ABSTRACTS

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Editors:
A. Hasanoglu (Hasanov)
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G.S. Dulikravich
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S. Kabanikhin
Sobolev Institute of Mathematics, Russia
D. Lesnic
University of Leeds, UK
A. Neubauer
University of Linz, Austria

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An inverse problem for the Stokes equations

Gullazzat Dairbayeva
Al-Farabi Kazakh National University, Almaty, Kazakhstan
E-mail: Lazzat.Dairbaeva@kaznu.kz

In this work we consider the ill-posed problem for the Stokes equations. The initial problem is reduced to the inverse problem in regard to some well-posed direct problem. The inverse problem is numerically solved using the optimization method and finite element method. The estimate of convergence is given.

In the domain we consider the following problem for the Stokes equations

$$\Delta u - \nabla p = 0 \tag{1}$$

$$divu = 0 \tag{2}$$

$$u|_{\Gamma_0} = \begin{cases} 0, \text{ if } (x,y) \in \Gamma_{01} \cup \Gamma_{03} \\ \varphi(y), \text{ if } (x,y) \in \Gamma_{02} \end{cases} \tag{3}$$

$$\left(pn - \frac{\partial u}{\partial n} \right)\bigg|_{\Gamma_0} = f(x,y), \quad (x,y) \in \Gamma_0 \tag{4}$$

where $\varphi = (\varphi_1, \varphi_2)$, $f = (f_1, f_2)$ are given, $\Gamma_0 = \Gamma_{01} \cup \Gamma_{02} \cup \Gamma_{03}$, $\Gamma_{01} = \{(x,0) : x \in [0,1]\}$

$\Gamma_{02} = \{(0,y) : y \in [0,1]\}$, $\Gamma_{03} = \{(x,1) : x \in [0,1]\}$.

The problem (1)-(4) is ill-posed. Its solving is reduced to the solving of the inverse problem in regard to the well-posed direct problem for the Stokes equations. Analogous approach for other problems may be found in [1,2].

The direct problem. Find (u, p) solution of the equations (1)-(2) with (3) and

$$\left(pn - \frac{\partial u}{\partial n} \right)\bigg|_{\Gamma_1} = q \tag{5}$$

conditions, where q is given, $\partial\Omega$ is boundary of Ω. This problem is well-posed.

The inverse problem. Find the function q from (1)-(5) if the functions φ, f are given.

Consider the spaces $L_2(\Omega), W_1^2(\Omega), W_1^{1/2}(\partial\Omega), \left(W_1^{1/2}(\partial\Omega)\right)^*$, analogous spaces are entered in [2].

The inverse problem can be written in the operator form

$$Aq = f, \tag{6}$$

where $A:\ q := \left(pn - \frac{\partial u}{\partial n} \right)\bigg|_{\Gamma_1} \to f := \left(pn - \frac{\partial u}{\partial n} \right)\bigg|_{\Gamma_0}$, (u, p) is the solution of the direct problem (1)-(3), (5), $A: \left(W_2^{1/2}(\Gamma_1)\right)^* \to \left(W_2^{1/2}(\Gamma_0)\right)^*$

Consider the objective functional $J(q)=\|Aq-f\|^2_{\Gamma_0}$. For minimizing this function Landveber's method is applied. The following theorem is proved.

Theorem *Let operator* $A:\left(W_2^{1/2}(\Gamma_1)\right)^* \to \left(W_2^{1/2}(\Gamma_0)\right)^*$ *be linear and boundary operator* $f\in\left(W_2^{1/2}(\Gamma_0)\right)^*$ *and the solution* q_* *of the inverse problem (6) exists. Then if the initial guess* $q_0\in\left(W_2^{1/2}(\Gamma_1)\right)^*$ *and* $\alpha\in(A,\|A\|^{-2})$*, the sequence* $q_{n+1}=q_n-\alpha J'(q_n)$ converge and the following estimate holds true

$$J(q_n)\le\frac{\|q_0-q_*\|^2}{n\alpha(1-\alpha\|A\|^2)}.$$

References

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2. G. Bastay, T. Johansson, D. Lesnic., V. Kozlov. An Alternating Method for the Stationary Stokes System. ZAMM (Z. Angew. Math. Mech) 86, 268-280 (2006).

Short-Bio

G. Dairbayeva has a scientific degree of the candidate of physical and mathematical sciences, the scientific rank of the senior lecturer. She works at the al-Farabi Kazakh National University, is the scientific secretary of Research Institute of Mathematics and Mechanics and a member of editorial board of scientific journal "Vestnik KazNU. Seriya Mathematics, Mechanics and Informatics"