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PHYSICS AND MATHEMATICS

AN APPROACH FOR SOLVING THE INTERVAL SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

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Abstract. The article discusses the interval system of linear algebraic equations. For these systems, a solution algorithm is proposed, using introduced interval mathematics, where the intervals are presentedin terms of the center (expected value) and the spreading respectto the center (variance). The application of this algorithm and its efficiencyare illustrated by several examples.

Keywords: *interval system of linear algebraic equations, interval mathematics, mathematical expectation, variance, Gaussian elimination.*

Introduction

In work [1], the author suggestedinterval mathematics that works with intervals that are independent normally distributed variables. "Classical" interval arithmetic [2] assumes that all values on interval are equally probable. Therefore, all the results obtained with it cover all possible values and are "super sufficient".

In [1] there is a formal concept of the interval that is introducedin the following form:

$a = \left[\bar{a} - \varepsilon_a, \bar{a} + \varepsilon_a\right] = \left(\bar{a}, \varepsilon_a\right);$

where $\bar{a}$ - the middle of the interval (or expectation) - ε_a width of the interval (or variance). We denote the set of all such intervals as $I_{aep}(R)$.

Let *a, b, c* - intervals from $I_{aep}(R)$. Then proposed the following interval arithmetic:

1. the addition of the two intervals $a,\ b \in I_{aep}(R)$: $c = a + b$

$\bar{c} = \bar{a} + \bar{b};\ \ \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2};$

2. subtraction of two intervals$a,\ b \in I_{aep}(R)$: $c = a - b$,

$\bar{c} = \bar{a} - \bar{b};\ \ \varepsilon_c = \sqrt{\varepsilon_a^2 + \varepsilon_b^2};$

3. multiplying two intervals$a,\ b \in I_{aep}(R)$: $c = a * b$,

$\bar{c} = \bar{a} \cdot \bar{b};\ \ \ \varepsilon_c = \sqrt{\bar{a}^2 \cdot \varepsilon_b^2 + \bar{b}^2 \cdot \varepsilon_a^2};$

4. inverse interval$a \in I_{aep}(R)$: $c = \frac{1}{a}$;

$\bar{c} = \frac{1}{\bar{a}};\ \ \varepsilon_c = \frac{\varepsilon_a}{\bar{a}^2};$

5. division two intervals $a,\ b \in I_{aep}(R)$: $c = \frac{1}{a}$;

$\bar{c} = \frac{\bar{a}}{\bar{b}};\ \ \varepsilon_c = \sqrt{\frac{\bar{a}^2 \cdot \varepsilon_b^2}{\bar{b}^4} + \frac{\varepsilon_a^2}{\bar{b}^2}}.$

In [1] considered and proved properties of these arithmetic operations, and provides examples and functions.

Algorithm for solving interval systems of linear algebraic equations

Next, consider the interval system of linear algebraic equations (ISLAE)

$$Ax = b, \tag{1}$$

here $A = \{a_{ij}\}$ - nxn - matrix and $b = \{b_i\}$ - n - vector whose elements are the intervals and can be represented as

$$a_{ij} = \{a_{ij}^{min}, a_{ij}^{max}\} = \left(\bar{a}_{ij}, \varepsilon_{ij}^{a}\right),$$

$$b_i = \{b_i^{min}, b_i^{max}\} = \left(\bar{b}_i, \varepsilon_i^{b}\right).$$

$\mathbf{A} = \{A_p = \{a_{ij}^p\} \mid a_{ij}^p \in \{a_{ij}^{min}, a_{ij}^{max}\}\}$ - set of dot matrices A_p whose elements belong to their corresponding interval elements of the original matrix.

In [3], there is a theorem on the applicability of the method of Gauss: Let $1 \le n \le 2$ and the set $\mathbf{A}$ doesn't contain non-degenerate "point" matrices $A_p \in \mathbf{A}$. ThenGauss's methodcan be applied.

This approach is not constructive because of the restrictive properties of the classical interval mathematics.

We introduce interval dot matrix $\bar{A} = \{\bar{a}_{ij}\}$ whose elements are the midpoints of intervals corresponding to the original interval matrix $\bar{A} \in \mathbf{A}$.

