Theoretical and Numerical Prediction of the Permeability of Fibrous Porous Media

Aziz Kudaikulov[1](✉), Christophe Josserand[2], and Aidarkhan Kaltayev[1](✉)

[1] Al-Farabi Kazakh National University, Almaty, Kazakhstan
aziz.kudaikulov@gmail.com,
aidarkhan.kaltayev@kaznu.kz
[2] UPMC, CNRS, UMR 7190, Inst Jean Le Rond DAlembert, 75005 Paris, France
christophe.josserand@gmail.com

Abstract. In this paper, the permeability of ordered fibrous porous media for normal flows is predicted theoretically and numerically. Moreover, microscopic velocity profiles in the "unit cell" are investigated in detail for normal flows. Porous material is represented by a "unit cell" which is assumed to be repeated throughout the media and 1D fibers are modeled. Fibers are presented as cylinders with the same radii. Planar flow that perpendicular to the axes of cylinders is considered in this paper. All numerical calculations are performed using Gerris program [6]. The quantitative comparison of numerical and theoretical results of computation of the permeability of ordered fibrous media is reasonably good and is about 10–15%. AQ1

Keywords: Fibrous porous media with periodic structure · Navier-stokes equations · Darcys law · Permeability of fibrous porous media

1 Introduction

Fibrous porous materials are widely used in modern industry and engineering applications, such as heat exchangers, filters, catalysts, and fuel cell electrodes. The main technical challenge for the fibrous porous medium is to determine the velocity of the flow in the media. If we know the velocity of the fluid flow in the fibrous porous media, we can determine the important technical features of the media, such as the rate of change of temperature of the medium, the rate of change of concentration of substance, etc.. In most cases, the flow in the fibrous porous media is very slow and obeys Darcy's law [1], that relates the flow rate to the pressure gradient:

$$\boldsymbol{u}_d = \frac{K}{\mu} \nabla (p + \rho g z), \tag{1}$$

where K - permeability of fibrous porous medium, μ - fluid viscosity, $\boldsymbol{u}_d$ - flow rate, p - pressure in the porous medium and $\rho g z$ - hydrostatic pressure. Calculating of the flow rate from the formula (1) is very difficult problem, because we don't know

N. Danaev et al. (Eds.): CITech 2015, CCIS 549, pp. 1–9, 2015.
DOI: 10.1007/978-3-319-25058-8_9

the permeability of fibrous porous medium generally. Basically permeability of the porous medium K is determined empirically, however new experimental technologies and high-resolution imaging of porous media can provide three-dimensional structural details of porous materials with resolution in one micron. There exist many pore-scale models of the fluid flows in the porous medium such as Lattice-Boltzmann [3], pore network models [2], discrete particle methods (smoothed particle hydrodynamics) [4] and direct discretization methods (finite difference, finite element, finite volume, immersed boundary methods) [5]. All of these methods require high computational power. In the direct discretization methods the Navier-Stokes equations are discretized and solved for domain with complex geometries. The main advantages of the direct discretization methods is that can be applied for the domains with complex geometries and simulate fluid flow in the porous medium more exactly than others. Also these methods have disadvantages, such as: it requires high computational power and these methods are available for very small domains (about 1 micron). From the above listed models, the most preferred is pore network models [2]. In this model, the porous medium is represented as a system of straight channels. Nevertheless, in many engineering designs the geometry of the porous medium is very simple. Also, in many engineering calculations no need to accurately calculate the parameters of fluid flow in porous media, but it is sufficient to calculate their average values. The fibrous porous medium with a periodic structure is considered in this paper (see Fig. 1). Fibers are presented as cylinders with the same radii. Planar flow that perpendicular to the axes of cylinders is considered in this paper. We numerically predicted the permeability of fibrous porous media and compared with existing theoretical predictions in this study. All numerical calculations are performed using Gerris program [6].

