta}/r_D)^2 < 0$. It is easy to show by rewriting $\sqrt{1-(2\lambda_{\alpha\beta}/r_D)^2} = \sqrt{-1}\sqrt{(2\lambda_{\alpha\beta}/r_D)^2-1}$ and after algebraic transformations, that in this case, the effective potential (3) transforms into the form:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta}{r}\frac{d_{\alpha\beta}}{\sqrt{\left(2\lambda_{\alpha\beta}/r_D\right)^2-1}}\sin\left(r/\sqrt{r_D\lambda_{\alpha\beta}}\sin(\omega/2)\right)\exp\left(-r/\sqrt{r_D\lambda_{\alpha\beta}}\cos(\omega/2)\right), \tag{5}$$

where $d_{\alpha\beta} = 4 - \frac{r_D^2}{2\lambda_{\alpha\beta}^2}$, $\omega = \arctan\left(\sqrt{\left(2\lambda_{\alpha\beta}/r_D\right)^2-1}\right)$.

If $1-\left(2\lambda_{\alpha\beta}/r_D\right)^2 = 0$, what is equivalent to the condition $2\lambda_{\alpha\beta}/r_D = 1$ the effective interaction potential transforms into:

$$\Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta}{\lambda_{\alpha\beta}\sqrt{2}}\exp(-\frac{r}{\lambda_{\alpha\beta}\sqrt{2}}). \tag{6}$$

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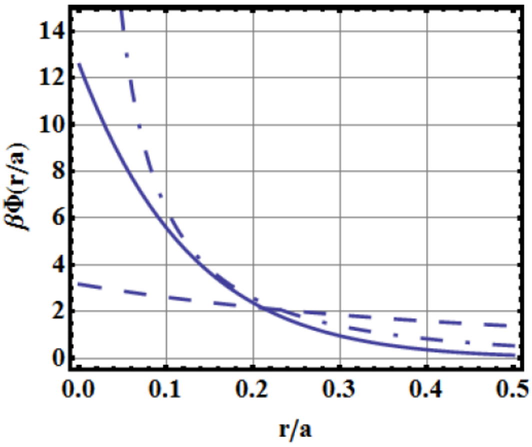

Fig. 1 Effective interaction potential of electrons for $\Gamma = 0.8$, $r_s = 1$. Solid line is potential (3), dot line is Deutsch potential, dashed line is Debye potential.

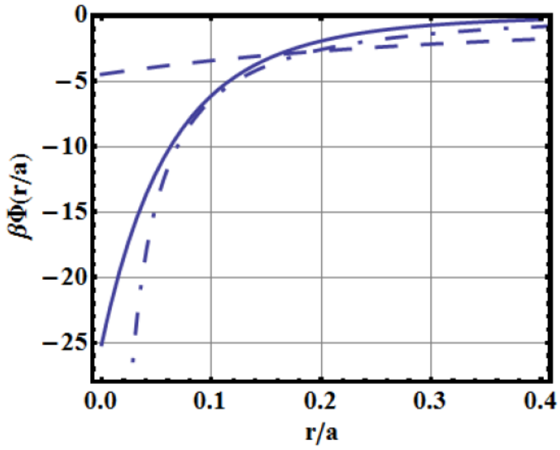

Fig. 2 Electron-ion effective interaction potential for $\Gamma = 0.8$, $r_s = 1$. Solid line is potential (3), dot line is Deutsch potential, dashed line is Debye potential.

3 Coulomb logarithm on the basis of effective interaction potential

In this work the Coulomb logarithm is determined by the center-of mass scattering angle of particles [20-22]:

$$\lambda = \frac{1}{b_\perp^2} \int_0^\infty \sin^2\left(\frac{\theta_c}{2}\right) b db, \tag{7}$$

where the center-of-mass scattering angle can be obtained by the formula [20]:

$$\theta_c = \pi - 2b \int_{r_0}^\infty \frac{dr}{r^2} \left(1 - \frac{\Phi(r)}{E_c} - \frac{b^2}{r^2}\right)^{-1/2}, \tag{8}$$

here $E_c = \frac{1}{2} m_{\alpha\beta} v^2$ is the energy of the center of mass,$m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass of α and β pair; $b_\perp = Z_\alpha Z_\beta/(m_{\alpha\beta} v^2)$, $b_{\min} = \max b_\perp, \lambda_{\alpha\beta}$ is taken as the minimum impact parameter.

