Analysis of Drill String Motion in a Gas Stream

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Summary. Nonlinear dynamics of drill strings used in the drilling of oil and gas wells was researched. Transverse vibrations of the compressed and twisted drill string in the gas stream were investigated. The drill string was subject to operation of an axial load and torque. Nonlinearity of the dynamic model was caused by an overpressure of the gas stream. The numerical analysis of the model was carried out. The effect of bar parameters on the amplitude of its vibrations was analyzed.

The technique of mechanical drilling includes operations on rock fracture, its feeding on a surface, security of stability of borehole walls, etc.
In recent years operations the mode of mechanical rotary drilling involving a drift purge with air has been applied. At this mode the destroyed rock (slime) is output on a surface under the effect of compressed air at high pressure and speed. Application of this method for well drilling in the conditions of permafrost and in desert anhydrous regions is especially perspective, as in the course of drilling water is not required. In this connection, research of drill string motion taking into account the effect of the gas stream is important and has practical interest.
The work aims at study of the dynamic model of transverse vibrations of the compressed and twisted drill string moving in the gas stream, and its numerical analysis.

Motion Model with Small Strains
The rotary motion of a drill string with an angular velocity ω in the gas stream under the effect of compressing force *N(t)* and torque *M(t)* is considered. Transverse vibrations of the bar are examined. The vibrations are assumed to be small. The mathematical model of the compressed and twisted drill string motion in the stream of gas (air) is constructed on the basis of a known linear model [1] taking into account an overpressure of the gas stream ΔP. The bar is represented as a one-dimensional rod of length *l*. The x-axis is directed along the rod axis. The displacement vector of a bar cross section is expanded into two components *U(x,t)* and *V(x,t)*. Therefore, the mathematical model of drill string motion is given as the two-equation system with the distributed parameters:

$$\begin{aligned} &\frac{\partial^2}{\partial x^2}\left[EJ_U\frac{\partial^2 U}{\partial x^2}\right]-\frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial V}{\partial x}\right]+\frac{\partial}{\partial x}\left[N(x,t)\frac{\partial U}{\partial x}\right]+K_1U=\frac{\gamma F}{g}\frac{\partial^2 U}{\partial t^2}+\Delta P_U, \\ &\frac{\partial^2}{\partial x^2}\left[EJ_V\frac{\partial^2 V}{\partial x^2}\right]+\frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial U}{\partial x}\right]+\frac{\partial}{\partial x}\left[N(x,t)\frac{\partial V}{\partial x}\right]+K_1V=\frac{\gamma F}{g}\frac{\partial^2 V}{\partial t^2}+\Delta P_V, \end{aligned} \tag{1}$$

where EJ_U, EJ_V are bending rigidity of the bar, γ is specific weight of the material, F is cross-sectional area, $K_1=\frac{\gamma F\omega^2}{g}$; $\Delta P_U, \Delta P_V$ are transverse effects describing an overpressure of the stream of air in the xOy and xOz planes.
Nonlinearity of the dynamic model is caused by the gas overpressure. Plane-sections hypothesis [2] is accepted. In this case after some transformations the gas pressure at high velocities exceeding speed of sound is defined as follows:

$$\begin{aligned} \Delta P_U &= -P_\infty\kappa\left(\frac{U_u}{C_\infty}+\frac{(\kappa+1)}{12}\cdot\left(\frac{U_u}{C_\infty}\right)^3\right), \\ \Delta P_V &= -P_\infty\kappa\left(\frac{U_v}{C_\infty}+\frac{(\kappa+1)}{12}\cdot\left(\frac{U_v}{C_\infty}\right)^3\right), \end{aligned} \tag{2}$$

where $U_{u,} U_v$ are the components of the gas stream velocity on the bar surface, C_∞ is the sound velocity for unperturbed gas, P_∞ is pressure of the unperturbed gas; κ is the polytropic exponent.
Assuming the rod as hinge clamped at the ends, the boundary conditions are set as:

$$\begin{aligned} &U=V\big|_{x=0}=0, \quad U=V\big|_{x=L}=0, \\ &EJ_U\frac{\partial^2 U}{\partial x^2}=EJ_V\frac{\partial^2 V}{\partial x^2}\bigg|_{x=0}=0, \quad EJ_U\frac{\partial^2 U}{\partial x^2}=EJ_V\frac{\partial^2 V}{\partial x^2}\bigg|_{x=L}=0, \end{aligned} \tag{3}$$