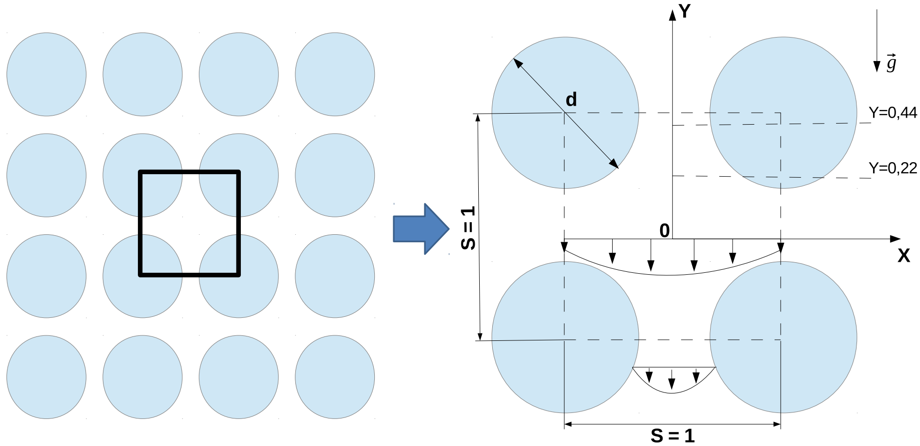

Fig. 1. Two dimensional rectangular area with sizes s x s in that the cylinders are periodically arranged

2 Modeling Approach

There exist many theoretical predictions of the permeability of fibrous porous media in the literatures [7] - [10]. From early works on the theoretical predictions of the permeability of fibrous porous medium we can emphasize the works of John Happel (1959) [7] and Hasimoto (1959) [8]. John Happel [7] found the theoretical prediction of the permeability of fibrous porous media by solving the Stokes equation for a fluid flow in fibrous porous medium. The flow around a cylinder investigated in his work (see Fig. 2). His theoretical prediction of the permeability of fibrous porous medium:

$$K_1^* = \frac{K_1}{d^2} = \frac{1}{32\phi}[ln(\frac{1}{\phi}) - \frac{1-\phi^2}{1+\phi^2}], \tag{2}$$

where K_1 - permeability of fibrous porous media, d - diameter of the cylinders and $\phi = \frac{d^2}{s^2}$, where s - the distance between centers of the cylinders.

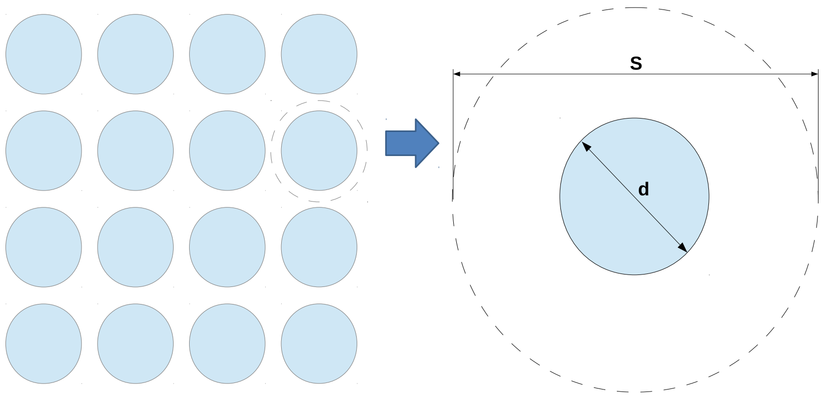

Fig. 2. The periodic structure of the fibrous porous medium

In the work of Hasimoto [8] the exact solution of the Stokes equation for the fluid flow in fibrous porous medium in the form of the infinite series is used to predict the permeability of fibrous porous media. He found the theoretical prediction of the permeability of fibrous porous media using only the terms of lowest order of this series:

$$K_2^* = \frac{K_2}{d^2} = \frac{1}{32\phi'}[ln(\frac{1}{\phi'}) - 1,476], \tag{3}$$

where $\phi' = \frac{\pi d^2}{4s^2}$. Later Sangani and Acrivos (1982) [9] improved the theoretical prediction of fibrous porous media using the terms of highest order of the series that presented in the work of Hasimoto [8]:

$$K_3^* = \frac{K_3}{d^2} = \frac{1}{32\phi'}[ln(\frac{1}{\phi'}) - 1,476 + 2\phi' - 1,774\phi'^2 + 4,076\phi'^3]. \quad (4)$$