The following dimensionless variables are used: the coupling parameter $\Gamma = Z_\alpha Z_\beta e^2/ak_B T$, where $Z_\alpha e$, $Z_\beta e$ are electric charges of α and β of particles; the average distance between the particles $a = (3/4\pi n)^{1/3}$,the density parameter $r_s = a/a_B$, plasma frequency $\omega_p = \sqrt{4\pi n e^2/m_e}$. In formula (8) $\Phi_{\alpha\beta}(r)$ is the interaction potential and r_0 is the distance of the closest approach for a given impact parameter b:

$$1 - \frac{\Phi_{\alpha\beta}(r_0)}{E_c} - \frac{b^2}{r_0^2} = 0. \tag{9}$$

Figures 3-4 show the dependence of the Coulomb logarithm for electron-electron $\lambda_{ee}(\Gamma)$ and electron-ion $\lambda_{ei}(\Gamma)$ pairs as a function of the coupling parameter Γ for $r_s = 1$.

The obtained numerical results are compared with the results obtained using the Debye potential without quantum effects and with a conventional formula $\lambda = \ln(\Lambda)$. It is seen that for $\Gamma << 1$ the results obtained with the effective potential tend to the conventional formula. For small values of coupling parameter Γ, i.e. at high temperatures, the thermal wavelength of particles becomes much smaller than other characteristic distances in plasma, such as the average interparticle distance a and the distance of closest approach r_0, hence, the influence of quantum effects becomes insignificant. In the case $\Gamma < 1$ the results lie below the curve obtained based using the Debye potential. In this area, the thermal wavelength of particles, and the distance of closest approach become comparable $\lambda_{\alpha\beta} \sim r_0$ and quantum effects at small interparticle distances have a significant influence on the value of the Coulomb logarithm, as it can be seen from Fig. 3 and 4. In the limit $\Gamma \to 1$ the obtained results have the asymptotics, which is consistent with the results found by the Debye potential (see fig. 3-4). This is explained by

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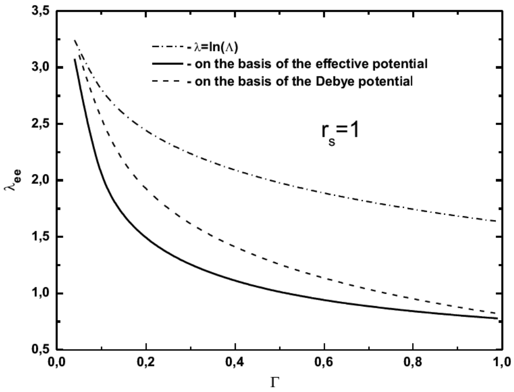

Fig. 3 The Coulomb logarithm for electron-electron pair at $r_s = 1$.

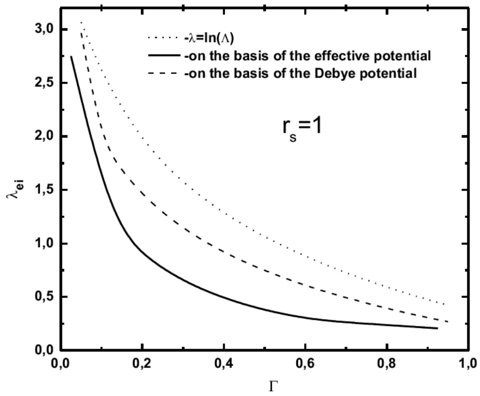

Fig. 4 The Coulomb logarithm for ion-electron pair at $r_s = 1$.

the fact that the distance of closest approach becomes larger than the thermal wavelength of particles $r_0 >> \lambda_{\alpha\beta}$ and scattering occurs at large interparticle distances, where quantum effects are not significant.

Figures 5 and 6 shows a comparison of the calculated data of the Coulomb logarithm in a dense plasma with the theoretical results of other authors [5-13]. The results obtained on the basis of the effective potential are presented by the solid curve (fig. 5-6). The obtained results agree with the results of MD simulation [5-7].

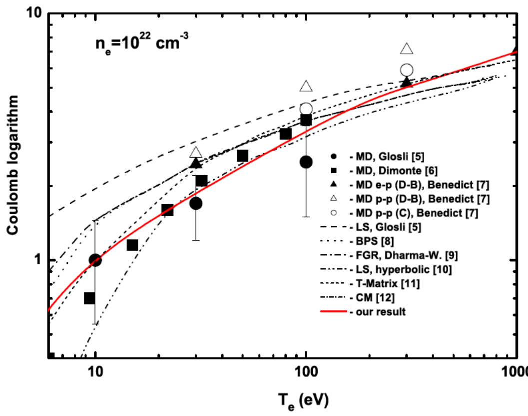

Fig. 5 The Coulomb logarithm in comparison with other theoretical results at $n_e = 10^{22}\text{cm}^{-3}$.

