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To cite this article: F B Belissarova *et al* 2020 *J. Phys.: Conf. Ser.* **1690** 012143

Special dyon-like black hole solution in the model with two Abelian gauge fields and two scalar fields

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Abstract. Dilatonic black hole dyon-like solution in the gravitational $4d$ model with two scalar fields and two 2-forms, governed by two 2-dimensional dilatonic coupling vectors $\vec{\lambda}_i$ obeying $\vec{\lambda}_i(\vec{\lambda}_1 + \vec{\lambda}_2) > 0$, $i = 1, 2$, is found. Some physical parameters of the solutions are obtained: gravitational mass, scalar charge, Hawking temperature, black hole area entropy and parametrized post-Newtonian (PPN) parameters β and γ. The PPN parameters do not depend on the coupling vectors $\vec{\lambda}_i$. A bound on the gravitational mass is proved.

1. Introduction

In this paper we continue our previous works [1–3] devoted to dilatonic dyon black hole solutions. We note that at present there exists a certain interest in spherically symmetric solutions, e.g. dilatonic black hole ones, see [4–10] and the references therein. These solutions appear in gravitational models with scalar fields and antisymmetric forms.

Here we consider a special dilatonic black hole solutions with electric and magnetic charges Q_1 and Q_2, respectively, in the $4d$ model with metric g, two scalar fields φ^1, φ^2, two 2-forms $F^{(1)}$ and $F^{(2)}$, corresponding to two vectors of dilatonic coupling constants $\vec{\lambda}_1$ and $\vec{\lambda}_2$, belonging to $\mathbb{R}^2$ respectively. All fields are defined on an oriented manifold $\mathcal{M}$.

Here we present a black hole solution with a dyon-like configuration for fields of 2-forms:

$$F^{(1)} = Q_1 \tau_1, \qquad F^{(2)} = Q_2 \tau_2, \tag{1}$$

where $\tau_2 = \mathrm{vol}[S^2]$ is magnetic 2-form, which is volume form on 2-dimensional sphere and τ_1 is an "electric" 2-form on $\mathcal{M}$. (We call this noncomposite configuration a dyon-like one while the original dyon configuration in theory with one 2-form F is composite, i.e. $F = Q_1\tau_1 + Q_2\tau_2$.) Due to (1) we deal here with a charged black hole with two color charges: Q_1 and Q_2. The charge Q_1 is the electric, while the charge Q_2 is the magnetic.

We note that in the case of one scalar field φ and two coupling constants λ_1, λ_2 the dyon-like ansatz was considered recently in [2,3,9,10]. For $\lambda_1 = \lambda_2 = \lambda$ our result from [2] was dealing with a trivial noncomposite generalization of dilatonic dyon black hole solutions in the model with one 2-form and one scalar field which was considered in [1], see also [7, 8, 11–14] and references therein.

The solutions with one scalar field from [2, 3] may be imbedded to the solutions under consideration by considering the case of collinear dilatonic coupling vectors:

$$\vec{\lambda}_1 = \lambda_1 \vec{e}, \qquad \vec{\lambda}_2 = \lambda_2 \vec{e}, \tag{2}$$

where $\vec{e}^2 = 1$, $\lambda_1 + \lambda_2 \neq 0$.

Here we find relations for the physical parameters of dyonic-like black holes, e.g. bound on the gravitational mass M and the vector of scalar charges $\vec{Q}_\varphi$. We note that in [2] the bound on the gravitational mass was found (in theory with one scalar field) up to a conjecture, which states a one-to-one (smooth) correspondence between the pair (Q_1^2, Q_2^2), where Q_1 is the electric charge and Q_2 is the magnetic charge, and the pair of positive parameters (P_1, P_2), which appear in decomposition of moduli functions at large distances.

2. Black hole dyon solutions

Let us consider a model governed by the action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{|g|} \Big\{ R[g] - g^{\mu\nu} \partial_\mu \vec{\varphi} \partial_\nu \vec{\varphi} - \frac{1}{2} e^{2\vec{\lambda}_1 \vec{\varphi}} F^{(1)}_{\mu\nu} F^{(1)\mu\nu} - \frac{1}{2} e^{2\vec{\lambda}_2 \vec{\varphi}} F^{(2)}_{\mu\nu} F^{(2)\mu\nu} \Big\}, \tag{3}$$

where $g = g_{\mu\nu}(x) dx^\mu \otimes dx^\nu$ is the metric, $\vec{\varphi} = (\varphi^1, \varphi^2)$ is the vector of scalar fields belonging to $\mathbb{R}^2$, $F^{(i)} = dA^{(i)} = \frac{1}{2} F^{(i)}_{\mu\nu} dx^\mu \wedge dx^\nu$ is the 2-form with $A^{(i)} = A^{(i)}_\mu dx^\mu$, $i = 1, 2$, G is the gravitational constant, $\vec{\lambda}_1 = (\lambda_{1i}) \neq \vec{0}$, $\vec{\lambda}_2 = (\lambda_{2i}) \neq \vec{0}$ are the dilatonic coupling vectors obeying

$$\vec{\lambda_1} \neq -\vec{\lambda_2} \tag{4}$$

and $|g| = |\det(g_{\mu\nu})|$.

