WEAK APPROXIMATE SOLUTION OF OPTIMIZATION CONTROL PROBLEMS FOR NONLINEAR INFINITE DIMENSIONAL SYSTEMS

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Extended Abstract

The approximate solution of optimization control problem is an admissible control, which is similar enough to the optimal one. However this property is very strong. Therefore the approximate solution of the problem is interpreted frequently as an admissible control such as the corresponding value of the minimizing functional is similar enough to its lower bound. These notions are equivalent for Tikhonov well-posed problems. But the second notion is applicable for the larger class of optimization problems. However it can be inapplicable too for difficult enough problems. So we proposed weaker form of the approximate solution. The state equation and constraints can be realized no exactly, but with small enough errors in this case.

We consider a set *V*, its subset *U* and the functional *I* on *U*. It is necessary to minimize the functional *I* on the set *U*. The point $u \in V$ is called (τ, O, ε) *-approximate solution* of this problem if $u \in O$ and $I(u) \le \inf I(U) + \varepsilon$, where τ is a topology of *V*, *O* is a small in the sense of τ neighborhood of the set *U*, and ε is a small positive number. This approximate solution is similar enough to the admissible control, and the corresponding value of the functional is similar enough to its lower bound on *U*.

An approximate solution is found by means of some algorithm, which determine a sequence of *V*. The sequence $\{u_k\}$ from *V* is called τ*-approximate minimizing sequence*, if it exist its subsequence, which converges in the sense of τ to the point of *U*, and the corresponding functional sequence converges to $\inf I(U)$. If $\{u_k\}$ is τ-approximate minimizing sequence then the number *k* can be chosen large enough such as u_k is (τ, O, ε) -approximate solution of the problem for all small neighborhood *O* of the set *U* and all small value ε. So it is necessary to find some approximate minimizing sequence. It can be obtain as a solution of an auxiliary optimization control problem.

The sequence $\{(U_k, I_k, u_k)\}$ is called τ*-approximation* of our problem, if $\{u_k\}$ is its τ-approximate minimizing sequence, where U_k is subset of *V*, I_k is the functional in U_k, and $\{u_k\}$ is a point of minimum of I_k in U_k. The solving of the initial problem is reduced to the finding of its τ-approximation. It is desirable that the topology τ is not very weak, and the approximation is easy enough for using of known optimization methods. We consider some uninvestigated optimization problems for nonlinear infinite dimensional systems as applications.

1. We consider the equation

$$y' - \Delta y + a(x,t,y) = v$$

in a set *Q* with homogeneous boundary conditions. The functional

$$I(v) = \int_Q f\left(x,t,y,\nabla y\right) dQ + \frac{\gamma}{2}\int_Q v^2 dQ$$

is given, where $\gamma > 0,$ and the function f satisfies standard conditions. The first problem is the minimization of this functional on the set of controls v, which guaranty the realization of the equality $y(x,T) = w(x).$ We have the fixed final state and the nonsmoothness of the nonlinear terms as difficulties. The problem is solved by means of the penalty method with smooth approximation.

2. We consider the equation

$$y' - \Delta y + a(x,t,y) = v + f$$

in a set Q with infinite horizon with homogeneous boundary conditions and some initial condition. The control v is a point of a convex closed set U. The second optimization problem is the minimization of the functional

$$I(v) = \int_Q \sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} - \frac{\partial z}{\partial x_i} \right)^2 dQ + \frac{\gamma}{2} \int_Q v^2 dQ$$

on the given set, where z is a known function. Its general difficulty is an infinite time interval. This problem is approximated by the corresponding problem with finite horizon.

3. We consider the Dirichlet problem for the equation

$$\Delta y + y^3 = v \, .$$

This boundary problem is singular because its single-valued solvability is not guaranty for all control v. The functional

$$I(v,y) = \frac{1}{6} \int_\Omega (y-z)^6 dx + \frac{\gamma}{2} \int_\Omega v^2 dx$$

is given. The third problem is the minimization of this functional on the set of pairs (v, y) such as the inequalities

$$v(x) \geq 0, \;\; \alpha \leq y(x) \leq \beta, \;\; x \in \Omega \, ,$$

and the boundary problem are true. The difficulties of this problem are the singularity of the boundary problem and the given state constraint. We use the penalty method for this problem. It transforms the optimization problem to the problem of the conditional minimization of the penalty functional. The state equation is interpreted here as the constraint in the form of equality and the given conditions are interpreted as the constraint in the form of inequalities.

We prove the convergence of the used approximation methods for all considered optimization problems. It guaranty the finding of approximate minimizing sequences and approximate solutions of the given problems. We note that the known optimization methods are not applicable in this situation. The desired result is obtained because of weakening of requirement for the approximate solutions.

We can use the analogical means for optimization problems with local solvability of boundary problems, inexistence of the optimal control, ill-posedness in the sense of Tikhonov and Hadamard, problems with nonconvexe functional, and some other.