

Development of river flow modeling methodology

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Abstract—The article is devoted to the development of methodology of river runoff modeling. This paper reviews possible approaches to fluid flow modeling, analyzing existing solutions and formulations of the research problem. The Navier-Stoke equation is replaced by a difference analogue, which is subsequently solved by a numerical method. A general description of the problem is given, and the objectives of the study are formulated. Flood and inundation monitoring methods developed in Kazakhstan are described, the results of their practical use in some areas are discussed, and directions for further development are outlined. Possible ways of solving the problem of flooding of territories because of flooding are given. This information is then used to forecast emergencies.

Keywords— *flooding; flood waves; breakout waves; computer modeling*

I. INTRODUCTION

Many examples of flooding emergencies caused by the propagation of flood and breakthrough waves are now known. Currently, the Republic of Kazakhstan faces the task of protecting agricultural land and settlements from floods [1].

Floods are becoming a growing problem. Meanwhile, budgets are spent on flood relief rather than flood prevention. Spring flooding is a seasonal phenomenon, and indeed has been somewhat inevitable in recent years. To decide the feasibility of coastal land use, possible flood damage must be analyzed. The quality of the land deteriorates a lot when floods and waves pass through it [2-4].

Engineering measures to prevent damage from potential flooding include the following:

– Monitoring and regulating flood flows through various engineering structures, including dams,

embankments, bank stabilization and channel straightening.

– In areas prone to frequent flooding, houses can be built on stilts, or the first floors of buildings can be converted to non-residential use.

– Ensuring resilience of critical infrastructure elements (e.g. bridges, communication lines), taking into account possible emergencies.

II. RESEARCH METHODS

To solve problems and define flood inundation zones in floods, results can be obtained in the following ways: Physical models; Analytical calculation; Numerical modeling [5].

III. RESULTS

All existing software packages can be divided into one-, two-, and three-dimensional software packages. One-dimensional or two-dimensional numerical simulations greatly simplify the models under study and do not provide a complete understanding of the processes involved in wave breaking and flood wave propagation, as will be shown below. Therefore, the most accurate application of three-dimensional numerical modeling is the calculation of wave inundation and breaking.

In most cases, the basic hydrodynamic modeling system is a three-dimensional system of evolving Navier-Stokes equations [6-8].

Let's analyze the input data required for different modeling methods.

In mathematics, the Navier-Stokes equations are a system of nonlinear differential equations for abstract vector fields of arbitrary magnitude. This equation is a

formulation of Newton's second law. It is considered as the sum of forces in a viscous Newtonian fluid [9-11].

IV. DISCUSSION

$$\begin{cases} \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \\ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) \end{cases} \quad (1)$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (2)$$

- (1) Impulse
- (2) Continuity.

where:

- U, V – flow velocity components;
- ρ – density;
- p – pressure;
- $\mu=1/Re$ – kinematic viscosity;
- t – time.

The problem statement solves the problem by numerically approximating its first part using interference, fluid inlet and outlet positions (Fig. 1).

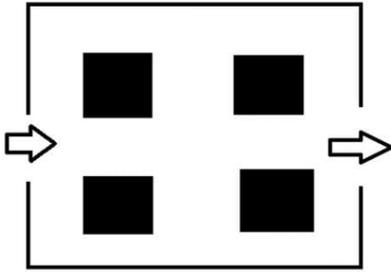


Figure 1. Initial condition

$$\begin{aligned} \frac{u_{ij}^{N+1} - u_{ij}^N}{\Delta t} + U_{ij}^N \frac{u_{i+1,j}^N - u_{ij}^N}{\Delta x} + V_{ij}^N \frac{u_{i,j+1}^N - u_{ij}^N}{\Delta y} = -\frac{1}{\rho} \frac{p_{i+1,j}^N - p_{ij}^N}{\Delta x} + \\ + \frac{1}{Re} \left(\frac{u_{i+1,j}^N - 2u_{ij}^N + u_{i-1,j}^N}{\Delta x^2} + \frac{u_{i,j+1}^N - 2u_{ij}^N + u_{i,j-1}^N}{\Delta y^2} \right) \end{aligned}$$

The next step is to replace continuous derivatives with their difference analogue according to formula (3):

$$\frac{\partial u}{\partial t} = \frac{u^{n+1} - u_j^n + u_j^* - u_{ij}^*}{\Delta t} \quad (3)$$

Adding the intermediate component U does not change the equation. Nevertheless, it shows how this problem can be solved.

$$\begin{aligned} \frac{u_{ij}^* - u_{ij}^n}{\Delta t} = -u_{ij}^n \frac{u_{i+1,j}^n - u_{ij}^n}{\Delta x} - v_{ij}^n \frac{u_{i,j+1}^n - u_{ij}^n}{\Delta y} - \\ - \frac{1}{Re} \left(\frac{u_{i+1,j}^n - 2u_{ij}^n + u_{i-1,j}^n}{\Delta x^2} + \frac{u_{i,j+1}^n - 2u_{ij}^n + u_{i,j-1}^n}{\Delta y^2} \right) \end{aligned}$$