From recent works on the theoretical predictions of the permeability of fibrous porous medium we can emphasize the work of Tamayol and Bahrami (2008) [10]. In this work, porous medium is considered as "unit cell" which is repeated throughout the media (see Fig. 1). Also the unidirectional flow with parabolic velocity profile is considered in this work. Their theoretical prediction of the permeability of fibrous porous medium:

$$K_4^* = \frac{K_4}{d^2} = \frac{1}{3\phi^2}\frac{(1-\phi)^{\frac{5}{2}}}{(2(\phi+2) + 4\frac{(1-\sqrt{\phi})(1-\phi)^2}{\sqrt{\phi}})\frac{\sqrt{1-\phi}}{\sqrt{\phi}} + 12arctan(\frac{1+\sqrt{\phi}}{\sqrt{1-\phi}})}. \quad (5)$$

In this paper, the permeability of ordered fibrous porous media for normal flows is calculated numerically and compared with the above theoretical predictions. Moreover, microscopic velocity profiles in the "unit cell" are numerically calculated and compared with the parabolic velocity profile which considered in the work of Tamayol and Bahrami (2008) [10]. Porous material is represented by a "unit cell" which is assumed to be repeated throughout the media and 1D fibers are modeled. Fibers are presented as cylinders with the same radii (see Fig. 1). Planar flow that perpendicular to the axes of cylinders is considered in this paper. All numerical calculations are performed using Gerris program [6].

3 Formulation of the Problem

The numerical simulation of single-phase fluid flow in fibrous porous medium is considered in this paper. The object of the study is the square domain which includes 4 cylinders (see Fig. 1). Planar flow that perpendicular to the axes of cylinders is considered in this paper. This model is based on the numerical solution of the Navier-Stokes equations for incompressible fluid flow:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \boldsymbol{g} - \nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}, \quad (6)$$

$$\nabla \cdot \boldsymbol{u} = 0, \quad (7)$$

where $\boldsymbol{u}$ - velocity of fluid flow, p - pressure and g - acceleration of gravity.

Initial condition for the velocity of fluid flow:

$$\boldsymbol{u}(0, x_k) = 0. \quad (8)$$

Boundary conditions for the velocity and pressure:

1) On the boundary of the domain (on the Fig. 1 showed as dashed lines):

$$\boldsymbol{u}(t,x_k)|_{x_k=-s/2} = \boldsymbol{u}(t,x_k)|_{x_k=s/2}, \tag{9}$$

$$p(t,x_k)|_{x_k=-s/2} = p(t,x_k)|_{x_k=s/2}, \tag{10}$$

$$\frac{\partial \boldsymbol{u}(t,x_k)}{\partial x_k}|_{x_k=-s/2} = \frac{\partial \boldsymbol{u}(t,x_k)}{\partial x_k}|_{x_k=s/2}. \tag{11}$$

2) On the surface of the cylinders (no-slip boundary condition):

$$\boldsymbol{u}(t,x_k) = 0, \tag{12}$$

where $k = 1{,}2$ (for two-dimensional case). We need to average the value of the velocity over the square domain (on the Fig. 1 showed as dashed lines) to find the flow rate - $\boldsymbol{u}_d$:

$$\boldsymbol{u}_d = \frac{\int\int \boldsymbol{u}(t,\boldsymbol{x})dxdy}{s^2}, \tag{13}$$

where s - the distance between centers of the cylinders. Since we only considered the flow, when $Re \ll 1$, then we can neglect the horizontal component of the velocity in comparison with the vertical component of the velocity and due to the incompressibility of the fluid we have:

$$\int U(t,\boldsymbol{x})dy + \int V(t,\boldsymbol{x})dx \approx \int V(t,\boldsymbol{x})dx = const, \tag{14}$$

where $U(t,\boldsymbol{x})$ - the horizontal component of the velocity, and $V(t,\boldsymbol{x})$ - the vertical component of the velocity:

$$\int V(t,\boldsymbol{x})dx = Q = const, \tag{15}$$

where Q - flow rate. From the equations (13) and (15) follows:

$$V_d = \frac{\int\int V(t,\boldsymbol{x})dxdy}{s^2} = \frac{\int Qdy}{s^2} = \frac{Q}{s}. \tag{16}$$

Further, from the equation (1) we can find the permeability of the porous medium:

$$K = |\frac{\mu \boldsymbol{u}_d}{\nabla(p+\rho g z)}| = \frac{\mu Q}{s\nabla(p+\rho g z)}. \tag{17}$$

4 Results

On the Fig. 3, 4 and 5 shows the comparison of the numerical and theoretical velocity profile which given in the work of Tamayol and Bahrami (2008) [10] for various values of the radius of the cylinders. The parabolic velocity profile is considered in the work of Tamayol and Bahrami (2008) (see Fig. 1):

$$V(x,y) = ax^2 + bx + c. \tag{18}$$

Since the fluid flow is incompressible, we have:

$$V(x,y) = -\frac{3Q}{4\delta^3}(x^2 - \delta^2) = \begin{cases} -\frac{3Q}{4(\frac{s}{2} - \sqrt{\frac{d^2}{4} - y^2})^3}(x^2 - (\frac{s}{2} - \sqrt{\frac{d^2}{4} - y^2})^2), 0 \leq x \leq \frac{d}{2} \\ -\frac{3Q}{2s^3}(4x^2 - s^2), \frac{d}{2} \leq x \leq \frac{s}{2}. \end{cases} \tag{19}$$

where δ - distance between the surfaces of the cylinders. Also comparison of the numerical value of the permeability of fibrous porous media with above theoretical predictions is showed on the Fig. 6 and Table 1.

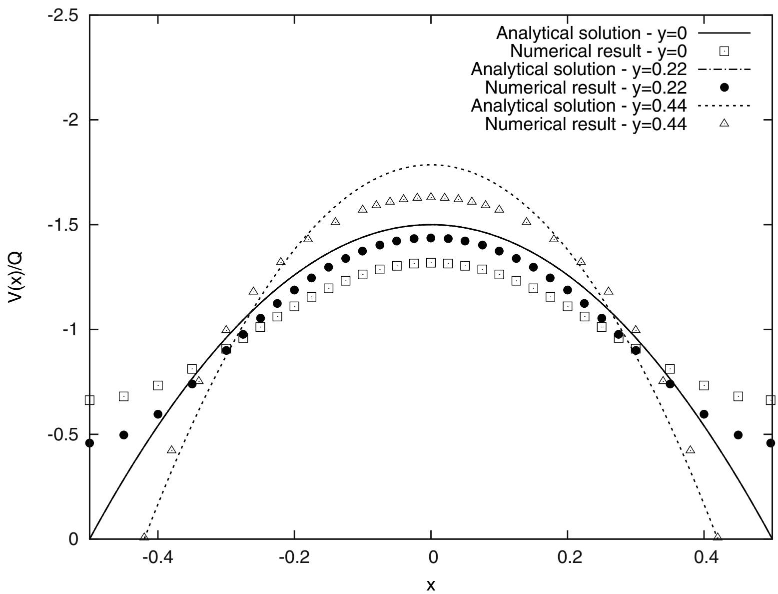

Fig. 3. The numerical and theoretical profile of the vertical component of the velocity when the diameter of the cylinders - d = 0,2