It is further proposed algorithm for solving ISLAE using the entered interval mathematics.

Theorem. For the applicability of the method of Gauss it is enough thatthe dot matrix be non-degenerate.

The Gauss algorithm with a choice of main element of solution of ISLAE based on the input interval mathematics:

Step 1. Let $i = 1$.

Step 2. If $i<n$, make the following steps, otherwise go to step 7.

Step 3. In the i-th row select j-th element with the highest absolute midpointvalue.

Step 4. If the selected element contains a zero, then the system is considered to be degenerate, which has no definite interval solutions and exits the algorithm. If the selected element does not contain zero, then swap the *i*-th and *j*-th columns.

Step 5. Conduct transformation of interval matrix *A* and vector *b* according to the formulas, since the *i*-th row:

$$a'_{jk} = a_{jk} - a_{ji} * \frac{a_{ik}}{a_{ii}}, \quad j = \overline{i+1, n}, \quad k = \overline{i, n},$$

$$a'_{ik} = \frac{a_{ik}}{a_{ii}}, \quad k = \overline{i, n}, \tag{2}$$

$$b'_j = b_j - a_{ji} * \frac{b_i}{a_{ii}}, \quad j = \overline{i, n}.$$

Step 6. Increase the value of *i* by one and go to step 2.

Step 7. As a result, we obtain an upper triangular intervalmatrix. We find the required interval solution according to the formulas:

$$x_n = \frac{b_n}{a_{nn}},$$

$$x_i = (b_i - \sum_{j=i+1}^{n} a_{ij} * b_j), \quad i = \overline{n-1, 1}. \tag{3}$$

Note: It is assumed that in the formulas (2) and (3) used input interval arithmetic [1].

Considering ISLAE of type:

$$x = Bx + c, \tag{4}$$

here $B = \{b_{ij}\}$ - *n*x*n*- matrix and $c = \{c_i\}$ - n - vector whose elements are the intervals.

We introduce a dot interval matrix $\bar{B} = \{\bar{b}_{ij}\}$, whose elements are the midpoints of the correspondingintervals of the original interval matrix $\bar{B} \in B$.

Theorem. In order to the method of successive approximations

$$x^{k+1} = Bx^k + c, \qquad k = 0,1,..., \tag{5}$$

beconverged to the solution of the system (4) it is sufficient that $\left\| \bar{B} \right\| < 1$.

The proof is obvious by virtue of the correctness of the theorem for the case of a point system of linear algebraic equations (SLAE) and properties of the introduced interval arithmetic does not produce displacement of the middle intervals multiplication and division.

Successive approximation algorithm for solving interval linear systems based on the input interval mathematics:

Step 1. Let $k = 0$, we choose the initial approximation x^0. In particular, we can take $x^0 = c$. We define the required accuracy of finding the solution ε, it can not be less than the specified accuracy of intervals:

$$\varepsilon \geq \max\left(\max_{ij} \varepsilon_{ij}^b, \max_i \varepsilon_i^c \right)$$

Step 2: Calculate the new approximation of the desired interval by the formula (5).

Step 3: Calculate the value

$$d = \max_{i=1,n} \left(abs\left(\bar{x}_i^{k+1} - \bar{x}_i^k \right) \right),$$

here $\bar{x}_i^k$ - the middle of the interval of *k*-th approximation.

Step 4. If the value $d > \varepsilon$, the value of $k = k + 1$ is incremented by one and go to step 2. Otherwise, $\bar{x}^k$ - the approximate interval solution.

Examples

Consider an example from [3]: Suppose we want to calculate the stress distribution in the R-circuit shown in Figure 1. In accordance with the method of nodal potentials conduction $r_i = R_i$ entered. The system of equations for determining the potential u_1, u_2, u_3 given by

$$\begin{pmatrix} r_1 + r_2 + r_3 & -r_3 & 0 \\ -r_3 & r_3 + r_4 + r_5 & -r_5 \\ 0 & -r_5 & r_5 + r_6 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} vr_1 \\ 0 \\ 0 \end{pmatrix}.$$