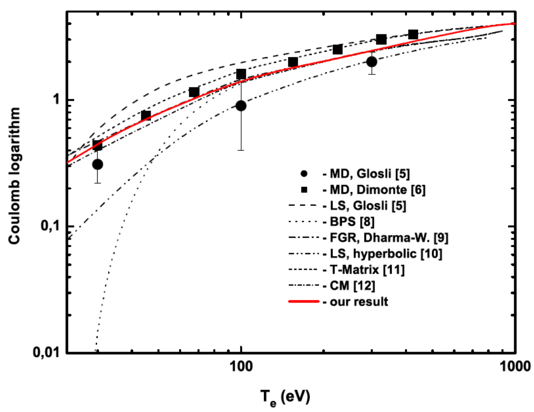

Fig. 6 The Coulomb logarithm in comparison with other theoretical results at $n_e = 10^{24}\text{cm}^{-3}$.

4 Relaxation properties of non-isothermal dense plasma

The relaxation rate of the electron-ion temperature, i.e., the rate of energy exchange, is determined by the difference of the average energy or temperature:

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{\tau_{ei}}, \frac{dT_i}{dt} = \frac{T_e - T_i}{\tau_{ie}}, \tag{10}$$

$$\tau_{ei} = \frac{3m_e m_i}{8\sqrt{2\pi n_i e^2 \lambda}}(\frac{k_B T_e}{m_e} + \frac{k_B T_i}{m_i})^{3/2}. \tag{11}$$

The relaxation times of the temperature in the plasma were calculated for different density values on the basis of the Coulomb logarithm using the effective potential (Fig. 7). It is seen that the equilibration rate increases with increasing density, which is caused by the increase in the frequency of collisions.

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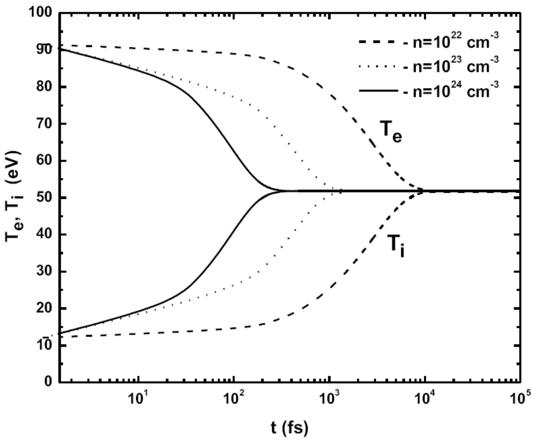

Fig. 7 The relaxation time of temperature between electrons and ions on the basis of the effective potential for different densities.

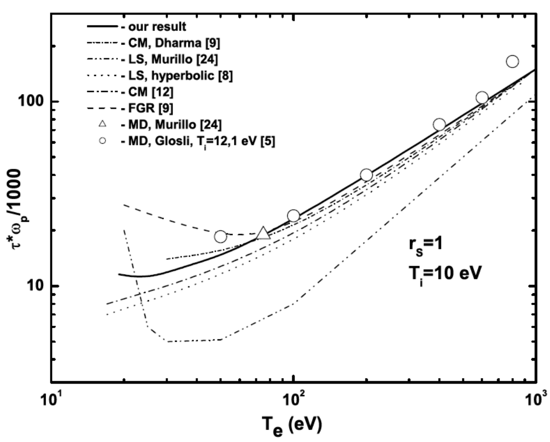

Fig. 8 The relaxation time in units of plasma frequency on the basis of the effective potential in comparison with theoretical results [5-13, 24].

It was found that the relaxation has two stages. The final stage is characterized by the exponential decrease of the temperature difference of components. Figure 8 shows the comparison of the temperature relaxation time obtained on the basis of the effective potential with the results of MD at $r_S = 1$, $T_i = 10\,\text{eV}$. It is seen that the relaxation time increases with increasing temperature. This is explained by the fact that the larger the temperature difference between electrons and ions, the more time is needed to come to an equilibrium state.

Figure 9 shows the values of e temperature of ions and electrons at $n_e = 10^{24}\text{cm}^{-3}$ in comparison with the theoretical results of other authors and with MD data.