The Burgers equation is characterized by the introduction of intermediate components and their classification by physical parameters. Balancing the rest of it:

$$\frac{u_{ij}^{n+1} - u_{ij}^*}{\Delta t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (4)$$

The second half of equation (1) performs the same operation:

$$\begin{aligned} \frac{v_{ij}^* - v_{ij}^n}{\Delta t} = -u_{ij}^n \frac{v_{i+1,j}^n - v_{ij}^n}{\Delta x} - v_{ij}^n \frac{v_{i,j+1}^n - v_{ij}^n}{\Delta y} - \\ - \frac{1}{Re} \left(\frac{v_{i+1,j}^n - 2v_{ij}^n + v_{i-1,j}^n}{\Delta x^2} + \frac{v_{i,j+1}^n - 2v_{ij}^n + v_{i,j-1}^n}{\Delta y^2} \right) \end{aligned}$$

$$\frac{v_{ij}^{n+1} - v}{\Delta t} = -\frac{1}{\rho} \frac{\partial p}{\partial y}$$

To satisfy equation (2), the classification is as follows:

$$\begin{cases} u_{ij}^{n+1} = u_{ij}^* - \frac{\Delta t}{\rho} \frac{\partial p}{\partial x} \\ v_{ij}^{n+1} = v_{ij}^* - \frac{\Delta t}{\rho} \frac{\partial p}{\partial y} \end{cases}$$

$$\text{execute: } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \left(u_{ij}^* - \frac{\Delta t}{\rho} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(v_{ij}^* - \frac{\Delta t}{\rho} \frac{\partial p}{\partial y} \right) = 0$$

$$\frac{\partial u^*}{\partial x} - \frac{\Delta t}{\rho} \frac{\partial^2 p}{\partial x^2} + \frac{\partial v^*}{\partial y} - \frac{\Delta t}{\rho} \frac{\partial^2 p}{\partial y^2} = 0$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{\rho}{\Delta t} \left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right)$$

The results show that the Poisson's equation is obtained. The Poisson equation can be solved in several ways. Of these, the Gauss-Seidel method is considered to be relatively acceptable:

$$\begin{aligned} \frac{p_{i+1,j}^n - 2 * p_{ij}^{n+1} + p_{i-1,j}^{n+1}}{\Delta x^2} + \frac{p_{i,j+1}^n - 2 * p_{ij}^{n+1} + p_{i,j-1}^{n+1}}{\Delta y^2} = \\ = f(x, y, t) \end{aligned}$$

$$\begin{aligned} p_{ij}^{n+1} = \frac{1}{4} (p_{i+1,j}^n + p_{i-1,j}^n + p_{i,j+1}^n + p_{i,j-1}^n) \\ - \frac{\rho * \Delta x^2}{\Delta t} \left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} \right) \end{aligned}$$

To solve the equations, simply calculate each one separately and substitute the values in the appropriate places.

The general algorithm can be derived as follows:

1. To determine the displacement at the beginning of the problem, a simple iterative calculation is performed using the values u^* , v^* obtained earlier from the Burgers tender:

$$\frac{u_{ij}^* - u_{ij}^n}{\Delta t} = -u \frac{\partial u}{\partial x} - V \frac{\partial u}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

$$\frac{v_{ij}^* - v_{ij}^n}{\Delta t} = -u \frac{\partial v}{\partial x} - V \frac{\partial v}{\partial y} + \frac{1}{Re} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right).$$

2. The pressure is then determined by the Burgers method. For this purpose, the above-mentioned Gauss-Seidel method is calculated using the Poisson equation.

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \frac{\rho}{\Delta t} \left(\frac{\partial u^*}{\partial x} + \frac{\partial V^*}{\partial y} \right)$$

3. The pressure gradient is achieved by displacement after accounting for the first and second stages.

$$\frac{u_{ij}^{n+1} - u_{ij}^*}{\Delta t} = -\frac{1}{\rho} \frac{\partial P}{\partial x}$$

$$\frac{V_{ij}^{n+1} - V_{ij}^*}{\Delta t} = -\frac{1}{\rho} \frac{\partial P}{\partial y}$$

4. Computation of u^{n+1}, V^{n+1} with accuracy to ε must satisfy the convergence conditions.

$$(u^{n+1} - u^n) < \varepsilon$$

$$(V^{n+1} - V^n) < \varepsilon$$

The computer models are focused on the dam, considering the absence of velocity and pressure fields at the dam and the pressure behavior at its boundary [12-14]. The above proposed algorithm is implemented in C++.