We present a dyonic-like black hole solution to the field equations corresponding to the action (3) which is defined on the manifold

$$\mathcal{M} = (2\mu, +\infty) \times S^2 \times \mathbb{R}, \tag{5}$$

and have the following form

$$ds^2 = H^a \Big\{ -H^{-2a} \left(1 - \frac{2\mu}{R}\right) dt^2 + \frac{dR^2}{1 - \frac{2\mu}{R}} + R^2 d\Omega_2^2 \Big\}, \tag{6}$$

$$\varphi^i = \nu^i \ln H, \tag{7}$$

$$F^{(1)} = \frac{Q_1}{H^2 R^2} dt \wedge dR, \qquad F^{(2)} = Q_2 \tau. \tag{8}$$

Here Q_1 and Q_2 are (colored) charges - electric and magnetic, respectively, $\mu > 0$ is the extremality parameter, $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the canonical metric on the unit sphere S^2 ($0 < \theta < \pi$, $0 < \phi < 2\pi$), $\tau = \sin\theta d\theta \wedge d\phi$ is the standard volume form on S^2,

$$H = 1 + \frac{P}{R}, \tag{9}$$

with $P > 0$ obeying

$$P(P + 2\mu) = \frac{1}{2}Q^2 \tag{10}$$

or

$$P = -\mu + \sqrt{\mu^2 + \frac{1}{2}Q^2}, \tag{11}$$

$$a = \frac{(\vec{\lambda}_1 + \vec{\lambda}_2)^2}{\Delta}, \tag{12}$$

$$\nu^i = \frac{\lambda_{1i}\vec{\lambda}_2(\vec{\lambda}_1 + \vec{\lambda}_2) - \lambda_{2i}\vec{\lambda}_1(\vec{\lambda}_1 + \vec{\lambda}_2)}{\Delta}, \tag{13}$$

$$\Delta \equiv \frac{1}{2}(\vec{\lambda}_1 + \vec{\lambda}_2)^2 + \vec{\lambda}_1^2\vec{\lambda}_2^2 - (\vec{\lambda}_1\vec{\lambda}_2)^2, \tag{14}$$

$i = 1, 2$, and

$$Q_1^2 = \frac{\vec{\lambda}_2(\vec{\lambda}_1 + \vec{\lambda}_2)}{2\Delta}Q^2, \qquad Q_2^2 = \frac{\vec{\lambda}_1(\vec{\lambda}_1 + \vec{\lambda}_2)}{2\Delta}Q^2. \tag{15}$$

Here the following additional restrictions on dilatonic coupling vectors are imposed

$$\vec{\lambda}_i(\vec{\lambda}_1 + \vec{\lambda}_2) > 0, \tag{16}$$

$i = 1, 2$.

Due to relations (16) and (17) the Q_s^2 are well-defined. We note that the restrictions (16) imply relations $\vec{\lambda}_s \neq \vec{0}$, $s = 1, 2$, and relation (4).

We note that

$$\Delta > 0, \tag{17}$$

is valid for $\vec{\lambda}_1 \neq -\vec{\lambda}_2$. Indeed, in this case we have the sum of two non-negative terms in (14): $(\vec{\lambda}_1 + \vec{\lambda}_2)^2 > 0$ and

$$C = \vec{\lambda}_1^2\vec{\lambda}_2^2 - (\vec{\lambda}_1\vec{\lambda}_2)^2 \geq 0, \tag{18}$$

due to the Cauchy-Schwarz inequality. Moreover, $C = 0$ if and only if vectors $\vec{\lambda}_1$ and $\vec{\lambda}_2$ are collinear. Relation (18) implies

$$0 < a \leq 2. \tag{19}$$

For non-collinear vectors $\vec{\lambda}_1$ and $\vec{\lambda}_2$ we get $0 < a < 2$ while $a = 2$ for collinear ones.

This solution may be verified just by a straightforward substitution into equations of motion.

The calculation of scalar curvature for the metric $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ in (6) gives us

$$R[g] = \frac{a(2-a)P^2(R - 2\mu)}{2R^{4-a}(R + P)^{1+a}}. \tag{20}$$

Non-collinear case. For non-collinear vectors $\vec{\lambda}_1$ and $\vec{\lambda}_2$ $(0 < a < 2)$ we obtain

$$R[g] \to -\infty, \tag{21}$$

as $R \to +0$ and hence we have a black hole with a horizon at $R = 2\mu$ and singularity at $R = 0$.

