EUROPEAN MECHANICAL SOCIETY

RECENT TRENDS IN MODELING OF MOVING LOADS ON ELASTIC STRUCTURES

BOOK OF ABSTRACTS

15—17 April, 2015

Eskişehir, Turkey

Boundary Value Problems of Elastodynamics at Action of Subsonic Transport Loads and Their Generalised Solutions

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In articles [1,2] the generalized and classical decisions of system of the Lama equations in moving coordinates system are constructed which describe the motion of isotropic elastic medium at subsonic, transonic, sonic and supersonic speeds of the movement of disturbance sources of various type. It is shown how the type of these equations is depending on two Mach numbers (the relation of speed of the source movement to the speeds of longitudinal and transverse elastic waves). The equations are elliptic at subsonic speeds, have mix hyperbolic-elliptic type at transonic speeds and they are hyperbolic at a hypersonic case.

Here we considered the first and second boundary value problems (BVPs) for elastic medium, which is limited cylindrical surface. On it there is *transport load* which moves with a constant subsonic speed along the cylindrical boundary.

This class of BVP is mathematical model for dynamics of massif in vicinity of underground constructions as transport tunnels, and also land road for transport load which speed of the movement is still much less than speed of disturbances waves in the environment. In this case we have an elliptic BVP. For its solving the method of the generalized functions (GFM) is used which allows to build dynamic analogs of Green formulas for the elliptic equations and systems, and on their basis to receive a decisions and to build the singular boundary integral equations resolving the set tasks. The main ideas of this method for a class of the transport solutions of the wave equation at sub - and supersonic speeds in spaces of different dimension are stated in [3]. Uniqueness of the solution of both BVPs is proved, the dynamic analogs of a Somigliana formula of and singular boundary integral equations resolving BVPs are constructed [4].

Calculations by method of consecutive approximations of decisions for the first BVP for a tunnel of a circular and vaulted profile are performed which are illustrated by diffraction pictures of vector fields of movements of the elastic medium near the tunnels.

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Dynamics of Two-Component Medium M. Biot For Subsonic Transport Load

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Among acting sources of waves generation in continuous media the transport loads are especially wide-spread, suppose their shape does not change over time during motion. By this their motion velocity significantly affects to the type of differential equations, which describe dynamics of medium. These loads are called transport. This class of problems is a model in the investigation of the environmental impact of various transport or transported objects. By this an important role is played by the ratio between the velocities of propagation of various kinds of perturbation in the environment, which can be several, and the velocity of transport load.

Here a homogeneous isotropic medium M. Biot, consisting of solid and liquid components, in the case of actions transport loads, moving at a constant velocity, is considered. Biot medium has three sound velocities [1]: c_1, c_2 describe the propagation velocity of longitudinal waves, the third c_3 - a shear wave ($c_2 < c_3 < c_1$). It is assumed that the mass forces acting in the medium, move with c constant velocity along the axis z and represented as $G_i = G_i(x_1, x_2, z + ct)$ and motion equations of such medium are

$$(\lambda+\mu)u_{sj},_{ji} + Qu_{fj},_{ji} + \mu u_{si},_{jj} + G_{si} = c^2(\rho_{11}u_{si},_{33} + \rho_{12}u_{fi},_{33}),$$

$$Qu_{si},_{ji} + Ru_{fj},_{ji} + G_{fi} = c^2(\rho_{12}u_{si},_{33} + \rho_{22}u_{fi},_{33})$$

where u_{si}, u_{fi} are the elastic and fluid components of the displacement vector, G_{si}, G_{fi} are the body forces acting respectively on the solid and fluid components, $u_{i},_{j} = \partial u_i/\partial x_j'$. The constants λ, μ, Q, R have the dimension of stress, $\rho_{11}, \rho_{12}, \rho_{22}$ are related to the particle mass density of the elastic component (ρ_s) and fluid (ρ_f) by relations: $\rho_{11} = (1-m)\rho_s - \rho_{12}$, $\rho_{22} = m\rho_f - \rho_{12}$, m is the porosity of the medium.

For equations M. Biot under the action of subsonic loads the Green tensor is written. Also generalized solutions of equations of motion of Biot medium for arbitrary transport mass forces recorded. The results of the computer implementation of the Green tensor in the form of graphs and pictures of vector fields displacement liquid and solid components of the environment is presented.

Solutions obtained here can be used to investigations the massif's dynamics in the neighborhood of underground constructions such as tunnels, transport pipelines depending on the properties of water saturation of the medium, the velocity and type of existing transport loads.

References

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