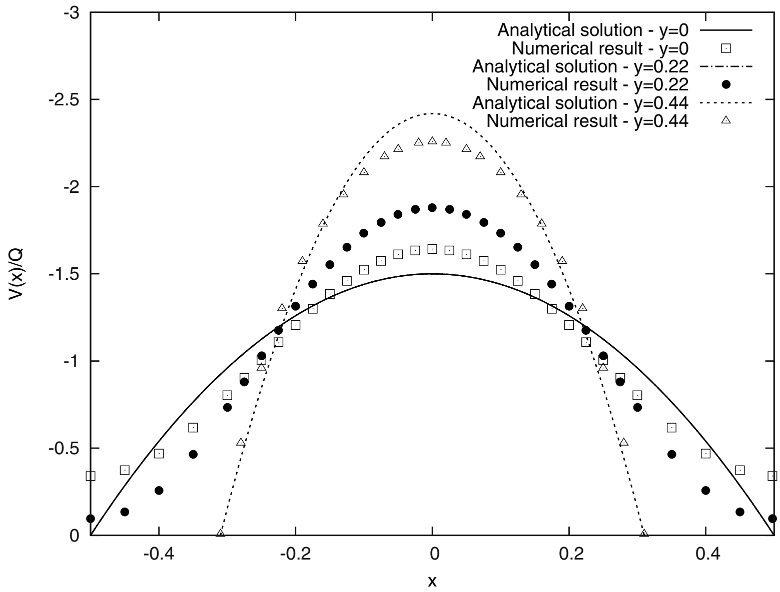

Fig. 4. The numerical and theoretical profile of the vertical component of the velocity when the diameter of the cylinders - d = 0,4

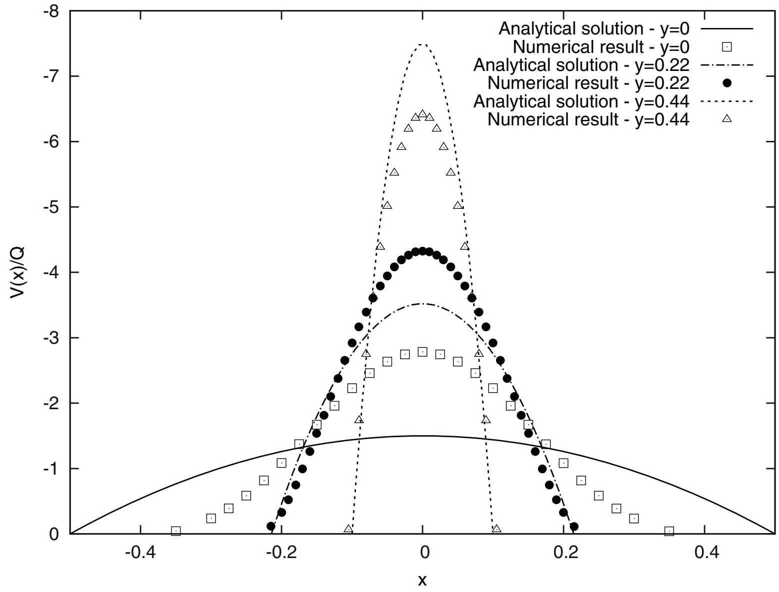

Fig. 5. The numerical and theoretical profile of the vertical component of the velocity when the diameter of the cylinders - d = 0,8