Collinear case. For collinear vectors $\vec{\lambda}_1$, $\vec{\lambda}_2$ from (2) obeying $\vec{\lambda}_1 + \vec{\lambda}_2 \neq \vec{0}$ we obtain $\nu^i = 0$, $a = 2$ and

$$Q_1^2 = \frac{\lambda_2}{\lambda_1 + \lambda_2}Q^2, \qquad Q_2^2 = \frac{\lambda_1}{\lambda_1 + \lambda_2}Q^2, \tag{22}$$

where $\lambda_1\lambda_2 > 0$. By changing the radial variable, $R = r - P$, we get a little extension of the solution from [2]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_2^2, \tag{23}$$

$$F^{(1)} = \frac{Q_1}{r^2} dt \wedge dr, \quad F^{(2)} = Q_2 \tau, \qquad \vec{\varphi} = \vec{0}, \tag{24}$$

where $f(r) = 1 - \frac{2GM}{r} + \frac{Q^2}{2r^2}$, $Q^2 = Q_1^2 + Q_2^2$ and $GM = P + \mu = \sqrt{\mu^2 + \frac{1}{2}Q^2} > \frac{1}{\sqrt{2}}|Q|$.

The metric in these variables coincides with the well-known Reissner-Nordström metric governed by two parameters: $GM > 0$ and $Q^2 < 2(GM)^2$. We have two horizons in this case. Electric and magnetic charges are not independent but obey eqs. (22).

3. Physical parameters

Here we consider certain physical parameters corresponding to the solutions under consideration.

3.1. Gravitational mass and scalar charges

For ADM gravitational mass we get from (6) (and $g_{00} = -(1 - 2GM/R + o(1/R))$

$$GM = \mu + \frac{a}{2}P. \tag{25}$$

The scalar charge vector $\vec{Q}_\varphi = (Q_\varphi^1, Q_\varphi^2)$ just follows from (7) and the definition: $\varphi^i = Q_\varphi^i/R + o(1/R)$:

$$\vec{Q}_\varphi = \vec{\nu} P. \tag{26}$$

By using relations (25) and (26) we obtain the following identity

$$2(GM)^2 + \vec{Q}_\varphi^2 = Q_1^2 + Q_2^2 + 2\mu^2. \tag{27}$$

This formula does not contain the vectors $\vec{\lambda}_s$.

The identity (27) may be verified by using (12), (15) and the following relation

$$\vec{\nu}^2 = \frac{(\vec{\lambda}_1^2 + \vec{\lambda}_2^2)(\vec{\lambda}_1^2 \vec{\lambda}_2^2 - (\vec{\lambda}_1 \vec{\lambda}_2)^2)}{\Delta^2}. \tag{28}$$

3.2. The Hawking temperature and entropy

The Hawking temperature corresponding to the solution (9) with $P > 0$ has the following form

$$T_H = \frac{1}{8\pi\mu} \left(1 + \frac{P}{2\mu}\right)^{-a}. \tag{29}$$

In this case the Hawking temperature T_H does not depend upon λ_s, when μ and P (or Q^2) are fixed.

The Bekenstein-Hawking (area) entropy $S = A/(4G)$, corresponding to the horizon at $R = 2\mu$, where A is the horizon area, reads

$$S_{BH} = \frac{4\pi\mu^2}{G} \left(1 + \frac{P}{2\mu}\right)^{a}. \tag{30}$$

It follows from (29) and (30) that the product

$$T_H S_{BH} = \frac{\mu}{2G} \tag{31}$$

does not depend upon $\vec{\lambda}_s$ and the charges Q_s. This product does not use an explicit form of the moduli functions $H_s(R)$.

3.3. PPN parameters

Introducing a new radial variable ρ by the relation $R = \rho(1 + (\mu/2\rho))^2$ $(\rho > \mu/2)$, we obtain the 3-dimensionally conformally flat form of the metric (6)

$$g = H^a \left\{ -H^{-2a} \frac{(1 - (\mu/2\rho))^2}{(1 + (\mu/2\rho))^2} dt \otimes dt + \left(1 + \frac{\mu}{2\rho}\right)^4 \delta_{ij} dx^i \otimes dx^j \right\}, \tag{32}$$

where $\rho^2 = |x|^2 = \delta_{ij} x^i x^j$ $(i, j = 1, 2, 3)$

The parametrized post-Newtonian (PPN) parameters β and γ are defined by the following standard relations:

$$g_{00} = -(1 - 2V + 2\beta V^2) + O(V^3), \tag{33}$$

$$g_{ij} = \delta_{ij}(1 + 2\gamma V) + O(V^2), \tag{34}$$

$i, j = 1, 2, 3$, where $V = GM/\rho$ is Newton's potential, G is the gravitational constant and M is the gravitational mass (for our case see (25)).