The Bubnov-Galerkin method is applied to the numerical analysis of the model (1). According to that displacements *U(x,t)* and *V(x,t)* are given as:

$$U(x,t) = f(t)\sum_{i=1}^{n}\varphi_i(x),$$
$$V(x,t) = g(t)\sum_{i=1}^{n}\varphi_i(x), \tag{4}$$

where $\varphi_i(x) = \sin\frac{i\pi x}{l}, i = 1,\ldots,n$. Taking into consideration first two terms of the sum in (4), the model (1) can be reduced to the system of two ordinary differential equations:

$$\begin{cases} a\ddot{f} + bf(t) + cg(t) + df^3(t) = 0, \\ a\ddot{g} + bg(t) - cf(t) + dg^3(t) = 0, \end{cases} \tag{5}$$

Motion Model with Finite Strains

Removing limitations for the magnitude of strain [3], the nonlinear model of drill string vibrations in the supersonic stream of gas is considered:

$$\frac{\partial^2}{\partial x^2}\left[EJ_U\frac{\partial^2 U}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial U}{\partial x}\right)^2\right)\right] - \frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial V}{\partial x}\right] + \frac{\partial}{\partial x}\left[N(x,t)\frac{\partial U}{\partial x}\right] + K_1 U = \frac{\gamma F}{g}\frac{\partial^2 U}{\partial t^2} + \Delta P_U,$$
$$\frac{\partial^2}{\partial x^2}\left[EJ_V\frac{\partial^2 V}{\partial x^2}\left(1-\frac{3}{2}\left(\frac{\partial V}{\partial x}\right)^2\right)\right] + \frac{\partial^2}{\partial x^2}\left[M(x,t)\frac{\partial U}{\partial x}\right] + \frac{\partial}{\partial x}\left[N(x,t)\frac{\partial V}{\partial x}\right] + K_1 V = \frac{\gamma F}{g}\frac{\partial^2 V}{\partial t^2} + \Delta P_V. \tag{6}$$

The system (6) is based on the model of compressed and twisted drill string vibrations taking into account finite strains [4]. Here both nonlinearity at the expense of an overpressure of the gas and geometrical nonlinearity nature of the model is involved.

Using the Bubnov-Galerkin method as in the previous section, the numerical solution of the derived nonlinear system of ordinary differential equations is conducted. The outcomes of the research are presented in the graphic package Tecplot. Drill strings of length less than 500 meters is considered.

The comparative analysis of the linear and nonlinear models of drill string motion in the supersonic gas stream and without it was carried out. It was established that nonlinearity of the model led to a solution improvement at the expense of magnitude diminution of the amplitude vibrations. The research results displayed that an increase in the bar length parameter in the air stream made an essential impact on its oscillatory process and resulted in significant rise of amplitude vibrations and their wave-length. In both cases (rod motion in the stream of gas and without it) transverse vibrations of duralumin bars were smaller than vibrations of steel bars (Fig.1). It means that the range of operating frequencies of duralumin bars is higher than for steel ones. An increase in rotational speed of the drill string allowing for the stream of air leads to a small increase of transverse vibrations. Thus, the bar length is of great importance and it has to be taken into consideration while computing operating frequencies. Cases of constancy and variability of the compressing axial load and torque were also researched. The effect of their constant and variable components on steel and duralumin drill string vibrations was revealed.

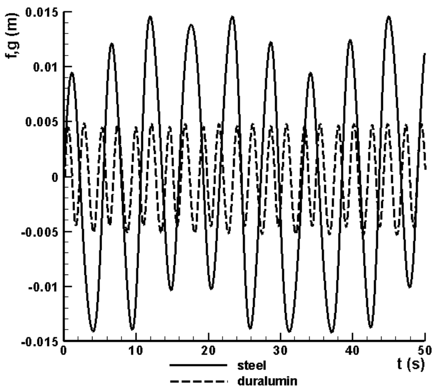

Fig.1. Drill string vibrations under
$l = 300$ *m*, $\omega = 80$ *turns/min*,
$N = 1.8\cdot10^6 + 0.4\cdot10^6\sin(\omega t)N,\ M = 10^4\ Nm$

Conclusions

It was established that the drill string length had the greatest influence on amplitude vibrations of the bar moving in the gas stream. Research of the effect of axial compressing load and torque variability showed the fact that the variable axial force led to considerable qualitative and quantitative changes in the oscillatory process. The operation of the variable torque affected only the drill strings of small length.

References

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