There are four cases:

1. Entry

In this case $U=U_0, V=0, \frac{\partial p}{\partial n} = 0$;

2. Wall

$U=0, V=0, \frac{\partial p}{\partial n} = 0$;

3. Barrier (Dam)

3.1 resistance at $U=0, V=0, p = 0$;

3.2 at the resistance boundary, the components $\frac{\partial p}{\partial n} = 0$; U, V are calculated.

4. Output

4.1 $\frac{\partial U}{\partial n}=0, \frac{\partial V}{\partial n}=0, p = 0$;

Let's determine the locations of the obstacles (Fig. 2-6):

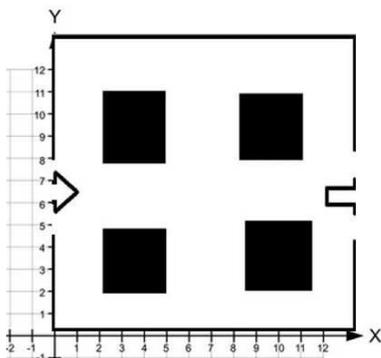


Figure 2. Determination of resistance points

```
18411 9.99972e-06 3.71216e-06
k =1513705
iter =18411
Для продолжения нажмите любую клавишу . . .
C:\Users\22807\source\repos\ConsoleApplication10\Deb
0.
```

Figure 3. Number of iterations. Value of u^{n+1}, V^{n+1} to ε accuracy

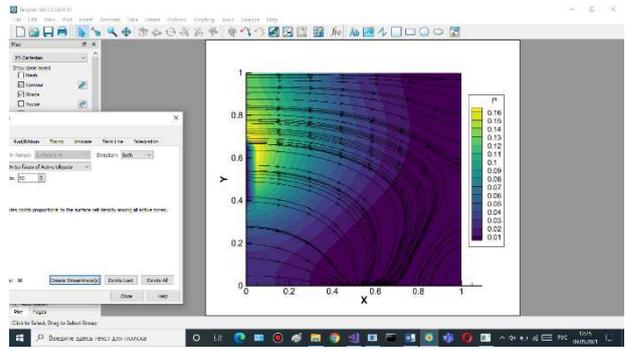


Figure 4. P - graph of the pressure component

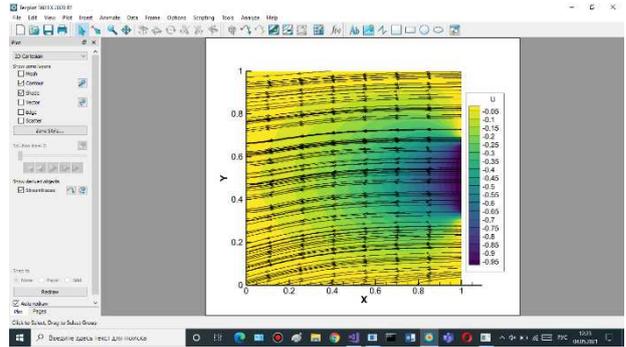


Figure 5. U - plot of the flow velocity component along the x direction

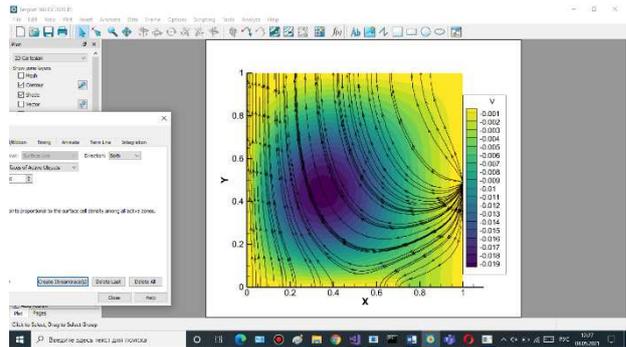


Figure 6. V - plot of flow velocity component along y direction

The results of the above graph [15] show that the value of Reynolds number first becomes small, which indicates that the flow is laminar. Therefore, a mathematical model of the Navier-Stokes equations was constructed, and a computer model was developed for this purpose. The plots clearly show the actual output and input data as well as the effect of interference on the laminar flow distribution.

River flow modeling is an important task. One-dimensional streamflow simulation models can only be used for preliminary calculations. Higher level models should be applied for a more accurate study [13].

V. CONCLUSION

Obstacle finding and component mapping were identified. The results show that the flow is laminar if the Reynolds number is selected from the range 1000-2000. In this problem, all barriers and boundary conditions are disclosed, and a complete analysis is performed. The choice of the C++ programming language was determined by the high speed of calculations and the possibility of

dynamically reserving RAM. In this regard, it is sufficient to use a standard personal computer.

This paper shows that numerical computer modeling is necessary to determine the propagation parameters of flood and breaking waves.

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