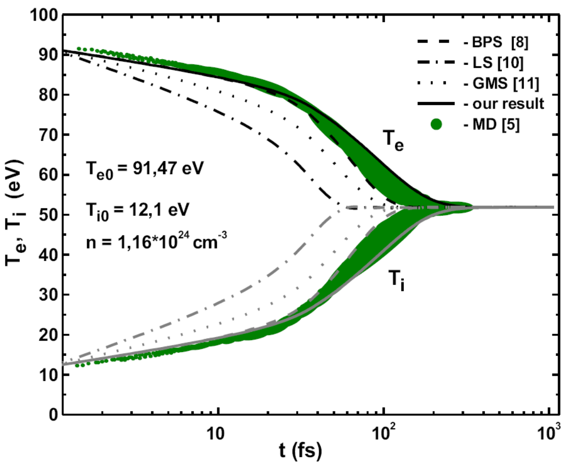

Fig. 9 A comparison of the values of temperature relaxation calculated on the basis of the effective potential with theoretical BPS [8], LS [10], GMS [11] and MD results [5].

5 Conclusion

The relaxation processes in dense plasmas were studied on the basis of effective interaction potentials taking into account quantum effects of diffraction at short distances and screening at large distances. The results obtained for the Coulomb logarithm and temperature relaxation times for different plasma parameters are consistent with the results of other authors.

The investigation of the Coulomb logarithm of dense, non-ideal semiclassical plasma showed that the quantum effect of diffraction is important in the plasma with moderate coupling ($\Gamma < 1$) (see Fig. 3-4).

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References

[1] N.A. Tahir, et al., Phys. Rev. STAB. **17**, 041003 (2014).
[2] W. Ebeling, R. Redmer, H. Reinholz, G. Roepke, Contrib. Plasma Phys. **48**, 670 (2008).
[3] N.A. Tahir, Th. Stöhlker, A. Shutov, I.V. Lomonosov, V.E. Fortov, M. French, N. Nettelmann, R. Redmer, A.R. Piriz, C. Deutsch, Y. Zhao, H. Xu, G. Xio and P. Zhan, New Journal of Physics **12**, 073022 (2010).
[4] S. Minaev, N. Alexeev, A. Golubev, D.H.H. Hoffmann, T. Kulevoy, B. Sharkov, A. Sitnikov, N.A. Tahir and D. Varentsov, Nucl. Inst. Meth. A **620**, 99104 (2010).
[5] J.N. Glosli, et al., Phys. Rev. E **78**, 025401(R) (2008).
[6] G. Dimonte, J. Daligault, Phys. Rev. Lett. **101**, 135001 (2008).
[7] L.X. Benedict, et al., Phys. Rev. Lett. **102**, 205004 (2009).
[8] L. S. Brown, D. L. Preston, and R. L. Singleton Jr., Phys. Rep. **410**, 237 (2005).
[9] M.W.C. Dharma-wardana, Phys. Rev. Lett. **101**, 035002 (2008).
[10] L. Spitzer, Physics of Fully Ionized Gases, Interscience, N.Y. (1967).
[11] D.O. Gericke, M.S. Murillo, M. Schlanges, Phys. Rev. E **65**, 036418 (2002).
[12] J. Vorberger, D.O. Gericke, Phys. Plasmas **16**, 082702 (2009).
[13] J. Vorberger, D.O. Gericke, High Energy Density Physics **10**, 1 (2014).
[14] T.S. Ramazanov, K.N. Dzhumagulova, Phys. Plasmas **9**, 3758 (2002).
[15] C. Deutsch, Phys. Lett. A 60 **4**, 317 (1977).
[16] P. Seuferling, J. Vogel, and C. Toepffer, Phys. Rev. A **40**, 323 (1989).
[17] R. Bredow, Th. Bornath, W.D. Kraeft, R.Redmer, Contrib.Plasma Phys. **53**, 276 (2013).
[18] C. Deutsch, Ann.Phys. N.Y. **115**, 404 (1978).
[19] J.C. Hansen and I.R. McDonald, Phys. Lett. A **97**, 42 (1983).
[20] C.A. Ordonez, M.I. Molina, Phys. Plasmas **1**, 2515 (1994).
[21] T.S. Ramazanov, S.K. Kodanova, Phys. Plasmas **8**, 5049 (2001).
[22] T.S. Ramazanov, S.K. Kodanova, Zh.A. Moldabekov, M.K. Issanova, Phys. Plasmas **20**,112702 (2013).
[23] T.S. Ramazanov, Zh.A. Moldabekov, M.T. Gabdullin, T.N. Ismagambetova, Phys. Plasmas **21**,012706 (2014).
[24] M.S. Murillo, M.W.C. Dharma-wardana, Phys. Rev. Lett. **100**, 205005 (2008).

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