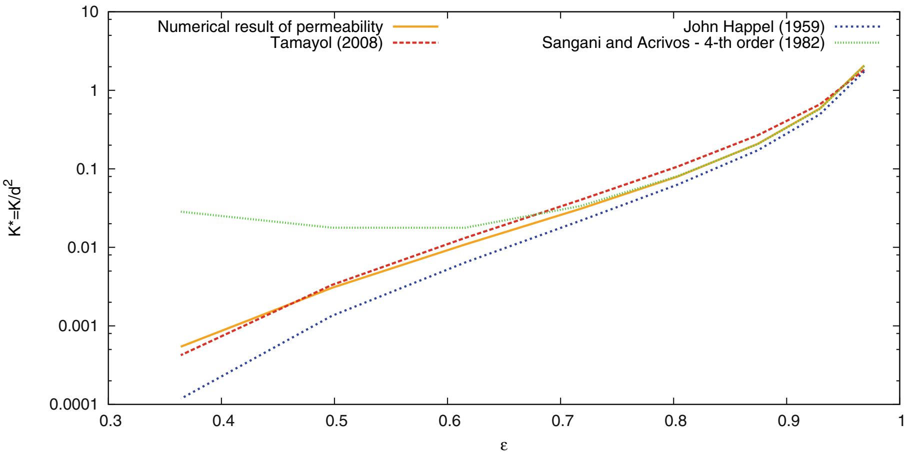

Fig. 6. A comparison of numerical and theoretical values of the permeability of fibrous porous medium

Table 1. A comparison of numerical and theoretical values of the permeability of fibrous porous medium

Porosity - ϵ	**Numerical value of the permeability -** K^*	**Tamayol and Bahrami (2008) -** K_4^*	**John Happel (1959) -** K_1^*	**Sangani and Acrivos (1982) -** K_3^*
0.9686	2.076525	1.828917	1.735993	2.036350
0.9294	0.584678	0.661806	0.494450	0.578512
0.8744	0.207850	0.267379	0.172364	0.206495
0.8037	0.080000	0.107103	0.062993	0.080788
0.7174	0.030931	0.040066	0.021798	0.033532
0.6154	0.010859	0.013148	0.006414	0.017729
0.4976	0.003048	0.003323	0.001340	0.017806
0.3641	0.000543	0.000424	0.000118	0.028461

5 Conclusions

As can be seen from Fig. 6 and Table 1, the theoretical prediction of the permeability of the fibrous porous medium which given in the work of Tamayol and Bahrami (2008) [10] is the most accurate in comparison with other theoretical predictions. Also in Fig. 3, 4 and 5 showed that the error of the theoretical prediction of the velocity profile is not so large and allows to investigate in detail the fluid flow in pore scale or micro scale.

References

1. Bear, J., Cheng, A.H.-D.: Theory and Applications of Transport in Porous Media. Modeling Groundwater Flow and Contaminant Transport, vol. 23. Springer (2010)
2. Blunt, M., King, P.: Relative permeabilities from two- and three-dimensional pore-scale network modelling. Transport in Porous Media **6**(4), 407–433 (1991)
3. Pan, C., Hilpert, M., Miller, C.T.: Lattice-Boltzmann simulation of two-phase flow in porous media. Water Resources Research **40**(1), W01501 (2004)
4. Tartakovsky, A.M., Meakin, P.: Pore scale modeling of immiscible and miscible fluid flows using smoothed particle hydrodynamics. Advances in Water Resources **29**(10), 1464–1478 (2006)
5. Huang, H., Meakin, P., Liu, M.B.: Computer simulation of two-phase immiscible fluid motion in unsaturated complex fractures using a volume of fluid method. Water Resources Research **41**(12), W12413 (2005)
6. Popinet, S.: The Gerris Flow Solver. http://gfs.sourceforge.net
7. Happel, J.: Viscous flow relative to arrays of cylinders. AIChE **5**, 174–177 (1959)
8. Hasimoto, H.: On the periodic fundamental solutions of the Stokes equations and their application to viscous flow past a cubic array of spheres. J. Fluid Mech. **5**, 317–328 (1959)
9. Sangani, A.S., Acrivos, A.: Slow flow past periodic arrays of cylinders with application to heat transfer. Int. J. Multiphase Flow **8**, 193–206 (1982)
10. Tamayol, A., Bahrami, M.: Analytical determination of viscous permeability of fibrous porous media. International Journal of Heat and Mass Transfer **52**, 2407–2414 (2009)

Author Queries

Chapter 9

Query Refs.	Details Required	Author's response
AQ1	Please confirm if the corresponding author is correctly identified. Amend if necessary.	