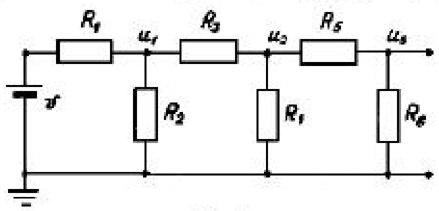

Fig. 1

Let circuit elements have the following ratings:

$r_1 = r_2 = r_4 = 0.1; \quad r_3 = r_5 = 2.0; \quad v = 6.3.$

Technologically, these values can be maintained not exact but with some error. Moreover, they are often dependent on temperature and other operating conditions. Suppose that as a result, denominations are hold within $\pm 10\%$. Thus, the actual:

$$r_1, r_2, r_4 \in [0.09, 0.11], \quad r_3, r_5 \in [1.8, 2.2], \quad v \in [5.67, 6.93].$$

With any combination of these data system matrix M is non-degenerate matrix. Denoted by A - interval matrix, b - an interval vector of right side of equations:

$$A = \begin{bmatrix} [1.98, 2.42] & [-2.2, -1.8] & [0,0] \\ [-2.2, -1.8] & [3.69, 4.51] & [-2.2, -1.8] \\ [0,0] & [-2.2, -1.8] & [1.89, 2.31] \end{bmatrix},$$

$b = \{[0.5103, 0.7623], [0,0], [0,0]\}$

According to the classical definition of arithmetic, interval matrix is a degenerate.

Using introduced in [1] definition of arithmetic, we obtain the following values of the potentials:

$u_1 = [-0.108, 3.443] = (1.667, 1.775);$

$u_2 = [-0.422, 3.460] = (1.519, 1.941);$

$u_3 = [-0.737, 3.630] = (1.447, 2.183).$

Consider the example of Raikhman [3]. Given interval matrix

$$S(\alpha) = \begin{pmatrix} 1 & [0,\alpha] & [0,\alpha] \\ [0,\alpha] & 1 & [0,\alpha] \\ [0,\alpha] & [0,\alpha] & 1 \end{pmatrix}.$$

For $(\sqrt{5}-1)/2 \le \alpha < 1$,using classical interval arithmetic, the calculation of the determinant of the Gauss method we find that the interval matrix $S(\alpha)$ is a degenerate. This example was given in [3] to show that in case of transition from the conventional method of Gauss to the interval version, its properties, in general, gets worse.

When using the input interval arithmetic operations, implemented as a library of routines and functions, this restriction has been cancelled, and we got a solution of a system of linearinterval algebraic equations

$$S(\alpha)x(\alpha) = \begin{pmatrix} [1,1] \\ [1,1] \\ [1,1] \end{pmatrix}$$

For different α :

$$x(0.65) = \begin{pmatrix} [0.244, 0.968] \\ [0.132, 1.080] \\ [0.075, 1.137] \end{pmatrix} = \begin{pmatrix} (0.606, 0.362) \\ (0.606, 0.474) \\ (0.606, 0.531) \end{pmatrix},$$

$$x(0.70) = \begin{pmatrix} [0.187, 0.989] \\ [0.065, 1.111] \\ [-0.002, 1.178] \end{pmatrix} = \begin{pmatrix} (0.588, 0.401) \\ (0.588, 0.523) \\ (0.588, 0.590) \end{pmatrix},$$

$$x(0.80) = \begin{pmatrix} [0.060, 1.052] \\ [-0.077, 1.188] \\ [-0.166, 1.277] \end{pmatrix} = \begin{pmatrix} (0.556, 0.496) \\ (0.556, 0.533) \\ (0.556, 0.721) \end{pmatrix},$$

$$x(0.90) = \begin{pmatrix} [-0.095, 1.147] \\ [-0.236, 1.289] \\ [-0.350, 1.403] \end{pmatrix} = \begin{pmatrix} (0.526, 0.621) \\ (0.526, 0.762) \\ (0.526, 0.877) \end{pmatrix}.$$

Conclusion

The proposed algorithm in the article gives a solution for ISLAE in the case when intervals represented as its middle -it is the expected value, and the spread - is the variance of independently normally distributed variables. The article gives examples, for which the resulting algorithmis used for solving ISALE and shows the effectiveness of the introduced interval mathematics.

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