The calculations of PPN (or Eddington) parameters for the metric (32) give

$$\beta = 1 + \frac{1}{4(GM)^2}(Q_1^2 + Q_2^2), \qquad \gamma = 1. \tag{35}$$

These parameters do not depend upon vectors $\vec{\lambda}_s$.

3.4. Bound on the mass

Here we outline the following proposition.

Proposition. *Let $Q_1 \neq 0$, $Q_2 \neq 0$ and $\mu > 0$. Then the following bounds on the mass are valid for all $\mu > 0$, $P > 0$ and $\vec{\lambda}_s$ obeying (16):*

$$\frac{1}{2}\sqrt{h_m(Q_1^2 + Q_2^2)} < GM. \tag{36}$$

Here $h_m = (\frac{1}{2} + |\vec{\lambda}|^2_{max})^{-1}$, where $|\vec{\lambda}|_{max} = \max(|\vec{\lambda}|_1, |\vec{\lambda}|_2)$.

Proof. The inequality (36) may written as

$$h_m(Q_1^2 + Q_2^2) < (2GM)^2 = (2\mu + aP)^2. \tag{37}$$

Due to relations (10), (12) and (15) we obtain

$$Q_1^2 + Q_2^2 = \frac{(\vec{\lambda}_1 + \vec{\lambda}_2)^2}{2\Delta} Q^2 = aP(P + 2\mu). \tag{38}$$

Hence the inequality (37) reads as

$$h_m aP(P + 2\mu) < (2\mu + aP)^2, \tag{39}$$

or, equivalently,

$$(a - h_m)aP^2 + 2\mu a(2 - h_m)P + 4\mu^2 > 0. \tag{40}$$

This relation is valid due to inequalities: a) $0 < h_m < 2$ and b) $h_m < a$. The first inequality is trivial while the second one may be redily verified. The proposition is proved.

In the case of collinear vectors $\vec{\lambda_1}$ and $\vec{\lambda_2}$ $(\vec{\lambda_1} \neq -\vec{\lambda_2})$ the Proposition was proved in fact (up to certain conjecture) for general setup in [2]. The case $\vec{\lambda_1} = \vec{\lambda_2}$ was considered earlier in [1] and also in [13] (BPS-like inequality), where the bound on mass was proved in general setup by using certain spinor techniques.

4. Conclusions

In this paper we have presented a non-extremal black hole dyon-like solution in a 4-dimensional gravitational model with two scalar fields and two Abelian vector fields. The model contains two vectors of dilatonic coupling constants $\vec{\lambda}_s \neq \vec{0}$, $s = 1, 2$, obeying relations (16) (e.g. $\vec{\lambda}_1 \neq -\vec{\lambda}_2$). In fact this is a special solution with dependent electric and magnetic charges, see (15). In case of non-collinear vectors $\vec{\lambda}_1$, $\vec{\lambda}_2$ the metric of the solution describes a black hole with one (external horizon) and singularity hidden by it. For collinear vectors $\vec{\lambda}_1$, $\vec{\lambda}_2$ the metric coincides with the Reissner-Nordström metric possessing two horizons and hidden singularity.

Here we have also calculated some physical parameters of the solutions: gravitational mass M, scalar charges Q^i_φ, Hawking temperature, black hole area entropy and post-Newtonian parameters β, γ. The PPN parameters $\gamma = 1$ and β do not depend upon $\vec{\lambda}_s$, if the values of M and Q^i_φ are fixed.

We have also obtained a formula, which relates M, Q_φ, the dyon charges Q_1, Q_2, and the extremality parameter μ for all values of admissible $\vec{\lambda}_s$. This formula does not contain $\vec{\lambda}_s$ and coincides with that of [2] in case of collinear dilatonic coupling vectors. As in the case of [2], the product of the Hawking temperature and the Bekenstein-Hawking entropy does not depend upon vectors $\vec{\lambda}_s$.

Here we have proved the lower bound on the gravitational mass, which is in agreement with our previous results from [2] for collinear coupling vectors. For $\vec{\lambda}_1 = \vec{\lambda}_2$ the lower bound on the gravitational mass is in agreement with that obtained (for composite case) earlier by Gibbons et al. [13] by using certain spinor techniques, see also [1].

Acknowledgments

This paper has been supported by the RUDN University Strategic Academic Leadership Program (recipient V.D.I., mathematical model development). The reported study was funded by the Ministry of Education and Science of the Republic of Kazakhstan, program identification registration number (IRN): BR05236454 and project IRN: AP08052311 (recipient K.A.B., simulation model development and numerical checking of results).

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