A perfect-fluid spacetime for a slightly deformed mass

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We present approximate exterior and interior solutions of Einstein's equations which describe the gravitational field of a static deformed mass distribution. The deformation of the source is taken into account up to the first order in the quadrupole.

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To describe the gravitational field of a static axially symmetric mass distribution in general relativity, it is necessary to consider the multipole moments of the source. From a physical point of view, one expects that the quadrupole is the largest contributor and higher multipoles can be neglected in a first approximation. In this case, to describe the exterior field one can use, for instance, the exact quadrupole metric ($q-$metric).[1,2]

$$ds^2 = A^{1+q}dt^2 - A^{-q} \times \left[\left(1+\frac{m^2\sin^2\theta}{r^2A}\right)^{-q(2+q)}\left(\frac{dr^2}{A}+r^2d\theta^2\right)+r^2\sin^2\theta d\varphi^2\right], \quad (1)$$

with $A = 1 - 2m/r$, which has been shown to be the simplest generalization of the Schwarzschild metric containing a quadrupole parameter q. Considering the quadrupole up to the first order only, we obtain

$$ds^2 = A\left(1+q\ln A\right)dt^2 - r^2\sin^2\theta\left(1-q\ln A\right)d\varphi^2 - \left[1+q\ln A - 2q\ln\left(A+\frac{m^2}{r^2}\sin^2\theta\right)\right]\left(\frac{dr^2}{A}+r^2d\theta^2\right) . \quad (2)$$

This is an approximate solution of Einstein's vacuum equations up to the first order in q. The total mass of the spacetime turns out to be $M_0 = m(1+q)$ and the quadrupole moment is $M_2 = -(2/3)qm^3$.

The interior solution can be generated by using the method proposed recently in Ref. 3. We obtain

$$ds^2 = e^{2\psi_0}(1+2\tilde{q}\psi_0)dt^2 - e^{-2\psi_0}(1-2\tilde{q}\psi_0)$$
$$\times\left[e^{2\gamma_0}(1+4\tilde{q}\gamma_0+\tilde{q}\gamma_1)\left(\frac{dr^2}{r^2f^2(r)}+r^2(\sin^2\theta-\tilde{q}\sin\theta\cos\theta)d\varphi^2\right)\right], \quad (3)$$

$$e^{2\psi_0} = \frac{3}{2}f(R)-\frac{1}{2}f(r),\ \ f(r)=\sqrt{1-\frac{2\tilde{m}r^2}{R^3}},\ \ e^{\gamma_0}=re^{2\psi_0}\ , \quad (4)$$

$$\gamma_1 = -2\int\frac{1+4\pi\sin^2\theta r^2p_0}{rf^2(r)(1+r\psi_{0,r})\sin^2\theta+\frac{r}{1+r\psi_{0,r}}\cos^2\theta}dr+\kappa\ , \quad (5)$$

$$\psi_{0,r} = \frac{2\tilde{m}r}{R^3f(r)[3f(R)+f(r)]}\ , \quad (6)$$

where $\tilde{m}$, $\tilde{q}$, R and κ are real constants. This is an interior solution up to the first order in $\tilde{q}$ for a perfect fluid with density and pressure

$$\rho\ = \rho_0[1+\tilde{q}(1+\psi_0-4\gamma_0-\gamma_1)],\ \ \rho_0 = const. \quad (7)$$

$$p\ = p_0[1+\tilde{q}(1+\psi_0-4\gamma_0-\gamma_1)],\ \ p_0=\rho_0\frac{f(r)-f(R)}{3f(R)-f(r)}, \quad (8)$$

respectively. In the limiting case $\tilde{q}\to 0$, the metric (3) represents a perfect fluid with constant density ρ_0 and pressure p_0 as given in Eq.(8). If $\tilde{m}=m$, this particular solution can be matched with the exterior Schwarzschild metric along a sphere of radius R.

In the general case $\tilde{q}\neq 0$, a more detailed analysis must be carried out in order to match the above approximate interior solution with the approximate exterior $q-$metric given in Eq.(2). First, the matching surface must be established. Then, the matching conditions must be imposed for all metric components. This would imply a relationship between the exterior parameters m and q and the interior parameters $\tilde{m}$, $\tilde{q}$, ρ_0 and κ. This result will be presented elsewhere.

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